HARMONIC MITIGATION IN POWER TRANSFORMERS THROUGH THIRD WINDING CURRENT INJECTION

by

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Abdullah Hamadi

ABSTRACT

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Harmonic distortions in power distribution systems present a substantial challenge to electric power systems by lowering power quality. This leads to increased power losses, reduced system efficiency, and even damage to sensitive electrical equipment. The increasing use of power electronic-based nonlinear loads and renewable energy sources has increased these challenges, prompting the development of improved harmonic reduction solutions. Traditional mitigation approaches, such as passive and active filters, suffer from efficiency limitations, implementation complexity, and high costs. This research proposes a novel transformer-based harmonic mitigation strategy that leverages current injections into the tertiary winding of a three-winding transformer to dynamically suppress harmonic distortions while maintaining stable voltage and current waveforms.

The proposed method exploits the magnetic flux in the transformer's iron core to mitigate harmonics at the distribution level, reducing reliance on external filtering components. A key innovation of this research is developing a controlled current injection technique that introduces compensating currents into the tertiary winding, effectively canceling phase-shifted harmonics in the primary and secondary windings. This technique reduces THD and improves overall power quality. Despite traditional filtering systems, which passively absorb or block harmonics, this active compensation method neutralizes undesired harmonic components inside the system, making it a more efficient and scalable option. A comprehensive analysis of harmonic sources and their impact on power distribution networks was conducted to establish the foundation for this research. Conventional mitigation techniques, including passive filters, active filters, hybrid filtering solutions, and phase-shifting transformers, were reviewed to highlight their benefits and limitations. A detailed MATLAB/Simulink model was developed to simulate the behavior of a three-winding transformer under varying nonlinear load conditions, incorporating core nonlinearities such as saturation and hysteresis effects. The proposed injection technique was tested in simulations to evaluate its effectiveness in suppressing harmonics across multiple frequency orders. Experiments were conducted in a controlled laboratory environment to verify the modeling results. The results indicated a considerable reduction in harmonic content, with primary winding current harmonics decreasing from 6.79% to 2.9% in simulations and 4.61% to 2.51% in laboratory testing. These findings demonstrate the proposed approach's feasibility and practical application to real-world power distribution networks. This study and its theoretical contributions provide beneficial insights into the practical application of transformer-based harmonic mitigation strategies. The approach is especially well-suited for renewable energy integration, electric car charging stations, and smart grid applications since it provides a cost-effective and scalable alternative to traditional filtering systems. This research

advances transformer-based harmonic suppression solutions, which improve power quality, enhance reliability, and eliminate harmonic-related losses in current electric power systems.

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LIST OF ABBREVIATIONS

DER	Distribution energy resources
EV	Electric Vehicle
HM	Harmonic mitigation technique
VFD	Variable frequency drive
THD	Total harmonic distortion
THDv	Total harmonic distortion voltage
THDi	Total harmonic distortion current
PQM	Power quality monitoring
ANN	Artificial neural network
cRIO	Compact RIO
DVR	Dynamic voltage drive
HPF	High pass filter
AHF	Active harmonic filter
EMF	Electromagnetic field
RC	Resistive-capacitive
UPS	Uninterruptible power supply
PWM	Pulse width modulation
FFT	Fast Fourier Transformer

CHAPTER ONE

INTRODUCTION

1.1 Background and Motivation Background and Motivation

Modern power distribution systems are increasingly affected by nonlinear loads, which introduce significant power quality challenges due to their impact on voltage and current waveforms. These nonlinear loads are primarily caused by rectifiers, inverters, and variable-speed drives, which distort voltage and current waveforms, leading to degraded system performance. Harmonics result in increased power losses, overheating of electrical equipment, reduced transformer efficiency, excessive neutral currents, and potential malfunctions in sensitive industrial and commercial loads.

The fast expansion of DERs, and the increasing penetration of EV charging infrastructure have heightened harmonic-related power quality problems. These distortions reduce system efficiency and contribute to grid instability, voltage fluctuations, and a lower power factor. Advanced HMTs are essential for maintaining power quality and grid stability to ensure reliable and efficient operation.

1.2 Power Quality

Power quality directly affects electrical equipment's performance, dependability, and resilience, which is the consistency, stability, and purity of the electrical power delivered to consumers. Electrical systems would produce a pure sinusoidal voltage waveform with a constant frequency and amplitude in an ideal state. However, in real-world situations, various variables lead to departures from this ideal, which are referred to as power quality disturbances. These disruptions include voltage sags and swells, frequency fluctuations, transients, flicker, notching, and most importantly harmonic distortion. One of the leading causes of the decline in power quality is the growing penetration of nonlinear loads. VFDs, switched-mode power supplies, solar inverters, industrial rectifiers, and electric vehicle chargers are examples of nonlinear devices that draw currents that are not proportionate to the voltage applied. This non-proportional behavior distorts the overall current and voltage waveform by introducing harmonic currents into the system. In addition to decreasing energy efficiency, these distortions negatively impact electricity infrastructure. For instance, transformer overheating, generator derating, premature cable aging, and faults in delicate digital equipment can all be caused by harmonic currents. The THD index quantifies harmonic distortion concerning the fundamental frequency. Low-quality high THD values indicate poor power quality, often a sign of insufficient mitigation techniques or excessive nonlinear load operation. Maintaining THD under acceptable levels is crucial for reducing losses, preventing resonance circumstances, and guaranteeing adherence to grid regulations and interconnection requirements, as per IEEE 519 and other international standards [1]. Transformers are particularly susceptible to the effects of harmonics since they are passive parts of the power delivery system. Elevated temperatures and decreased operational efficiency result from harmonic-rich currents increasing eddy current and hysteresis losses in the core as they pass through the transformer windings. These heat stressors can limit transformer lifespan, deteriorate insulation, and increase acoustic noise. In the worst situations, extended exposure to harmonics can result in force derating or transformer failure, which reduces the transformer's ability to manage nominal loads [2]. Historically, external solutions like passive, active, or hybrid filtering systems have mitigated the harmonics. However, these methods frequently increase complexity, expense, and space requirements, especially in retrofit applications. Transformer-based harmonic mitigation presents a viable substitute given these difficulties. Engineers may greatly enhance power quality at the distribution level without exclusively depending on external filtering equipment by employing internal transformer characteristics, such as tertiary winding, to introduce compensatory currents that cancel out harmonic components. This study suggests a cutting-edge transformer-based strategy that incorporates harmonic reduction straight into the electrical and magnetic design of the transformer. In addition to making the system design simpler, this approach offers a scalable, economical, and energy-efficient solution to one of the most urgent power quality issues facing contemporary electrical networks.

In addition, it becomes increasingly important to maintain power quality at every node as electric networks grow more decentralized and integrated with renewable energy sources. Similarly, the network experiences extra nonlinear loads due to the increasing need for infrastructure for electric car charging. These trends highlight the necessity of localized power quality solutions that can adjust to changing system demands. Considering the benefits of passive operation, low maintenance, and smooth integration into current grid infrastructure, transformer-based mitigation solutions are ideally suited for this changing environment. In summary, power quality is a fundamental prerequisite for the secure, effective, and dependable functioning of contemporary power systems and is not only a technical statistic. In an electrical environment that is becoming more complicated, addressing harmonic distortion at the transformer level is a new and innovative way to improve power quality. This work advances that goal by investigating the use of controlled tertiary winding current injections as a technique for reducing harmonic distortion and reaffirming transformers' active role in improving power quality.

1.3 Power Quality Monitoring

Maintaining stable and dependable electrical networks requires PQM, especially in systems increasingly impacted by dispersed energy supplies and nonlinear loads. PQM measures detect, classify, and analyze a variety of disturbances that impact the waveform integrity of voltage and current in the grid, including harmonics, voltage sags, transients, flicker, and frequency deviations. In addition to identifying these deviations, monitoring aims to evaluate their effects and provide the information required to direct mitigation and system enhancements. In the past, traditional power systems utilized post-event diagnostics or customer complaints to determine problems with power quality. However, given how quickly networks are changing nowadays, this reactive strategy is no longer adequate. The intricacy of power electronic demands and the incorporation of renewable energy have necessitated the shift to a more proactive and distributed monitoring architecture. By using strategic data estimate techniques, Power Quality Information Nodes allow for the optimal placement of monitors across a system, lowering costs while preserving complete observability, as mentioned by Eldery et al. [3]. Wide area monitoring is made possible by these nodes, which detect a portion of the system voltages and currents and estimate the remainder based on the system architecture. This eliminates the need for sensors at every location. Technology has advanced in PQM systems, in addition to spatial optimization. ANN and electromagnetic sensing-based portable monitoring system are presented in the work of Xu et al. [4]. The accuracy and flexibility of monitoring systems are improved by this technique, especially in microgrids where voltage and current characteristics change often. Because of a normalization approach that standardizes characteristics across various contexts, the ANN-based classifier only needs one training session. This portable design shows how artificial intelligence may improve the intelligence and scalability of monitoring systems in various grid applications. PQ monitoring has been further enhanced by smart grid technologies, which offer centralized visibility and real-time data collecting. A smart grid architecture uses hundreds of sensors in distribution transformers, smart meters, and substations to detect and categorize voltage issues in advance, as explained by McBee and Simões [5]. Unlike traditional systems that respond to complaints, smart grids automatically notify utilities of voltage quality violations and continually monitor voltage quality. Along with lowering operating expenses and preventing equipment damage, this proactive approach also increases customer satisfaction.

The proposed method relies on PQ monitoring to validate the efficacy of harmonic mitigation techniques implemented through tertiary winding current injection, which necessitates high-resolution, synchronized measurements at the transformer's primary, secondary, and tertiary windings. Using hardware like cRIO for real-time acquisition in conjunction with FFT analysis in Simulink enables accurate quantification of harmonic distortion before and after implementing the mitigation strategy. Monitoring becomes crucial in this feedback loop to validate the concept and adjust the injection control parameters based on real-time system response. Predictive maintenance and resilience analysis benefit significantly from the present monitoring systems. Utilities can forecast equipment failures, schedule condition-based maintenance, and ascertain how external factors, such as load switching or variations in solar generation, impact system stability by continuously monitoring waveform distortion trends. As Hossain et al. [6] pointed out, monitoring systems also aid in the deployment and performance assessment of custom power devices such as D-STATCOM and UPQC, particularly in renewable-integrated networks.

1.4 Power Quality Equipment

Power quality equipment refers to various instruments and devices used to monitor, maintain, and improve the integrity of voltage and current waveforms in power systems. The two main types of these devices are monitoring devices, which identify and document power outages, and mitigation devices, which either actively or passively fix such outages. Power quality equipment selection and integration have become crucial for stable operation, regulatory compliance, and long-term system resilience as electrical systems grow more crowded with sensitive devices and nonlinear loads. Traditional power quality mitigation technologies include passive harmonic filters, active power filters, UPS, DVR, surge protectors, and static VAR compensators. Each of these devices addresses a distinct sort of power quality issue. Passive filters are designed to block or absorb harmonic frequencies, whereas active filters utilize electrical switching to inject compensatory currents that cancel harmonics in real-time. DVRs and UPS units contribute to voltage stability during transient events or outages, whereas SVCs control reactive power to maintain voltage levels. Many of these traditional solutions, albeit shown to be efficient, have drawbacks in terms of cost, space needs, scalability, and integration complexity, especially when it comes to current infrastructure. For instance, passive filters may lose effectiveness when load circumstances change because of their set frequency response. Although more adaptable, active filters can be costly and need ongoing upkeep and control coordination. Furthermore, these devices are often outside, expanding the system footprint and necessitating careful coordination with the current grid. On the other hand, this thesis explores a transformer-integrated harmonic mitigation strategy that uses a three-winding transformer's tertiary winding as an internal, system-embedded technique to lower harmonic distortion. Without large external filtering devices, harmonic components in the main power flow may be actively canceled by introducing regulated current into the tertiary winding at particular harmonic frequencies. This offers a small, affordable, and naturally scalable approach, marking a paradigm change from external mitigation to transformer-level harmonic suppression. This method's laboratory use necessitates specific power-quality tools for simulation and real-time testing. The essential elements consist of:

a. The key component of the experimental system is the three-winding transformer. The supply is linked to one winding, the load to the second, and harmonic injection is saved for the tertiary winding. The magnetic interaction between these windings makes harmonic cancellation within the transformer itself possible.

b. Function Generator: Used to generate harmonic signals corresponding to the system's harmonic distortion phase and amplitude. In order to introduce compensatory currents that oppose and cancel the current harmonics, the current is linked to the tertiary winding.

c. HPF is placed in series with the tertiary winding to guarantee that only harmonic frequencies- not the fundamental frequency are injected. This selective filtering makes targeted harmonic suppression possible and keeps interference with the regular power flow at bay.

d. National Instruments cRIO Data Acquisition System is a real-time embedded control and monitoring device that records high-resolution voltage and current waveforms. It facilitates real-time data processing and allows synchronized monitoring across all transformer windings.

e. The LabVIEW software records, displays, and analyzes waveform data by interacting with the cRIO hardware. It makes it easier to compare performance before and after mitigation, do real-time FFT analysis, and compute THD.

f. MATLAB/Simulink models and simulates the transformer, harmonic behavior, and mitigating effects. The simulation environment also supports the harmonic injection strategy's control design.

g. An optional power quality analyzer can independently measure transient events, voltage imbalance, THD, and harmonic suppression.

This unique incorporation of technology enables the suggested harmonic mitigation approach to be evaluated under controlled settings, allowing for a thorough examination of its performance over a wide range of load scenarios. This method may be easily included in new transformer designs or retrofitted into old ones with just modest modifications, unlike the majority of commercial filtering devices, which need installation outside the transformer. Transformer-based solutions provide a more comprehensive approach to power quality control from a larger system architect perspective. Harmonics are handled in the device that transforms voltage, as opposed to being treated as a downstream problem that needs to be filtered after the fact. This creates new opportunities for integrating intelligence and flexibility into the distribution network's essential elements.

1.5 Problem Statement

Traditional harmonic mitigation approaches, such as passive, active, and hybrid filtering systems, are widely utilized in power distribution networks. However, these methods possess several limitations, including high implementation costs, complexity, power losses, and development challenges. Passive filters are commonly employed due to their simplicity and cost-effectiveness, but they suffer from resonance issues, fixed compensation characteristics, and dependency on grid impedance variations. AHFs provide real-time harmonic compensation, but they require complex control strategies, additional power electronics, and increased energy consumption, making them less practical for large-scale deployment [7]. Transformers play a fundamental role in power distribution by providing voltage conversion, isolation, and impedance matching. Despite their essential function, transformers are susceptible to harmonic distortions due to core nonlinearity, saturation, and hysteresis effects. Existing harmonic mitigation solutions often focus on external filtering techniques rather than transformer-based mitigation strategies.

This gap in research presents an opportunity to explore innovative transformer-based approaches that integrate harmonic mitigation within the power distribution infrastructure.

1.6 Objectives of the Thesis

Maintaining good power quality has become increasingly difficult as nonlinear load sources are increasingly included in contemporary electrical networks, mainly because of harmonic distortion. Even though conventional solutions like passive and active filters provide some respite, they frequently have disadvantages, including higher system complexity, expense, and space requirements. This thesis investigates a new method that uses the transformer's tertiary winding for controlled harmonic current injections, embedding harmonic mitigation capabilities directly inside the transformer. The study aims to offer a technically sound and reasonably priced solution to harmonic distortion problems supported by simulation and experimentation. The main goal of this study is to design, create, and validate a transformer-based harmonic mitigation technique that actively employs tertiary winding to cancel harmonic components in the power distribution system. The technique seeks to increase system efficiency and lessen reliance on external filtering equipment by addressing harmonics at the transformer level. The thesis aims to accomplish this goal by pursuing the following goals:

1. The purpose of this study is to examine how harmonic distortion affects power distribution transformer performance and longevity, especially when nonlinear loads are present. This entails researching harmonic current-induced voltage waveform distortion, core losses, and thermal stress.

2. To create a thorough simulation model of a three-winding transformer that considers nonlinear magnetic phenomena like hysteresis and core saturation. Under regulated and changing load settings, the model provides a basis for researching harmonic behavior and assessing the mitigation approach.
3. To create and implement a harmonic current injection method that targets the prominent harmonics found in the system (such as the third, fifth, and seventh) by utilizing the transformer's tertiary winding. This method counteracts harmonic components seen on the primary and secondary windings by generating a coordinated injection signal using a function generator and high-pass filter.
4. Hardware, including a three-winding transformer, nonlinear loads, a cRIO data gathering system, a LabVIEW monitoring interface, and harmonic injection circuitry, will be used to construct a lab testbed replicating authentic harmonic-rich loading circumstances.
5. To compare transformer responsiveness, voltage waveform quality, and THD levels before and after injection to assess and evaluate the effectiveness of the suggested harmonic mitigation technique. Both the time and frequency domains are used for this evaluation.
6. Verify the method's efficacy through experimental and simulation testing, ensuring the outcomes are reliable and scalable for practical use. Testing under various loading scenarios and harmonic characteristics is part of this.
7. Evaluate the suggested transformer-integrated system's benefits, drawbacks, and potential for future improvements compared to traditional harmonic reduction

strategies. The comparison considers elements including operational robustness, cost, system footprint, and simplicity of deployment.

8. To provide a universal framework or design standard for transformer-integrated harmonic mitigation that may be used by utility companies or manufacturers seeking small and effective power quality solutions.

By achieving these goals, the thesis hopes to significantly advance the subject of power systems engineering by presenting a novel transformer-level technique for harmonic suppression that is easy to use, scalable, and efficient in contemporary distribution settings.

1.7 Contributions

This thesis introduces a novel transformer-based harmonic mitigation strategy that utilizes controlled current injection into the tertiary winding to cancel phase-shifted harmonics in the primary and secondary windings effectively. The proposed approach enhances power quality by addressing harmonic distortion at its source, reducing reliance on external filtering methods. The key contributions of this research include:

- A comprehensive review of harmonic mitigation techniques and transformer modeling is presented, covering methods such as passive filters, active filters, and transformer-based approaches used in power systems. The review also explores transformer harmonic modeling techniques, with a focus on those that account for magnetic core nonlinearity, including hysteresis and saturation effects. These insights highlight the limitations of existing methods and provide the motivation for developing the proposed approach.

- Development of an advanced harmonic suppression method – This work directly integrates a harmonic mitigation technique into transformer design rather than relying solely on external passive or active filters. The approach neutralizes nonlinear harmonics created by strategically injecting a regulated compensatory current into the tertiary winding, considerably enhancing the transformer's overall harmonic performance.
- Simulation-based validation is conducted using extensive MATLAB/Simulink simulations to evaluate the effectiveness of the proposed method. Multiple case studies and sensitivity analyses are performed to assess the influence of key parameters such as injection current magnitude, frequency content, and transformer loading on harmonic suppression performance.
- Experimental evaluation is conducted using LabVIEW with cRIO to confirm the real-world applicability of the proposed method. The results demonstrate significant reductions in THD, validating both the effectiveness and practical feasibility of the approach in actual transformer configurations.

1.8 Thesis Organization

This research focuses on implementing a tertiary winding current injection method for harmonic mitigation in power distribution systems. It involves insights into transformer-based solutions for enhanced power quality and recommendations on using sophisticated harmonic control tactics to increase the adaptability and efficacy of future large-scale and complex power systems.

Chapter 2 provides a detailed analysis of harmonic reduction strategies and their significance in enhancing power quality in power distribution networks. It discusses passive, active, hybrid, and transformer-based filtering systems, addressing their principles, benefits, limits, and practical applications. This review establishes the foundation for the dissertation and identifies research gaps addressed in this study.

Chapter 3 introduces the proposed harmonic mitigation method using tertiary winding current injection. It covers key aspects such as design, mathematical modeling, and theoretical principles, explaining how this approach reduces harmonic distortion and improves system performance.

