TWELVE COINCIDENCES

by Alberto G. Rojo

The purpose of Physics is to decipher–perhaps to impose–a causal skeleton to the regularities of Nature. But the paths of that search are sprinkled with fortuitous coincidences, false leads that are fertile soil for the esoteric and the pseudoscientific. The history of science teaches us that, even if there is no established procedure to discern the false from the true leads, every coincidence is an invitation to decipher clues that many times lead astray and, occasionally, to great discoveries.

In a fortuitous coincidence, the coexistence of two phenomena insinuates a casual tie between them that in fact does not exist. A caricature of this fallacy is the story, told by Edmond Rostand (author of the famous *Cyrano de Bergerac*) in his satirical comedy Chantecler, of a barnyard rooster who believes that his song makes the sun rise. And, not too feebly, the disc of the Moon and of the Sun has the same size in the sky: the Moon is four hundred times smaller than the Sun but is four hundred times closer. Thanks to this beautiful coincidence, the Moons covers the Sun completely in an eclipse.

The second lunar coincidence is meaningful: its period of rotation around its axis is the same as its period of revolution around the Earth. This is due to tidal forces that tend to align a slightly oblong moon in a direction that points towards the Earth. As a result, the Moon always shows us the same side. The third refers to the correspondence between human menstrual cycles and the lunar month; they are both roughly 28 days. The

very term "menstruation" comes from the Latin *mensis* (month), which in turn relates to the Greek *mene* (moon). But the menstrual periods of other primates are widely removed from the lunar cycle and the coincidence appears to be accidental. I can't resist mentioning a fourth, ancient connection between Moon and insanity, now considered to be nonsense, but nevertheless enshrined in our word "lunacy."

A celebrated coincidence is what the astronomer Johannes Kepler called the "Cosmic Mystery". In 1595 Kepler was haunted by a question that he considered profound: Why are there six planets? He reached his answer following the premise that God is a geometer, and invoking a correspondence between the regular solids (or Platonic solids) and planetary orbits. Regular solids (the cube is one of them) are bodies whose faces, all identical, are polygons of equal sides that can be circumscribed by a circle (the equilateral triangle, the square, the pentagon, etc.) Curiously, there are only five regular solids: besides the cube, there is the tetrahedron, the octahedron, the dodecahedron, and the icosahedron. To Kepler, they corresponded to the spaces between planets; that is why there were only six. He pushed his construction to claim that it explained the sizes of the orbits. First embed the orbit of the Earth in a sphere. Fit a dodecahedron around it and put a sphere over that and you get Mars' orbit. Repeat the procedure with a tetrahedron and you get Jupiter's. Then use the cube to get Saturn's orbit. And inside the Earth Kepler placed an icosahedron to obtain Venus's orbit. And within Venus, the octahedron, for Mercury. The amazing part of the story is that the ratios of diameters of the orbits were in nice (although not perfect) quantitative agreement with the real ones. Now we know that there are more than six planets and that the correspondence was merely accidental.

A related planetary coincidence is the Titius-Bode "law". It was first announced in 1766 by the German astronomer Johan Daniel Titius but was popularized only from 1772 by his countryman Johann Bode. Titius devised a numerical se-

quence that was obeyed by the distances of the planets to the Sun. Start with the sequence 0, 3, 6, 12, 24, in which each number after 3 is twice the previous one. To each number is add 4, and divide each result by 10. Of the first seven answers—0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0—six of them (2.8 being the exception) closely approximate the distances from the Sun, expressed in astronomical units (AU; the mean Sun-Earth distance), of the six planets known when Titius devised the rule: Mercury, Venus, Earth, Mars, Jupiter, and Saturn. What is interesting is that the exception (2.8) was later, in 1801, identified as Ceres, and a series of asteroids at roughly that distance to the Sun. More amazingly, if the law is extrapolated to an eight planet it predicts a distance of 19.6 AU, almost exactly the distance to Uranus, discovered in 1781. According to the latest edition of the Encyclopedia Britannica, Bode's law is now generally regarded as a numerological curiosity with no known justification. However, according to astrophysicist Fred Adams from the University of Michigan (in personal communication to the author) states that the spirit of Bode's law is both useful and correct. The numerical simulations (and some hand waving arguments) of his team show something like Bode's law, i.e., a factor by which each successive planet's orbit must be larger than the last. One might classify Titius-Bode's law as a meaningful coincidence.

An intriguing coincidence I heard about at a dinner table in an interdisciplinary conference refers to the number of neurons in an ant colony, which is roughly the same as the number of neurons in a human brain. I asked Deborah Gordon, an expert on ants from Stanford and she replied that there are 12,000 species of ants, and colonies of different species vary in size from 10 to many millions of workers. So, this would mean that there isn't really any such number as the number of neurons in an ant's brain or the number of ants per colony. But when I asked her whether more "complex" ants (with more neurons) form smaller colonies, she gracefully responded "Good question—no one knows".

The following coincidence I heard from Marc Ross, who

researches the efficiency with which a society uses energy: the number of turns of a car motor during its lifetime is roughly the same as the total number of heartbeats of a mammal. I did the calculation and in fact they coincide, but nothing profound is behind it.

