A Computational Analysis of Black Hole Interiors

Submitted by

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Abstract

This project will explore the interiors of black holes using computer simulations. Currently, photographs are being released of black holes, but because of their powerful singularities where even light cannot escape, the interiors of these black holes cannot be seen. As a result, the project takes an approach of computational analysis to determine if the smoothing of the current universe compares to the smoothing of a black hole interior. If the cyclic universe theories are correct, does the interior of a black hole smooth with the same mechanism that controls the smoothing of the current universe?

The research will use the programming language of Fortran 95 and supporting visualizer programs to observe the smoothing mechanism over time. With enough time evolution for the simulation, the research predicts that near the singularity of the black hole, the smoothing results in a Robertson-Walker metric (isotropy and homogeneity).

Possible benefits of this research include a better understanding of the origins of this universe and a deeper explanation of how black holes smooth and collapse (Garfinkle, Lim, Pretorius, & Steinhardt, 2008). Because the inflationary model does not completely explain the collapse, the cyclical model is an alternative explanation of the current universe.

Introduction

The event horizon of a black hole is so strong that not even light can escape due to general relativity. While telescopes can capture images of a black hole, the interior of a black hole is still a mystery. Since general relativity has been tested through many trials, the research can consider how general relativity predicts the interiors of black holes to operate.

The smoothing mechanism inside a black hole is included for the Bouncing Universe Theory (Brandenberger & Peter, 2017). While the widely-used model of this universe is inflation, the most likely explanation is that this inflation model occurs by chance (Erikson, Steinhardt, & Turok, 2007). An alternative model of the universe is the **Big Bounce** in which the period of expansion is followed by a period of contraction. This contraction is so powerful that when collapsed to a black hole, a new universe begins. However, this model falls short when tested against inflation and is currently missing mechanisms to explain such behavior.

The Big Bounce model is supported by the collapse of a black hole. The black hole's interior mechanisms are the **bouncing** and **smoothing** mechanisms (Garfinkle, Lim, Pretorius, & Steinhardt, 2008). When a black hole collapses, the bouncing mechanism causes a contraction period followed by an expansion period (Steinhardt & Turok, 2005). However, the smoothing mechanism occurs in between the two periods, explaining the isotropic interior of the black hole.

This collapse near the singularity is expected to behave with regard to the Robertson-Walker metric (Garfinkle, Lim, Pretorius, & Steinhardt, 2008), an equation that illustrates an isotropic, homogenous, and expanding universe. The metric is an exact solution of Einstein's field equations. The development of the time-dependent mechanisms using the collapse method provides grounds that a collapsing black hole could re-expand into a new universe (Steinhardt & Turok, 2005).

Aims

1. To create programs in the Fortran language designed to simulate the behavior of black holes.
2. To analyze the behavior of simulated black holes using Einstein's field equations.
3. To compare the simulated black hole data to current research in the field, making sure that the equations and numbers used in the simulations are accurate enough for complete data

Objectives

1. In this study, there are multiple steps from creating data to understanding it. First, the black holes need to be simulated to collect data regarding their behavior.
2. Analyzing the simulated black holes will give some visualization (graphically) to the origins and behavior of black holes.
3. After the analyzed simulations are checked for errors against already-known equations, they can be compared to the current universe in terms of the smoothing mechanism behavior

Methods and Materials

This research began with simplified programs in **Fortran 95**, a programming language designed for scientists. These programs simulated the interiors of black holes. Then, these programs were adjusted to approximate the more complex situations in which more accurate data could be established. Fortran 95 was used for this research primarily because of its physics and mathematics design. This programming language includes a variety of native packages to deal with matrices, tensors, physical time step evolutions, and others native to Linux operating systems. Figure 1 illustrates one section of the code used for the simulations.

In order to visualize the process of the black hole collapse, each time step of the data was processed using **Gnuplot** (Williams et al., 2019), a scientific-based graphing utility. Gnuplot is simple to use and extremely effective in plotting 2D curves; Figure 5 illustrates the abilities of Gnuplot with 2D curves, and Figure 2 is the entire program in use. For the simplified code, Gnuplot was the primary choice since only time steps were necessary as opposed to full animations. Additionally, the simplified code does not have the limitations of outer boundary conditions unlike the collapse code. However, the interior mechanisms vary with time and space, so a more extensive program was needed that could deal with both time and special-dependent data. Visit was the choice for the more complicated code.