Chapter 4 focuses on the simulation and sensitivity analysis of the proposed harmonic mitigation strategy using MATLAB/Simulink. Various scenarios are simulated to assess harmonic reduction, system stability, and power quality improvements. The chapter also explores the impact of key parameters such as transformer winding configurations and injected current magnitudes on system performance, providing a comprehensive sensitivity analysis to evaluate the method's robustness and adaptability.

Chapter 5 presents the experimental verification of the proposed strategy. It describes the hardware setup, measurement tools, and testing procedures used to validate simulation outcomes. Experimental results are compared with simulated data to demonstrate the practical feasibility and effectiveness of the approach in reducing harmonics and enhancing power quality in real-world conditions.

Chapter 6 concludes this work and provides suggestions for future research. It focuses on adaptive harmonic control strategies and extends the method to three-phase power systems for broader applicability and improved mitigation performance.

CHAPTER TWO

LITERTURE REVIEW

2.1 Introduction

Harmonic distortion has become a serious issue in electric systems due to the growing use of nonlinear loads, such as power electronic converters and variable frequency drives. These nonlinear devices introduce harmonic voltages and currents into the system, creating power quality action, including power losses, overheating of electric equipment, reduced efficiency, and potential malfunction of sensitive loads. Various mitigation methods have been developed to mitigate the influence of harmonics, such as passive and active filters, transformer-based methods, and control approaches. Hence, these methods face limitations such as high cost, complex design, and reduced efficiency due to different conditions. To overcome these limitations, harmonic mitigation by injecting current through the tertiary winding has been proposed to highlight its capability to cancel harmonic distortion before it separates into the entity system. This chapter reviews harmonic distortion in power systems, its causes, effects, and measurement methods. Moreover, it discussed the existing mitigation approaches in terms of the design. The limitations of the existing mitigation approaches are analyzed to highlight the need for more efficient methods. This chapter introduces the motivation for proposing a novel migration method through the thirty winding, aiming to improve harmonic distortion, improve power quality, and optimize overall system performance.

2.2 Harmonics in Power Systems

Harmonics, defined as voltage or current waveforms, occur at multiple integers of the fundamental frequency of the power system. These distortions are caused by the nonlinear behavior of particular loads that draw non sinusoidal current from the power supply. The existence of harmonics can induce power quality issues, leading to inefficiencies and potential damage to electric components [8]. From a mathematical aspect, any periodic non sinusoidal waveform can be represented using Fourier series decomposition, which is expressed as a sum of sinusoidal components at different frequencies [9].

$$x(t) = p_o + \sum_{n=1}^{N} p_n \cos(n\omega t) \tag{2.1}$$

Where p_o is the DC component (or average value) of the function over a period. $\sum_{n=1}^{N} p_n \cos(n\omega t)$ is a sum of cosine terms, where: p_n is the Fourier coefficients, which determine the contribution of each cosine term. n is the harmonic index (integer multiples of the fundamental frequency). ω is the fundamental angular frequency (rad/s). t is the time. N is the number of harmonics included in the approximation. Harmonic distortion is a deviation from the ideal sinusoidal waveform in alternating current systems caused by various factors. Companies that supply electric power systems must analyze their data on harmonics to identify potential problems and compare them with power quality standards. The voltage magnitudes with harmonic orders determine the level of harmonic voltage distortion, which is based on the level of harmonic current injected by nonlinear load components into the distribution grid. To evaluate the influence of harmonics on the distribution system, it is important to use THD measurements. THD is a metric that measures waveform distortion proportional to the fundamental frequency. This calculation is based on data collected while powering a nonlinear load, and it assesses the level of harmonic distortion in both voltage and current. Equation (1), shown below [9], defines THD.

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} U_n^2}}{U1} \times 100 \tag{2.2}$$

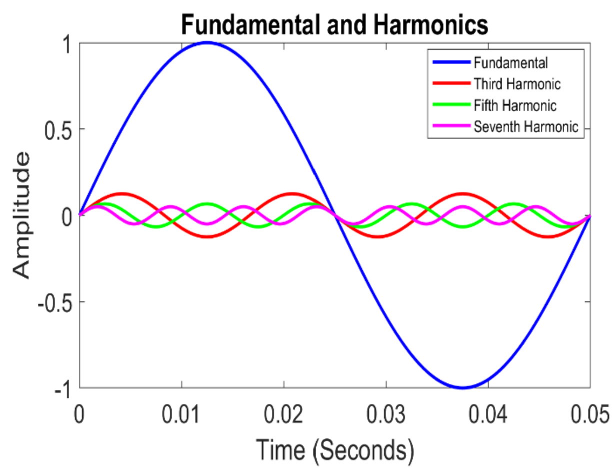
THD is defined by equation (2.2), as seen in [9]. When calculating the sum for harmonic order, it usually begins from the second order ($n = 2$) up to the 50th order. The letter *U* can represent either voltage or current. When calculating the sum for harmonic order, it usually begins from the second order ($n = 2$) up to the 50th order. *U* can represent either voltage or current. A further approach is to compute the THD for current and voltage, as shown in (2.3) and (2.4). The following expression expresses the amplitude of the harmonic current as:

$$THD_I = \frac{\sqrt{I_{rms-}^2 I_{1rms}^2}}{I_{1rms}} \times 100 \tag{2.3}$$

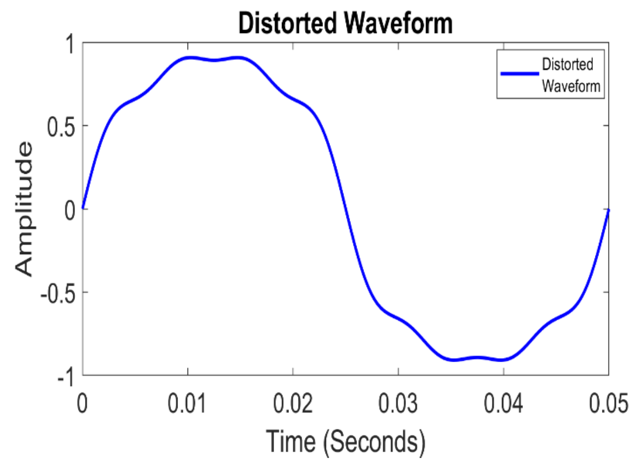
$$THD_V = \frac{\sqrt{V_{rms-}^2 V_{1rms}^2}}{V_{1rms}} \times 100 \tag{2.4}$$

where *I* subscript represents the fundamental load current and voltage.

Upon analyzing the output voltage and current in the frequency domain using Discrete Fourier Transform, it is seen that the nonlinear loads produce harmonics at multiple frequencies, including 180 Hz, 300 Hz, 420 Hz, 540 Hz, 660 Hz, 780 Hz, and 900 Hz, respectively, in the third, fifth, seventh, ninth, eleventh, thirteenth, and fifteenth orders. The proposed harmonic injection method can significantly reduce high-order harmonics as explained in the next section. Additionally, harmonic levels can be manually adjusted by introducing a current source with a specific frequency. Figure 1 shows and compares the ideal AC waveform, as well as a distorted waveform, representing real-world harmonics and non-linearities. In order to analyze this problem, it is necessary to create a model of an oscillating harmonic waveform to calculate non-sinusoidal voltages and currents. This study is limited to the scenario of smoothly varying waveforms.



(a)



(b)

Figure 1. Electrical waveform with HD (a) Extracted harmonics, (b) Distorted waveform

2.2.1 Sources of Harmonics in Power Distribution Systems

Harmonics in power distribution systems are mainly created from nonlinear loads that draw currents not proportional to the applied voltage. This nonlinear distorted waveform introduces harmonic components throughout the system. A classic example of such a nonlinear load is the circuit in Figure 1, where a diode only conducts current for one half-cycle of the AC waveform (usually the positive half). As a result, although the voltage waveform remains sinusoidal, the current waveform becomes distorted and non-sinusoidal. This non-sinusoidal current can cause power quality issues such as increased system losses, overheating of equipment, and interference with communication systems due to the presence of harmonics.

Power electronic converters are among the primary sources of harmonics due to their switching operations. These converters include rectifiers and inverters, both of which produce non-sinusoidal current waveforms because of high-frequency switching. Rectifiers (AC to DC converters), especially single-phase and three-phase uncontrolled types, generate mainly odd harmonics. Inverters (DC to AC converters), on the other hand, contribute harmonics as a result of PWM techniques used in their operation. Switching power supplies found in devices like computers and LED drivers also introduce high-frequency harmonics due to their rapid switching characteristics.

In addition to these, nonlinear industrial loads such as arc furnaces and welding machines introduce significant harmonic distortion through rapid fluctuations in arc impedance. Likewise, VFDs, which are used to control electric motor speeds by adjusting supply frequency, are another substantial source of harmonics in modern power distribution systems [10], [11].

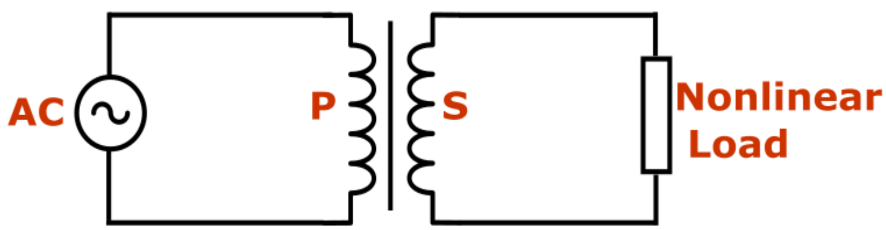


Figure 2. AC circuit with nonlinear load

2.2.2 Effects of Harmonics on Power Systems

Harmonics decrease power system performance, leading to efficient operation, overheating, and potential equipment failure. One of the essential influences of harmonics is to cause power losses and decrease system efficiency. Transformers and other electrical components suffer significant heating, leading to increased power losses, which increase eddy currents and hysteresis in transformers, and rotating machines deteriorate inefficiencies. An additional significant impact is voltage distortion, where harmonic producing loads distort voltage waveforms, influencing the operation of sensitive equipment. Distorted voltage contributes to relay operations, flickering in lighting systems, and inflated meter readings [12]. Future, harmonic distortion induces neutral conductor overheating in three phase systems, typically due to tripled harmonics. Another primary concern of harmonic currents is the overheating of equipment that increases thermal stress on electric devices such as transformers, motors, and cables. Excessive heating deteriorates insulation breakdown, reducing the lifespan of the equipment. Furthermore, harmonics can induce resonance by affecting system impedance and amplifying harmonic frequencies. Resonance issues can generate high voltages and currents, especially destroying capacitors, transformers, and switchgear. Additionally, power factor correction capacitors specially exposed to harmonic resonance raise the risk of failure [13].

2.3 Harmonic Mitigation Techniques

Through the existing strategies, passive filtering is considered one of the most widely employed techniques due to its simplicity, reliability, and cost-effectiveness. Passive filters use inductors and capacitors to construct tuned circuits that selectively block or absorb particular harmonic frequencies by providing a low-impedance path. These filters are classified into single-tuned filters (notch filters), high-pass filters, and bandpass filters, each designed to address different harmonic components. Single-tuned filters effectively attenuate targeted harmonics, such as the 5th or 7th, by leveraging LC circuits tuned to frequencies. High-pass filters eliminate high-order harmonics over a specific cutoff frequency, an essential requirement in industrial power systems where high-frequency harmonics can obstruct sensitive electronic equipment [14]. Bandpass filters provide a more inclusive solution and introduce the attenuation of diverse harmonic frequencies into a particular order, making them suitable for applications that desire broader spectrum harmonic elimination [15]. While passive filters provide various features, inclusive ease of implementation, low cost, and operation without external power sources, their efficiency is limited by their fixed frequency response, susceptibility to resonance issues, and critical physical footprint due to the bulkiness of inductors and capacitors. These limitations usually make passive filters insufficient for environments that dynamically alter harmonic content and constrain more adaptive solutions deployment [16].

In contrast, active filtering has appeared as a more flexible and accurate approach to harmonic mitigation, leveraging power electronics to dynamically introduce compensating currents that counteract harmonic distortions in real-time. Active filters are classified into shunt active, series active, and hybrid active filters, each offering unique operational benefits. Shunt active power filters, which are connected in parallel with the load, operate by injecting currents equal in magnitude but opposite in phase to the harmonic components, effectively neutralizing them at the point of common coupling. On the other hand, Series active filters are installed in series with the supply line and function as controlled voltage sources, preventing harmonics from spreading into the power network. Hybrid active filters combine the strengths of both passive and active filtering techniques, offering improved harmonic suppression while optimizing cost and system complexity. The primary characteristics of active filters include dynamic adaptability to varying harmonic conditions, compact design, and the ability to mitigate both voltage and current harmonics simultaneously. However, these benefits come at the cost of higher initial investment, increased energy consumption due to the need for external power, and greater system complexity, requiring sophisticated control algorithms and continuous maintenance. In spite of these challenges, active filtering remains a vital solution in applications that require accurate and adaptive harmonic mitigation, including industrial motor drives, renewable energy systems, and smart grids [17],[18],[19].

Beyond filtering techniques, transformer-based solutions provide a robust and passive means of mitigating harmonics by leveraging specialized winding configurations and core designs. These approaches include zigzag autoconnected transformers, magnetic-integrated multipulse rectifier transformers, and space-shifted wye-delta windings tailored to particular harmonic mitigation challenges. Zigzag autoconnected transformers are incredibly efficient in 12-pulse rectifier systems, where they eliminate triplen harmonics and improve power quality, making them well-suited for industrial applications [20]. Magnetic-integrated multipulse rectifier transformers employ impedance equalization strategies to optimize harmonic elimination in DC traction power supply systems, significantly decreasing THD and enhancing overall system efficiency [21]. Space-shifted wye-delta windings provide a novel process to reduce space harmonics in fractional-slot winding configurations by strategically adjusting winding placements, thereby canceling both subharmonics and super harmonics [22]. Transformer-based solutions are specifically advantageous in industrial and large-scale applications due to their passive operation, efficiency in mitigating low-order harmonics, and improved reliability compared to active systems. However, there are also existing challenges, including large physical size, higher upfront investment costs, and limited adaptability to dynamically changing harmonic conditions. Beyond these core methodologies, various advanced harmonic mitigation mechanisms have been developed to handle particular industrial and transportation sector challenges. For instance, C-type filters have been successfully performed in high-speed railway networks to counteract harmonic resonance and enhance power quality [23]. Similarly, model predictive control techniques have been utilized in marine power systems, such as diesel-electric ships, enabling real-time optimization of harmonic mitigation across multi-bus electrical grids. In high-speed permanent magnet synchronous machines, electromagnetic vibration analysis methods have been proposed to eliminate the adverse influence of current harmonics on machine performance, decreasing core losses and improving operational efficiency [24]. These specialized mitigation approaches demonstrate the increasing sophistication of harmonic control technologies and their increasing importance in complex electrical networks. An optimal harmonic mitigation approach should be selected based on various factors, including the severity of harmonic distortion, the particular application requirements, and economic limitations. Passive filters are most appropriate for systems with well-defined, stable harmonic frequencies and low power quality demands. At the same time, more expensive and complex active filters offer higher performance in dynamic environments where harmonic levels fluctuate significantly. Although most efficient for low-order harmonics, transformer-based solutions are best suited for large-scale industrial applications where passive, long-term mitigation is preferred. In many situations, the most efficient approach comprises a hybrid approach that combines multiple mitigation techniques, integrating the cost-effectiveness of passive filters, the adaptability of active filters, and the robustness of transformer-based solutions to obtain optimal power quality and long-term system reliability.

2.4 A Review Power Transformer Harmonic Modeling

Transformers are critical to effectively running power systems since they regulate voltage and ensure the dependable transfer of electrical energy across networks. As the energy sector evolves, power networks get more complicated as renewable energy sources are integrated, nonlinear demands increase and grid technology progresses. These developments present new problems to the electric grid, such as harmonic distortions, core saturation, and nonlinear interactions, which can degrade transformer performance and reduce system reliability due to nonlinear loads that need to be resolved. Transformer modeling is necessary to handle these challenges, including dealing with a high frequency to simulate and optimize these vital components accurately under various operating conditions. Researchers implemented optimized transformer models to address these challenges involving nonlinearities, hysteresis effects, and dynamic harmonic behavior. These models aim to offer a further understanding of transformer behavior under diverse settings, from steady-state operations to transient phenomena such as inrush currents and ferroresonance [25],[26]. Harmonic-domain modeling is an effective technique for investigating the generation and transfer of harmonics in transformers [27]-[30]. By representing harmonic currents and voltages as a sequence of harmonic components, these models can predict how they will interact within the transformer and propagate across the network. Another exciting development in transformer modeling is the application of the Jiles–Atherton hysteresis model to represent the magnetic behavior of transformer cores [31]-[34]. This method considers the nonlinear relationship between magnetic flux and magnetizing current, as well as core saturation and remnant magnetization.

The Jiles-Atherton model has improved the accuracy of transient assessments, such as estimating inrush currents during transformer energization or assessing residual flux after de-energization [35]. These improvements are critical for minimizing the effects of harmonic distortions and enhancing transformer reliability in power systems. In addition to low-frequency effects, researchers have increasingly focused on transformers' high-frequency behavior, particularly in the context of power electronic applications and electromagnetic interference [36]. High-frequency models integrate wideband frequency response characteristics with core nonlinearities to comprehensively understand transformer performance across a broad spectrum of operating conditions [38]. Research studies have proven the usefulness of these hybrid models in reproducing high-frequency transients, such as lightning strikes and switching surges while maintaining accuracy for low-frequency phenomena [38]. Furthermore, parameter identification has played an important role in improving transformer modeling. For particular transformer topologies, core and winding parameters have been precisely determined using genetic algorithms, finite element analysis, and harmonic injection tests [39]. Parameter identification has also greatly aided in enhancing transformer modeling. Accurate core and winding parameters for transformer designs have been determined using techniques including finite element analysis, harmonic injection experiments, and genetic algorithms [40]. Even with these developments, problems still exist. Ensuring power system reliability requires accurate transformer modeling, especially in situations with a high harmonic content when performance is significantly impacted by nonlinear magnetization effects. Traditional models usually fail to detect inrush current transients, necessitating numerical approaches that account for hysteresis, saturation, and eddy current losses. To improve efficiency and decrease losses, researchers have investigated hybrid transformer core architectures, including various materials, optimizing flux distribution and improving overall transformer performance in current power grids [41]. Future studies should also investigate how transformer modeling is affected by new technologies like solid-state transformers and hybrid grid configurations. Considering the critical role of transformers in power systems and the growing challenges associated with harmonic distortions, several review papers have been published focusing on methodologies, modeling techniques, and the impacts of harmonics on transformer performance. A comprehensive review of harmonic modeling techniques, including linear and nonlinear approaches, is presented in [42]. The complexities of representing transformer behavior under low- and mid-frequency transients are emphasized, highlighting the need for accurate core and winding models. Amoiralis et al. conducted a thorough literature review of transformer design and optimization strategies, identifying shortcomings in standard approaches and proposing innovative ways to overcome harmonic issues [43].

Furthermore, Raichura et al. reviewed approaches for distinguishing transformer magnetizing inrush from fault circumstances, providing valuable insights into harmonic interactions during transient occurrences [44]. The authors in [45] include core and coil configurations, mutual and self-inductances, leakage fluxes, magnetic core saturation, hysteresis, eddy current losses, and capacitive effects. Recent research thoroughly investigated nonlinear harmonic modeling approaches, evaluating their efficacy in addressing transformer losses, thermal performance, and operational dependability in harmonic distortion situations. Recent research thoroughly investigated nonlinear harmonic modeling approaches, evaluating their efficacy in addressing transformer losses, thermal performance, and operational dependability in harmonic situations. Given the increasing complexity of power systems and the prevalence of harmonic distortions, this work seeks to give a new viewpoint by addressing the key concerns in harmonic transformer modeling. Figure 3. shows the number of recent papers published in various articles. On the other hand, Figure 4. illustrates the number of published papers from 2014 to 2024. This research considered data from the last ten years for analysis. Most publications occurred in 2024, continuing an upward trend over the past decade. It categorizes modeling approaches according to methodology, application areas, and important issues, highlighting gaps in current research and recommending future paths. This paper aims to help academics and engineers improve transformer performance and reliability in harmonic power systems by synthesizing current advances. This review study addresses over 130 transformer modeling research findings, emphasizing techniques that handle nonlinearities, harmonic interactions, and transient events. The study categorizes these models according to their aims, methodology, and application areas and comprehensively analyzes their strengths and weaknesses. Since it integrates existing information and identifies gaps in the current research environment, this study provides a complete resource for academics, engineers, and policymakers working to improve transformer reliability and efficiency in modern power networks.

