

ALGEBRAIC STRUCTURES OF POLYNOMIAL DATA: APPLICATIONS TO
GALOIS THEORY AND QUANTUM CODES

by

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*To my family, for their love and support throughout this
journey.*

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Jurgen Mezinaj

ABSTRACT

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Adviser: Tanush Shaska, Ph.D.

We develop a computational framework for classifying Galois groups of irreducible degree-7 polynomials over $\mathbb{Q}$, combining explicit resolvent constructions with modern machine-learning techniques. A database of over one million normalized projective septics is constructed, each annotated with algebraic invariants $J_0, \dots, J_4$ obtained via binary transvections. For each polynomial, carefully designed resolvents are factored to determine its Galois group among the seven transitive subgroups of S_7 . Using this dataset, we train neuro-symbolic classifiers that integrate invariant-theoretic features with supervised learning, achieving improved accuracy in detecting rare solvable groups compared to coefficient-based and purely numerical baselines. The resulting database provides a reproducible resource for constructive Galois theory and supports empirical investigations into the distribution of Galois groups under height and discriminant constraints.

Building on the same polynomial data, we construct families of superelliptic curves on weighted projective spaces and derive algebraic-geometry and quantum error-correcting codes from them. We use a weighted projective framework to control genus, singularities, and rational points, and we introduce graded evaluation codes adapted to weighted embeddings. Within the CSS construction, we obtain

criteria for self-orthogonality and establish refined Singleton-type bounds for the resulting quantum weighted algebraic-geometry (QWAG) codes. Numerical experiments over finite fields show how Galois-theoretic labels, weighted models, and graded evaluation spaces interact to produce quantum codes with competitive parameters and support a systematic, database-driven search for good classical and quantum codes.

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CHAPTER ONE

Introduction

The study of polynomial equations has occupied a central position in mathematics since antiquity, serving as a crucible for many of its deepest developments. From the quadratic formula of the Babylonians and the geometric constructions of the ancient Greeks, through the discovery of the cubic and quartic solutions in the Italian Renaissance, to the groundbreaking work of Abel and Galois in the nineteenth century establishing the unsolvability by radicals of the general quintic, polynomial equations have driven the creation of group theory, field theory, and modern algebra. In the twentieth and twenty-first centuries, this classical theme has ramified into algebraic geometry (via the study of algebraic curves and their Jacobians), arithmetic geometry, coding theory, and, more recently, computational and data-driven mathematics, including machine learning.

Irreducible polynomials encode profound arithmetic and geometric information. The Galois group of an irreducible polynomial governs the symmetries of its splitting field, determines solvability by radicals, and influences the distribution of roots and the arithmetic of associated number fields. For higher-degree polynomials these Galois groups reflect the rich subgroup structure of the symmetric group S_n . The geometric objects attached to such polynomials—especially superelliptic curves—provide explicit models for studying moduli spaces, rational points, and applications in error-correcting codes.

This thesis focuses on irreducible septic (degree-seven) polynomials over $\mathbb{Q}$ and their associated geometric and coding-theoretic structures. A central innovation is the construction and systematic analysis of a large-scale *canonical database* of more than one million irreducible septic polynomials, enriched with arithmetic,

Galois-theoretic, and invariant-theoretic data. This database serves as a unified computational laboratory connecting computational Galois theory, classical invariant theory, superelliptic and weighted projective geometry, and coding theory.

The geometric and coding-theoretic framework of the thesis rests heavily on the foundational theory of superelliptic curves and weighted projective geometry developed primarily by Shaska and his collaborators; see [5, 17, 4, 19, 23], which has transformed the study of curves given by equations of the form $y^r = f(x)$ (where f is a polynomial of degree 7 in the septic case) from isolated examples into a systematic arithmetic theory. His introduction and development of weighted projective models, weighted heights, and graded structures have provided the natural setting in which superelliptic curves can be studied with controlled singularities, rich automorphism groups, and explicit arithmetic invariants.

Degree seven occupies a privileged position for this investigation. Septic polynomials realize a rich variety of transitive subgroups of S_7 , including solvable, imprimitive, primitive, and exceptional groups such as $\mathrm{PSL}_2(7)$. At the same time, they remain computationally tractable for large-scale enumeration. The classical invariant theory of binary septic forms supplies natural fixed-dimensional feature vectors, while the associated superelliptic curves $y^r = f(x)$ (with f septic) embed naturally into weighted projective spaces.

A major theme is the organization of large algebraic datasets using projective and invariant-theoretic structures. Polynomials are interpreted as rational points in projective parameter spaces of binary septic forms, respecting the natural GL_2 -action. This canonical perspective, combined with Shaska's theory of weighted heights and graded invariants, facilitates systematic enumeration, normalization, and feature extraction.

The coding-theoretic component builds directly on Shaska’s pioneering constructions of weighted (graded) algebraic-geometric (AG) codes and their quantum analogues. Traditional AG codes, introduced by Goppa, evaluate rational functions on algebraic curves. Shaska’s framework extends this paradigm to hypersurfaces in weighted projective spaces, incorporating orbifold singularities, graded coordinate rings, and weighted point-counting techniques. These graded AG-codes frequently exhibit improved parameters and natural self-orthogonality properties, making them especially suitable for quantum stabilizer codes via the CSS construction. His recent work on graded quantum codes unifies weighted AG-codes with homological approaches, providing a powerful bridge between weighted geometry and quantum information theory.

Machine learning enters the thesis as a complementary neurosymbolic tool. By combining classical invariants of binary septics (and weighted invariants in the geometric models) with supervised and hybrid learning methods on the large database, we investigate how algebraic invariants encode Galois-theoretic structure. This data-driven layer augments the symbolic and geometric foundations laid by Shaska and others, enabling rapid classification and pattern discovery that guide more expensive exact computations.

The principal contributions of this thesis are the following:

1. Construction of a comprehensive canonical database of more than one million irreducible septic polynomials over $\mathbb{Q}$, equipped with projective normalization, invariant-theoretic features, Galois data, and links to associated weighted superelliptic models (building on Shaska’s arithmetic framework).
2. Development of efficient computational pipelines for determining transitive Galois groups of septic polynomials using resolvents, invariants, and database-driven techniques.

3. Integration of classical invariant theory of binary septics with supervised and neurosymbolic machine-learning methods for Galois group classification and pattern discovery.
4. Systematic exploration of superelliptic curves and their weighted projective models, extending Shaska’s foundational arithmetic theory with new explicit families arising from the septic database.
5. Construction and analysis of graded algebraic–geometric codes and quantum graded AG-codes from these superelliptic and weighted curves, building upon and extending Shaska’s constructions.
6. Creation of integrated computational pipelines linking symbolic computation, invariant theory, weighted geometry, database methods, and machine learning.

The thesis is organized as follows.

Chapter 2 develops the algebraic and Galois-theoretic foundations, including splitting fields, transitive group actions, and resolvent methods for degree seven.

Chapter 3 presents the theory of superelliptic curves and weighted projective geometry, drawing heavily on Shaska’s foundational contributions: genus formulas, weighted embeddings, divisor theory, weighted heights, and arithmetic invariants.

Chapter 4 introduces algebraic–geometric codes, their weighted/graded extensions, and quantum stabilizer constructions, with particular emphasis on the framework pioneered by Shaska.

Chapter 5 details the construction and organization of the canonical septic database, including projective parameterization, normalization, height-bounded enumeration, and data structures.

Chapter 6 presents the machine-learning and neurosymbolic methodology using invariant vectors.

Chapter 7 examines explicit families of superelliptic and weighted projective curves arising from the database, together with the associated graded AG-codes and quantum codes.

The concluding chapter summarizes the main results, discusses limitations of the current framework, and outlines directions for future research. These include deeper arithmetic statistics on Galois groups of septics, more advanced invariant-theoretic learning paradigms, extensions of weighted constructions to other degrees or function fields, and further applications in quantum coding theory and post-quantum cryptography.

This thesis demonstrates the power of combining canonical large-scale algebraic datasets with the rich geometric and arithmetic structures developed by Shaska in the theory of superelliptic curves and weighted projective spaces. By uniting symbolic computation, invariant theory, weighted geometry, coding theory, and data-driven methods, the work seeks to advance both the understanding of septic extensions and the broader methodology of experimental mathematics in algebra and arithmetic geometry.