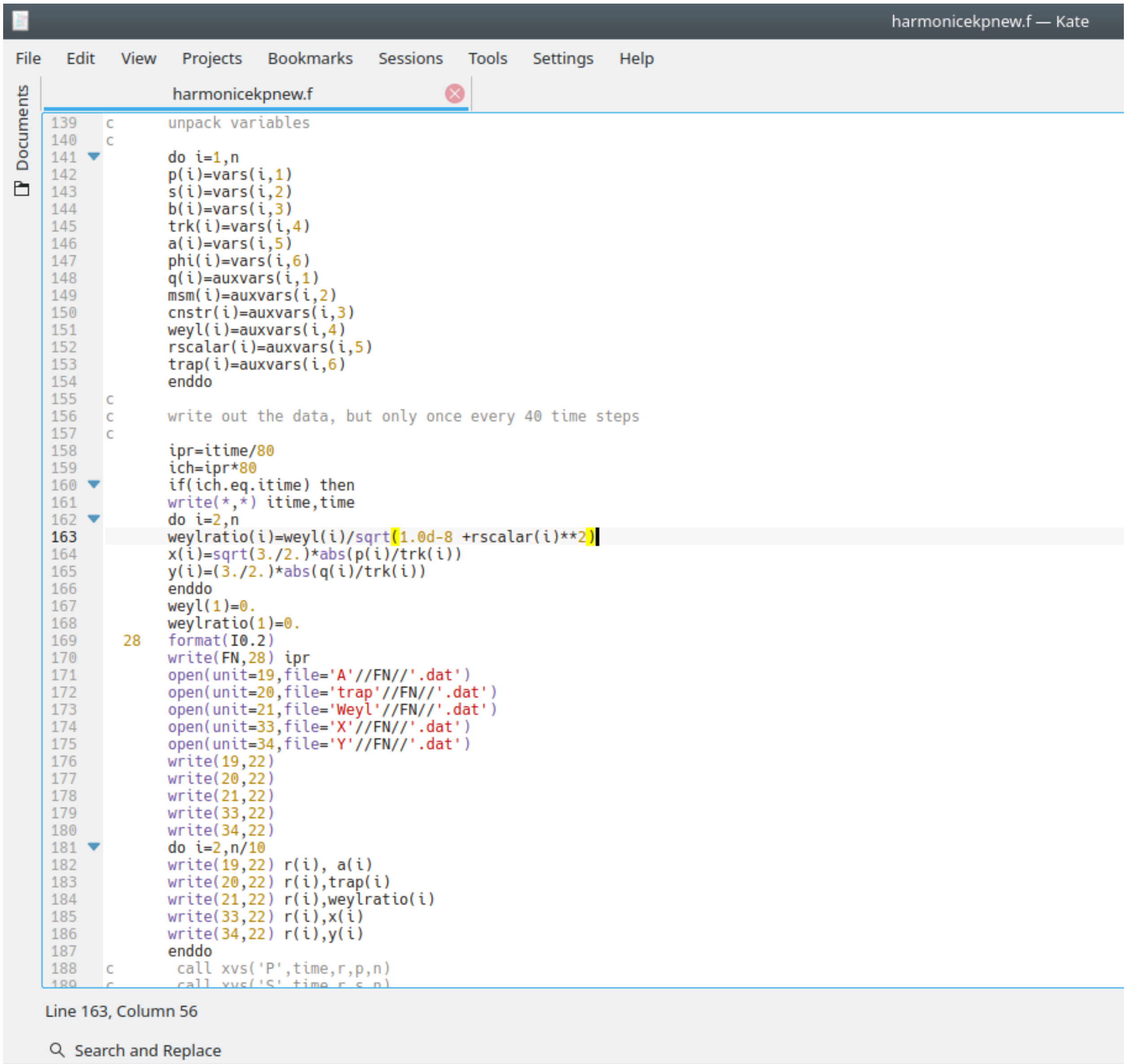

Figure 1. Fortran 95's Linux compiler, Kate, delineates functions, names, and numbers using various colors. Additionally, it numbers each line of code for ease of access. This section of the code unpacks the Robertson-Walker variables to use in the initial data (Lines 141-154). Lines 158-168 create a loop that write out the data into files once every 80 time steps, and lines 168-186 create the respective files for the data.

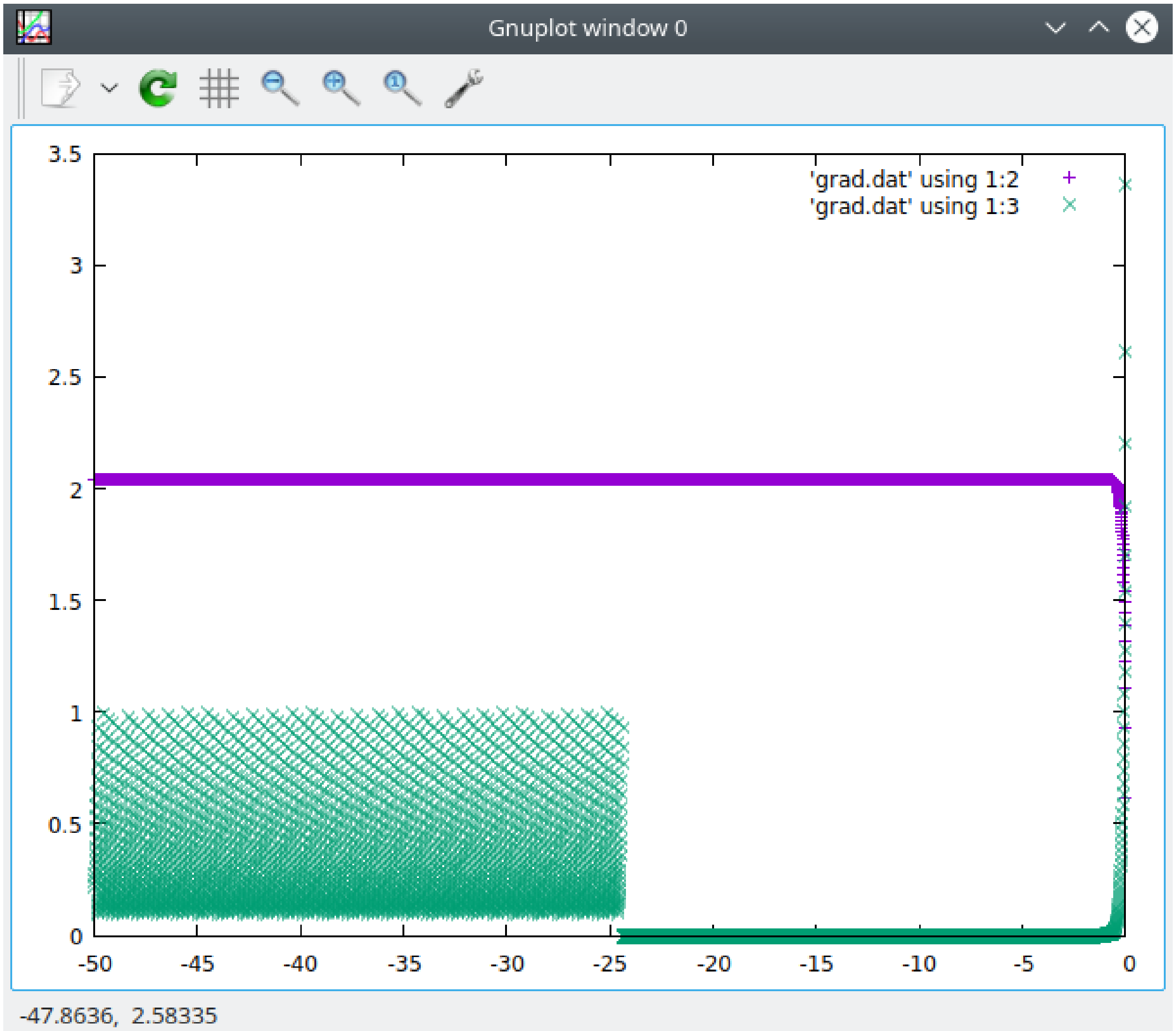

Figure 2. This illustrates the entire program of Gnuplot. Extensive zoom tools can be used to various levels when data is to be accurately examined. Additionally, a grid tool can place lines over the graph to help the user locate intersections.