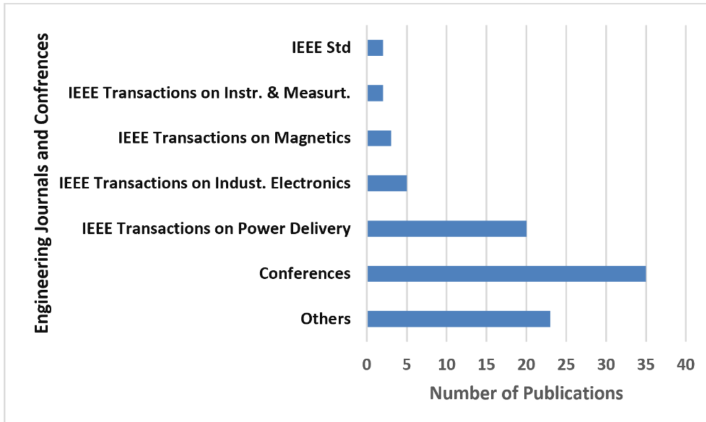


Figure 3. Distribution of Publications Across Engineering Journals and Conferences

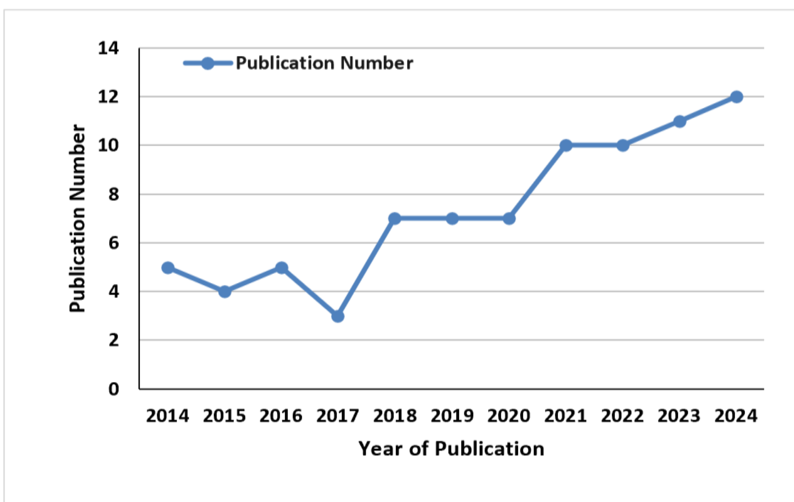


Figure 4. Trend of Transformer Harmonic Modeling Publications (2014–2024)

Recent transformer modeling techniques are categorized into several classes: linear models, nonlinear models, EMF models, frequency domain models, harmonic-domain models, and hybrid approaches.

2.4.1 Linear Models

Linear models are often employed in transformer research due to their simplicity and computational efficiency. These models benefit low-complexity studies requiring a basic understanding of transformer behavior under idealized conditions. They provide a foundational approach for analyzing steady-state processes and are frequently used as the initial approximation in system design and analysis [46], [47]. However, linear models have substantial limitations in representing the complexity imposed by nonlinear behaviors in transformers, particularly under harmonic distortion environments generated by distributed energy resources and electric vehicles. Transformers subjected to nonlinear loads exhibit phenomena such as core saturation, hysteresis, and harmonic interactions [48], [49]. For example, strong harmonic currents can cause the magnetizing properties of transformer cores to deviate significantly from linearity, resulting in inaccuracies in predictions made by linear models [50], [51]. Moreover, linear approximations fail to capture dynamic phenomena, such as inrush currents and DC bias effects, intensified in systems incorporating power electronic interfaces [52], [53]. These oversights may lead to underestimating core losses, thermal stresses, and aging effects that transformers experience in real-world operating environments [54], [55]. While linear models are a useful starting point, their inability to reflect nonlinear interactions highlights the need for more advanced modeling techniques. Methods such as nonlinear magnetic equivalent circuit modeling and harmonic domain modeling have been developed to address these limitations [56], [57]. These approaches provide a more accurate representation of transformer behavior, enabling reliable analysis and design advancements in modern power systems.

2.4.2 Nonlinear Models

Nonlinear models are widely utilized to capture the complex behaviors of transformers operating under realistic conditions. Unlike linear models, these approaches account for phenomena such as core saturation, hysteresis, and harmonic distortion, making them suitable for analyzing transformers subjected to nonlinear loads, such as those introduced by distributed energy resources and electric vehicles [58], [59]. These models effectively address issues arising from harmonic-rich environments by representing the magnetization characteristics and magnetic coupling effects with higher accuracy. For example, nonlinear magnetic equivalent circuit models and finite-element-based approaches have been developed to study dynamic behaviors, including inrush currents, saturation, and DC bias [60]. Such methods are particularly useful in evaluating transient performance and mitigating harmonic losses and thermal stress [61]. Advanced nonlinear modeling techniques also enable the study of high-frequency behaviors and localized saturation effects, which are critical in systems integrating power electronic interfaces [62].

Finite-element analysis and harmonic-domain modeling have been successfully applied to simulate harmonic propagation and losses in transformers under various loading scenarios [63]. Furthermore, recent developments in modeling nonlinear magnetization characteristics have facilitated real-time analysis and condition monitoring of transformers. These approaches contain dynamic characteristics, allowing for precise forecasts of core losses and efficiency under harmonic situations [64]. Nonlinear modeling methods have also been shown to help study high-frequency issues in distributed energy systems applications, such as electromagnetic interference and transformer saturation [65]. While nonlinear models provide greater accuracy and flexibility, they require higher computational resources and parameter identification efforts than linear models. Nevertheless, their ability to simulate complex behaviors makes them indispensable for modern power system analysis, particularly in environments dominated by distributed energy resources and electric vehicles [49]- [68].

2.4.3 Electromagnetic Field Models

EMF models provide a highly detailed representation of transformer behavior by directly solving Maxwell's equations, enabling accurate analysis of phenomena such as flux leakage, eddy currents, and localized saturation effects. These models are particularly effective in studying harmonic propagation, electromagnetic interference, and transient behaviors, which are prevalent in systems integrating EMF models. They provide a highly detailed representation of transformer behavior by directly solving Maxwell's equations, enabling accurate analysis of phenomena such as flux leakage, eddy currents, and localized saturation effects [60]. These models are particularly effective in studying harmonic propagation, electromagnetic interference, and transient behaviors, which are prevalent in systems integrating DERs and EVs. FEM is widely used in EMF modeling, offering precise calculations of magnetic flux, thermal stresses, and core losses under both steady-state and dynamic conditions [64]. FEM-based approaches can simulate complex effects, such as inrush currents, saturation, and DC bias, enabling better design optimization and performance evaluation [59]. Recent advancements have also integrated real-time monitoring techniques for condition assessment, fault diagnostics, and thermal management, making these models valuable for evaluating transformer reliability in harmonic-rich environments [61]. Despite their high accuracy, EMF models demand significant computational resources and detailed material data, posing challenges for real-time simulations; however, advances in parallel computing and hardware acceleration have made them more accessible for modern power system applications [68]. Their ability to capture nonlinear effects and high-frequency responses makes them indispensable for designing transformers that operate efficiently under harmonic distortions and transient conditions introduced by DERs and EVs [63], [69], and [70].

2.4.4 Frequency-Domain Models

Frequency domain models provide a robust framework for analyzing transformer behavior under harmonic and non-sinusoidal conditions by focusing on steady-state responses rather than transient dynamics. These models transform time-domain differential equations into the frequency domain, allowing for the representation of system components as impedance and admittance matrices, which simplifies the evaluation of harmonic propagation, resonance effects, and frequency-dependent losses [71]. This transformation is especially useful in systems with DERs and EVs, where power electronic interfaces generate high harmonic content that impacts transformer performance [72]. Unlike time-domain models, frequency-domain approaches efficiently handle harmonic load flow analysis, enabling the evaluation of harmonic distortions and voltage/current interactions at each harmonic frequency [53]. Such models can also predict frequency-dependent losses, including eddy current effects and hysteresis, by incorporating wideband parameters that capture variations in magnetic permeability and resistivity [73]. One of the significant applications of frequency-domain modeling is harmonic resonance analysis, which can identify resonant frequencies caused by interactions between transformers, capacitors, and harmonic sources [74]. This is particularly significant for modern electrical grids, where the widespread use of variable-frequency motors and inverters raises the potential of resonance circumstances, resulting in elevated harmonic levels and equipment damage. [51]. Furthermore, these models provide mitigation techniques for harmonic losses and voltage distortions by offering insights into reactive power needs and filter design [55]. In order to improve accuracy in high-frequency applications, recent developments have expanded frequency-domain algorithms by considering proximity effects, skin effects, and leakage fluxes in transformer windings into consideration [75].

These enhancements make frequency-domain models valuable for evaluating insulation performance, thermal management, and electromagnetic interference in transformers subjected to high-frequency harmonics [76]. Despite their strengths in steady-state analysis, frequency-domain models have inherent limitations in capturing dynamic and transient phenomena such as inrush currents, ferroresonance, and switching surges, which require complementary time-domain or electromagnetic field simulations for detailed assessments. However, their computational efficiency and ability to focus on frequency-specific interactions make them indispensable for designing transformers that operate efficiently under harmonic and non-linear loading environments [69],[77].

2.4.5 Transient Models

Transient models are essential for analyzing the dynamic behavior of transformers under rapidly changing operating conditions, including switching events, fault transients, inrush currents, and ferroresonance phenomena. Unlike steady-state frequency-domain approaches, transient models operate in the time domain, enabling the study of nonlinear and time-varying responses caused by DERs and EVs [52]. These models simulate electromagnetic transients by solving differential equations that describe flux linkage, winding currents, and magnetic saturation effects, making them highly effective for investigating sudden voltage surges, harmonic resonances, and switching transients [78]. Transient models are often used to analyze inrush currents, which occur during transformer energization and are impacted by core saturation and residual flux [58]. Such investigations aid in determining the strains placed on insulation and windings, offering information on surge suppression and transformer protection strategies [50]. Assessment of ferroresonance, a nonlinear oscillatory phenomenon that can result in overvoltage and magnetic saturation, particularly in systems with a high penetration of power electronic devices, requires transient models.

These models incorporate nonlinear magnetization curves, hysteresis, and frequency-dependent losses to accurately simulate core dynamics during switching operations and short circuits [68]. Advanced techniques, such as EMTP and finite-element-based time-domain methods, enable detailed simulations of electromagnetic interference, eddy currents, and leakage fluxes under transient conditions [77]. Furthermore, transient models have been adapted to study hybrid systems involving renewable energy resources, capturing interactions between transformers, inverters, and grid-tied converters [69]. Such applications are essential for evaluating transformer stability and reliability in modern smart grids [76]. While transient models provide a detailed analysis of time-varying behaviors, they are computationally intensive and often require precise material data, including magnetic characteristics and core geometry [63]. Despite these requirements, numerical approaches like adaptive meshing and FDTD algorithms have significantly increased simulation efficiency and accuracy [75]. These tools are increasingly used to assess electromagnetic compatibility, insulation performance, and fault response, ensuring transformers can withstand transient disturbances in grids dominated by DERs and EVs [55], [65]. By capturing nonlinear dynamics and short-term interactions, transient models remain indispensable for designing transformers that meet modern power systems' performance and reliability requirements [59].

Table 1. Comparison of transformer modeling techniques

Modeling Technique	Features	Advantages	Limitations	Applications	References
Linear Models	Simplified, computationally efficient, steady-state analysis.	Easy to implement, suitable for low-complexity and initial studies.	Cannot capture nonlinear effects (core saturation, hysteresis) or dynamic behaviors (inrush, DC bias).	Basic transformer design, steady-state analysis, and initial approximation.	[50], [51], [54], [55], [59]
Nonlinear Models	Incorporates nonlinear effects (core saturation, hysteresis) and harmonic distortion.	High accuracy for non-linear loads, transient performance, and harmonic analysis.	Computationally intensive and require complex parameter identification.	Harmonic propagation, localized saturation effects, and real-time monitoring in DERs and EV systems.	[31], [39], [48], [61], [71], [75]-[81], [53]-[89]
Hysteresis Models	Models' magnetic hysteresis and core losses.	Captures hysteresis behavior accurately, improves simulation of nonlinear magnetic properties.	Requires extensive experimental data, computationally complex.	Used in electromagnetic interference analysis, core material optimization.	[25], [27], [28], [37], [42], [48], [49], [51], [54], [58], [88], [72], [90], [91], [92], [93], [94], [95]-[119]
Finite Element Method Models	Using Maxwell's equations, discretizes the geometry for high-fidelity simulations.	Highly detailed, suitable for transient simulations and structural analysis.	Computationally expensive requires detailed material properties.	Thermal analysis, condition monitoring, and transient studies.	[35], [50], [59], [63], [113], [117], [120]-[124]
Frequency and Harmonic Domain Models	Analyzes steady-state harmonic response, impedance, and resonance effects.	Efficient for harmonic analysis, resonance identification, and frequency-dependent loss studies.	Limited for transient or time-varying phenomena like inrush and ferroresonance.	Harmonic load-flow studies, filter design, and harmonic resonance analysis in DERs and EV applications.	[29], [32], [33], [46], [53], [55], [57],[58], [62], [65], [69], [70], [73], [79], [125]-[133]
Hybrid Models	Combines multiple modeling approaches for improved accuracy.	Flexible, can balance complexity and efficiency.	Requires expertise in multiple modeling techniques.	High-fidelity transformer simulations and multi-domain analysis.	[38], [67], [134],[135]
Saturation and Saturable Core Nonlinear Models	Captures core saturation effects, nonlinear magnetization, and saturation-related harmonics.	Improves transformer behavior under high saturation conditions, enhances harmonic analysis.	Computationally heavy, requires detailed nonlinear parameters, may have numerical convergence issues.	Power system analysis, real-time simulation, harmonic distortion studies, DC bias effects, inrush current modeling.	[25], [30], [36], [40], [47], [56],[57], [74], [102]-[104], [114], [120], [124], [136]-[156]

2.5 Magnetic Core Mathematical Modeling Approaches

This section focuses on the mathematical frameworks and equations essential for accurately modeling the magnetic core of transformers, a key factor in determining their operational performance. This section focuses on sophisticated methodologies for representing the core's nonlinear characteristics, such as magnetic saturation, hysteresis, and harmonic interactions, which substantially impact efficiency and dependability. Given the complex nature of these phenomena, the section explores methods for capturing the intricate relationship between magnetic flux density and magnetic field intensity and accounting for losses due to eddy currents and hysteresis effects. From basic linear models applicable in low-frequency, unsaturated conditions to sophisticated nonlinear models like Jiles-Atherton and Preisach, a range of approaches are discussed to address the diverse requirements of modern power systems. The practical uses of these approaches are highlighted, including how they may be used to optimize transformer design and mitigate performance difficulties in high harmonic distortion and transient dynamics conditions.

2.5.1 Linear Model

The linear model is fundamental to modeling magnetic behavior in transformers and other magnetic devices. This method is particularly suitable for applications operating at low frequencies and under linear operating conditions, where the magnetic core functions well below saturation levels. It assumes that the relationship between magnetic flux density and magnetic field intensity follows a linear magnetization curve, implying that the core material exhibits a constant permeability throughout its operation. This assumption makes it easier to analyze and build magnetic circuits by simplifying the mathematical description of magnetic characteristics, particularly when saturation effects, hysteresis, and core losses are minimal. However, this method naturally ignores saturation effects and non-linearities related to realistic magnetic materials. This assumption makes it easier to analyze and build magnetic circuits by simplifying the mathematical description of magnetic characteristics, particularly when saturation effects, hysteresis, and core losses are minimal. The relationship between the magnetic flux density B and the magnetic field intensity H is expressed as:

$$B = uH \tag{2.5}$$

where $B$is Magnetic flux density (T), H is Magnetic field intensity (A/m), and μ present Permeability (H/m).

In this equation, μ denotes the capacity of a material to facilitate the establishment of a magnetic field. For the majority of materials, μ may be regarded as constant at low field intensities, hence simplifying calculations and modeling. In practical situations, permeability may fluctuate with field strength, resulting in departures from this theoretical linear correlation. For more complex scenarios involving core saturation, hysteresis, and frequency-dependent losses, advanced models such as the Preisach model are employed to capture the non-linear hysteresis behavior of magnetic materials accurately. The Preisach model represents hysteresis as a superposition of many unit hysteresis operators, effectively modeling the complex magnetic behavior observed in real-world applications. Recent studies have focused on improving the accuracy of these models. An enhanced Preisach model has been presented for simulating magnetic hysteresis in grain-oriented silicon steel under pulse width modulation stimulation, improving the accuracy of magnetic field estimates in transformer cores [32]. Additionally, research has been conducted on soft magnetic materials' hysteresis and loss characteristics, further contributing to the development of more accurate magnetic models [106].

2.5.2 Saturable Core Model

Saturation in nonlinear transformers is a complicated phenomenon caused by the nonlinear relationship between magnetic flux density and magnetic field intensity. As transformers operate under varied loads and transient conditions, their cores can become saturated, leading to distortions in magnetic response and harmonic production. This behavior is influenced by core material properties, magnetic coupling, and structural design, which affect energy storage and flux distribution within the core [90],[25]. To model this behavior, saturation models are employed to describe the performance of transformer cores as they approach magnetic saturation under high magnetic fields. The Piecewise Linear Model simplifies the magnetization curve using multiple linear segments, enabling quick calculations but with limited precision for complex systems [28]. This approach is particularly useful for preliminary designs where computational efficiency is prioritized. In contrast, the Exponential Saturation Model provides a more accurate representation by capturing the smooth transition from linear to saturated states through exponential equations, making it suitable for harmonic studies and current-limiting scenarios [47], [140].

This model is especially effective in applications involving variable-frequency motors and renewable energy systems, where harmonic distortions can result in excessive saturation. These models are essential for assessing transformer performance under non-sinusoidal loads, ensuring efficient design and operation in power systems with high harmonic content [142]. Figure 3 depicts a transformer's per-phase electrical equivalent circuit, which is required to assess steady-state and transient performance. The model incorporates primary and secondary winding resistances R_1, R_2 and leakage inductances L_1, L_2, which reflect copper losses and leakage flux, respectively. The magnetizing branch, consisting of a nonlinear inductance L_m and shunt resistance R_c, captures the core's nonlinear behavior, including saturation and core losses (hysteresis and eddy currents). The relationship between magnetic flux density and magnetic field intensity is often expressed as:

$$B = u_0 H + X(H)H \tag{2.6}$$

Where u_0 is the permeability of free space, and $X(H)$ is the nonlinear susceptibility based on H [38]. Upon saturation, this relationship markedly diverges from linearity, necessitating the consideration of the fundamental geometry and material properties. The magnetic flux Φ inside the core, contingent upon its cross-sectional area A_{core}, may be represented as:

$$\phi = A_{core} B \tag{2.7}$$

These models emphasize the influence of saturation on fundamental performance, particularly during transients such inrush currents and abrupt load variations [39]. The magnetic energy density quantifies the energy contained inside the magnetic core.

$$W = \frac{1}{2}\int_0^B HdB \tag{2.8}$$

The nonlinear behavior of the core is illustrated by this energy equation, which serves as a basis for evaluating harmonic losses and enhancing core materials [34], [40]. Recent research has focused on novel core designs, such as amorphous alloys and composite architectures, to improve performance and minimize losses in nonlinear environments [41]. These attempt to solve saturation-related difficulties by allowing more accurate transformer models and enhancing their dependability in power systems. Advances in modeling techniques and material engineering continue to push the frontiers of transformer design, opening new avenues for reducing saturation effects and ensuring efficient power transmission [91],[46],[47].