CHAPTER TWO

Polynomials and Galois Theory

This chapter develops the Galois-theoretic background for the database and machine-learning chapter 8 that follow: basic theory, the septic setup, and the resolvent polynomials used to distinguish transitive subgroups of S_7 .

2.1 Introduction

Galois theory provides a foundational framework for understanding the solvability of polynomial equations in terms of field extensions and their automorphism groups. Originating in the early 19th century, the theory explains why radical solutions exist for quadratic, cubic, and quartic equations, but not in general for quintics or higher degrees, due to the non-solvability of the symmetric group S_n for $n \geq 5$.

Subsequent developments by Lagrange, Jordan, Klein, and others introduced computational tools such as resolvent polynomials to study the structure of Galois groups. These tools have become central to both theoretical investigations and practical applications in number theory and algebraic geometry. A longstanding open problem in the area, the *Inverse Galois Problem*, asks whether every finite group arises as the Galois group of a field extension over $\mathbb{Q}$. While the problem remains unresolved in general, progress has been made for several classes, including abelian and many solvable groups. The problem is closely related to Hilbert's 12th problem, which seeks explicit descriptions of abelian extensions via transcendental methods. Although certain abelian cases have been addressed through class field theory and complex multiplication, non-abelian generalizations are still not well understood.

This work investigates the Galois groups of irreducible septic polynomials over $\mathbb{Q}$, integrating classical algebraic techniques with modern computational tools. In particular, we construct a large-scale database of over 1.18 million irreducible septics, enriched with algebraic invariants derived from transvections of binary forms. Each polynomial is analyzed through explicit resolvent constructions to identify its Galois group among the seven transitive subgroups of S_7 : S_7 , A_7 , $\text{PSL}(3, 2)$, $C_7 \rtimes C_6$, $C_7 \rtimes C_3$, D_7 , and C_7 .

Our method combines symbolic invariant computations [12] with supervised learning via neurosymbolic networks [22], trained to classify Galois groups using both raw coefficients and invariant-based features. The use of projective normalization and height bounds follows the approach of [21], enabling scalable enumeration while ensuring algebraic diversity. Inspired by connections to monodromy groups of superelliptic curves and their Igusa invariants, we explore how invariant-theoretic features correspond to root symmetries and impact classification accuracy.

Experimental results indicate that symbolic invariants enhance the identification of rare Galois groups (e.g., solvable groups such as $C_7 \rtimes C_6$), particularly when applied to datasets where the dominant class S_7 is removed. The inclusion of invariants significantly improves classification performance in imbalanced settings. These findings support the viability of hybrid symbolic-ML pipelines for automating group identification in computational Galois theory.

This study builds on and extends previous work in invariant-based arithmetic statistics [12], hybrid symbolic/ML models [22], and moduli space enumeration [21]. By explicitly constructing resolvent polynomials tailored for distinguishing subgroups of S_7 , we enable large-scale validation of classical methods and uncover empirical patterns in Galois group distributions.

Recent research has demonstrated the potential of machine learning in number-theoretic contexts, including graded neural architectures [24], rational function classification [3], and cryptographic moduli spaces [21]. Our contribution fits within this broader direction, applying computational techniques to the explicit realization of Galois groups and the analysis of their statistical properties.

The remainder of the thesis falls into two connected threads. The first develops computational and machine-learning tools for classifying Galois groups of irreducible septic polynomials over $\mathbb{Q}$: Chapter 2 introduces Galois extensions, septic polynomials and their invariants, and the resolvent polynomials used for septic Galois group classification (Section 2.3); Chapter 6 describes the construction and augmentation of the septic polynomial database; Section 2.4 discusses the realization of cyclic and solvable groups through constructive methods; Chapter 8 presents machine-learning classification results, including the neurosymbolic network of Section 8.6; and Chapter 7 develops the Galois applications, explicit constructions, and statistical analysis of the septic database.

The second thread turns to algebraic geometry and quantum error-correcting codes: it introduces algebraic-geometry and quantum codes, develops background on finite fields, algebraic curves, and superelliptic curves on weighted projective spaces, and constructs graded algebraic-geometry codes and quantum codes from weighted models, with emphasis on families arising from the septic polynomial database. A brief summary of contributions and a discussion of open problems appear in Section 9.6 of Chapter 9.

Throughout, the overarching aim is to develop scalable and interpretable approaches to classifying Galois groups and organizing polynomial data, with applications to arithmetic statistics, inverse Galois realizations, and the construction of algebraic-geometry and quantum codes.

2.2 Galois Theory Preliminaries

Let $\mathbb{F}$ be a perfect field with characteristic $\text{char}(\mathbb{F}) = 0$. Consider a polynomial $f(x) \in \mathbb{F}[x]$. We say that $f(x)$ is *irreducible* over $\mathbb{F}$ if it cannot be expressed as the product of two non-constant polynomials with coefficients in $\mathbb{F}$; that is, if there do not exist non-constant polynomials $g(x), h(x) \in \mathbb{F}[x]$ such that $f(x) = g(x)h(x)$. For any polynomial $f(x) \in \mathbb{F}[x]$, there exists an extension field K of $\mathbb{F}$ containing at least one root α of $f(x)$, and the smallest such extension E_f in which $f(x)$ factors completely into linear factors is called its *splitting field* over $\mathbb{F}$.

2.2.1 Galois Groups of Irreducible Polynomials

A polynomial $f(x) \in \mathbb{F}[x]$ of degree n is *separable* if it has n distinct roots in its splitting field, or equivalently, if it factors as $f(x) = (x - \alpha_1) \cdots (x - \alpha_n)$ with distinct α_i in E_f . An extension E of $\mathbb{F}$ is a *separable extension* if every element of E is a root of some separable polynomial in $\mathbb{F}[x]$. Given a finite field extension $K/\mathbb{F}$, we say that K is *Galois* over $\mathbb{F}$, or that $K/\mathbb{F}$ is a Galois extension, if

$$|\text{Aut}(K/\mathbb{F})| = [K : \mathbb{F}],$$

where $\text{Aut}(K/\mathbb{F})$ denotes the group of automorphisms of K fixing $\mathbb{F}$, and $[K : \mathbb{F}]$ is the degree of the extension. In this case, $\text{Aut}(K/\mathbb{F})$ is called the *Galois group* of $K/\mathbb{F}$, denoted $\text{Gal}(K/\mathbb{F})$.

A field extension E/F is called *normal* if every irreducible polynomial in $F[x]$ with a root in E splits completely over E . A finite extension E/F is called a *Galois extension* if it is both normal and separable.

Theorem 2.1 (Fundamental Theorem of Galois Theory). *Let E/F be a finite Galois extension with Galois group $G = \text{Gal}(E/F)$. Then:*

1. There is a bijection between the set of intermediate fields $F \subseteq K \subseteq E$ and the set of subgroups $H \leq G$, given by $K \mapsto \text{Gal}(E/K)$ and $H \mapsto E^H$.
2. For any intermediate field K , we have

$$[E : K] = |\text{Gal}(E/K)|, \quad [K : F] = [G : \text{Gal}(E/K)].$$

3. The extension K/F is normal if and only if $\text{Gal}(E/K)$ is a normal subgroup of G . In this case,

$$\text{Gal}(K/F) \cong G / \text{Gal}(E/K).$$

Proof. The proof follows from the properties of fixed fields and the fact that for a Galois extension, the Galois group acts faithfully. The bijection is anti-inclusion: larger subgroups fix smaller fields. The degree relations come from Lagrange's theorem in the group. Normality of the subgroup corresponds to normality of the extension via the quotient group isomorphism. $\square$

Let L/F be a finite field extension. For $\theta \in L$, the *norm* and *trace* of θ over F are defined as

$$N_{L/F}(\theta) = \prod_{\sigma \in \text{Gal}(L/F)} \sigma(\theta), \quad T_{L/F}(\theta) = \sum_{\sigma \in \text{Gal}(L/F)} \sigma(\theta),$$

when L/F is Galois. A Galois extension L/F is called *cyclic* if its Galois group $\text{Gal}(L/F)$ is a cyclic group.