Visit (Childs et al., 2012) was the last step in the research to visualize the full data. Visit is another scientific-based graphing utility, but it accepts groups of files to process into an animated database. The data from the mechanisms were processed and fitted into a video format. This processed video presents the complete time-dependence of black hole interiors. Figures 6-9 are processed time steps from the Visit program, and Figures 3 and 4 are the complicated Visit user interface. Since Visit requires strict parameters for program animations, each timestep requires a file; as a result, the entire program contains hundreds of files.

Visit was developed by Lawrence Livermore National Laboratory but is widely used around the world by renowned universities. Additionally, when processing videos out of Visit, the user can use remote processors hosted by Visit to create the animations. This collaboration is crucial in every field of science and is the main reason why Visit achieved its presence in Physics and Mathematics.

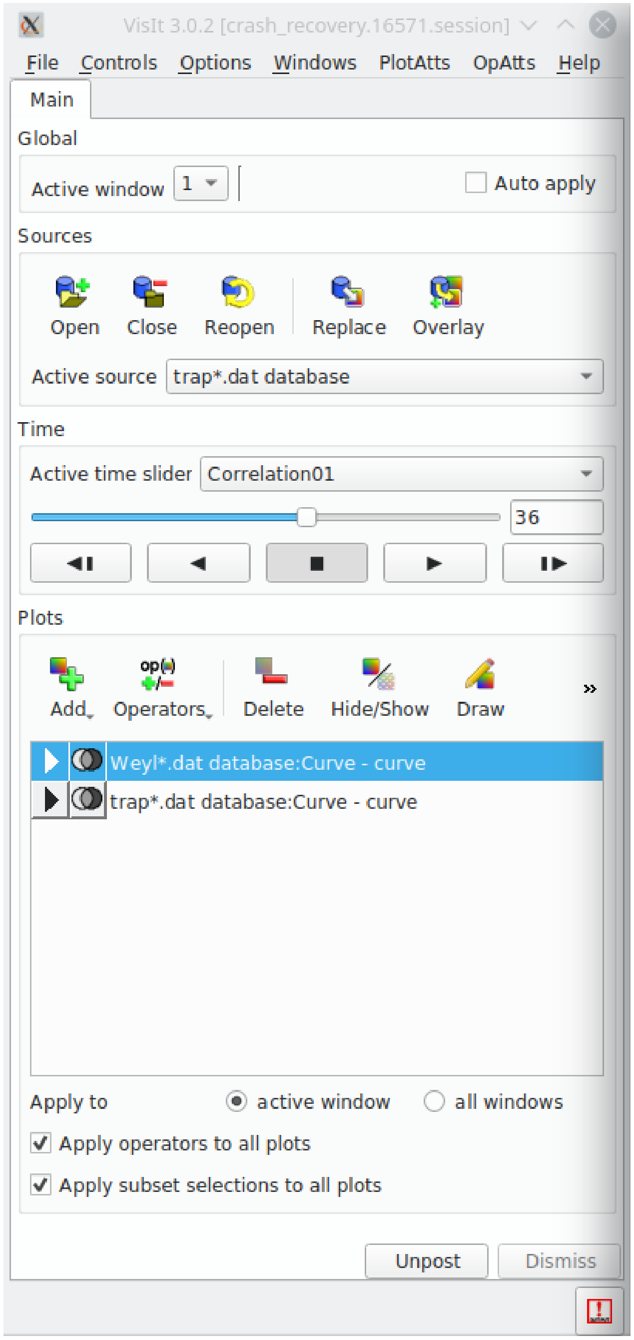

Figure 3. The smaller window of Visit provides a detailed selection of tools for the user. The top of the display hosts options and pre-selections for the user, but what's more important is the data selection. After active sources are selected, the user may add plots (2D Curves for this data) and draw those plots in the Figure 4 window. Once drawn, a grouping of files can be animated using the tools near the middle of the window.