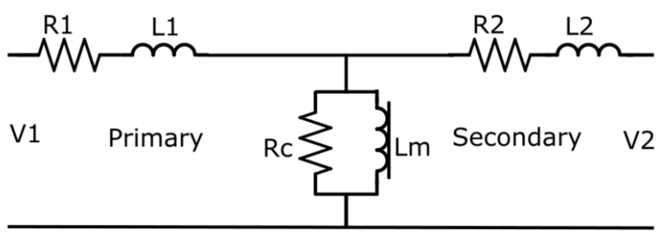


Figure 5. Equivalent electric circuit for Saturable Core Model

2.5.3 Hysteresis Models

The nonlinear magnetic characteristics of transformer cores are simulated using magnetic hysteresis models, particularly indicating the connection between magnetic flux density and magnetic field strength. These models account for hysteresis effects, including remanence and coercivity, which influence the magnetization process and contribute to hysteresis losses, eddy currents, and excess losses under dynamic and transient excitations. The Jiles-Atherton Model provides a physics-based approach for modeling hysteresis loops and is widely applied in transient analyses, such as inrush current simulations and core loss estimations [25],[38]. It models magnetic domains and their interactions, accurately capturing magnetization processes under varying conditions, making it highly effective for steady-state and transient studies [64]. On the other hand, the Preisach Model captures memory-dependent behavior, effectively modeling minor loops within the significant hysteresis loop, making it suitable for analyzing ferromagnetic materials [139]. These models enhance transformer performance assessments in nonlinear and harmonic-rich settings by boosting the accuracy of magnetic behavior forecasts under diverse operating conditions. Figure 6. represents the circuit diagram displaying the equivalent model of a transformer, highlighting key components for understanding its behavior. The primary side comprises resistance R_1 and leakage inductance L_1, whereas the secondary side has resistance R_2 and leakage inductance L_2. The parallel branch represents the magnetizing behavior of the core, modeled by the magnetizing inductance L_m and core-loss resistance R_c , which account for core hysteresis and eddy current losses. This model is crucial for analyzing transformer performance under various loading conditions, including nonlinearities such as saturation and losses. These elements enable the model to accurately predict power loss, efficiency, and harmonic behavior in power systems, consistent with the ideas presented in the referenced article on power losses and hysteresis in transformer laminations. Among the most widely used approaches is the Preisach Model, which represents the magnetization as a weighted sum of elementary loops known as hysterons, mathematically expressed as [107],[108]:

$$M(H) = \iint u(\alpha, \beta) d\alpha d\beta \quad (2.9)$$

where M represents the magnetization, and the distribution of the hysterons is defined by $u(\alpha, \beta)$. This model accurately represents tiny loops and anhysteretic behavior, making it ideal for static and low-frequency simulations. The Jiles-Atherton Model is another popular model that describes hysteresis in magnetic materials using energy principles and domain wall motion. It simulates magnetization with an effective magnetic field [109],[110]:

$$H_{eff} = H + \alpha M \quad (2.10)$$

where α stand for the interaction between magnetic domains, and dM donate change in magnetization is given by [110]:

$$dM = \frac{M_s - M}{k} dH \quad (2.11)$$

Where M_s is saturation magnetization, and k controls the ease of domain motion. This model benefits transient simulations and dynamic analysis in systems involving switching transients and high-frequency excitations. Both models enable the computation of hysteresis loops, capturing the irreversible magnetic behavior and frequency-dependent losses, which are critical for evaluating harmonics, core losses, and voltage distortions in transformers.

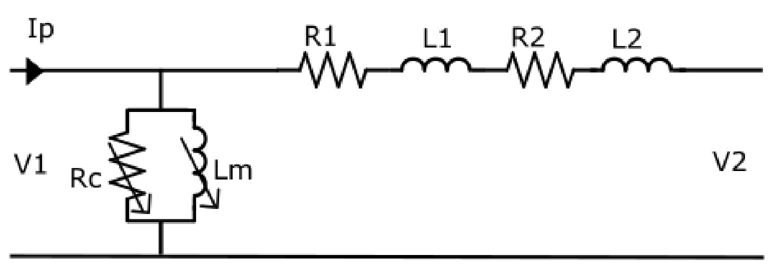


Figure 6. The equivalent circuit of modeling Hysteresis

2.5.4 Finite Element Method Models

FEA models divide the transformer core into small elements, enabling localized computation of magnetic fields and electromagnetic interactions. FEM provides a precise and thorough approach to modeling the magnetic behavior of transformer cores by solving Maxwell's equations within each finite element, capturing spatial fluctuations in the magnetic field. It incorporates nonlinearities, geometric complexities, and localized effects such as skin effects, eddy currents, hysteresis, and saturation, ensuring highly accurate simulations [59],[63]. Figure 5 shows the transformer's equivalent circuit, highlighting electromagnetic interactions and complementing FEM analysis by illustrating magnetic coupling and harmonic distortion effects for optimized design. FEM-based models deliver detailed insights into magnetic flux distributions, core losses, and thermal effects, aiding in the design of high-performance transformers [35]. These computationally intensive models are particularly advantageous for applications requiring high-frequency performance, such as inverters and power electronic transformers, where broad frequency response characteristics must be considered [134]. FEM techniques have also been combined with optimization algorithms to fine-tune design parameters, thereby reducing costs and improving reliability. With governing equations derived from Maxwell's principles, such as Faraday's Law for calculating induced voltages [121], FEM guarantees robust performance for advanced design and diagnostic applications in transformer engineering.

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.12}$$

where E is the electric field and B is the magnetic flux density. The model uses Ampere's Law to calculate magnetic field strength (H) and current (J) [118]:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \tag{2.13}$$

where H is the magnetic field strength, J is the current density, and ∂D is the electric displacement field. The equations are solved numerically within each finite element using boundary conditions and material attributes (for instance, permeability (μ) and conductivity (σ). For materials with nonlinear magnetic behavior, FEM integrates the B-H curve as follows [122]:

$$B = \mu(H)H \tag{2.14}$$

where the nonlinear permeability that changes with field intensity is represented by $\mu(H)$. FEM models include eddy current losses through Ohm's Law.

$$J = \sigma E \tag{2.15}$$

Coupling with heat transfer equations allows you to capture the thermal consequences of these losses. This makes FEM suitable for studying frequency-dependent effects, harmonic production, and localized saturation zones in transformer cores. FEM models are implemented in software tools like ANSYS Maxwell, COMSOL Multiphysics, and MATLAB, allowing for both steady-state and transient analysis. While FEM is extremely accurate, it takes significant computer resources and exact geometry inputs, making it more suited for core design optimization, material testing, and localized research than large-scale power system modeling. FEM models are frequently used in transformer performance evaluation, harmonic mitigation studies, renewable energy systems, and electric car charging stations, since they provide exceptional precision in capturing spatial variations and dynamic effects in current electrical engineering applications.

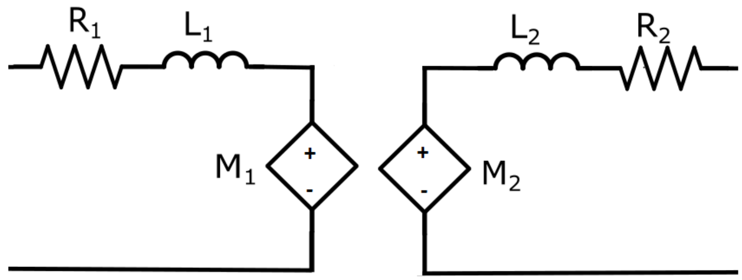


Figure 7. Finite Element Model Equivalent Circuit

2.5.5 Harmonic Domain Models

Analyzing transformers under non-sinusoidal and harmonic distortion conditions require advanced modeling techniques [47]. These models enable detailed studies of transformer performance by incorporating the effects of magnetic saturation, hysteresis, eddy currents, and other nonlinearities that impact electromagnetic behavior [46]. Figure 8 demonstrates a power transformer's equivalent circuit in the harmonic domain. This model uses a current source I_m to simulate the transformer core's nonlinearities and inject harmonic currents into the network. This method considers the effects of core saturation, eddy currents, and hysteresis losses. The circuit's inductive L_m component is responsible for the core's magnetizing action. The harmonic current injection enables detailed modeling of the transformer's influence on the network, especially under distorted or nonlinear operating conditions. Harmonic domain techniques represent voltages, currents, and flux linkages as Fourier series, allowing for frequency-domain analysis of harmonic interactions [140]. Unlike time-domain methods, harmonic domain approaches facilitate efficient computation of steady-state and quasi-steady-state responses under distorted waveforms by solving Maxwell's equations in the frequency domain, expressed as:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \tag{2.16}$$

$$\nabla \cdot B = 0 \tag{2.17}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.18}$$

$$\nabla \cdot D = \rho \tag{2.19}$$

where H is the magnetic field intensity, J is the current density, D is the electric displacement field, and B is the magnetic flux density [120]. These equations enable accurate modeling of nonlinear magnetic coupling and core saturation effects, including inrush currents and harmonic losses [53].

Advanced methods, such as the Preisach and Jiles-Atherton models, are incorporated to account for hysteresis loops [57], while Fourier transforms handle harmonic decomposition, represented as:

$$X(t) = \sum_{n=-\infty}^{\infty} X_n e^{j n \omega t} \tag{2.20}$$

where $X(t)$ is the time-domain signal, X_n are the Fourier coefficients, ω is the fundamental angular frequency, and n is the harmonic order [92]. Harmonic domain models are applied to assess core loss, thermal effects, and inrush current behaviors, aiding in transformer optimization and predictive maintenance [93]. Hybrid approaches, combining harmonic models with finite element methods, enhance spatial accuracy for high-frequency harmonics and electromagnetic interference [141]. Emerging trends leverage machine learning for real-time fault diagnosis and adaptive modeling, addressing challenges in modern power systems and renewable energy applications [59].

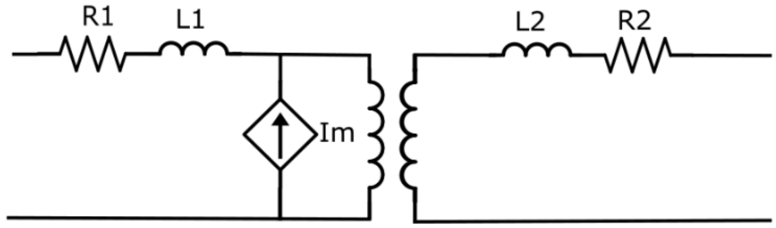


Figure 8. Harmonic Domain Modelling of Transformer Core

2.5.6 Hybrid Models

Modern transformer modeling necessitates sophisticated methodologies that combine analytical and numerical techniques to handle the complicated behaviors caused by magnetic saturation, hysteresis, eddy currents, and frequency-dependent losses [32], [93]. Hybrid models have evolved as practical tools for capturing steady-state and transient responses by combining frequency-domain approaches like Fourier analysis with time-domain techniques like differential equations and finite element analysis [98],[127]. These models help analyze harmonic propagation and electromagnetic interference because they resolve localized flux distributions and nonlinear magnetic coupling [62]. Figure 9 illustrates a typical waveform of a radiated electric field and highlights major field characteristics utilized in the investigation. The graph depicts changes in electric field strength over time, highlighting two notable shifts: ΔE1 and ΔE2. These fluctuations reflect the impact of external electromagnetic disturbances and the internal behavior of electrical insulation systems. The waveform contains crucial qualities such as rising time, peak values, and transient characteristics required for understanding electromagnetic interactions in high-voltage situations. Examining this waveform allows for a better assessment of electrical insulation performance and transient response characteristics, contributing to enhanced design and reliability of power system components. Mathematical formulations using Maxwell's equations describe the spatial and temporal fluctuations of flux density and magnetic field intensity [82], whereas differential equations, such as :

$$V(t) = RI(t) + L\frac{dI(t)}{dt} \tag{2.21}$$

Model dynamic behavior in transformer windings [127]. To handle harmonic distortion, Fourier series expansions such as

$$X(t) = \sum_{n=0}^{\infty} X_{n\cos(n\,\omega\,t+\phi_n)} \tag{2.22}$$

Complex waveforms are simplified into harmonic components, which provide insights into nonlinear interactions and resonance [62], [67]. Hybrid models also use transfer functions.

$$H(s) = \frac{Vout(s)}{Vin(s)} \tag{2.23}$$

for frequency-domain assessments, allowing for accurate characterization of harmonic responses and filtering effects [94]. Finite element approaches improve spatial resolution, allowing the simulation of core losses and eddy currents under distorted situations. Advanced frameworks incorporate real-time electromagnetic simulations employing magnetic equivalent circuits based on FPGA hardware, allowing for adaptive fault detection via hardware-in-the-loop testing [79]. Wideband models expand on these capabilities by integrating frequency-dependent inductance expressions.

$$L(f) = L_0 + \sum_{n=1}^{\infty} L_n \cos(2\pi f t + \phi_n) \tag{2.24}$$

To analyze high-frequency harmonics and EMI propagation. Such hybrid models not only improve transformer design and performance, but also allow for predictive maintenance and problem diagnostics, assuring dependability in modern power systems that face harmonic-rich conditions and renewable energy integration [92].

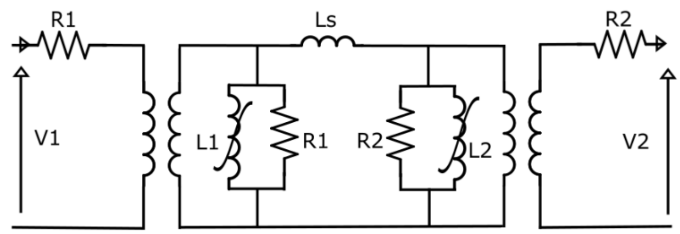


Figure 9. Dual Model of a Single-Phase Transformer Circuit

2.6 Classification Based on Design

This section examines transformer design classes, emphasizing three essential aspects including core design, winding design, and economic considerations. The core design emphasizes the importance of materials and shapes in maximizing transformer performance, with advances such as amorphous alloys and hybrid cores lowering losses and increasing efficiency in harmonic settings. Finite element analysis and real-time monitoring systems reduce flux leakage, mitigate magnetic saturation, and enable predictive maintenance. The winding design investigates interleaved and zigzag windings that improve harmonic suppression, thermal management, and load balancing. Advanced modeling tools, including FEM and harmonic domain models, help to address skin and proximity effects, resulting in enhanced electromagnetic performance. Lastly, economic considerations focus on balancing initial costs and lifecycle expenses through material selection, predictive maintenance systems, and life cycle cost analysis.

2.6.1 Core Design

The design and material composition of the core has a significant impact on efficient transformer performance. Traditional laminated silicon steel cores have long been preferred for their combination of magnetic characteristics and cost-effectiveness. However, contemporary improvements increasingly focus on amorphous alloy cores, which perform better in harmonic-rich environments to lower core losses and greater resistivity [27], [42]. Amorphous alloys considerably minimize hysteresis losses and eddy currents, which improves energy efficiency, particularly for non-sinusoidal waveforms [25]. Hybrid cores, which blend silicon steel and amorphous materials, enhance magnetic performance while preserving structural integrity and lowering costs [38]. The core's geometric structure is also important. Step-lap joints and tailored laminations reduce flux leakage, increase magnetic path homogeneity, and reduce inrush currents induced by magnetic saturation [26], [39]. Numerical simulations using finite element analysis are commonly used to simulate magnetic flux distributions and forecast saturation effects, providing information on localized heating zones and flux density changes [34]. Such approaches assist engineers in identifying areas of high electromagnetic stress and optimizing core shapes to reduce losses [40]. In addition to static design concerns, dynamic characteristics such as hysteresis loops, saturation, and residual flux must be considered. The Preisach and Jiles-Atherton models [41], [28] have been used to replicate these nonlinear tendencies properly. Harmonic domain techniques are built on this analysis by describing core flux connections and magnetic fields as Fourier series, allowing for the evaluation of harmonic interactions and EMI under distorted waveforms

[87], [47]. Modern transformers increasingly use real-time condition monitoring systems to measure temperature fluctuations, magnetic flux changes, and core losses, allowing for early fault diagnosis and predictive maintenance techniques [46], [140]. These advancements increase dependability, minimize downtime, and extend equipment life. Integrating core designs optimized for harmonic abatement and real-time monitoring has become critical in applications involving renewable energy sources and distributed generation systems, where harmonic distortion is common [120].

2.6.2 Winding Design

Transformer winding arrangement directly impacts impedance, electromagnetic performance, and the ability to reduce harmonic distortion. Advanced designs include interleaved, disc, and continuous windings to reduce leakage inductance and stray capacitances, resulting in improved harmonic attenuation and fewer voltage spikes during transient operations [53], [57]. Distributed windings improve thermal performance by increasing heat dissipation and decreasing hotspot formation induced by harmonic currents [92]. Winding electromagnetic modeling must consider proximity effects, in which currents concentrate near conductor surfaces, and skin effects, in which current density changes with frequency. These effects are most prominent under harmonic distribution settings, resulting in uneven heating and higher losses [93], [141]. Numerical approaches, like FEM and coupled circuit analysis, provide realistic modeling of these phenomena, allowing engineers to design winding shapes for lower losses and better harmonic suppression [59], [70]. Specialized winding configurations, such as delta and zigzag windings, are widely implemented to address harmonic propagation and load imbalances. Delta connections effectively suppress zero-sequence harmonics, while zigzag windings improve phase balancing and grounding performance [126], [142]. The harmonic response of such configurations has been analyzed using harmonic domain models, which provide frequency-domain insights into resonance effects and EMI interactions [73], [65]. High-frequency effects caused by harmonic distortion are addressed using shielding techniques, dielectric optimization, and flux barriers to minimize interference and protect sensitive components [50], [55]. Advanced winding technologies use smart materials and integrated sensors to measure temperature, current, and voltage in real time. These sensors provide adaptive control and fault detection, increasing dependability and a longer operating life [69], [157]. Recent studies also explore machine learning-based optimization techniques for winding design, focusing on minimizing losses and improving efficiency under dynamic load conditions. Such advancements allow transformers to adapt to modern grid demands, including renewable energy integration and electric vehicle charging stations, where harmonic content is high,[49], [48],[94], [75].

2.6.3 Economic Considerations

Economic optimization in transformer design involves balancing initial costs, operational efficiency, and lifecycle expenses. Materials like amorphous alloys and hybrid core designs reduce operational losses but have higher manufacturing costs than traditional silicon steel cores [74], [76]. However, their long-term savings due to lower energy losses and improved harmonic handling often justify the initial investment, especially in applications involving renewable energy sources and variable frequency drives [77]. Cost-effective solutions frequently incorporate composite designs that combine high-performance materials for critical components with standard materials for structural elements, maintaining both performance and affordability [63], [78]. Similarly, winding materials such as copper and aluminum present trade-offs; copper offers lower resistive losses and greater durability, while aluminum provides cost advantages at slightly higher losses [79], [58]. Economic feasibility also depends on integrating predictive maintenance systems, which minimize downtime and prevent catastrophic failures. Operators may watch the condition of transformers in real time with online monitoring technologies like thermal imaging and partial discharge detection, which enables preventative maintenance and lowers lifetime costs [95], [80]. Life Cycle cost analysis is essential when choosing and optimizing transformers. This involves assessing initial material and manufacturing costs, long-term operational expenses, maintenance requirements, and energy efficiency. Computational tools employing machine learning algorithms assist in multi-objective optimization, enabling engineers to balance cost, performance, and reliability in transformer designs [52]. Transformers are made to satisfy demanding performance standards without sacrificing economic feasibility in response to the growing need for harmonic mitigation and renewable energy integration. In order to provide affordable solutions for intricate power systems, future designs are anticipated to use sophisticated simulation models and intelligent materials.