Theorem 2.2. *Let F be a field containing a primitive n -th root of unity. Assume $\text{char}(F) = p$ with $(n, p) = 1$. Then the following are equivalent:*

1. L/F is a cyclic extension of degree $d \mid n$.
2. $L = F(\theta)$ where θ satisfies $\theta^d = a \in F$.
3. L is the splitting field of $x^d - a$ over F , for some $a \in F$.

Proof. The equivalence is from Kummer theory: the cyclic extension is generated by a d -th root, and the splitting field of the Kummer equation $x^d - a = 0$ has Galois group cyclic of order d when the base field contains the d -th roots of unity. $\square$

Let $n \in \mathbb{Z}_{>0}$. The n -th *cyclotomic extension* of F is the splitting field of $x^n - 1$ over F , denoted $F(\zeta_n)$, where ζ_n is a primitive n -th root of unity. Now, consider an irreducible and separable polynomial $f(x) \in \mathbb{F}[x]$ of degree n , factoring in its splitting field E_f as

$$f(x) = (x - \alpha_1) \cdots (x - \alpha_n),$$

where $\alpha_1, \dots, \alpha_n$ are distinct roots. Since E_f is both normal (as it is a splitting field) and separable over $\mathbb{F}$, it is a Galois extension. The Galois group of the polynomial, $\text{Gal}_{\mathbb{F}}(f) = \text{Gal}(E_f/\mathbb{F})$, consists of automorphisms permuting the roots $\alpha_1, \dots, \alpha_n$. For any distinct roots α_i and α_j , there exists $\sigma \in \text{Gal}_{\mathbb{F}}(f)$ such that $\sigma(\alpha_i) = \alpha_j$, implying that $\text{Gal}_{\mathbb{F}}(f)$ acts transitively on the roots. Thus, $\text{Gal}_{\mathbb{F}}(f)$ embeds naturally into the symmetric group S_n , the group of all permutations on $\{1, 2, \dots, n\}$, with this embedding defined uniquely up to conjugacy due to the arbitrary ordering of the roots.

2.3 Resolvents

To aid in computing $\text{Gal}(f)$, we introduce the *resolvent polynomial* for a degree n polynomial and then apply the results for $n = 7$.

In practice, the roots of $f(x)$ are typically not known explicitly, while the database computations developed later in the thesis are performed entirely in terms of polynomial coefficients. Consequently, the central computational problem is to convert symmetric expressions in the roots into explicit polynomial expressions in the coefficients of $f(x)$.

Consider a generic irreducible polynomial

$$f(x) = \prod_{i=1}^n (x - x_i) = x^n + a_{n-1}x^{n-1} + \cdots + a_0 \in \mathbb{Q}[x], \quad (2.1)$$

where the roots $x_1, \dots, x_n$ are regarded as variables. The symmetric group S_n acts on $\mathbb{Q}[x_1, \dots, x_n]$ by permuting the variables:

$$\begin{aligned} S_n \times \mathbb{Q}[x_1, \dots, x_n] &\rightarrow \mathbb{Q}[x_1, \dots, x_n], \\ (\tau, F(x_1, \dots, x_n)) &\mapsto F(\tau(x_1), \dots, \tau(x_n)) =: F^\tau. \end{aligned}$$

Let $G \subseteq S_n$, and let $F(x_1, \dots, x_n) \in \mathbb{Q}[x_1, \dots, x_n]$. Define the stabilizer of F in G by

$$H = \{\tau \in G \mid F^\tau = F\}.$$

Definition 2.3. The resolvent polynomial associated to $f(x)$, G , and F is defined by

$$R_F(f)(x) = \prod_{\sigma \in G/H} (x - F^\sigma(x_1, \dots, x_n)).$$

The product is taken over coset representatives of G/H , and the degree of the resolvent is

$$k = [G : H] = |G|/|H|.$$

The quantities

$$\theta_\sigma = F(x_{\sigma(1)}, \dots, x_{\sigma(n)})$$

form the orbit of $F(x_1, \dots, x_n)$ under the action of G . The factorization pattern of $R_F(f)$ over $\mathbb{Q}$ reflects the action of the Galois group on these orbit elements.

For example:

1. If $F = x_1$ and $G = S_n$, then $H = S_{n-1}$, $k = n$, and

$$R_F(f) = f(x).$$

2. If F is fully symmetric, then $H = G$, $k = 1$, and the resolvent is linear.

Theorem 2.4. *The factorization of $R_F(f)$ over $\mathbb{Q}$ into irreducible factors has degrees equal to the orbit lengths under the left action of $\text{Gal}(f)$ on the right cosets G/H . The number of irreducible factors equals the number of double cosets*

$$\text{Gal}(f) \backslash G/H.$$

Corollary 2.5. *If $R_F(f)$ has a linear factor over $\mathbb{Q}$, then $\text{Gal}(f)$ is conjugate in S_n to a subgroup of H .*

For proofs and further details, see [25].

This principle is used later in the thesis to construct resolvents capable of distinguishing transitive subgroups of S_7 .

2.3.1 Computing Resolvents from Polynomial Coefficients

The preceding definition expresses the resolvent in terms of the roots $x_1, \dots, x_n$. However, for explicit computations and database construction, one must rewrite the coefficients of $R_F(f)$ entirely in terms of the coefficients

$$a_0, \dots, a_{n-1}$$

of the polynomial $f(x)$.

Write

$$R_F(f)(x) = x^k - e_1 x^{k-1} + e_2 x^{k-2} - \dots + (-1)^k e_k,$$

where $e_1, \dots, e_k$ are the elementary symmetric polynomials in the orbit values

$$\theta_\sigma = F(x_{\sigma(1)}, \dots, x_{\sigma(n)}).$$

Since the roots x_i are not explicitly known, we instead work with the elementary symmetric functions

$$s_1 = x_1 + \cdots + x_n = -a_{n-1},$$

$$s_2 = \sum_{i < j} x_i x_j = a_{n-2},$$

$$\vdots$$

$$s_n = x_1 \cdots x_n = (-1)^n a_0,$$

obtained from Vieta's formulas.

Define the power sums

$$p_m = \sum_{\sigma \in G/H} \theta_{\sigma}^m = \sum_{\sigma} \left(F(x_{\sigma(1)}, \dots, x_{\sigma(n)}) \right)^m.$$

In all applications considered in this thesis, we take $G = S_n$. Consequently, the quantities p_m are symmetric polynomials in the roots $x_1, \dots, x_n$. By the fundamental theorem of symmetric polynomials, they may therefore be written uniquely in terms of

$$s_1, \dots, s_n,$$

and hence in terms of the coefficients of $f(x)$.

Relate the power sums p_m to the coefficients e_j using Newton's identities:

$$e_1 = p_1,$$

$$e_2 = \frac{1}{2}(e_1 p_1 - p_2),$$

$$e_3 = \frac{1}{3}(e_2 p_1 - e_1 p_2 + p_3),$$

$$\vdots$$

$$e_j = \frac{1}{j} \left(\sum_{i=1}^{j-1} (-1)^{i-1} e_{j-i} p_i + (-1)^{j-1} p_j \right).$$

Solving recursively determines

$$e_1, \dots, e_k,$$

and hence the resolvent polynomial

$$R_F(f)(x) = x^k - e_1 x^{k-1} + e_2 x^{k-2} - \dots + (-1)^k e_k.$$

This procedure converts the root-theoretic definition of the resolvent into an explicit symbolic computation in the coefficients of $f(x)$. Consequently, the coefficients of $R_F(f)$ become polynomial expressions in

$$a_0, \dots, a_{n-1}.$$

For low-degree resolvents these computations may be carried out symbolically by hand. However, for septic polynomials the resulting expressions become extremely large, particularly for high-degree resolvents such as the 35-ic, 30-ic, and 120-ic resolvents used later in the thesis. In practice, symbolic computation systems such as SageMath or Magma are therefore required.

2.3.2 Numerical Computation

For high-degree resolvents or complex permutation representations, numerical methods are often more practical [9]. This approach approximates the roots of $f(x)$, computes the orbit values θ_σ , and reconstructs the corresponding resolvent polynomial numerically.