Turning back to Physics, a profound coincidence has to do with two definitions of mass. In our everyday experience we think of mass as something we can weigh, which in fact relates to one of the definitions, that of *gravitational* mass. The larger the gravitational mass of an object the larger it's attraction to the Earth. Newton found the law that relates the force of gravitational attraction between two bodies: the magnitude of the force is proportional to the product of the gravitational masses. The second definition relates to the resistance of a body to accelerate. Try pushing an elephant and you'll notice the magnitude of its "inertial mass". Newton found another law (his famous "second law") relating the force on an object and its acceleration: they are proportional, and the constant of proportionality is the inertial mass. The mass of an object can therefore measured in two ways: weighing it and using Newton's law of universal gravitation, or measuring its resistance to accelerate and using the second law. Many experiments of this kind have been done to measure masses and all lead to the same conclusion: the inertial mass is the same as the gravitational mass. Newton himself realized that the equality of the two masses was something his theory couldn't explain and considered this result as a simple coincidence. The equality of these two masses, in the case of Newtonian gravity, is an *accidental* fact. In contrast, the identity between inertial and gravitational mass is a necessary and unavoidable feature of any theory (such as general relativity) that conceives of gravitational motion as nothing other than "free fall" motion in curved space-time. In such a theory, inertial mass and gravitational mass are not just accidentally numerically equal, they are ontologically identical. This is a case where a coincidence is pointing to a true law behind it.

Newton's law of gravitational attraction also states that the

force between two masses decreases as the inverse of the square of the distance separating them: if we were to double the distance from the Earth to the Sun, the force of attraction would become four times smaller. The same law applies for the attraction (and repulsion) between electrical charges. There is another situation were the inverse square law emerges: consider a light bulb in your dining room. It irradiates energy in all directions. Draw an imaginary spherical surface centered at the bulb, a foot in diameter. All the energy emitted by the bulb flows through the imaginary surface. Now consider a second, similar sphere of twice the diameter, though which all the energy irradiated by the bulb flows as well. The surface of the new sphere is four times bigger than the first (the surface of a sphere is proportional to the square of its diameter). This means that the energy *flux* decreases as the square of the distance from the light bulb, the same distance law as found in gravitation and electricity. Is this a coincidence? Yes and no. The inverse square in the case of the light bulb is a consequence of the fact that space is three dimensional. And the three dimensionality of space is also crucial in determining the law of gravitation and electricity. In the modern, quantum mechanical, description of the universe, the forces result from the exchange of particles: when a charge attracts another, a myriad of invisible quantum particles is going back and forth between the charges. The subtlety of the coincidence comes about when one notices that the particles exchanged in electricity and gravity have zero mass. In electricity, the particle is the photon. And photons are what the light bulb emits when it is incandescent. There are however, other forces in Nature, the so called weak and strong force, relevant at nuclear scales, where the attraction does not decrease as the inverse square. But if a nuclear "light bulb" were to emit particles (not photons, but neutrinos, which are massive) their intensity would still decrease as the inverse square. In summary, the inverse square law personifies a delicate coincidence.

My favorite coincidence is behind the discovery by the Scottish scientist James Clerk Maxwell, in 1864, that light is at

the same time an electric and a magnetic phenomenon. By the mid eighteen hundreds it was known that magnetism was electricity in motion: the attractive and repulsive force between magnets is due to the motion of the electrical charges in its interior. A few years before Maxwell, the German physicist Wilhelm Weber asked himself how the magnitude of the force between charges in motion compares with the case when the charges are at rest. In other words, how fast do two charges have to move in order for the magnetic and the electric force to be the same? Weber designed an experiment and found that the velocity was very close to three hundred thousand kilometers per second, identical to the velocity of light. In 1855 he wrote: "One should not hold great expectations for establishing an inner connection between optics and electricity through this numerical coincidence". I asked Maxwell's biographer Francis Everitt about the way this coincidence was perceived before Maxwell's breakthrough. According to him Weber didn't have a physical interpretation of his velocity. In 1860, another German physicist Gustav Robert Kirchhoff made a calculation from Weber's theory of the velocity of propagation of signals along an ideal telegraph line made up of wires with no electrical resistance. The velocity turned out to be the velocity of light, very nearly. He, too, doesn't seem to have made nearly as much of this as one, retrospectively would expect. Everitt pointed me to a long article by William Thomson (the future Lord Kelvin) also written in 1860, The Velocity of Electricity. It is fascinating to read Kelvin's article and see him, so to speak, dancing around the question that we feel he should have asked and never quite asking it. When Maxwell wrote his four-part 1860 to 1861 paper On Physical Lines of Force, he found in the fourth part that a velocity emerged for the propagation of electrical signals through space but wrote the paper at his estate in Scotland where he didn't have the journal containing Weber's paper. When he got back to London and plugged in the numbers, he fell flat on his face upon discovering that with his assumptions and calculations, it came out exactly as the velocity of light. It would

seem, but Everitt does not know this for sure, that he was unaware of Kirchhoff's result. Maxwell concludes, in 1861 "This coincidence is not merely numerical . . . and I think we have now strong reason to believe, *whether my theory is a fact or not*, that the luminuferous and the electromagnetic medium are one".

To the Argentinean writer Jorge Luis Borges, coincidences obey the purpose of making us aware of a world order, that there is a divine that wants to be, perhaps not revered, but certainly suspected.