2.6.4 Modeling Challenges and Research Gaps

Despite major advances, difficulties remain. Many high-fidelity models are still computationally costly, and getting precise core parameters (such as B-H curves or eddy current constants) can be challenging. Furthermore, most existing models lack real-time flexibility or predictive power for early issue identification. The literature recognizes the need for integrated modeling and monitoring frameworks that combine transformer behavior with real-time data collecting and machine learning approaches for condition-based assessment.

CHAPTER THREE

MATHEMATICAL MODEL AND THEORY

Implementing the suggested three-winding transformer for harmonic mitigation relies on an accurate mathematical model that conducts its operation. This chapter establishes the theoretical principle, including transformer modeling, harmonic current injections, and fundamental mathematical derivations. The purpose is to define conditions where the tertiary winding effectively cancels harmonic currents, preventing their propagation into the power grid. By formulating the transformer's electrical behavior, analyzing harmonic interactions, and applying circuit theory principles, this section provides a framework for evaluating the system's performance. The mathematical model incorporates core nonlinearities, winding configurations, and current injection strategies for optimal harmonic suppression. This formulation presents the basis for simulations and experimental validation, ensuring the transformer operates as proposed in mitigating harmonic distortion and enhancing power quality in distribution networks.

3.1 Three-Winding Transformer Model

A three-winding transformer is designed to improve power quality by mitigating harmonic distortions created by nonlinear loads. It contains three magnetically coupled windings, either serving a particular function within the system. The primary winding (N_1) is connected to the AC power grid and is the fundamental input for power transmission. It ensures that fundamental frequency power flows from the grid to the load while ideally remaining free of harmonic components. The secondary winding (N_2) is connected to a nonlinear load that creates harmonic currents. These harmonics appear due to the switching characteristics of power electronics, saturation effects in electric machines, or other nonlinear conduct in industrial applications. The tertiary winding (N_3) injects a compensating current to offset the harmonic components generated by the secondary winding. Harmonic currents are neutralized before propagating into the grid by adequately controlling the phase and magnitude of the injected current. The main to efficient harmonic mitigation lies in precisely controlling current interactions through the windings. By leveraging the magnetic coupling between windings, the tertiary winding is designed to cancel out specific harmonics and ensure that the primary winding maintains a clean sinusoidal waveform. The capability to mitigate harmonics depends on selecting a suitable turns ratio and ensuring minimal phase deviation between the injected compensating current and the harmonic currents created by the nonlinear load. The theoretical framework of this transformer model forms the foundation for understanding how harmonic mitigation can be systematically accomplished in power distribution networks.

3.2 Magnetic Coupling and Equivalent Circuit

The three-winding transformer deploys mutual magnetic coupling, where all windings share a common core flux Φ. The primary and secondary windings elaborate as a classic transformer, while the tertiary winding obeys as an active harmonic compensator. The tertiary winding injects a compensating current that opposes the harmonics created by the nonlinear load connected to the secondary winding, thereby suppressing their propagation into the primary winding and therefore into the power grid. The fundamental voltage equations prevailing the transformer, assuming negligible core losses and leakage reactance, are given by:

$$V_1 = N_1 \frac{d\phi 1}{dt} \tag{3.25}$$

$$V_2 = N_2 \frac{d\phi 2}{dt} \tag{3.26}$$

$$V_3 = N_3 \frac{d\phi 3}{dt} \tag{3.27}$$

Where Φ represents the mutual magnetic flux linking all three windings. V_1, V_2, and V_3 are the induced voltages in the primary, secondary, and tertiary windings, respectively. N_1, N_2, and N_3 denote the number of turns in each winding. Since all windings share the same magnetic core, the induced voltages are proportional to their competent turn counts. The voltage equations indicate that any alternation in the flux will induce voltages in each winding in correspondence with their winding ratios. Applying Kirchhoff's Current Law at the transformer windings, the total magnetizing current in the core is given by:

$$N_1 I_1 + N_2 I_2 + N_3 I_3 + R_c \emptyset = 0 \tag{3.28}$$

Where I_1 is the current in the primary winding (grid-side), I_2 is the current drawn by the nonlinear load connected to the secondary winding, and I_3 is the compensating current injected by the tertiary winding. To ensure that harmonic currents are not transmitted to the power grid, the primary current must consist only of fundamental frequency components while being free of harmonics. This conducts to the harmonic compensation condition:

$$I_1 = 0 \tag{3.29}$$

Substituting this condition into the current equation yields:

$$N_2 I_2 + N_3 I_3 = 0 \tag{3.30}$$

which simplifies to:

$$I_3 = -\frac{N_2}{N_3} I_2 \tag{3.31}$$

This equation highlights that the tertiary winding should inject a current I_3 that is equivalent in magnitude but obverse in phase to the harmonic components of I_2. The appropriate selection of the turn's ratio N_2/N_3 ensures that harmonic currents are fully neutralized in the primary winding. This compensation strategy enables the transformer to function as a passive harmonic filter without imposing further external filtering circuits.

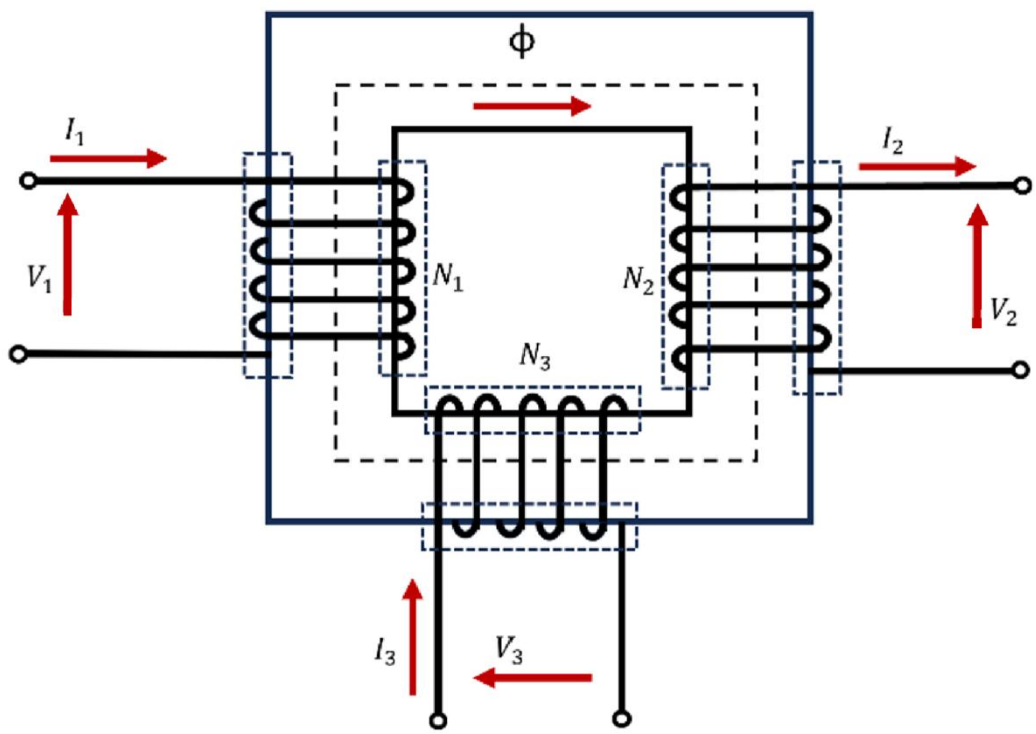


Figure 10. Magnetic Coupling in a Three-Winding Transformer

Furthermore, Figure 10 shows the equivalent circuit representation of the three-winding transformer's current relationship, depicting the interaction of magnetizing currents in the windings and the compensation effect of the tertiary winding concerning the nonlinear load.

Figure 10 shows the equivalent circuit of a three winding transformer utilized for harmonic cancellation. The figure represents the current interactions through the windings, highlighting how the tertiary winding counteracts the harmonic currents created by the nonlinear load. The resistor Rc in the circuit accounts for core losses, which are typically small but may impact overall transformer efficiency.

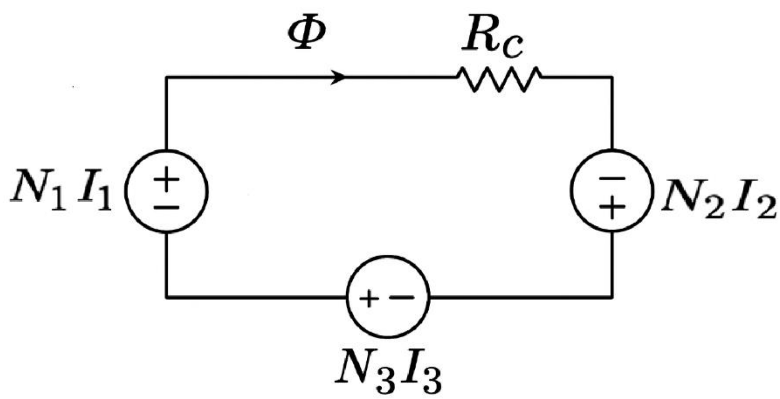


Figure 11. Equivalent Circuit of a Three Winding Transformer

The effectiveness of harmonic cancellation techniques relies on accurately identifying the harmonic spectrum of I_2 and designing I_3 accordingly. Since nonlinear loads produce varying harmonic profiles, transformer design must be optimized to target the predominant harmonics within the system. Several practical factors, including leakage reactance, core nonlinearity, and winding resistance, can influence the effectiveness of harmonic cancellation. These factors should be carefully considered in practical implementations to enhance system performance. The subsequent sections will explore these considerations in detail to establish a comprehensive and robust harmonic mitigation strategy.

3.3 Description of the Harmonic Mitigation Configuration

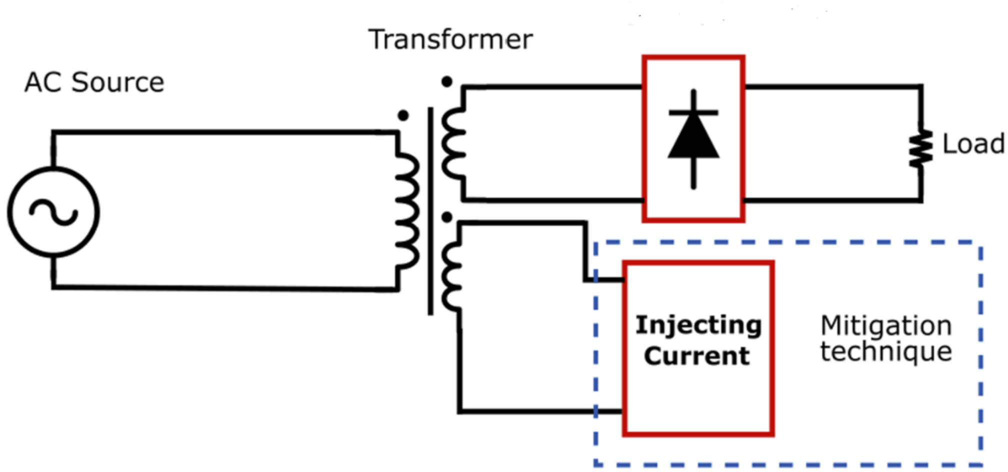


Figure 12. Harmonic mitigation technique

Figure 12. represents the harmonic mitigation technique that injects compensating current into the tertiary winding of a three-winding transformer to neutralize harmonic distortions generated by nonlinear loads, thereby ensuring the primary winding remains free of harmonic contamination and maintains power quality. The process initiates with a power source delivering sinusoidal voltage and current to the primary winding, which under ideal conditions should contain only the fundamental frequency component; however, nonlinear loads connected to the secondary winding, such as rectifiers, power electronic converters, and variable frequency drives that generate harmonic currents due

to their non-sinusoidal power consumption characteristics. When these harmonics interact with the transformer's magnetic circuit, they penetrate the primary winding and create significant power quality issues such as increased energy losses, thermal stress in electrical components, voltage irregularities, interference with sensitive equipment, and reduced system efficiency and reliability. The mitigation technique addresses these concerns through precise current injection via the tertiary winding, effectively intercepting harmonics before propagating into the power grid. To address this issue, the three winding transformer's tertiary winding functions as an injection circuit, creating a compensatory current of equivalent amplitude but opposite phase to the nonlinear load's harmonic components. This opposing current successfully cancels out the harmonic distortions, eliminating harmonic components in the transformer. The technique's performance depends on appropriately modifying the turns ratio and matching the phase between the injected current and the load-generated harmonics. This method reduces total harmonic distortion and prevents harmonic currents from obtaining the primary winding and power grid when adequately developed. The mitigation approach demonstrates the canceling mechanism enabled achievable by the tertiary winding's current injection. This procedure removes harmonics from their source before they may damage other system components. The technique's efficacy is enhanced by maximizing magnetic coupling between windings, lowering leakage reactance, and maintaining the transformer within its linear zone to avoid core saturation. The main benefit of this methodology is its ability to ensure compliance with power quality standards such as IEEE 519, which restricts the permitted harmonic distortions in electrical networks. By preserving harmonic levels below acceptable limits, the approach increases system efficiency, minimizes equipment stress, and increases component longevity. It performs exceptionally well in industrial and commercial environments where frequent nonlinear loads and harmonic reduction are critical for reliable operation. The transformer enhances power quality without needing external filters by carefully selecting the tertiary winding's turns ratio and adjusting the phase relationship of the injected current. This represents an efficient and cost-effective method for reducing harmonics in power distribution systems, thereby improving reliability and performance.

CHAPTER FOUR

SIMULATION AND SENSITIVITY ANALYSIS

This chapter provides the simulation model used to assess the performance of the suggested harmonic mitigation strategy in a three-winding transformer. The study uses MATLAB/Simulink and focuses on replicating realistic operating conditions of a distribution transformer subjected to nonlinear loads. The simulation aims to verify how harmonic components introduced by the nonlinear load can be suppressed using controlled current injection into the tertiary winding. The method is tested under different system configurations and parameters to assess its performance and adaptability. The model includes a magnetic coupling-based transformer with core nonlinearity, a full-wave bridge rectifier as a nonlinear load, and a harmonic injection circuit integrated into the tertiary winding path. A sensitivity analysis is conducted alongside the baseline simulations to evaluate the robustness of the mitigation technique. This analysis investigates the impact of varying system parameters such as load resistance, number of tertiary winding turns, and injection current magnitude. The results provide insight into how each parameter influences the harmonic suppression capability of the system and help determine the optimal conditions for implementation. This chapter is organized into two major sections:

- Simulation Testing: Describes the design of the model and presents results without and with the mitigation method active, highlighting the impact on waveform distortion and THD.

- Sensitivity Analysis: Explores the influence of key system parameters on harmonic reduction performance through a series of controlled variations and comparative results.

The combined findings establish the effectiveness of the proposed approach and demonstrate its potential for integration into real-world distribution systems.

4.1 Simulation Testing

4.1.1 Overview of Simulation Setup

The simulation design for testing the proposed harmonic reduction approach was built with MATLAB/Simulink, a popular tool for modeling and simulating power systems and electromagnetic components. The main goals of the simulation were to replicate real-world operating circumstances for a power distribution transformer exposed to harmonic-rich nonlinear loads and to examine how well tertiary winding current injection reduces harmonic distortion.

The core configuration of the simulated system consisted of a single-phase AC voltage source rated at 120V and 60Hz, representative of typical residential or commercial distribution levels. The power supply is connected to the primary winding of a special three-winding transformer. To better simulate the dynamic magnetic behavior of genuine transformer cores under harmonic stress, the transformer model was built employing magnetic coupling blocks with nonlinear core features such as saturation and flux hysteresis.

A nonlinear load, represented by a full-wave bridge rectifier feeding a resistive-capacitive filter, was connected to the secondary winding of the transformer. This load type is a great option for producing harmonically rich scenarios since it is known to produce significant waveform distortion as a result of abrupt current switching during conduction intervals. The load was the principal source of odd-order harmonics, including the 3rd, 5th, 7th, and 9th, which were identified and measured using FFT analysis. A harmonic injection circuit was added through the transformer's tertiary winding to reduce the harmonics produced by the nonlinear load. A regulated current source that was configured to inject a waveform with specific harmonic components was a part of this circuit. Only harmonic frequencies, not the fundamental, were applied to the tertiary winding after the injected signal was sent through a high-pass filter of capacitive and resistive components.

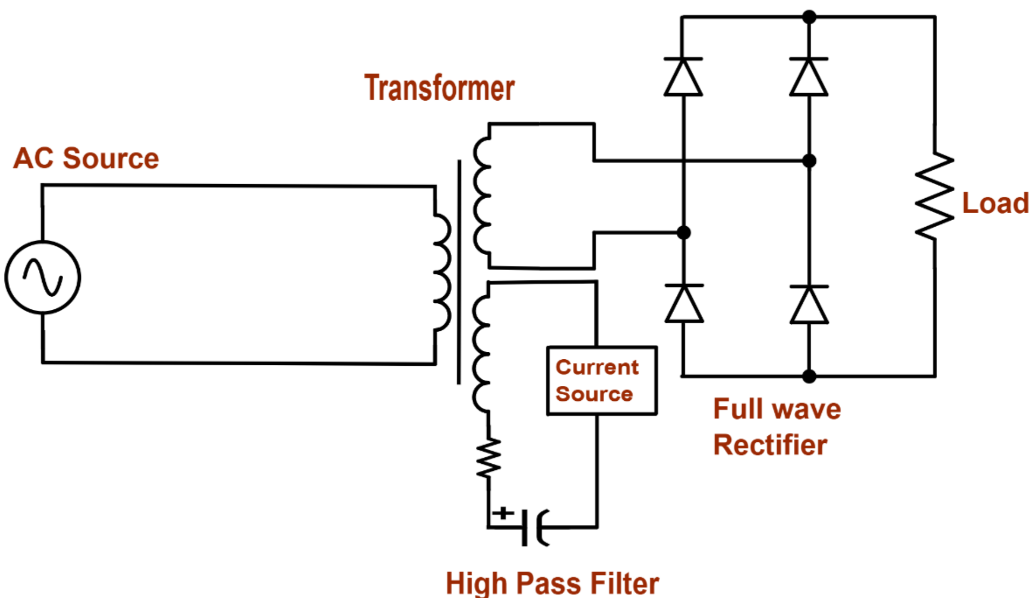


Figure 13. Simulation test setup of a mitigation method

The general framework of the simulation is shown in Figure 13. The AC source on the left powers the primary winding of the three-winding transformer. A full-wave bridge rectifier receives the secondary winding and transforms the AC signal into pulsing DC in order to drive a resistive load. Due to the nonlinear current draw generated by this rectifier, the system experiences harmonic distortion. The tertiary winding beneath the transformer is connected to a current source configured to produce a particular harmonic waveform. This signal is routed via a high-pass filter before injection, which neutralizes some harmonic orders by allowing only higher-frequency components to interact with the transformer's internal magnetic field.

This simulation setup enabled controlled experimentation with different injection frequencies, magnitudes, and phase angles. Sensitivity studies of transformer impedance, load resistance, and winding turn count were also possible. Using integrated analytic tools like the Powergui FFT block and the monitoring of voltage and current waveforms across all three windings, the model offered important insights into the system's harmonic behavior and the efficacy of the mitigation method.

4.1.2 High-Pass Filter Design

In the proposed harmonic mitigation strategy, an HPF plays a pivotal role in enabling the selective injection of harmonic currents into the transformer's tertiary winding. Its primary objective is to block the 60 Hz fundamental frequency component, thereby preventing unnecessary flux excitation while allowing higher-order harmonic components, such as the 3rd (180 Hz), fifth (300 Hz), seventh (420 Hz), and higher, to pass through with minimal attenuation. This targeted selectivity ensures that only distortion-causing harmonics interact with the winding, thereby maximizing the effectiveness of mitigation while preserving the integrity of the fundamental voltage and current flow between the primary and secondary windings. HPF is implemented using a first-order configuration that directs high-frequency harmonics into the tertiary winding. In this design, a capacitor is used to create a frequency-selective impedance, blocking low-frequency components, while allowing higher-frequency harmonics to pass. The resistor, representing the impedance of the tertiary winding, forms the parallel branch. This arrangement ensures that the fundamental component is largely rejected, minimizing its impact on the tertiary circuit, while enabling harmonic currents to flow through and be mitigated. The design enhances the system's harmonic filtering capability without affecting the fundamental power flow.