Compute the roots $x_1, \dots, x_n \in \mathbb{C}$ of $f(x)$ to high precision (e.g., 50–100 decimal places) using a root-finding algorithm such as Newton–Raphson or Laguerre’s method [9]. Once approximate roots are obtained, apply additional refinement steps to recover precision lost during deflation or polynomial division.

For ill-conditioned polynomials, increased working precision or suitable changes of variables may be required.

For each $\sigma \in G/H$, compute

$$\theta_\sigma = F(x_{\sigma(1)}, \dots, x_{\sigma(n)}).$$

If the values θ_σ are not distinct, then the resolvent polynomial will fail to be squarefree. This phenomenon may occur for special choices of $f(x)$ or F , and is detected by the squarefree test described below.

Form the resolvent

$$R_F(f)(x) = \prod_{\sigma \in G/H} (x - \theta_\sigma) = x^k - e_1 x^{k-1} + \dots + (-1)^k e_k.$$

The coefficients e_j may then be computed numerically from the power sums

$$p_m = \sum_{\sigma} \theta_\sigma^m$$

by applying Newton's identities from the previous subsection.

Round the resulting coefficients to nearby integers or rational numbers. To ensure correctness, sufficient precision must be used so that each coefficient lies unambiguously within the rounding tolerance.

Compute

$$V = \gcd(R_F(f), R_F(f)')$$

to determine whether the resolvent is squarefree. If $V \neq 1$, apply a Tschirnhausen transformation [9]: choose a polynomial $A \in \mathbb{Z}[x]$ of degree less than n , replace f by

$$\text{Res}_y(f(y), x - A(y)),$$

and restart the numerical computation with the transformed polynomial. Since

$$\mathbb{Q}(\theta) = \mathbb{Q}(A(\theta))$$

for generic A , the transformed polynomial defines the same number field while typically producing a squarefree resolvent.

Once a squarefree resolvent has been obtained, factor $R_F(f)$ over $\mathbb{Z}[x]$. The degrees of the irreducible factors determine the orbit structure of $\text{Gal}(f)$ acting on G/H , thereby identifying the Galois group among the candidate transitive subgroups of S_n .

The resolvent construction therefore provides a bridge between permutation-theoretic information about the roots of a polynomial and explicit symbolic computations in its coefficients. This coefficient-based framework is fundamental for the large-scale identification of Galois groups within the septic polynomial databases developed later in the thesis.

2.4 Constructive Galois Theory

Naive enumeration of irreducible septic polynomials by bounded coefficients produces datasets overwhelmingly dominated by the generic Galois group S_7 . Solvable and exceptional transitive subgroups occur with extremely low density and may therefore be statistically invisible in random sampling. Consequently, constructive methods are required to generate polynomial families with prescribed Galois groups. The geometric framework of branched covers, monodromy, Nielsen classes, and Hurwitz spaces provides a systematic method for producing such examples and is therefore essential for constructing balanced databases used in the later statistical and machine-learning investigations.

We investigate the realization of finite groups as Galois groups of extensions of $\mathbb{Q}$, employing geometric and arithmetic techniques involving branched coverings of the projective line and Hurwitz spaces. This section builds on foundational

results from MR2363329 [20] and Völklein [29], with particular emphasis on the inverse Galois problem and the construction of explicit septic families.

2.4.1 Coverings of $\mathbb{P}^1$ and Monodromy

Let K be a field of characteristic 0. A finite *covering* of the projective line is a finite morphism

$$\phi : X \longrightarrow \mathbb{P}_K^1$$

from a smooth projective curve X over K . Equivalently, ϕ induces a finite extension of function fields

$$K(\mathbb{P}^1) = K(t) \hookrightarrow K(X),$$

and it is often convenient to study the cover through this extension [1].

We say that ϕ is a *Galois cover* if the induced extension $K(X)/K(t)$ is Galois. In this case, the group of deck transformations identifies with the Galois group of function fields:

$$\text{Deck}(\phi) \cong \text{Gal}(K(X)/K(t)) =: G,$$

and the quotient recovers the base curve:

$$X/G \cong \mathbb{P}_K^1.$$

Thus finite Galois covers of $\mathbb{P}_K^1$ correspond to finite Galois extensions of $K(t)$ [1, 20, 16].

Let

$$S = \{p_1, \dots, p_r\} \subset \mathbb{P}^1(\overline{K})$$

denote the branch locus of ϕ . For $t \in \mathbb{P}^1(\overline{K}) \setminus S$, the fiber $\phi^{-1}(t)$ consists of $n = \deg(\phi)$ points, and the action of G on the fiber induces a faithful permutation representation $G \hookrightarrow S_n$, well-defined up to conjugacy in S_n [1, 29].

For each branch point $p_i \in S$ and each point $x \in X(\overline{K})$ satisfying $\phi(x) = p_i$, define the inertia group

$$G_x = \{\sigma \in G : \sigma(x) = x\}.$$

Since the characteristic is zero, each inertia group is cyclic, and its order equals the ramification index at x . Choosing a generator determines an element $g_i \in G$, well-defined up to conjugacy, describing the local monodromy around p_i [1, 29].

Thus a Galois cover $\phi : X \rightarrow \mathbb{P}_K^1$ with group G may be viewed equivalently as:

- (i) a smooth projective curve X equipped with a faithful G -action satisfying $X/G \simeq \mathbb{P}_K^1$, together with specified ramification data, or
- (ii) a finite Galois extension $K(X)/K(t)$ with Galois group G .

[1, 16]

The branch-cycle description provides a combinatorial encoding of the cover and forms the basis for the construction of Hurwitz spaces and explicit Galois realizations.

Let $\phi : X \rightarrow \mathbb{P}^1$ be a Galois cover with group G and branch locus

$S = \{p_1, \dots, p_r\}$. Choose a base point $t_0 \in \mathbb{P}^1(\overline{K}) \setminus S$ together with a geometric generating system of loops around the branch points. The resulting branch-cycle description determines a tuple $(g_1, \dots, g_r) \in G^r$, where each g_i is a local monodromy element generating the corresponding inertia group. Different choices of generators for $\pi_1(\mathbb{P}^1 \setminus S, t_0)$ produce braid-equivalent tuples, while relabeling the fiber corresponds to simultaneous conjugation in G [29, Ch. 2, §2.2].

These elements satisfy the relations

$$g_1 \cdots g_r = 1, \quad \langle g_1, \dots, g_r \rangle = G. \tag{2.2}$$

The first relation reflects the defining relation in $\pi_1(\mathbb{P}^1 \setminus S, t_0)$, while the second expresses connectedness of the cover, equivalently transitivity of the monodromy action [29, Ch. 2, §2.2].

Fix conjugacy classes $\mathcal{C} = (C_1, \dots, C_r)$ in G describing the ramification type. Following Völklein, the associated *Nielsen class* is

$$\text{Ni}(G, \mathcal{C}) = \left\{ (g_1, \dots, g_r) \in C_1 \times \dots \times C_r \mid g_1 \cdots g_r = 1, \langle g_1, \dots, g_r \rangle = G \right\} / \text{Inn}(G), \quad (2.3)$$

where $\text{Inn}(G)$ acts by simultaneous conjugation:

$$(g_1, \dots, g_r) \mapsto (hg_1h^{-1}, \dots, hgrh^{-1}), \quad h \in G.$$

[29, Ch. 2, §2.2]

The braid group $B_r = \langle \sigma_1, \dots, \sigma_{r-1} \rangle$ acts on branch-cycle descriptions by Hurwitz moves. The action is given by

$$\sigma_i \cdot (g_1, \dots, g_r) = (g_1, \dots, g_{i-1}, g_i g_{i+1} g_i^{-1}, g_i, g_{i+2}, \dots, g_r). \quad (2.4)$$

This action preserves both the product-one condition and the generated subgroup, and therefore induces an action on $\text{Ni}(G, \mathcal{C})$. Geometrically, the Hurwitz move corresponds to braiding the branch points p_i and p_{i+1} inside $\mathbb{P}^1 \setminus S$ [20, Ch. 5, §5.1].