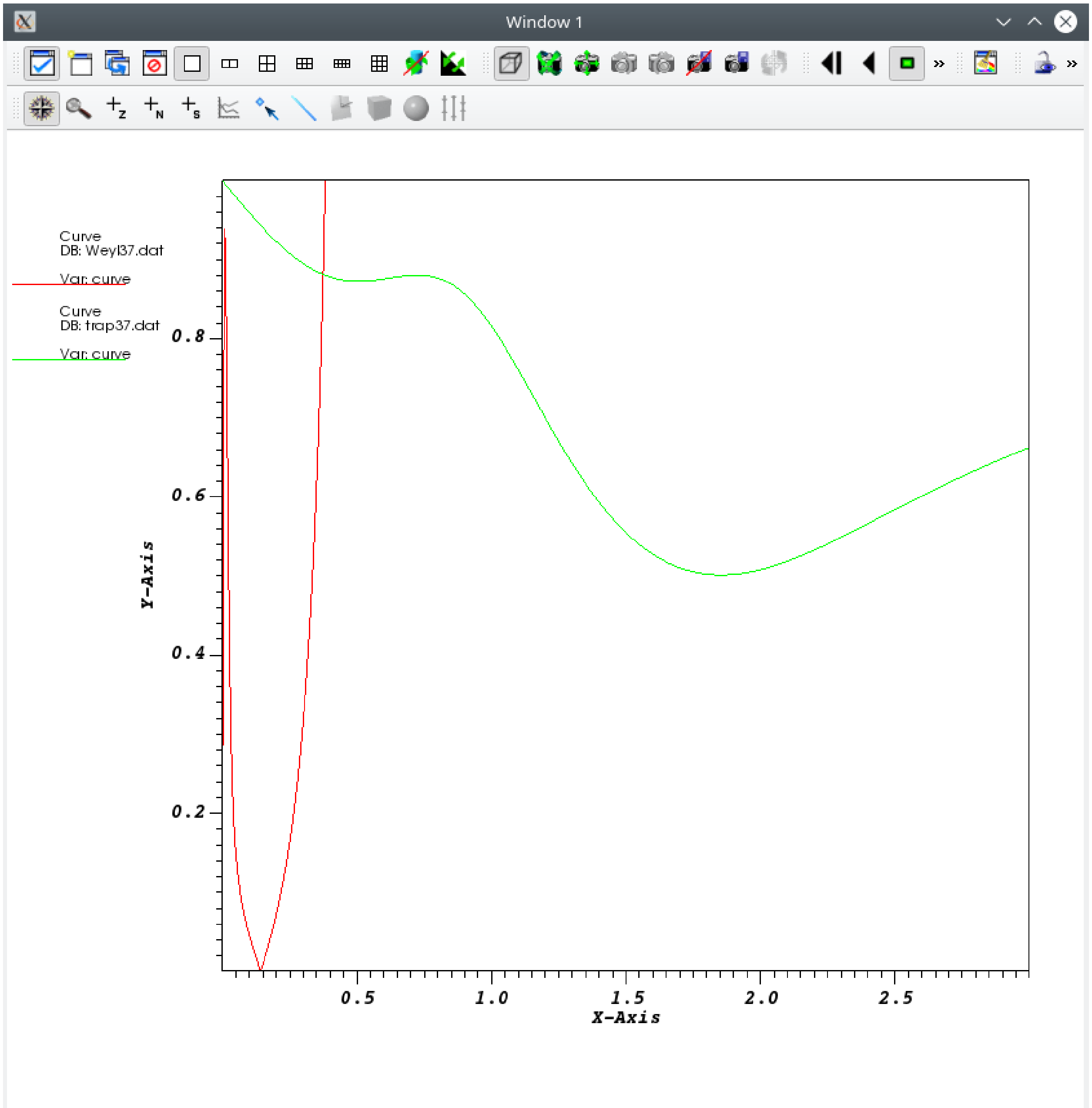

Figure 4. The larger window of Visit provides the user with a graphical representation of the data. For this research, 2D Curves were primarily used to plot various elements against the x-axis. Near the top of the window, a multitude of viewing options can be changed, and the top right provides another opportunity for the user to animate the data.

Results

Initial data of the simplified x and y coordinates (to find solutions to simplified equations) were plotted in Gnuplot, shown in Figure 5. The data runs from a time of 0 to a time of 20; this time is unitless and is only significant in the magnitude. This resulting data either flow to a "fixed point" ($\frac{c}{\sqrt{6}}$, 0) or a "fixed circle" ($x^2 + y^2 = 1$). The "fixed circle" corresponds to a circular scalar field, forcing the initial data into anisotropy and inhomogeneity. However, the initial data flowing to the "fixed point" results in **good behavior** for the universe; this means anisotropy and inhomogeneity vanish, and the spacetime approaches Robertson-Walker. When the spacetime approaches Robertson-Walker, the simplified data is an adequate approximation for the complex (collapse) code.

Figures 9 and 10 graph an example timestep (54) in the black hole evolution. This timestep is not arbitrary but rather a timestep after the collapse. In Figure 9, as the Weyl-R ratio approaches 0, the interior smoothing mechanism produces isotropy. When this ratio approaches 0, the R tensor grows extremely large; in contrast, the Weyl-R ratio approaching infinity indicates the Weyl tensor also approaches infinity.

Figure 10 outlines the Trap: when Trap is below 0, light rays are trapped inside the event horizon, illustrating the black hole's radius. As a result, the largest R-intercept point on the graph outlines the span of the event horizon (1.53). The R-intercept points are only present in the evolution after about the 40^{th} timestep due to the formation of a black hole at this timestep.