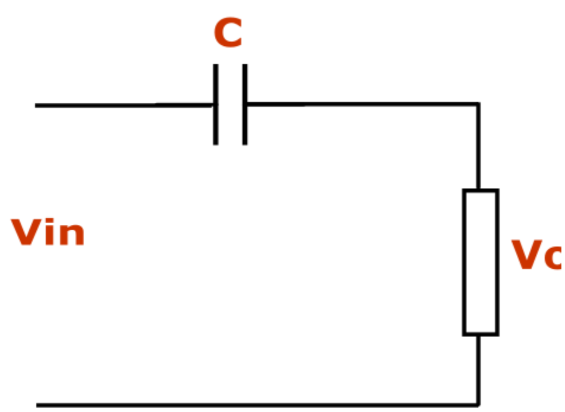


Figure 14. High pass filter circuit

The cutoff frequency f_c of the filter is defined by the equation:

$$f_c = \frac{1}{j\omega C} \tag{4.32}$$

For this application, a capacitor value of $C = 100\ \mu F$ was selected to create a high-pass filter with a cutoff frequency of approximately 118 Hz. This cutoff lies above the fundamental frequency (60 Hz) and below the third harmonic (180 Hz), ensuring effective attenuation of the fundamental component while allowing higher-order harmonics to pass into the tertiary winding. The filter's performance is illustrated in Figure 15, where the magnitude response shows strong suppression of low-frequency signals and minimal attenuation in the harmonic range. This targeted filtering enables accurate harmonic injection into the transformer, avoids low-frequency saturation, and improves flux cancellation. The passive filter design also enhances system reliability and reduces implementation complexity compared to active filtering methods.

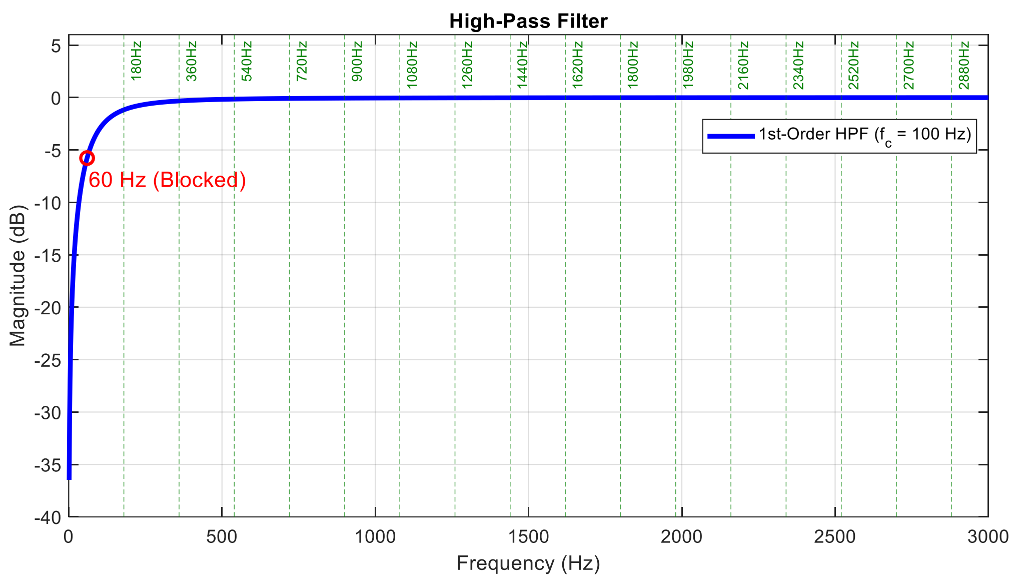
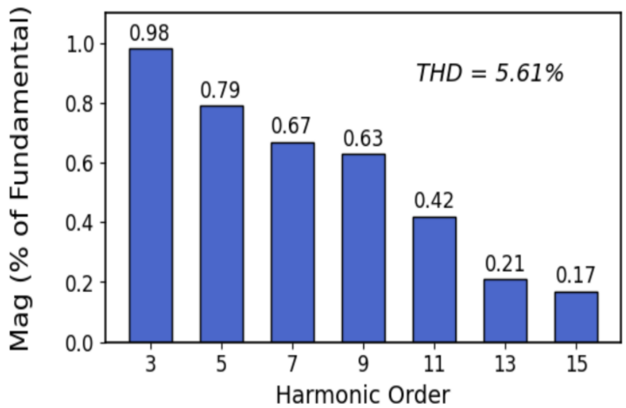


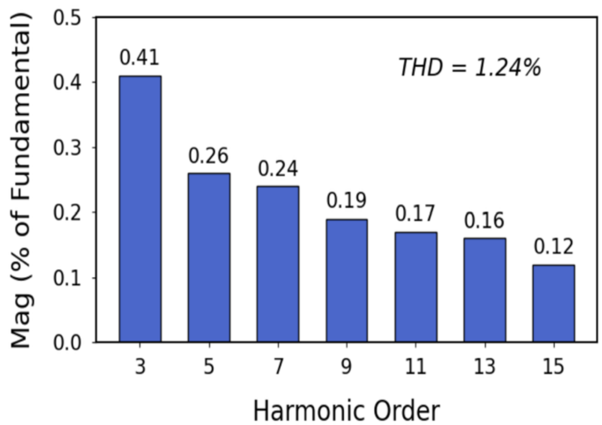
Figure 15. Typical magnitude response of a high-pass filter.

4.1.3 Simulation Results Without Mitigation

A baseline simulation was performed without activating the harmonic injection circuit to serve as a reference for assessing the effectiveness of the suggested mitigation technique. This allowed for observing harmonic behavior solely due to the nonlinear load, independent of any compensating current. The modeled system included a three-winding transformer supplying a full-wave rectifier connected to RC load. As expected, the rectification process introduced substantial waveform distortion, primarily caused by rapid switching and discontinuous current conduction. These characteristics resulted in various odd-order harmonics propagating through the transformer and back to the source. FFT analysis was used to quantify the THDi, yielding 5.61% at the primary and 7.25% at the secondary winding. These values exceed recommended thresholds set by power quality standards such as IEEE 519, particularly in low-voltage distribution systems. The harmonic spectrum revealed firm peaks at the 3rd, 5th, and 7th orders—typical of full-wave rectifier behavior. These results are illustrated in Figure 16 and 17, which show the harmonic voltage and current spectrums at the primary and secondary windings, respectively. Specifically, Figure 16 (a) displays the voltage harmonic spectrum, and Figure 14(b) shows the current distortion at the primary side. Similarly, Figures 17(a) and (b) depict the secondary winding's voltage and current harmonic profiles. Time-domain waveforms demonstrated noticeable distortion, including flattened voltage peaks, irregular zero-crossings, and sharp transitions in the current waveform. These effects are shown in Figure 16**,** which presents the voltage and current waveforms under uncompensated conditions. The results reflect the transformer's exposure to significant harmonic stress and non-sinusoidal load demands. Overall, the unmitigated simulation confirms the necessity of implementing a targeted harmonic mitigation method such as tertiary winding current injection to reduce harmonic propagation and improve power quality.

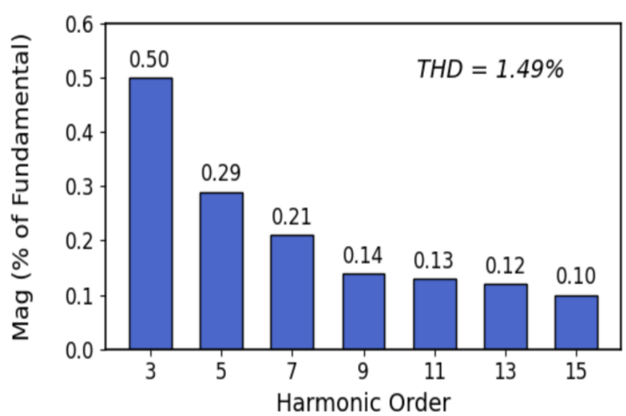


(a)

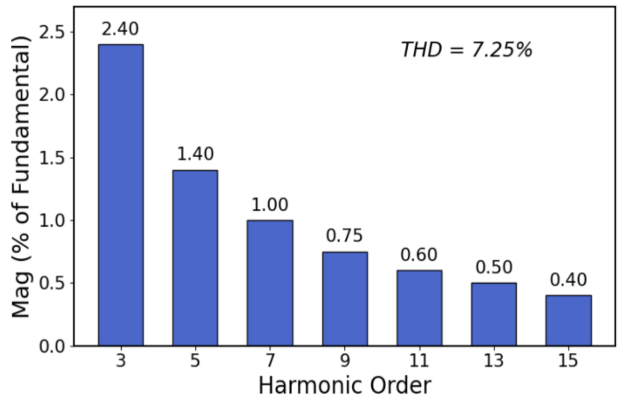


(b)

Figure 16. Simulation result for the investigation of harmonic created on primary winding, (a) Harmonic on voltage primary winding, (b) Harmonic current on primary winding



(a)

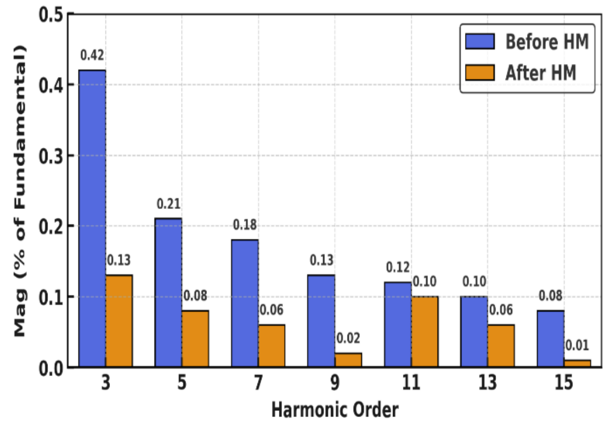


(b)

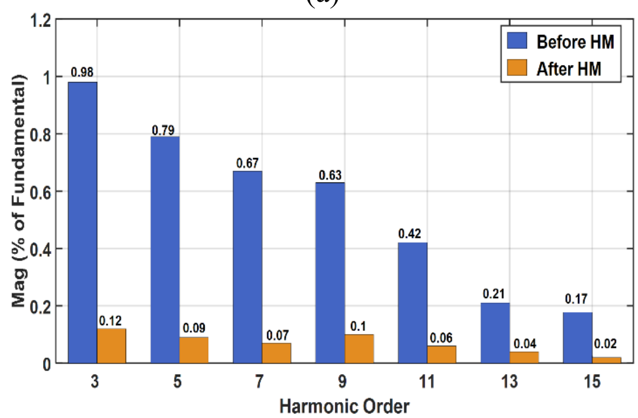
Figure 17. Simulation result for the investigation of harmonic created on secondary winding, (a) Harmonic on voltage secondary winding, (b) Harmonic current on secondary winding

4.1.4 Simulation Results after Inject Current

These harmonic distortions impair the grid current and overall system performance because the non-sinusoidal current from the nonlinear load reflects onto the primary side through magnetic coupling. The voltage and current waveforms improved significantly and measurably when the compensatory current was injected into the tertiary winding. Figures 18 and 19 compare the waveforms before and after the implementation of the tertiary injection. Figure 8(a) shows the voltage waveform of the primary winding, and Figure 18(b) shows the current waveform. A discernible shift from distorted to notably cleaner sinusoidal signals occur after mitigation. Similarly, Figure 19 shows comparable enhancements in the secondary winding. The voltage waveform appears considerably smoother after injection in subfigure (a), and subfigure (b) demonstrates that compensating current effectively neutralizes the harmonic components coming from the nonlinear load by confirming that the current waveform has also been purified.

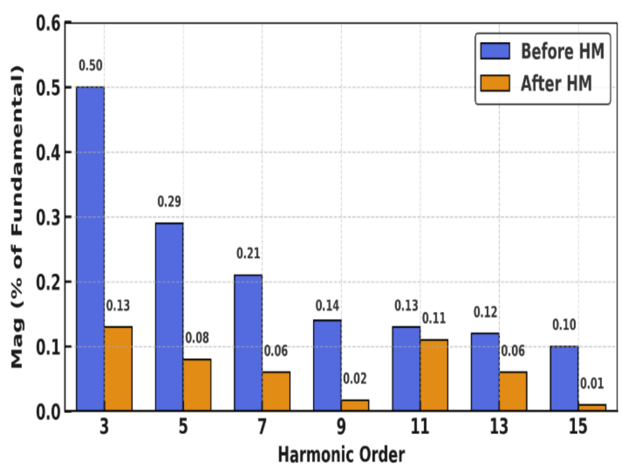


(a)

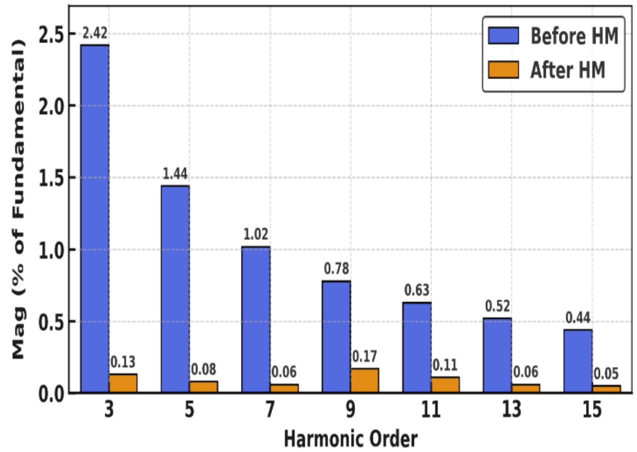


(b)

Figure 18. Voltage and current waveform on primary winding before and after injecting, (a) Voltage on primary winding, (b) Current on primary winding



(a)



(b)

Figure 19. Voltage and current waveform on secondary winding before and after injecting, (a) Voltage on secondary winding, (b) Current on secondary winding

The quality of the voltage and current waveforms greatly improved when the substitute current was injected using the tertiary winding. The waveforms before and after the tertiary injection are contrasted in Figures 18 and 19. The primary winding voltage waveform is shown in Figure 18(a), and the current waveform is shown in Figure 18(b). Both indicate an apparent change from distorted to seriously cleaner sinusoidal signals following mitigation. Figure 19 shows corresponding achievements in the secondary winding. The effectiveness of the compensating current in neutralizing the harmonic components created by the nonlinear load is demonstrated by subfigure (a), which shows that the voltage waveform is significantly smoother after injection, and subfigure (b), which shows that the current waveform has also been significantly filtered. According to the modeling findings, the tertiary winding's injected current efficiently eliminates a large number of harmonic orders, greatly lowering THD. It is also critical to keep in mind that while the converter's AC side gets better, the DC side's harmonic content changes. This is the outcome of electromagnetic interactions between the AC and

DC components of the converter system. Therefore, any decrease in AC harmonic distortion benefits the DC output and provides a system-wide benefit. A capacitor was connected in series with the tertiary winding to improve harmonic cancellation. Therefore, any decrease in AC harmonic distortion benefits the DC output and provides a system-wide benefit. A capacitor was connected in series with the tertiary winding to improve harmonic cancellation. This capacitor is a high impedance used for the fundamental frequency usually 60 Hz and a low impedance path for higher-order harmonics. This indicates that only those harmonic components pass through the tertiary winding, allowing the fundamental to remain untouched. This approach design directs the injected current to target and cancel harmonics without interfering with power transfer between the primary and secondary windings. The harmonic analysis performed for this study concentrated on harmonic orders up to the 15th, which is typically the most essential for systems with rectifier-based loads. Meanwhile, the model can be adapted to handle higher-order harmonics created by fast-switching converters and current power electronics. The simulation results generally verify the proposed method's central concept. Applying a tertiary winding in a three-winding transformer as an ineffective injection site for harmonic improvement. This approach significantly reduces harmonic content across both essential and secondary windings, ensuring minimum distortion is carried to the utility grid. The study demonstrates that this method might enhance system performance while staying reasonably straightforward by evaluating waveforms and measuring THD before and after correction. Moreover, while the technology is ineffective and does not rely on complex active filtering systems, it is less expensive and more reliable, with fewer points of failure. The reduced harmonic presence improves system stability, eliminates component overheating and undesirable resonance, and increases the operational lifespan of transformers and other associated equipment. Additionally, the method is consistent with IEEE 519 standards, making it acceptable for commercial and industrial power systems requiring compliance without investing in costly external filtering equipment.

4.2 Sensitivity Analysis

Sensitivity studies on specific components allow for a thorough investigation of harmonic reduction in power distribution networks. These tests help evaluate how harmonic mitigation techniques perform in real-world situations, which often involve erroneous but plausible scenarios. This part does sensitivity analysis by varying several factors, including the kind of nonlinear load, transformer winding layout, current injection amount, and load resistance. These changes clarify the relative contributions of each system component to harmonic generation and suppression. The investigation begins by analysis the system using a half-wave rectifier linked to the load. This sort of design causes severe harmonic distortion. These harmonics are common in both residential and commercial power systems and can have a significant influence on overall power quality. The rectifier's distorted waveform serves as a useful test case for determining how well various harmonic reduction strategies function in ensuing simulations. The first scenario is studies based on changes in the number of turns in the transformer's third winding. This winding generates harmonic current, which negates or minimizes distortion in the primary channel. The number of turns changes the voltage transformation ratio and magnetic flux in the transformer core. A decrease in the number of turns canon can result in faulty harmonic injections, whereas an increase in turns can cause core saturation, which increases harmonic content. Finding the optimal number of turns is thus critical to balancing flux control and harmonic suppression. The magnitude of the injected current through the third winding can influence system behavior and winding turns. Higher injection current magnitudes may suppress stronger harmonic components; however, nonlinearity in magnetic or overload system components may result. Conversely, lower injection currents cannot effectively remove harmonics. As a result, this parameter is adjusted to determine its effect on THD in both voltage and current waveforms.

The effect of load magnitude and resistance is also evaluated. Changing the load affects the total system impedance, modifying how harmonics propagate through the network. THD may be more predictable for resistive loads, but distortion may interact with impedance in more complex ways for reactive or nonlinear loads. Sensitivity to load variation is important in real-world applications where dynamic loading is standard. This analysis isolates the influence of each variable on system performance via varying each one independently while keeping the others constant. The ultimate goal is to learn how to tune each component for optimal harmonic mitigation under a variety operating condition. The results confirm that the harmonic injection technique using the third winding is robust and can be adapted to different system configurations.

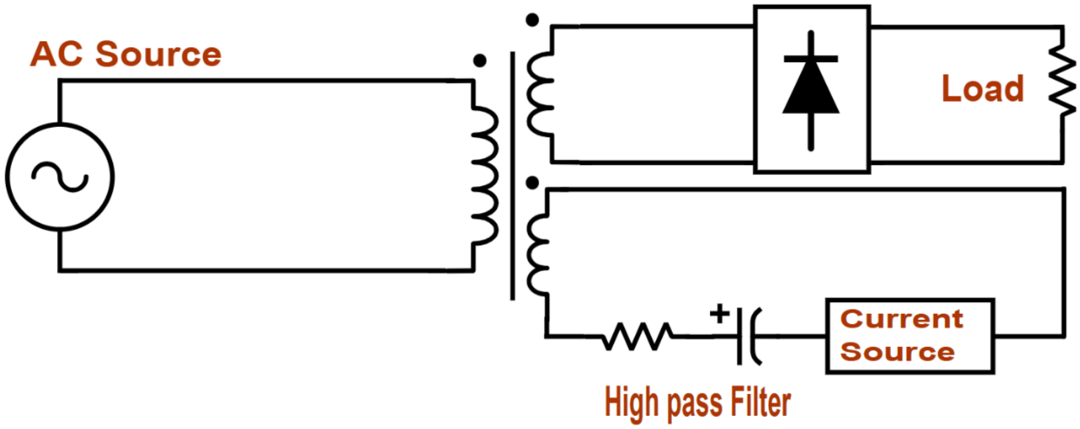


Figure 20. Simulation test setup of the mitigation method

Figure 20 shows the circuit used in this sensitivity analysis study. It includes an AC source, a transformer with tertiary winding, a half-wave rectifier, a nonlinear load, and a high-pass filter coupled with a current injection path. MATLAB Simulink is used to simulate different scenarios, analyze harmonic spectra, and verify the effectiveness of the proposed method under diverse operating conditions.