2.4.2 Hurwitz Spaces and Braid Action

The geometric data encoded by Nielsen classes naturally leads to Hurwitz spaces.

The Hurwitz space $\mathcal{H}(G, \mathcal{C})$ is the moduli space parametrizing isomorphism classes of degree- n Galois covers $\phi : X \rightarrow \mathbb{P}^1$ with Galois group G and ramification type $\mathcal{C}$. Völklein constructs $\mathcal{H}(G, \mathcal{C})$ as a complex algebraic variety whose connected components correspond to braid orbits in $\text{Ni}(G, \mathcal{C})$ [29, Ch. 3, §3.1].

If the cover has $r \geq 3$ branch points, then

$$\dim \mathcal{H}(G, \mathcal{C}) = r - 3,$$

since PGL_2 acts simply transitively on ordered triples of distinct points of $\mathbb{P}^1$, allowing three branch points to be normalized.

A fundamental result states that $\mathcal{H}(G, \mathcal{C})$ is irreducible whenever the braid group B_r acts transitively on $\mathrm{Ni}(G, \mathcal{C})$ [29, Thm. 3.2].

In particular, braid transitivity implies that all covers with the given ramification data lie in a single connected family. This plays an important role in constructing parametrized families of covers suitable for specialization and database generation. For specific groups and ramification types, braid transitivity must be verified case by case.

MR2363329 further shows that Hurwitz spaces admit models defined over $\mathbb{Q}$, allowing arithmetic questions concerning rational points and Galois realizations to be studied geometrically [20, Ch. 6, §6.2].

2.4.3 Realizing Covers over $\mathbb{Q}$

The geometric theory of Hurwitz spaces connects naturally to the inverse Galois problem through rational points and specialization.

The Riemann Existence Theorem provides the fundamental link between branch-cycle data and covers over $\mathbb{C}(t)$. As stated by MR2363329 [20, Ch. 4, §4.2], for any finite group G , ramification type $\mathcal{C}$, and tuple $(g_1, \dots, g_r) \in \mathrm{Ni}(G, \mathcal{C})$, there exists a Galois cover $\phi : X \rightarrow \mathbb{P}^1$ over $\mathbb{C}$ with branch points $\{p_1, \dots, p_r\} \subset \mathbb{P}^1(\mathbb{C})$, monodromy tuple $(g_1, \dots, g_r)$, and Galois group G .

The associated extension $\mathbb{C}(X)/\mathbb{C}(t)$ is Galois with group G , and the cover is determined up to isomorphism and braid equivalence by its monodromy data.

If $g_1, \dots, g_{r-1}$ generate G , then setting $g_r = (g_1 \cdots g_{r-1})^{-1}$ produces a generating tuple satisfying the product-one condition. Consequently every finite group occurs as the Galois group of a branched cover of $\mathbb{P}_{\mathbb{C}}^1$, equivalently over $\mathbb{C}(t)$.

To obtain realizations over $\mathbb{Q}(t)$, one seeks rational points on the corresponding Hurwitz space $\mathcal{H}(G, \mathcal{C})$. A rational point defined over $\mathbb{Q}$ corresponds to a Galois cover over $\mathbb{Q}(t)$ with group G and ramification type $\mathcal{C}$. Such a cover is represented by an irreducible polynomial $f(x, t) \in \mathbb{Q}(t)[x]$ satisfying $\text{Gal}(\mathbb{Q}(X)/\mathbb{Q}(t)) = G$.

Völklein identifies the existence of such rational points as a central problem in inverse Galois theory over $\mathbb{Q}(t)$ [29, Ch. 4, §4.1].

Arithmetic obstructions arise because the absolute Galois group $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acts on the connected components of $\mathcal{H}(G, \mathcal{C})$, equivalently on the braid orbits in $\text{Ni}(G, \mathcal{C})$. This action arises from the Galois action on the branch points and on the roots of unity used to identify local monodromy generators with elements of G . A connected component not fixed under this action need not descend to a component defined over $\mathbb{Q}$, and therefore may fail to contain rational points. MR2363329 analyzes these descent obstructions and their relation to rationality questions for Hurwitz spaces [20, Ch. 6, §6.3].

Once a rational point on $\mathcal{H}(G, \mathcal{C})$ has been obtained, Hilbert's Irreducibility Theorem provides the transition from covers over $\mathbb{Q}(t)$ to number fields.

Let $f(x, t) \in \mathbb{Q}[t][x]$ be irreducible over $\mathbb{Q}(t)$ with Galois group G . Then, for all but a thin set of specializations $t_0 \in \mathbb{Q}$, the specialized polynomial $f(x, t_0) \in \mathbb{Q}[x]$ remains irreducible and satisfies $\text{Gal}(f(x, t_0)/\mathbb{Q}) = G$ [20, Ch. 3, §3.1].

Thus rational points on Hurwitz spaces produce infinite families of number fields with prescribed Galois group.

In later chapters, explicit septic constructions realizing groups such as D_7 , $C_7 \rtimes C_3$, and $\mathrm{PSL}_3(2)$ are used to supplement bounded-height searches and produce more balanced datasets containing both generic and sparse transitive subgroups of S_7 .

The methods developed in this chapter serve two complementary purposes throughout the thesis. Resolvent constructions provide explicit computational tools for identifying transitive Galois groups of septic polynomials, while constructive realization methods generate polynomial families with prescribed monodromy. Together these techniques enable the construction of large-scale databases containing both generic and rare Galois groups, forming the algebraic foundation for the statistical, machine-learning, and coding-theoretic applications developed in later chapters.

CHAPTER THREE

Algebraic Curves

This chapter develops the algebraic and geometric framework underlying the explicit constructions used throughout the thesis. Particular emphasis is placed on superelliptic curves and their weighted generalizations, since these form one of the few large classes of higher genus curves for which equations, automorphism groups, ramification structures, and Riemann–Roch spaces remain computationally accessible.

Superelliptic curves provide a natural bridge between polynomial arithmetic, Galois theory, and algebraic geometry. Their defining equations $y^m = f(x)$ encode cyclic coverings of the projective line, permit explicit descriptions of automorphism groups and differential forms, and admit effective computational methods through the associated binary forms and weighted projective models. These features make superelliptic curves especially suitable for explicit arithmetic constructions developed throughout the thesis.

We begin with a concise account of finite fields and algebraic curves over finite fields. We then study rational points, zeta functions, and asymptotic bounds before turning to the geometry of superelliptic curves: ramification, genus formulas, pole semigroups, automorphisms, and point counting. The final sections extend these constructions to generalized superelliptic curves and weighted projective geometry.

Throughout, $\mathbb{F}_q$ denotes the finite field of cardinality $q = p^r$ (p prime, $r \geq 1$), and all curves are smooth, projective, and geometrically irreducible unless stated otherwise.

3.1 Finite Fields

A finite field $\mathbb{F}_q$, also written $\mathbb{F}(q)$, is a field containing finitely many elements. Its order is necessarily a prime power $q = p^r$, where $p = \text{char}(\mathbb{F}_q)$ and $r \geq 1$.

For every prime power $q = p^r$, there exists, up to isomorphism, a unique field $\mathbb{F}_q$. The multiplicative group $\mathbb{F}_q^\times$ is cyclic of order $q - 1$.

In characteristic p , the Frobenius automorphism

$$\Phi : \mathbb{F}_q \rightarrow \mathbb{F}_q, \quad x \mapsto x^p$$

generates the cyclic Galois group

$$\text{Gal}(\mathbb{F}_q/\mathbb{F}_p).$$

The iterates satisfy

$$\Phi^k(x) = x^{p^k}, \quad 0 \leq k \leq r - 1,$$

and Φ^r is the identity.

Frobenius automorphisms govern much of the arithmetic of algebraic curves over finite fields and later appear in point counting, zeta functions, and trace constructions.

For an extension $\mathbb{F}_{q^m}/\mathbb{F}_q$, the trace map

$$\text{Tr}_{\mathbb{F}_{q^m}/\mathbb{F}_q}(\alpha) = \sum_{i=0}^{m-1} \alpha^{q^i}$$

is a surjective $\mathbb{F}_q$ -linear map.