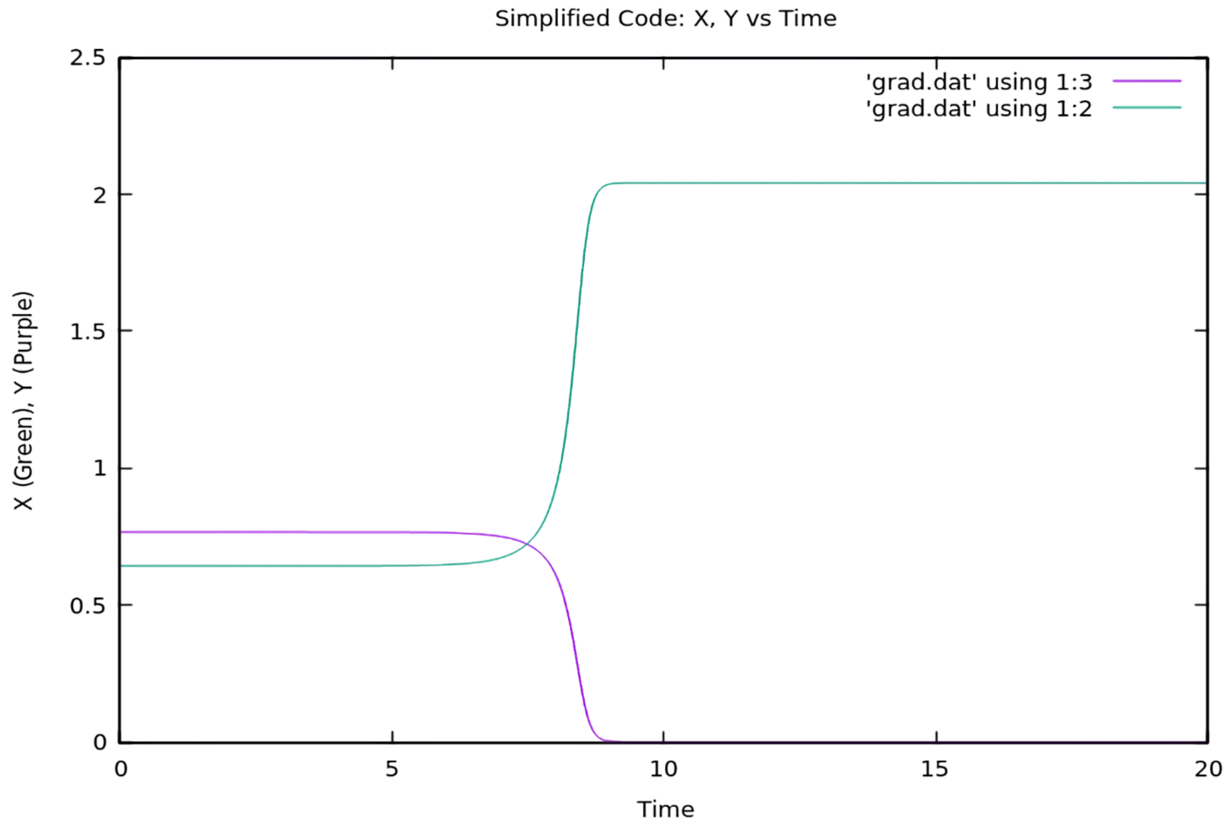

Figure 5. The simplified code produces a graph of X and Y vs Time. Over time, the x coordinate flows to ~ 2.04 ($\frac{5}{\sqrt{6}}$), and the y coordinate flows to 0. These coordinates represent the simplified code, and as such, are unitless. The end coordinates result in isotropy of the system and can be used to approximate the more complicated Robertson-Walker metric.

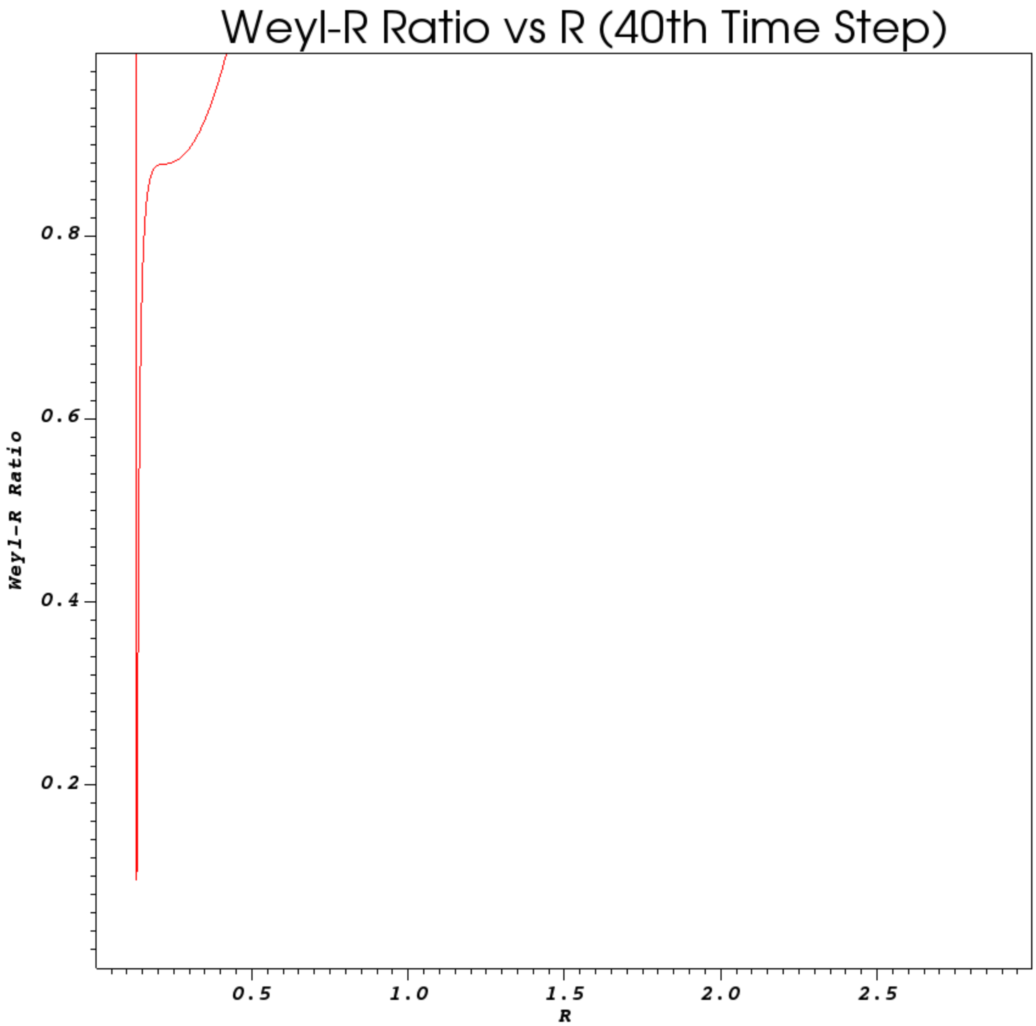

Figure 6. The 40th time step for the Weyl-R ratio is displayed. This graph plots the unitless Weyl-R tensor ratio against the unitless R, a coefficient of length for the event horizon. This exact time step was examined due to its extreme concavity near at $R = 0.045$, consistent with the following figures (7,8, and 9). This time step contains the most extreme concavity, and the research concludes that this time step is where the collapse occurred.