4.2.1 Testing the system using a half wave rectifier

This section focuses on specific harmonics and frequencies in the context of a half-wave bridge rectifier, as illustrated in Tables 2 and 3. The influence of these harmonic loads on the primary winding transformer is comprehensively investigated within the scope of half-wave rectifiers. The THD_v and THD_i are assessed on each simulation's primary side and subsequently compared. Table 2 presents the THD_i measured on the primary side of the circuit employing a half-wave rectifier. After implementing the mitigation method, the THD_v on the primary winding decreased significantly from 1.21% to 0.35%, indicating the method's effectiveness. Subsequently, Table 3 demonstrates that employing the same HM for the load resulted in a 6.79% decrease in THD_i to 2.9%, which resulted in the primary winding.

Table 2 Harmonic voltage contents on primary winding

H	Before HM (%)	After HM (%)
3	0.43	0.15
5	0.22	0.07
7	0.17	0.07
9	0.12	0.02
11	0.10	0.00
13	0.09	0.02
15	0.08	0.02
THD	1.21	0.35

Table 3 Harmonic current contents on primary winding

H	Before HM (%)	After HM (%)
3	1.45	0.79
5	1.05	0.56
7	0.93	0.43
9	0.87	0.38
11	0.85	0.31
13	0.83	0.26
15	0.81	0.19
THD	6.79	2.9

4.2.2 Change in the load magnitude

In the context of harmonic mitigation using tertiary winding injection, one of the most critical parameters influencing performance is nature and impedance of the connected load. A comprehensive sensitivity study is performed by changing the load resistance in the injection channel to determine its influence on harmonic distortion at both voltage and current waveforms at the transformer's primary winding. The findings, reported in Tables 4 and 5, provide quantitative proof of how varying resistance levels affect the efficiency of the mitigation strategy. The harmonic voltage contents of the primary winding under various resistance settings are shown in Table 4.

Table 4. Harmonic voltage contents on primary winding

H	Before HM (%)	R=10 Ω (%)	R=15 Ω (%)	R=20 Ω (%)	R=25 Ω (%)	R=30 Ω (%)	R=35 Ω (%)	R=40 Ω (%)
3	0.43	0.15	0.11	0.06	0.18	0.19	0.19	0.20
5	0.22	0.07	0.11	0.04	0.04	0.13	0.03	0.10
7	0.17	0.07	0.07	0.03	0.08	0.07	0.02	0.05
9	0.12	0.02	0.04	0.04	0.06	0.05	0.02	0.05
11	0.10	0.02	0.01	0.01	0.04	0.02	0.02	0.05
13	0.09	0.02	0.01	0.01	0.02	0.03	0.03	0.03
15	0.08	0.02	0.01	0.0	0.02	0.01	0.03	0.03

Table 5. Harmonic current contents on primary winding

H	Before HM (%)	R=10 Ω (%)	R=15 Ω (%)	R=20 Ω (%)	R=25 Ω (%)	R=30 Ω (%)	R=35 Ω (%)	R=40 Ω (%)
3	1.45	0.68	0.45	0.92	0.67	0.63	0.64	0.84
5	1.05	1.05	0.49	0.58	0.65	0.65	0.69	0.66
7	0.93	0.38	0.47	0.54	0.58	0.62	0.67	0.63
9	0.87	0.41	0.43	0.49	0.54	0.58	0.60	0.50
11	0.85	0.41	0.33	0.36	0.41	0.45	0.54	0.50
13	0.83	0.26	0.32	0.32	0.37	0.42	0.46	0.49
15	0.81	0.08	0.28	0.26	0.32	0.41	0.46	0.49

Before HM, the system shows a high voltage content of the third harmonic at 0.43%, with other harmonics, including the fifth and seventh, also present at notable levels. However, with the application of tertiary winding injection and varying load resistances, a marked reduction in harmonic voltage was observed across all orders. The 20 Ω resistor yielded the lowest overall harmonic voltage, particularly reducing the 3rd harmonic to just 0.06%, a reduction of over 85% from the unmitigated case. In addition to concentrating on dominant harmonics, impedance tuning also decreases higher-order distortion, as seen by similar patterns observed for various harmonic orders, including the fifth, seventh, ninth, and eleventh. Table 10 examines the harmonic current content on the primary side, which is directly related to concerns such as transformer overheating, higher copper losses, and lower efficiency. Using a 15 Ω resistor reduces the 3rd harmonic current from 1.45% to 0.45%. This indicates that 15 Ω provides the most efficient current harmonic reduction among the tested values. Interestingly, while current harmonics were generally lower with 15 Ω, voltage harmonics were best suppressed with 20 Ω. This highlights a trade-off system designers must navigate depending on their performance priorities voltage THD at the point of common coupling vs. transformer-side current distortion. This comparative analysis confirms the fact that harmonic suppression performance is non-linearly dependent on load resistance. Lower resistance values allow higher injection currents, which are useful for suppression but can also increase power loss or system stress if too low. Conversely, higher resistance values reduce injection current and limit the system's capacity to cancel harmonics. The results demonstrate an optimal resistance between 15 Ω and 20 Ω, where mitigation performance is maximized. This analysis illustrates the usefulness of resistance as a tuning parameter for harmonic suppression in addition to the numerical decreases in harmonic amplitudes. In real-world situations, loads are not constant and might change due to malfunctions, equipment switching, and demand throughout the day. A dynamic or programmable resistor or an adaptive control scheme based on real-time THD feedback could help continuously optimize the injection circuit to maintain power quality. Furthermore, the combined gains in voltage and current harmonics confirm that the tertiary winding strategy is a workable and feasible option for distribution system harmonic abatement, in addition to being a theoretical idea. These results highlight how crucial it is to incorporate hardware-level parameter management and simulation-based tuning into system design.

4.2.3 Change the Number of Turns in the Transformer

The tertiary-to-secondary winding turns ratio has significantly impacted the efficacy of the harmonic mitigation technique based on tertiary winding current injection. The amplitude of the injected current that causes harmonic cancellation is determined by the voltage essentially determined by this ratio in the tertiary winding. The ampere-turn balance between the windings directly impacts the flux distribution inside the core of magnetically linked transformers. In order to regulate the magnitude and phase of the opposing magnetomotive force produced by the injected current, precise adjustment of N_3 becomes crucial. This sensitivity analysis examined the effects of N_3 on harmonic suppression efficacy, ranging from 1.6 to 4.0. The research sought to determine the ideal arrangement for minimizing THDv and THDi at the transformer's primary winding. Harmonic orders from the third to the fifteenth were explored, resulting in a complete spectrum-level knowledge of how the turn ratio affects the system's harmonic profile. The simulation results, shown in Table 6 and Table 7, clearly reveal that the effectiveness of harmonic mitigation varies nonlinearly concerning N_3.

Table 6. Harmonic voltage contents on primary winding

H	Before HM (%)	N=4 (%)	N=3.2 (%)	N=2.67 (%)	N= 2.29 (%)	N=2 (%)	N= 1.78 (%)	N= 1.6 (%)
3	0.43	0.15	0.15	0.15	0.09	0.05	0.12	0.11
5	0.22	0.10	0.07	0.07	0.04	0.03	0.02	0.06
7	0.17	0.04	0.06	0.07	0.03	0.03	0.01	0.03
9	0.12	0.06	0.04	0.02	0.03	0.03	0.01	0.02
11	0.10	0.05	0.04	0.02	0.03	0.04	0.01	0
13	0.09	0.04	0.01	0.02	0.02	0.02	0.02	0.02
15	0.08	0.03	0.02	0.02	0.02	0.02	0.01	0.01

As shown in Table 6, the 3rd harmonic voltage, often the most dominant in rectifier-based nonlinear loads was initially 0.43% before mitigation. When N_3 was tuned to 1.75, the 3rd harmonic voltage dropped significantly to 0.12%, while other harmonics (e.g., 5th, 7th, and 9th) were also notably suppressed. Similar trends are visible in Table 6, where the 3rd harmonic current decreased from 1.45% before mitigation to 0.84% at N_3 =1.75.

Table 7. Harmonic current contents on primary winding

H	Before HM (%)	N=4 (%)	N=3.2 (%)	N=2.67 (%)	N= 2.29 (%)	N=2 (%)	N= 1.78 (%)	N= 1.6 (%)
3	1.45	0.97	0.70	0.68	0.89	0.85	0.84	0.70
5	1.05	0.55	0.62	0.49	0.49	0.49	0.53	0.46
7	0.93	0.51	0.43	0.48	0.39	0.43	0.39	0.37
9	0.87	0.4	0.41	0.38	0.37	0.33	0.36	0.29
11	0.85	0.38	0.34	0.32	0.32	0.30	0.29	0.25
13	0.83	0.34	0.31	0.26	0.24	0.26	0.26	0.22
15	0.81	0.3	0.27	0.26	0.21	0.24	0.20	0.19

This indicates a clear improvement in power quality due to more effective harmonic injections and cancellations. Moreover, analysis of the surrounding values shows that both increasing and decreasing N_3 from the optimal point led to performance degradation. For instance, at N_3=2.67, the 3rd harmonic current is higher at 0.68%, and at N_3=1.6, it rises again to 0.70%. This indicates that the system is extremely sensitive to the turn ratio and that even small deviations from the ideal point can decrease mitigation efficiency. Physically, this phenomenon is described by the coupling dynamics in the transformer core. When N_3 is too low, the voltage across the tertiary winding is insufficient to drive the necessary cancellation current, leading to under-compensation of harmonic flux. Conversely, when N_3 is too high, the excessive injection current can lead to overcompensation, core saturation, or resonance effects that distort the voltage waveform. These nonlinear phenomena reinforce the need for precise design when choosing the transformer's winding configuration. Additionally, the analysis reveals that the influence of N_3 is not limited to the 3rd harmonic alone. Other harmonic orders, including the 5th, 7th, and 9th, also show measurable reductions at the optimal winding ratio. For example, the 5th harmonic current decreased from 1.05% to 0.46%, and the 7th harmonic current reduced from 0.93% to 0.37% at N_3=1.75. These comprehensive improvements across the spectrum confirm that optimal tertiary winding design has a broadband effect on harmonic suppression, not just targeting individual components.

The findings from this section demonstrate that N_3=1.75 offers the best balance between injection strength and system stability. Effective harmonic cancellation is possible at this tuning point without overexcitation or undesirable side effects. It might be a helpful design guideline if engineers wish to use similar mitigation strategies in real-world systems. Furthermore, the suggested approach is passive and adaptive due to the ability to adjust harmonic mitigation using physical transformer factors like winding ratios. This lowers the cost, complexity, and maintenance needs by eliminating the need for external filtering devices or active feedback control. It also makes the method highly suitable for compact, embedded, or standalone systems such as electric vehicle chargers, renewable inverters, industrial motor drives, and distributed generation units. In summary, careful adjustment of the tertiary-to-secondary winding turns ratio is one of the most effective levers for enhancing the performance of harmonic mitigation systems. The clear quantitative improvements observed in both THDv and THDi across harmonic orders especially at $N_3 = 1.75$ underscore the importance of this design parameter and validate the underlying physics governing magnetic flux cancellation.

4.2.4 Change in the Current Magnitude

The amount of current injected into the tertiary winding, shown by the symbol I_3, has a significant impact on the effectiveness of harmonic reduction in transformer-based systems. The fundamental mechanism relies on infusing a precisely calibrated current that generates a magnetic flux in order to counteract the harmonics brought on by nonlinear loads, particularly in the transformer's core. As such, the amplitude of I_3 directly affects the strength and phase alignment of the cancellation flux, making it one of the most sensitive and critical parameters for achieving optimal harmonic suppression. In this sensitivity analysis, the system was evaluated by varying the injection current incrementally from 0.3 A to 4.5 A. The impact of THDv and THDi on the transformer’s primary side was monitored and documented. The results in Tables 8 and 9 present a clear correlation between I_3 and harmonic suppression effectiveness across different harmonic orders.

Table 8. Harmonic voltage contents on primary winding

H	Before HM (%)	I=0.3 A (%)	I=0.5 A (%)	I=0.75 A (%)	I=1.5 A (%)	I=2.25 A (%)	I=3 A (%)	I=3.75 A (%)	I=4.5 A (%)
3	0.43	0.03	0.13	0.15	0.12	0.04	0.11	0.14	0.19
5	0.22	0.04	0.04	0.07	0.09	0.05	0.03	0.08	0.07
7	0.17	0.06	0.01	0.07	0.07	0.02	0.03	0.02	0.05
9	0.12	0.02	0.01	0.04	0.03	0.02	0.04	0.02	0.01
11	0.10	0.02	0.01	0.02	0.01	0.04	0.01	0.01	0.02
13	0.09	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.0
15	0.08	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.0

According to Table 8, the 3rd harmonic voltage content without mitigation was 0.43%. This number fell sharply to 0.03%, or a 93% decrease, when the injected current was adjusted to 0.3 A. The system's ability to suppress various low-order harmonics was validated by considerable drops in other harmonic orders, such as the fifth and seventh, to which this current level also contributed. Interestingly, once the injection current exceeded 0.3 A, the THDv readings began to rise again. At 4.5 A, for example, the 3rd harmonic voltage climbed to 0.19%, suggesting that excessive injection could degrade waveform quality through overcompensation or secondary flux imbalances. Table 9 further supports this conclusion by showing how current harmonics responded to variations in I_3 . When the injection current reached 0.5 A, the untreated third harmonic current, initially 1.45%, decreased to 0.68%. With equivalent improvements in other harmonics, such as the 5th and 7th, which were decreased to 0.49% and 0.48%, respectively, this value was the most successful in mitigating current distortion.

Table 9. Harmonic current contents on primary winding

H	Before HM (%)	I=0.3 A (%)	I=0.5 A (%)	I=0.75 A (%)	I=1.5 A (%)	I=2.25 A (%)	I=3 A (%)	I=3.75 A (%)	I=4.5 A (%)
3	1.45	0.79	0.7	0.68	0.87	0.85	0.92	1.01	1.08
5	1.05	0.56	0.51	0.49	0.62	0.58	0.56	0.44	0.45
7	0.93	0.43	0.43	0.48	0.56	0.39	0.33	0.32	0.39
9	0.87	0.38	0.34	0.38	0.34	0.35	0.33	0.24	0.26
11	0.85	0.31	0.26	0.33	0.34	0.3	0.3	0.2	0.22
13	0.83	0.26	0.22	0.28	0.22	0.21	0.2	0.18	0.22
15	0.81	0.19	0.19	0.26	0.18	0.17	0.18	0.17	0.21

However, as I_3 increased beyond this optimal point, the current harmonics either stagnated or slightly increased, again highlighting a performance ceiling governed by nonlinear system dynamics. This behavior is consistent with established magnetic circuit theory: minimal suppression results from the MMF's inability to interact much with the harmonic flux at low injection levels. The cancellation is powerful and well-aligned at modest levels, producing destructive interference that reduces harmonic distortion. Waveform distortion, inefficiency, or system instability might result from the tertiary winding's excessive opposing flux introduced at high injection levels, which could cause it to misalign with the harmonic phase or excite parasitic resonances. This study demonstrates that the injected current magnitude must be carefully selected to match the harmonic profile and transformer characteristics. The dual findings 0.3 A being optimal for THDv and 0.5 A being optimal for THDi suggest that designers must balance minimizing voltage distortion at the PCC and reducing harmonic loading on the transformer itself.

CHAPTER FIVE

EXPERIMENTAL VERIFICATIONS

5.1 Introduction

This chapter focuses on the experimental validation of the harmonic mitigation approach reported previously in the dissertation. The method is based on injecting compensating harmonic currents through the tertiary winding of a specially designed three-winding transformer. This tertiary winding acts as a medium for harmonic cancellation by targeting specific frequency components generated by nonlinear elements, such as diode rectifiers and switching loads.

While simulation studies using MATLAB/Simulink provided strong evidence for the technique's effectiveness, theoretical validation alone is insufficient. Real-world systems have a variety of uncertainties, including component tolerances, parasitic impedances, signal noise, and non-ideal switching behavior. As a result, hardware-based testing is required to evaluate the suggested approach's feasibility, dependability, and limits.

This chapter uses a laboratory-scale testbed to replicate a typical power distribution scenario under harmonic distortion. The experimental system includes a single-phase AC supply, a nonlinear load, a three-winding transformer, and a harmonic injection circuit composed of a function generator, a high-pass filter, and a Darlington transistor amplifier. The experiments are designed to analyze voltage and current waveforms at different points in the system before and after applying the mitigation method. The results are compared with those obtained from simulation to highlight consistencies and deviations. Special attention is given to reducing individual harmonic orders and THD in the primary and secondary windings. The experimental study thus serves a dual purpose; it validates the suggested harmonic mitigation paradigm while also revealing practical difficulties for real-world applications.

This validation approach extensively evaluates the tertiary winding injection method's efficacy, efficiency, and adaptability. The findings in this chapter play a crucial role in confirming the method's readiness for future integration into industrial and utility-scale applications where harmonic suppression is critical.

5.2 Experimental Setup

This section provides a comprehensive examination of strategies for controlling harmonic distortion, which will be presented along with experimental validation of the proposed simulation design. The experimental setup, illustrated in Figure 21, includes a primary AC circuit, a three-winding transformer, a function generator, a high-pass filter, a resistive load, and cRIO hardware. Notably, the high-pass filter allows only high frequencies to pass through the third winding, effectively blocking lower-order harmonics. To facilitate harmonic analysis, probes are connected between the cRIO hardware and the transformer's primary and secondary windings, and data is processed using LabVIEW software. The third winding of the transformer is energized by a pulse signal generated by a function generator and a DC voltage source. This signal cancels out unwanted harmonics within the transformer core. A Darlington transistor pair is used to amplify this pulse signal before it is injected into the third winding.

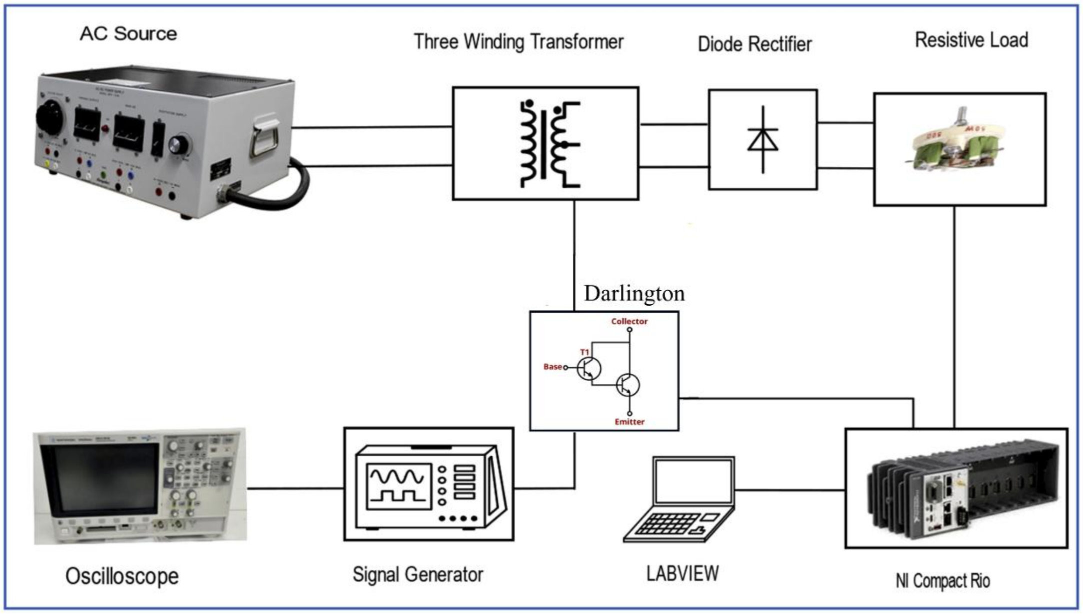


Figure 21. Experimental test setups of the mitigation method

5.3 Darlington Circuit Configuration and Role

To enable sufficient harmonic current injection into the transformer's tertiary winding, a Darlington transistor pair was introduced between the harmonic signal source and the winding. This high-gain configuration allows a low-level control signal to generate a significantly amplified injection current. The pair consists of two BJTs connected, such that the emitter of the first feeds the base of the second, resulting in an overall current gain equal to the product of their gains. In this setup, the harmonic control signal is applied to the base, the collector is biased via a DC power supply, and the emitter of T2 is connected through a current-limiting resistor to the tertiary winding. Figure 22 illustrates the internal structure of this configuration. Simulation and experimental results demonstrate that this Darlington-based injection circuit reduces THD in both the primary and secondary windings. By functioning as a high-gain power amplifier, the Darlington configuration ensures that a small control signal from the function generator can drive strong harmonic currents into the tertiary winding. This improves harmonic cancellation without altering the signal's frequency content, enabling robust harmonic control under diverse operating conditions.