3.2 Algebraic Curves over Finite Fields

An algebraic curve over $\mathbb{F}_q$ is a smooth, projective, geometrically irreducible variety of dimension one defined over $\mathbb{F}_q$.

Its function field $\mathbb{F}_q(\mathcal{X})$ consists of rational functions on $\mathcal{X}$.

The genus $g = g(\mathcal{X})$ measures the geometric complexity of the curve. The cases

$$g = 0, \quad g = 1, \quad g \geq 2$$

correspond respectively to rational, elliptic, and higher genus curves.

The set of rational points

$$\mathcal{X}(\mathbb{F}_q) = \{P \in \mathcal{X} : P \text{ defined over } \mathbb{F}_q\}$$

encodes the arithmetic of the curve.

3.2.1 Divisors and the Riemann–Roch theorem

A divisor on $\mathcal{X}$ is a formal finite sum

$$D = \sum_{P \in \mathcal{X}} n_P P, \quad n_P \in \mathbb{Z}.$$

The degree of D is

$$\deg(D) = \sum n_P \deg(P).$$

The Riemann–Roch space associated to D is

$$\mathcal{L}(D) = \{f \in \mathbb{F}_q(\mathcal{X})^\times : \div(f) + D \geq 0\} \cup \{0\},$$

with dimension

$$\ell(D) = \dim_{\mathbb{F}_q} \mathcal{L}(D).$$

Theorem 3.1 (Riemann–Roch). *For any divisor D and canonical divisor $K_{\mathcal{X}}$,*

$$\ell(D) - \ell(K_{\mathcal{X}} - D) = \deg(D) - g + 1.$$

The Riemann–Roch theorem links geometric invariants of the curve to dimensions of spaces of rational functions. In explicit constructions, a central

computational problem is to determine concrete monomial bases for the spaces $\mathcal{L}(D)$.

3.3 Rational Points on Curves over Finite Fields

One of the central arithmetic problems concerning algebraic curves over finite fields is the determination of the number of rational points

$$N_q(\mathcal{X}) = \#\mathcal{X}(\mathbb{F}_q).$$

The distribution of rational points reflects deep interactions between the geometry of the curve, the Frobenius endomorphism, and the arithmetic of the underlying field.

3.3.1 The Hasse–Weil theorem

The Hasse–Weil theorem bounds the deviation of $N_q(\mathcal{X})$ from the value $q + 1$ expected for the projective line.

Theorem 3.2 (Hasse–Weil). *Let $\mathcal{X}/\mathbb{F}_q$ be a smooth projective curve of genus g . Then*

$$|N_q(\mathcal{X}) - (q + 1)| \leq 2g\sqrt{q}.$$

Curves attaining the upper bound are called maximal, while those attaining the lower bound are called minimal.

3.3.2 Zeta functions and Frobenius

The arithmetic of rational points over finite extensions of $\mathbb{F}_q$ is encoded in the zeta function

$$Z(\mathcal{X}, T) = \exp \left(\sum_{n \geq 1} \#\mathcal{X}(\mathbb{F}_{q^n}) \frac{T^n}{n} \right).$$

A theorem of Weil states that

$$Z(\mathcal{X}, T) = \frac{P_{\mathcal{X}}(T)}{(1-T)(1-qT)},$$

where $P_{\mathcal{X}}(T) \in \mathbb{Z}[T]$ has degree $2g$.

Writing

$$P_{\mathcal{X}}(T) = \prod_{i=1}^{2g} (1 - \alpha_i T),$$

the roots satisfy $|\alpha_i| = \sqrt{q}$.

Consequently,

$$\#\mathcal{X}(\mathbb{F}_{q^n}) = q^n + 1 - \sum_{i=1}^{2g} \alpha_i^n.$$

The Hasse–Weil theorem follows immediately when $n = 1$.

Thus the zeta function connects:

- (i) the geometry of the curve,
- (ii) the Frobenius endomorphism,
- (iii) point counts over finite extensions,
- (iv) the arithmetic of the Jacobian.

3.3.3 Asymptotic towers of curves

Rather than studying individual curves, one may consider sequences

$$\{\mathcal{X}_i/\mathbb{F}_q\}_{i \geq 1}$$

with genera tending to infinity.

The asymptotic quantity

$$A(q) = \limsup_{g(\mathcal{X}) \rightarrow \infty} \frac{\#\mathcal{X}(\mathbb{F}_q)}{g(\mathcal{X})}$$

measures the maximal possible ratio of rational points to genus.

Theorem 3.3 (Drinfeld–Vladut). *One has*

$$A(q) \leq \sqrt{q} - 1.$$

Garcia and Stichtenoth constructed explicit towers over $\mathbb{F}_{q^2}$ attaining this bound.

The existence of such towers illustrates the importance of explicit families of curves admitting computable equations, ramification structures, and point counts.

3.4 Superelliptic Curves

Superelliptic curves occupy a distinguished position in explicit algebraic geometry. Outside special families such as hyperelliptic, superelliptic, and cyclic n -gonal curves, very little is known effectively about equations, automorphism groups, moduli invariants, or explicit Riemann–Roch bases of higher genus curves.

Superelliptic curves remain computationally tractable because their cyclic covering structure imposes strong constraints on the geometry while preserving rich arithmetic and automorphism structures.

The defining equation $y^m = f(x)$ simultaneously determines:

- (i) a cyclic cover of $\mathbb{P}^1$,
- (ii) a Kummer extension of function fields,
- (iii) a binary form associated to $f(x)$,
- (iv) explicit automorphisms,
- (v) concrete Riemann–Roch spaces and differential forms.

The computational theory of superelliptic curves has been developed extensively in the work of Shaska and collaborators, particularly in connection with automorphism groups, invariants of binary forms, generalized superelliptic curves, and explicit moduli computations.

Fix $m \geq 2$, $\text{char}(\mathbb{F}_q) \nmid m$, and let

$$f(x) \in \mathbb{F}_q[x]$$

be separable of degree n .

Consider the affine curve

$$y^m = f(x). \tag{3.1}$$

Let

$$\mathcal{X}/\mathbb{F}_q$$

denote its smooth projective model.

3.4.1 Kummer extensions and cyclic coverings

The coordinate projection induces a degree- m morphism

$$x : \mathcal{X} \rightarrow \mathbb{P}^1.$$

Because

$$\text{char}(\mathbb{F}_q) \nmid m,$$

the extension

$$\mathbb{F}_q(x) \hookrightarrow \mathbb{F}_q(\mathcal{X})$$

is a Kummer extension.

Over

$$\overline{\mathbb{F}}_q,$$

the curve admits a cyclic automorphism

$$\sigma(x, y) = (x, \omega_m y),$$

where

$$\omega_m$$

is a primitive m th root of unity.

Consequently,

$$\mathbb{P}^1 \cong (\mathcal{X} \otimes \overline{\mathbb{F}}_q) / \langle \sigma \rangle.$$

Set

$$\vartheta = \gcd(m, n).$$

3.4.2 Ramification and genus

Each simple root of

$$f(x)$$

determines a totally ramified point above the branch locus.

Above infinity there are

$$\vartheta$$

points, each having ramification index

$$m/\vartheta.$$

Theorem 3.4 (Genus formula). *Let*

$$\mathcal{X} : y^m = f(x)$$

with

$$\deg(f) = n$$

and

$$\vartheta = \gcd(m, n).$$

Then

$$g(\mathcal{X}) = \frac{(m-1)(n-1) - (\vartheta-1)}{2}.$$

When

$$\gcd(m, n) = 1,$$

the point at infinity is unique and totally ramified.

3.4.3 Pole semigroups and monomial bases

Assume $\gcd(m, n) = 1$, and let

$$P_\infty$$

denote the point at infinity.

Then

$$\text{ord}_{P_\infty}(x) = -m, \quad \text{ord}_{P_\infty}(y) = -n.$$

Consequently,

$$\text{ord}_{P_\infty}(x^i y^j) = -(im + jn).$$

The pole orders therefore form the numerical semigroup

$$H(P_\infty) = \langle m, n \rangle.$$

This semigroup governs admissible monomials in Riemann–Roch spaces and encodes the geometry at infinity combinatorially.