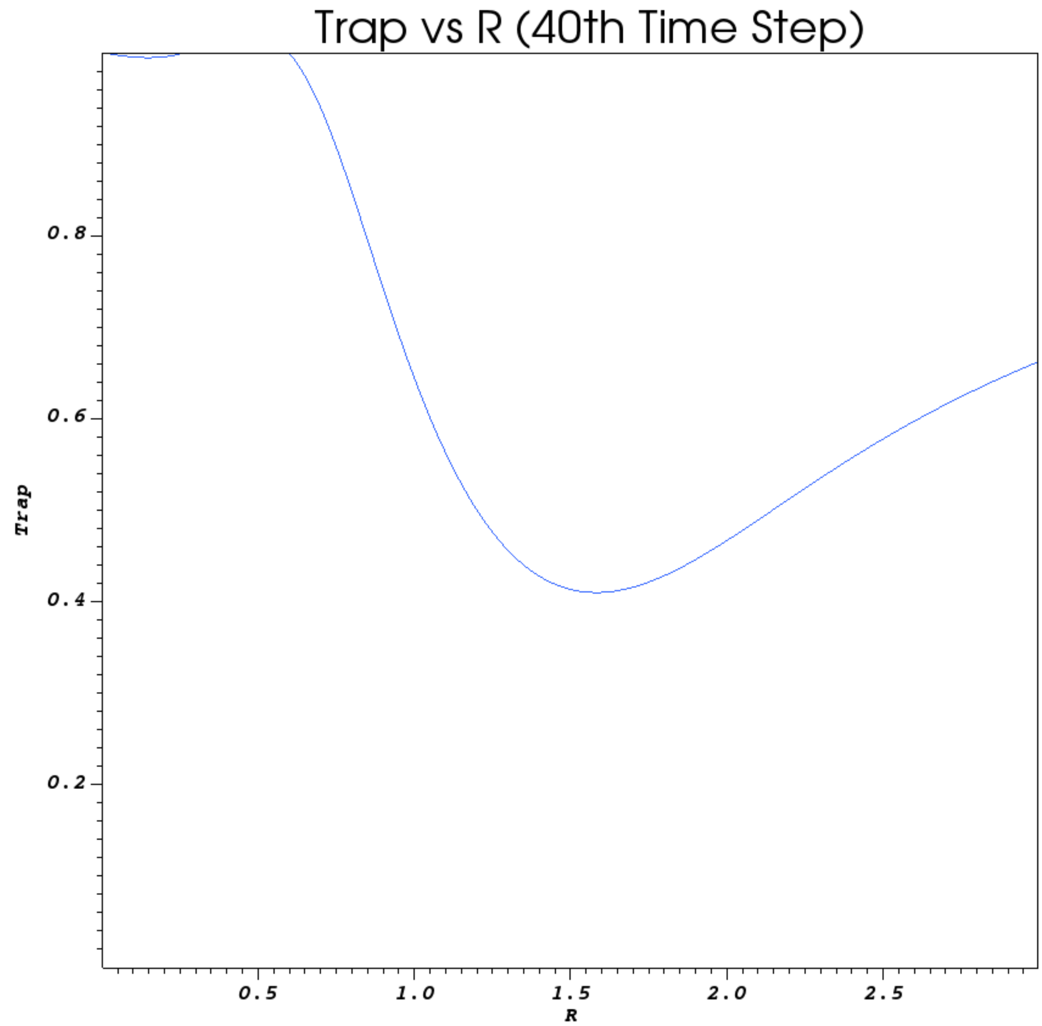

Figure 7. The graph above is the 40th time step of the Trap plotted against R (both unitless). In comparison with the previous figure (6), this time step is where the collapse occurred. However, due to the time needed for re-expansion, the Trap does not begin to cross the R-intercept until the 47th time step.

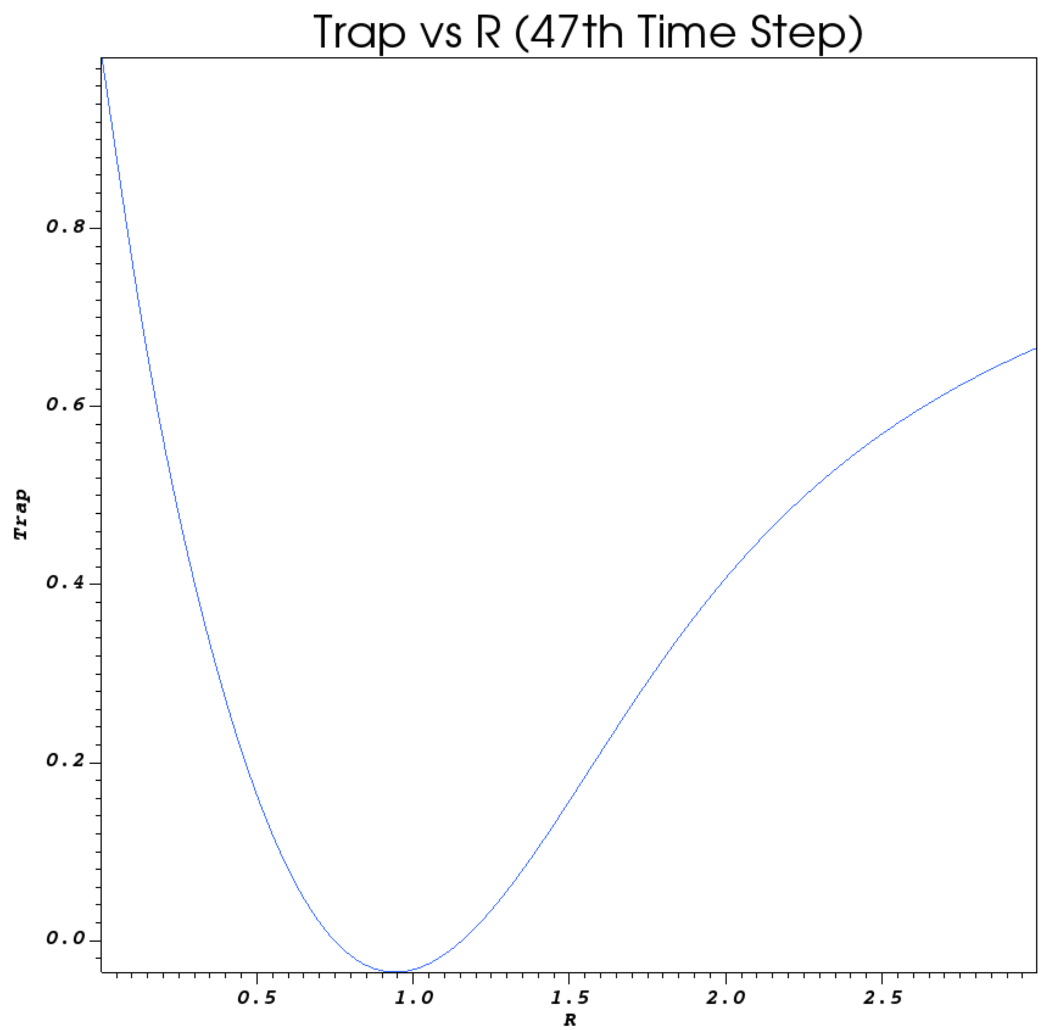

Figure 8. The above graph depicts the 47th time step of the unitless Trap against the unitless R. This time step was examined due to its correlation with the collapse. When the collapse occurs at the 40th time step (figures 6 and 7), the Weyl-R ratio reaches maximum concavity. Due to a lag between collapse and re-expansion, the black hole begins to form at the 47th time step. This graph illustrates the first R-intercept cross of the Trap, indicating the smallest possible width of the black hole.