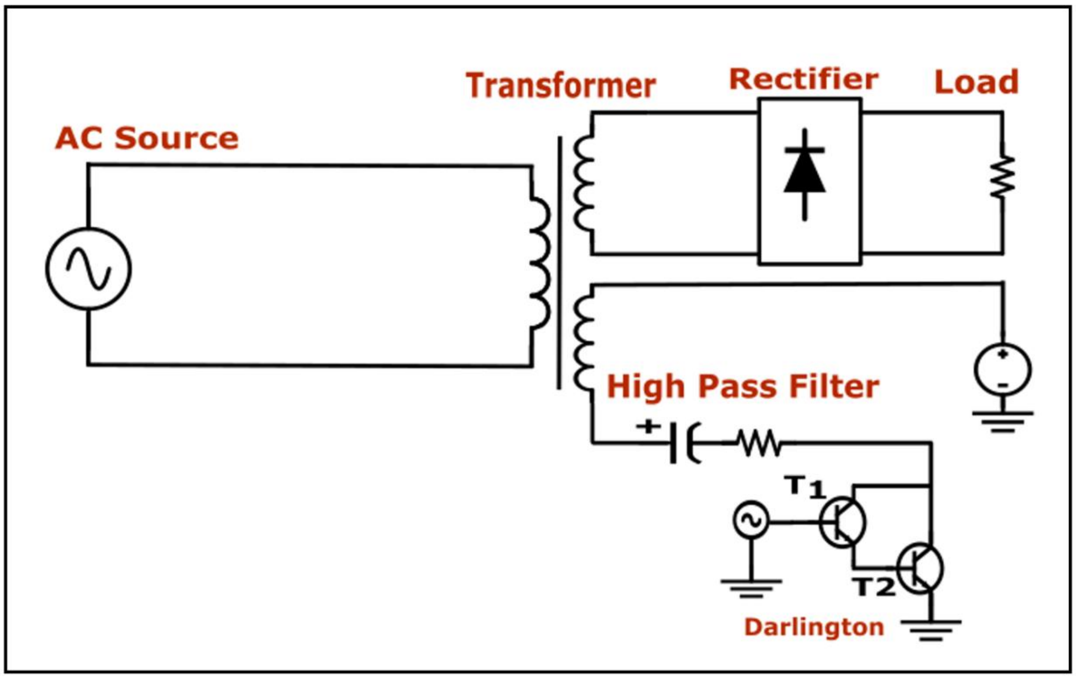


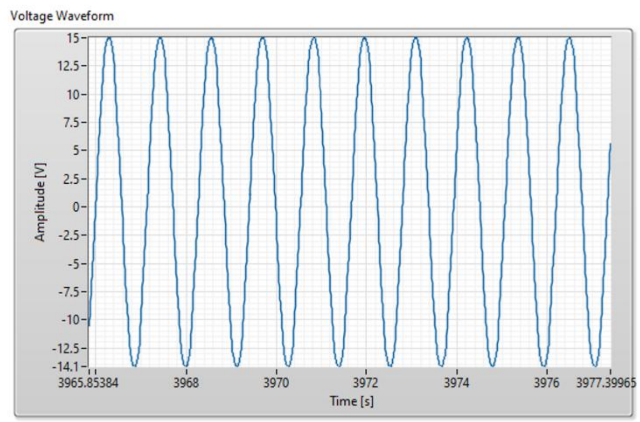
Figure 22. Harmonic Injection Circuit Using Darlington Pair and High-Pass Filter

5.4 Experimental Procedure

A controlled set of experimental procedures was used to assess the effectiveness of the suggested harmonic mitigation approach. Initially, the system was operated without activating the harmonic injection circuit to get baseline readings. Voltage and current waveforms were recorded at the primary and secondary windings as a reference for future comparison. Under these baseline conditions, the diode rectifier and the load produced sharp current pulses and noticeable voltage distortion, particularly evident in the primary and secondary currents. FFT analysis was performed to identify the dominant harmonic components. The harmonic injection procedure was then initiated by configuring a function generator to produce a square waveform at selected harmonic frequencies. A DC bias was applied to shift the signal to an appropriate operating level before amplification. The biased signal was then amplified using a Darlington pair circuit and routed through a high-pass filter before being injected into the transformer's tertiary winding. After the injection circuit was activated, voltage and current waveforms were re-recorded at the same points to observe changes in harmonic content. These post-injection measurements were analyzed to assess the reduction in harmonic amplitudes and the overall THDi. Finally, LabVIEW software was used to extract and analyze the harmonic data from pre- and post-injection conditions. The laboratory results were then compared to simulated data collected from MATLAB/Simulink to assess the mitigation system's accuracy, consistency, and practical performance.

5.5 Results and Analysis

The efficiency of the suggested harmonic mitigation method was evaluated by analyzing the voltage and current waveforms on both the primary and secondary windings of the transformer before and after the harmonic injection circuit was activated. The system was first operated under normal conditions with a nonlinear load (diode bridge rectifier with resister) to establish a baseline for harmonic distortion. As expected, the uncompensated system exhibited significant waveform distortion caused by high-frequency harmonic currents. This distortion was visible in both voltage and current waveforms, with flattened peaks, waveform ripples, and irregular zero crossings.



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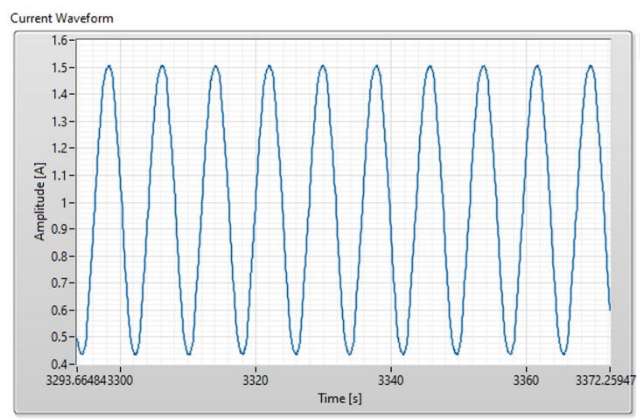
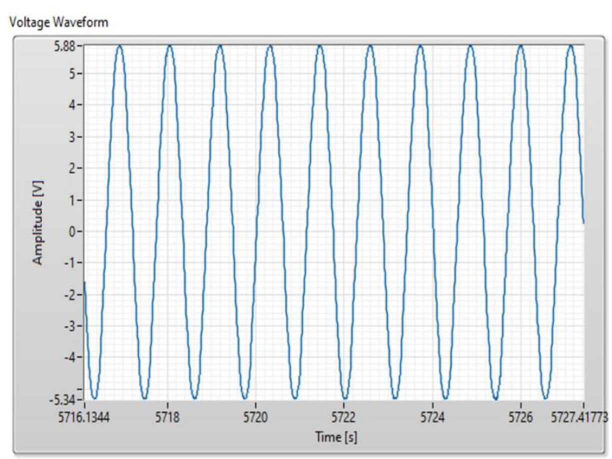
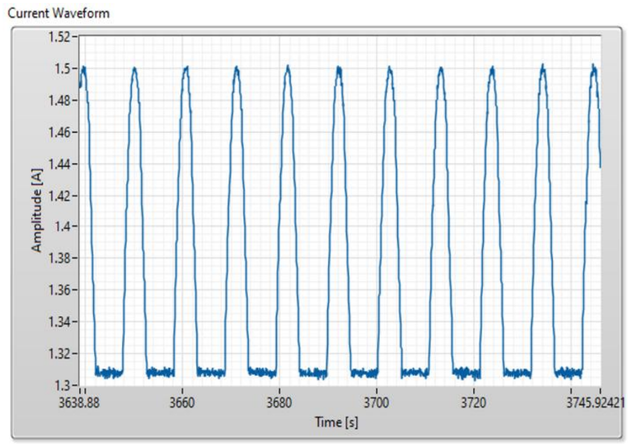


(b)



(c)



(d)

Figure 23. The voltage and current waveforms for primary and secondary winding, (a) Voltage on primary winding, (b) Current on primary winding, (c) Voltage on secondary winding, (d) Current on secondary winding

Figures 23 a and b show the voltage and current waveforms on the primary winding before mitigation, while Figures 24 a and b present the corresponding waveforms after the injection circuit was applied. FFT analysis revealed notable reductions in several dominant harmonics. The 3rd harmonic was reduced from 0.13% to 0.102%, the 5th from 1.64% to 1.02%, and the 7th from 1.35% to 1.12%. While higher-order harmonics such as the 9th and 11th showed more minor changes, the overall improvement was reflected in the reduction of THDi from 4.607% to 2.5067%. Similarly, the secondary winding experienced significant harmonic reduction even though the injection was not directly applied on that side. Due to magnetic coupling within the transformer core, the tertiary injection indirectly affected the secondary waveform. As illustrated in Figure 25, the 3rd harmonic was reduced from 3.14% to 0.115%, the 5th from 1.7% to 0.876%, the 7th from 1.45% to 1.18%, and additional harmonics such as the 11th, 13th, and 15th showed comparable decreases. Most importantly, the THDi in the secondary winding dropped from 15.842% to 3.69%, demonstrating a substantial improvement in power quality. The combined results confirm that the proposed tertiary winding injection method reduces harmonic content across both windings. Even without direct compensation on the secondary side, the mutual effects within the transformer allowed for a system-wide reduction in harmonics. This validates the method's capability to target specific harmonic orders and improve the overall waveform quality throughout the distribution system.

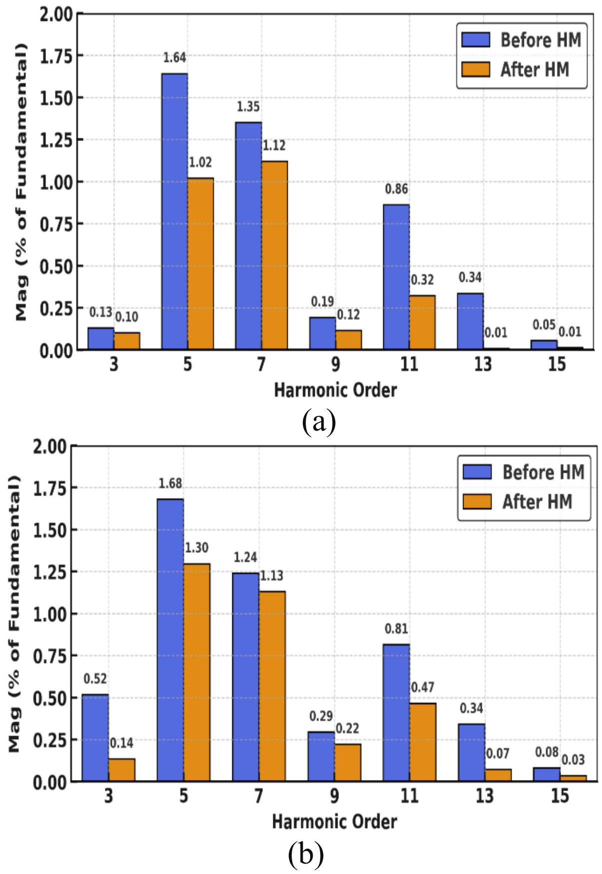
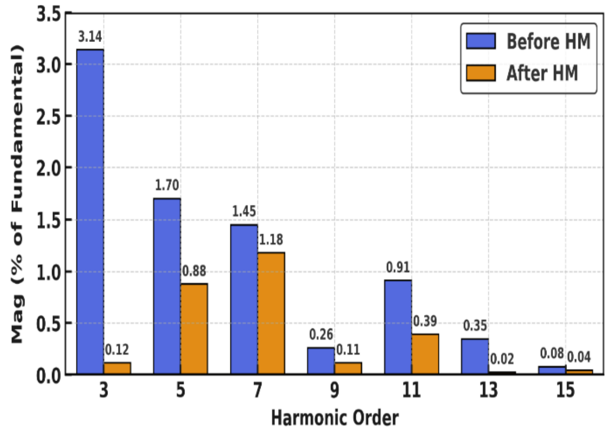
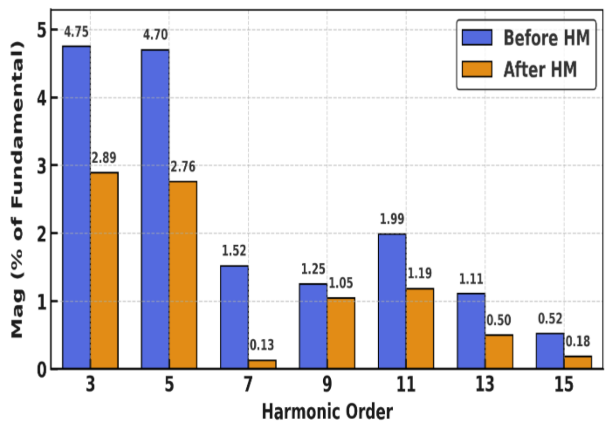


Figure 24. The results of the experiment on primary winding for voltage before and after injecting, (a) Voltage on primary winding, (b) Current on primary winding



(a)



(b)

Figure 25. The results of the experiment on secondary winding for voltage before and after injecting, (a) Voltage on secondary winding, (b) Current on secondary winding

5.6 Comparison with Simulation Results

The experimental results were compared with those obtained from MATLAB/Simulink simulations to assess the accuracy and reliability of the proposed harmonic mitigation technique. Overall, the trends observed in both domains showed strong consistency, particularly in terms of harmonic reduction patterns and the effectiveness of the tertiary winding injection method. However, some differences were identified. The experimental setup exhibited a slightly higher THDi than the simulation, which can be attributed to real-world parasitic elements such as wiring resistance, stray capacitance, and electromagnetic interference. Furthermore, the hardware's Darlington amplifier circuit had actual constraints in bandwidth and gain, resulting in tiny aberrations that did not exist in the idealized simulation model. Measurement limits also led to variance; for example, the cRIO hardware's resolution and filtering capabilities caused modest differences in signal collection and processing. Despite these considerations, the overall harmonic suppression trends in the experimental data matched those in the simulation, demonstrating the resilience of the suggested model. These results demonstrate that the harmonic mitigation strategy is still successful under practical settings, and that the simulation properly represents the system's real-world performance.

CHAPTER SIX

SUMMARY

6.1 Conclusion

This thesis has presented a novel transformer-integrated approach for mitigating harmonic distortion in power distribution systems by utilizing the tertiary winding of a three-winding transformer. Harmonic distortion particularly from nonlinear loads such as rectifiers and converters has emerged as one of modern networks' most persistent power quality issues. While successful in many circumstances, traditional methods, such as passive and active filters, can need large installation space, involve sophisticated control systems, and may be unable to adapt to changing system conditions. By integrating the mitigation strategy directly into the transformer design, the technology presented in this study provides a more compact and economical option, reducing the need for external components while preserving good performance. By introducing a regulated harmonic current into the tertiary winding, the suggested technique creates a counteracting magnetic flux inside the transformer core. The harmonic fluxes generated by the nonlinear load are destructively interfered with by this flux, which lessens their presence in the primary and secondary windings. The design was implemented and tested through both simulation and experimental frameworks. The MATLAB/Simulink simulation models provided a controlled environment in which to design, refine, and evaluate the method under varying conditions. At the same time, the hardware-based validation confirmed the approach's real-world feasibility using a physical transformer, a full-wave rectifier load, and a programmable harmonic injection system monitored via cRIO and LabVIEW interfaces.

Both the simulation and experimental results demonstrated significant improvements in power quality metrics. THDi and THDv were reduced substantially across various load conditions and operating scenarios. The harmonic content of orders such as the 3rd, 5th, 7th, and even higher frequencies were suppressed effectively through proper tuning of the injection waveform. Time-domain waveforms showed smoother sinusoidal shapes, and FFT analysis confirmed reductions in harmonic amplitudes at key frequencies. In addition, the technique proved resilient under challenging scenarios such as half-wave rectification, where waveform asymmetry and even-order harmonics are more severe. The sensitivity analysis conducted in this study added further value by demonstrating the method's adaptability. It explored how performance responds to variations in load type, winding turns ratio, injection current magnitude, and load resistance. These tests confirmed that the mitigation strategy can be fine-tuned through design parameters or control strategies to perform optimally under diverse real-world conditions. According to the results, the suggested method shows promise as a substitute for traditional harmonic filters, particularly in distribution networks where integration flexibility, cost, and space are important considerations.

In conclusion, this study has shown a versatile, small, and scalable harmonic mitigation method that can be easily included in transformer design. It contributes not only a new method for addressing power quality challenges but also to a potential framework for the future of transformer-based smart power components.

6.2 Future Work

While the performance of the proposed method has been validated in single-phase systems under controlled conditions, there remains significant scope for future research aimed at expanding, optimizing, and enhancing its applicability and adaptability. One major direction involves the development of adaptive harmonic control systems, which would replace fixed waveform injection with a real-time feedback mechanism. Such a system would utilize continuous harmonic monitoring to detect variations in load-generated distortion and adjust the magnitude, frequency content, and phase of the injected current accordingly. Implementing closed-loop control would make the method more resilient in environments where nonlinear loads vary over time such as commercial buildings, data centers, or electric vehicle charging stations.

The control framework for adaptive injections could leverage modern digital signal processing platforms or real-time embedded systems such as FPGA-based controllers. Additionally, with the increasing role of data-driven technologies in power systems, machine learning algorithms could be integrated to predict harmonic behavior and optimize injection signals preemptively. This would allow the system to adapt reactively and proactively, enhancing power quality before significant degradation occurs. Another essential area for future investigation is extending the proposed method to three-phase power systems. Three-phase systems dominate industrial and utility-scale networks, and unbalanced loads, zero-sequence harmonics, and interharmonics present additional challenges not encountered in single-phase systems. Applying tertiary injection in this context requires coordinated control across all three phases and advanced transformer winding configurations such as delta or zigzag. These arrangements could cancel triplen harmonics and enhance zero-sequence suppression, making the approach even more powerful in practical grid environments. Further research could also focus on hardware integration and optimization. The harmonic injection circuitry and control interface must be compact, modular, and easily embedded into commercial transformer designs for widespread adoption. Exploring power electronic converters that can act as programmable current sources with real-time harmonic injection capabilities would streamline implementation. Additionally, long-term field testing in live distribution networks would provide valuable data on system durability, energy efficiency, and economic feasibility, including lifecycle cost savings compared to traditional harmonic mitigation equipment.

Finally, aligning the solution with international standards such as IEEE 519, IEC 61000, and national grid codes will enable compliance and interoperability in regulated environments. Collaborations with transformer manufacturers and utilities could pave the way for next generation of smart transformers that deliver voltage transformation and actively enhance power quality at the source. In conclusion, while this research establishes a strong foundation for transformer-based harmonic mitigation, it also opens multiple pathways for innovation in control, design, and application ultimately contributing to the development of smarter, cleaner, and more adaptable power systems.

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