When

$$\gcd(m, n) > 1,$$

the fibre above infinity splits into several points, and weighted projective methods provide a natural framework for describing the resulting geometry.

3.4.4 Holomorphic differentials

For integers

$$i \geq 0, \quad 1 \leq j \leq m-1,$$

define

$$\omega_{i,j} = \frac{x^i dx}{y^j}.$$

Proposition 3.5. *Assume*

$$\gcd(m, n) = 1.$$

Then the differentials

$$\omega_{i,j} = \frac{x^i dx}{y^j}$$

form a basis of

$$H^0(\mathcal{X}, \Omega_{\mathcal{X}}^1)$$

for all pairs satisfying

$$0 \leq i \leq n-2 - \left\lfloor \frac{jn}{m} \right\rfloor.$$

The cyclic automorphism acts diagonally:

$$\sigma^*(\omega_{i,j}) = \omega_m^{-j} \omega_{i,j}.$$

Thus the space of holomorphic differentials decomposes into eigenspaces indexed by characters of the cyclic automorphism group.

3.4.5 Automorphisms and binary forms

One of the most important features of superelliptic curves is that automorphism computations reduce to transformations of the defining polynomial $f(x)$.

Homogenizing

$$f(x)$$

produces the binary form

$$F(X, Z) = Z^n f(X/Z).$$

Automorphisms of the curve induce projective transformations preserving the branch locus of

$$F(X, Z).$$

Consequently, automorphism groups of superelliptic curves are closely related to invariants and covariants of binary forms.

This interaction between coverings, automorphisms, and binary forms plays a central role in explicit moduli computations and constructive approaches to algebraic curves.

3.4.6 Point counting on superelliptic curves

Superelliptic curves admit explicit point-counting formulas because their equations reduce arithmetic questions to polynomial value distributions over finite fields.

Let

$$\mathcal{X} : y^m = f(x).$$

Then

$$N_q(\mathcal{X}) = 1 + \sum_{a \in \mathbb{F}_q} \#\{y \in \mathbb{F}_q : y^m = f(a)\}.$$

If

$$\chi : \mathbb{F}_q^\times \rightarrow \mathbb{C}^\times$$

is a multiplicative character of order dividing m , then character sums yield formulas of the form

$$N_q(\mathcal{X}) = q + 1 + \sum_{\chi^m=1} \sum_{a \in \mathbb{F}_q} \chi(f(a)).$$

Thus the arithmetic of the curve is governed by character sums associated to the polynomial $f(x)$.

This explicit relation between polynomial arithmetic and rational points is one of the principal reasons that superelliptic curves remain computationally tractable.

3.4.7 Example: Hermitian curves

Over

$$\mathbb{F}_{q^2},$$

the Hermitian curve

$$\mathcal{H} : y^{q+1} = x^q + x$$

is superelliptic with genus

$$g = \frac{q(q-1)}{2}.$$

Moreover,

$$\#\mathcal{H}(\mathbb{F}_{q^2}) = q^3 + 1,$$

so the curve is maximal.

The pole semigroup at infinity is generated by

$$q \quad \text{and} \quad q + 1.$$

The Hermitian curve simultaneously exhibits:

- (i) maximality,
- (ii) large automorphism groups,
- (iii) explicit semigroup structure,
- (iv) computable Riemann–Roch bases.

It is therefore one of the most important explicit examples in arithmetic geometry.

3.5 Generalized Superelliptic Curves

Weighted and orbifold geometry naturally lead to singular affine models. This motivates the broader class of generalized superelliptic curves introduced by Shaska and collaborators.

A generalized superelliptic curve is a cyclic n -gonal curve whose quotient by a central automorphism of order n has genus zero.

Unlike the classical case, the affine model may possess singularities, and weighted projective methods become necessary to study the geometry uniformly.

Generalized superelliptic curves therefore form a natural bridge between:

- (i) cyclic coverings,
- (ii) automorphism groups,
- (iii) weighted projective geometry,

(iv) orbifold Riemann–Roch theory.

They provide an intermediate framework between ordinary superelliptic curves and the weighted constructions developed in the next chapter.

3.5.1 Transition to weighted geometry

Classical projective models often force unnecessarily large genus for fixed polynomial degree. In many situations, especially when singularities or nontrivial ramification structures occur at infinity, weighted projective embeddings provide a more natural geometric framework.

Generalized superelliptic curves frequently admit simpler weighted models than ordinary projective embeddings, and weighted geometry permits singular and orbifold structures to be treated systematically. In particular, weighted projective methods extend the explicit computational framework of superelliptic curves while preserving much of their arithmetic and automorphism structure.

The next chapter develops the geometry of weighted projective spaces and weighted algebraic curves in detail, with particular emphasis on weighted Riemann–Roch theory, weighted hypersurfaces, and explicit weighted models arising from superelliptic constructions.

CHAPTER FOUR

Weighted Projective curves

4.1 Weighted Projective Spaces

The theory of weighted projective spaces as a systematic framework for algebraic geometry codes was developed in [23], to which we refer for full details. We recall here the definitions and structural results required for the code constructions of Section 5.4.

In this section we define weighted projective spaces and recall their basic properties, following [13, 15]. Fix a tuple of positive integers $\mathbf{w} = (w_0, \dots, w_n)$, called the *weights*, and let k be a field.

4.1.1 Definitions and algebraic structure

Definition 4.1 ([23, Def. 5.1]). The *weighted projective space* $\mathbb{P}(\mathbf{w})$ over k is the set of equivalence classes of points in $\mathbb{A}^{n+1} \setminus \{0\}$ under the weighted action of the multiplicative group $k^\times$. Two points $(x_0, \dots, x_n)$ and $(x'_0, \dots, x'_n)$ are equivalent if there exists $\lambda \in k^\times$ such that

$$(x'_0, \dots, x'_n) = (\lambda^{w_0} x_0, \dots, \lambda^{w_n} x_n).$$

We denote the equivalence class of a point by $[x_0 : \dots : x_n]_w$.

Algebraically, $\mathbb{P}(\mathbf{w})$ is realised as the projective scheme $\text{Proj } S$, where $S = k[x_0, \dots, x_n]$ is the graded polynomial ring with $\deg x_i = w_i$. A polynomial $F \in S$ is *weighted homogeneous of degree d* if every monomial $x_0^{a_0} \cdots x_n^{a_n}$ in its support satisfies $\sum_{i=0}^n a_i w_i = d$. Unlike standard projective space, $\mathbb{P}(\mathbf{w})$ generally possesses singularities; however, these can be controlled by imposing a well-formedness condition on the weights.

Definition 4.2 ([23, Def. 5.2]). The space $\mathbb{P}(\mathbf{w})$ is *well-formed* if, for every $i \in \{0, \dots, n\}$,

$$\gcd(w_0, \dots, \widehat{w_i}, \dots, w_n) = 1,$$

where the hat denotes omission of the i th weight.

If $\mathbb{P}(\mathbf{w})$ is not well-formed, it is isomorphic to a well-formed weighted projective space obtained by dividing the weights by their common divisors [13]. Throughout we assume $\mathbb{P}(\mathbf{w})$ is well-formed; this guarantees that the singular locus has codimension at least 2.

4.1.2 Singularities and stratification

General weighted projective spaces are normal, irreducible projective varieties with cyclic quotient singularities. The structure of these singularities is governed by the greatest common divisors of subsets of weights.

Definition 4.3 (Weighted GCD [23, Def. 5.3]). For a set of weights $\mathbf{w}$ and a subset of indices $I \subseteq \{0, \dots, n\}$, let

$$k_I = \gcd(\{w_i : i \in I\}).$$

These integers k_I determine the stratification of $\mathbb{P}(\mathbf{w})$. Specifically, a point $P = [x_0 : \dots : x_n]_w$ is singular if and only if the index set $I = \{i : x_i \neq 0\}$ satisfies $k_I > 1$. The singular locus is the union of the linear subspaces defined by the coordinate axes where these GCD conditions fail.

Example 4.4 ([23, Ex. 5.4]). Consider $\mathbb{P}(1, 1, 2)$ with coordinates $[x : y : z]_w$. The weights are well-formed.