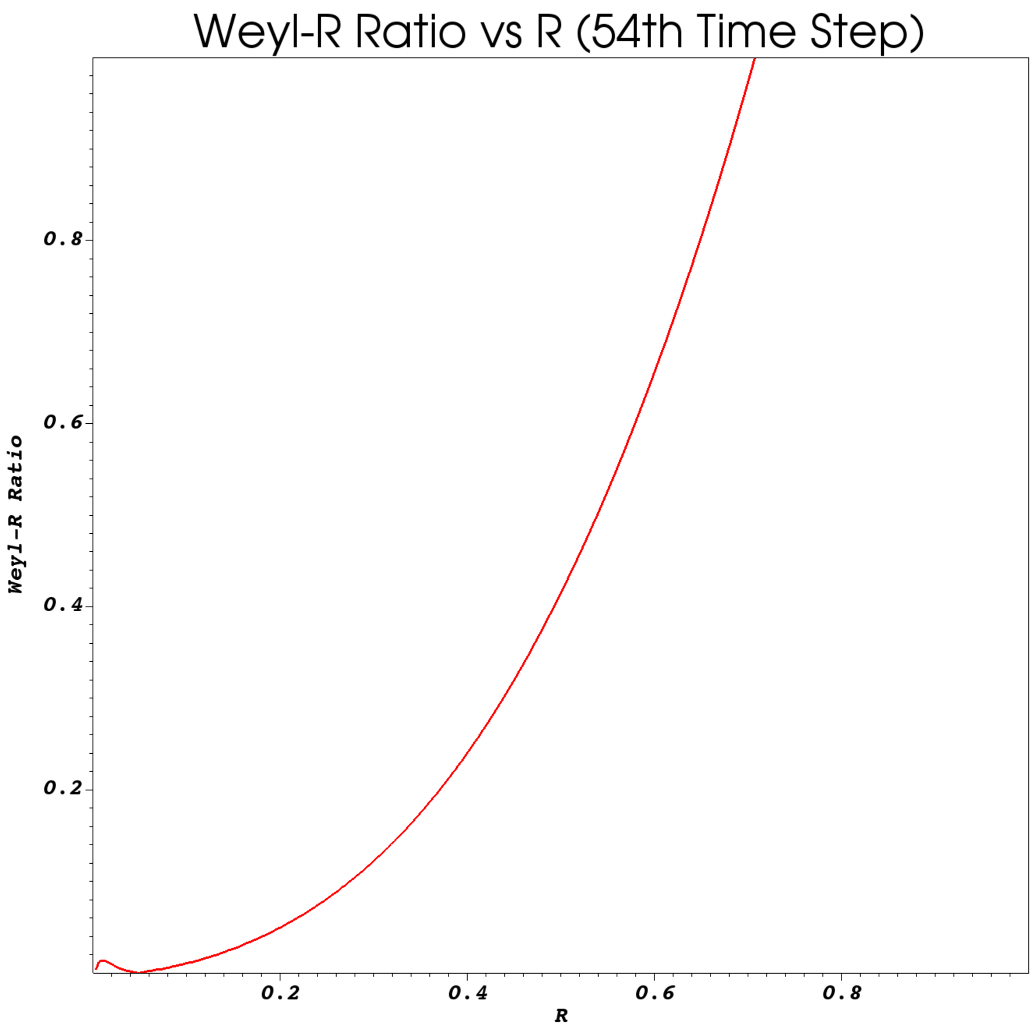

Figure 9. The data illustrate the behavior of the unitless Weyl-R ratio to the unitless radius, R. For this specific time step, when $R = 0.045$, isotropy occurs in the interior of the black hole as the Weyl-R ratio approaches 0. This is consistent with the data in Figure 10.

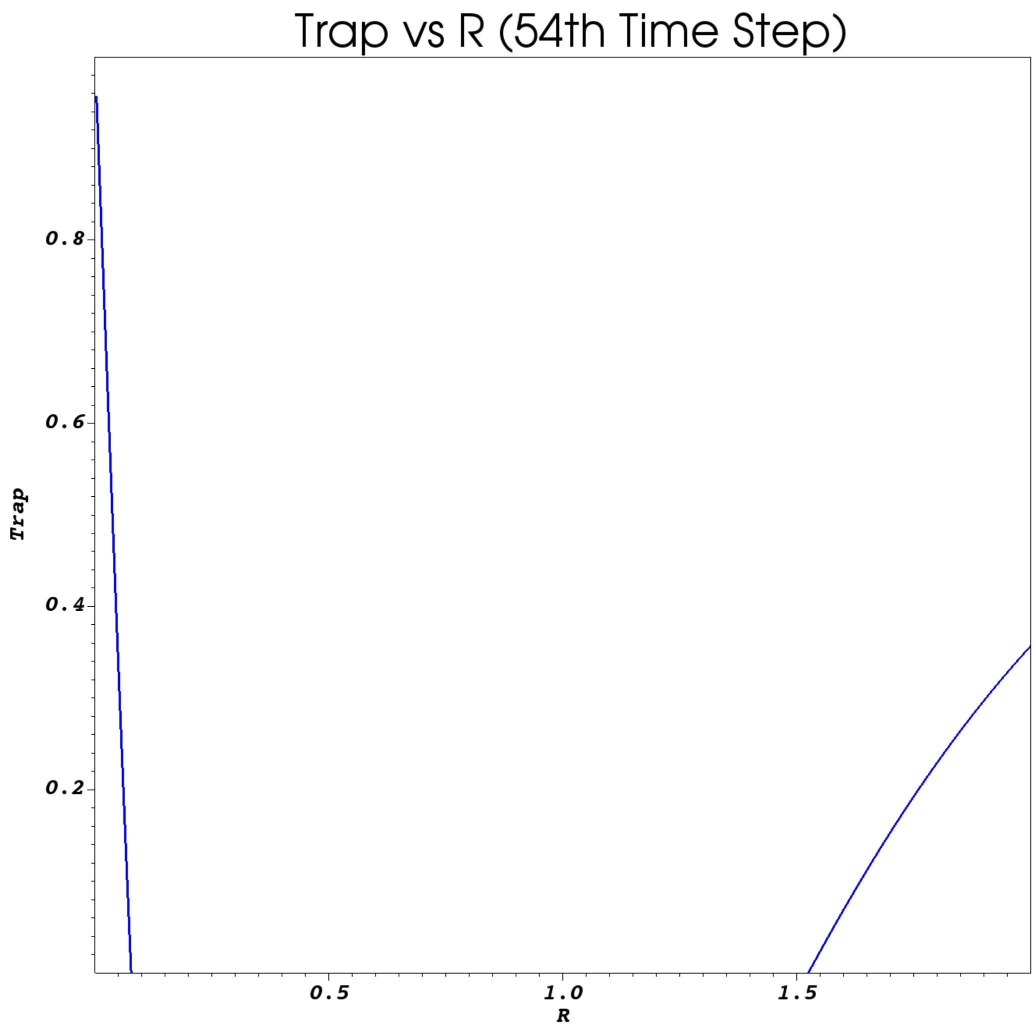

Figure 10. The data present the relationship between the unitless Trap and the unitless radius, R. Trap determines if a projected light ray is undisturbed ($Trap > 0$) or if the light ray is absorbed by the event horizon ($Trap < 0$). For this data, the event horizon (and by extension, the radius of the black hole) is between 0.58 and 1.53, consistent with Figure 9.

Discussion

Most scientific discoveries are created with some level of approximation. The simplified code created a graph indicative of the initial data flowing to isotropic behavior, but this was not enough to guarantee the smoothing of black hole interiors. The simplified code was then compared with the more complicated collapse code.

Figure 1 is just one scenario of the simplified code. In reality, this data was generated from an extensive list of initial data points. The data flow was approximated using 3 decimal points with respect to the accuracy of the research. This data flow illustrated the initial data to flow to the fixed point in an overwhelming majority of cases. Most of the initial data resided near a circle of $(x^2 + y^2 = 1)$ but still flowed to the fixed point of $(\frac{c}{\sqrt{6}}, 0)$. This is an interesting discovery as even though the initial data was placed directly very near the fixed circle (anisotropy), the data flowed to isotropy with enough evolution.

The evolution of the simple code began with the Runge-Kutta method, a numerical analysis designed for time-dependent wave equations. Specifically, the method uses an Euler analysis which uses the previous time step to advance the next time step. In order to create the Runge-Kutta evolution, only the wave equations and initial data are needed. Once these are inputted into the program, the program will evolve the equations until the end time.

The collapse code required a much greater magnitude of time to analyze than the simplified code, but once the analysis was completed (Figures 6-10), this behavior also corresponded with isotropy after the black hole collapse. The Robertson-Walker metric is an exact solution to Einstein’s field equations.

Figures 6 and 7 explain the detailed research of the 40th time step. When the collapse occurs at the 40th time step, the Weyl-R ratio occurs with the maximal concavity. This concavity begins to decrease after the 40th time step (figure 8). At the collapse (Weyl-Ratio approaching 0), the interior of the black hole smooths to a Robertson-Walker metric (isotropy and homogeneity).

Figure 8 illustrates the 47th time step. After the collapse occurs, a period between collapse and re-expansion occurs. During this time, the black hole has yet to form up until the 47th time step. At this time step, the smallest possible width of the black hole is observed, and after the 47th time step, the width of the black hole continually increases (figure 10).

The Trap in the begins to evolve with R-intercepts at the 47th time step (see figure 8), then begins to expand the event horizon until the end of the run. For the 40th time step, the trap has not yet crossed the R-intercepts. This is due to a lag in time between the collapse and the re-expansion.

Figures 9 and 10 display the Weyl-R ratio and Trap against R, respectively, at the 54th time step. As stated in the results, this time step was not chosen arbitrarily but rather a time between the collapse and the end of the run. Due to these parameters, the graphs illustrate that the black hole has increased in width since the collapse (figure 10), and the Weyl-R ratio has since decreased in concavity, indicating that the collapse occurred before this time step (figure 9).

Unfortunately, the collapse code is not designed to run for exceedingly-long times (with respect to the collapse). Because of this, further research will need to be performed, and a change of variables is suggested to end the problem. A change of variables or coordinate system can be made in the program, but this complicates the code entirely and is counter-intuitive to the goal of understanding the collapse. The cyclical model is interpreted with ‘clean’ equations, but with a change of coordinates, the system would be difficult to interpret.

Conclusion

The data find the interior mechanism (smoothing) of the black hole to correlate with the current smoothing of the universe. A black hole, in theory, can collapse, smooth, and re-expand into a new universe. The figures demonstrate that a collapsing black hole (if the bouncing mechanism is also valid with the current universe) can and will re-expand.

Both mechanisms contribute to the collapse, and by extension, a collapsing black hole can explain the big bang. The smoothing of the current universe can be attributed not to inflation, but to a smoothing mechanism present in a black hole. Additionally, the cyclical behavior (Steinhardt & Turok, 2005) of the current universe can be attributed to the bouncing mechanism inside the black hole. The comparison of the current universe with the interior of a black hole supports the Big Bounce model.

Since the more complex code is a valid extension of the simplified code, the same initial data $(\frac{c}{\sqrt{6}}, 0)$ was used in the more complex code in order to accurately simulate the interior of the black hole.

In the future, the research will attempt to determine if the bouncing mechanism present in the early universe is also supported by the interior of a black hole. The two mechanisms combined can provide ample evidence that the current universe is the result of a collapsed black hole.

Biographical Note

My name is Maxx Haehn, and I am currently finishing my last semester as a physics major and in the Honors College at Oakland University. For most of my life, I have been fascinated with the natural sciences, figuring out or wanting to see how things work. For the latter half of my life, I have looked toward the stars in hopes that one day, I can figure out how the universe behaves in terms of black holes and their collapse. When I graduate from Oakland University, I hope to attend graduate school pursuing research in astrophysics. That education will give me a chance to build a profession in observatories and universities.

This research will provide me with a deeper understanding of the cosmos, how black holes function, and the nature of gravitational waves. Furthermore, it will provide more details on the behavior of black holes which might change the scope of the fields of astrophysics and cosmology. Lastly, the project will give me an opportunity to contribute to a published paper and the ability to present the project’s findings.

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