1. If $z \neq 0$, one may scale to $[x : y : 1]_w$, which is a smooth point.

2. The point $P = [0 : 0 : 1]_w$ corresponds to the index set $I = \{2\}$ with $k_I = \gcd(2) = 2$.

Thus $\mathbb{P}(1, 1, 2)$ has a single singular point at $[0 : 0 : 1]_w$, which is a quotient singularity of type $\frac{1}{2}(1, 1)$. This space is isomorphic to the cone over a smooth conic in $\mathbb{P}^3$.

4.1.3 Arithmetic properties

In the context of coding theory and Diophantine geometry, one requires a notion of complexity for points in $\mathbb{P}(\mathbf{w})$.

Definition 4.5 (Weighted height [23, Def. 5.5]). Let k be a global field. The *weighted height* of a point $P = [x_0 : \cdots : x_n]_w \in \mathbb{P}(\mathbf{w})(\bar{k})$ is defined as

$$H_w(P) = \prod_{v \in M_k} \max_i \{|x_i|_v^{1/w_i}\},$$

where M_k is the set of places of k and coordinates are chosen in k . For finite fields or specific applications this may be adapted to a logarithmic height or restricted to selected valuations.

The weighted height admits Northcott-type finiteness theorems, which are essential for algorithms that enumerate rational points or compute code lengths via GCD-based sums, as implemented for instance in the QWAG framework of [23].

4.2 Weighted Projective Curves

Let $\mathbf{w} = (w_0, w_1, w_2)$ be a weight vector with $w_i \geq 1$. Set $S = \mathbb{F}_q[x_0, x_1, x_2]$ graded by $\deg(x_i) = w_i$ and define $\mathbb{P}(\mathbf{w}) = \text{Proj}(S)$. A polynomial $F \in S$ is *weighted homogeneous of degree d* if every monomial $x_0^{a_0} x_1^{a_1} x_2^{a_2}$ in its support satisfies $a_0 w_0 + a_1 w_1 + a_2 w_2 = d$. A *weighted plane curve* is $X = V(F) \subset \mathbb{P}(\mathbf{w})$; see [13, 15, 23].

Definition 4.6. $\mathbb{P}(w_0, w_1, w_2)$ is *well-formed* if $\gcd(w_i, w_j) = 1$ for all $i \neq j$.

Definition 4.7. $X = V(F) \subset \mathbb{P}(\mathbf{w})$ is *quasi-smooth* if the only common zero of F , $\partial F/\partial x_0$, $\partial F/\partial x_1$, $\partial F/\partial x_2$ in $\mathbb{A}^3$ is the origin.

Quasi-smoothness guarantees that X is a V -manifold (orbifold): its only singularities are inherited from the ambient stack structure of $\mathbb{P}(\mathbf{w})$, so cohomological methods behave predictably [23, Ch. 5].

For $I \subseteq \{0, 1, 2\}$ the *class* $\mathbb{P}(\mathbf{w})_I = \{[x_0 : x_1 : x_2] : x_i \neq 0 \Leftrightarrow i \in I\}$ has stabiliser order $r_I = \gcd\{w_i : i \in I\}$. A point P is singular if and only if $r_P := \gcd\{w_i : x_i(P) \neq 0\} > 1$, giving the stratification $\mathbb{P}(\mathbf{w}) = WP(\mathbf{w}) \sqcup \text{Sing}(\mathbb{P}(\mathbf{w}))$.

Example 4.8. In $\mathbb{P}(1, 1, 2)$ the only singular point is $[0 : 0 : 1]$ with stabiliser μ_2 , making it a quotient singularity of type $\frac{1}{2}(1, 1)$.

Assume $X \subset \mathbb{P}(\mathbf{w})$ is reduced, well-formed, and quasi-smooth of degree d . Weighted adjunction [13, 15] gives

$$\omega_X \cong \mathcal{O}_X(d - w_0 - w_1 - w_2). \quad (4.1)$$

The genus is computed from $\deg K_X = 2g - 2$ by combining the virtual weighted contribution with local correction terms at singular points; see [13, 8].

Theorem 4.9 (Virtual genus formula [23, Thm. 5.7]). *Let X be a quasi-smooth curve of degree d in a well-formed weighted projective space $\mathbb{P}(w_0, w_1, w_2)$. If X avoids the singular points of $\mathbb{P}(\mathbf{w})$, its genus is*

$$2g - 2 = \frac{d(d - (w_0 + w_1 + w_2))}{w_0 w_1 w_2}. \quad (4.2)$$

Example 4.10. A quasi-smooth curve of degree $d = 6$ in $\mathbb{P}(1, 2, 3)$ that avoids the ambient singularities has

$$2g - 2 = \frac{6(6 - 6)}{6} = 0,$$

hence $g = 1$: it is an elliptic curve.

Example 4.11 (Fermat septic in $\mathbb{P}(1, 1, 1)$). The curve

$\mathcal{X} : x_0^7 + x_1^7 + x_2^7 = 0 \subset \mathbb{P}(1, 1, 1)$ over $\mathbb{F}_q$ with $\text{char}(\mathbb{F}_q) \neq 7$ is quasi-smooth: the partial derivatives $7x_i^6$ have no common zero in $\mathbb{A}^3 \setminus \{0\}$. Formula (4.2) gives

$$2g - 2 = \frac{7(7-3)}{1} = 28, \quad g = 15.$$

Example 4.12 (Rational curve in $\mathbb{P}(1, 1, 7)$). In $\mathbb{P}(1, 1, 7)$ with weights $\mathbf{w} = (1, 1, 7)$, the only singular point is $[0 : 0 : 1]$ with isotropy μ_7 . The quasi-smooth curve

$$\mathcal{X} : x_0^7 + x_1^7 + x_2 = 0 \subset \mathbb{P}(1, 1, 7)$$

has degree $d = 7$. Since $\partial F / \partial x_2 \equiv 1$, the Jacobian never vanishes, so $\mathcal{X}$ is quasi-smooth. Furthermore $F(0, 0, 1) = 1 \neq 0$, so $\mathcal{X}$ avoids the singular locus.

Formula (4.2) gives

$$2g - 2 = \frac{7(7-9)}{7} = -2, \quad g = 0.$$

Indeed $\mathcal{X} \cong \mathbb{P}^1$: the map $[s : t] \mapsto [s : t : -(s^7 + t^7)]$ is an isomorphism.

Comparing Examples Thm. 4.11 and Thm. 4.12 illustrates *genus compression*: the same polynomial degree $d = 7$ yields $g = 15$ in ordinary projective space but $g = 0$ in $\mathbb{P}(1, 1, 7)$.

Remark 4.13. Since 7 is prime, every w_i dividing 7 belongs to $\{1, 7\}$. Among well-formed spaces $\mathbb{P}(w_0, w_1, w_2)$, the virtual genus formula (4.2) yields an integer genus and the curve generically avoids all ambient singularities *only* in the two cases above: $\mathbb{P}(1, 1, 1)$ with $g = 15$ and $\mathbb{P}(1, 1, 7)$ with $g = 0$. Any intermediate genus at degree 7 requires the curve to pass through an orbifold point, necessitating the correction terms of the explicit genus formula [23, Thm. 5.8].

Table 4.1: Genera of generic quasi-smooth curves in $\mathbb{P}(\mathbf{w})$

Weights $\mathbf{w}$	Degree d	$2g - 2$	Genus g
(1, 1, 1)	3	0	1
(1, 1, 1)	4	4	3
(1, 1, 2)	4	0	1
(1, 2, 3)	6	0	1
(1, 1, 1)	7	28	15
(1, 1, 7)	7	-2	0
(2, 3, 5)	30	20	11

4.3 Weighted Divisors and the Orbifold Riemann-Roch Theorem

Let $X \subset \mathbb{P}(w_0, w_1, w_2)$ be a quasi-smooth curve. For a geometric point $P = [x_0 : x_1 : x_2] \in X$, the *isotropy group* (or stabiliser) is

$$\Gamma_P = \{\lambda \in \mathbb{F}_q^\times : (\lambda^{w_0}x_0, \lambda^{w_1}x_1, \lambda^{w_2}x_2) = (x_0, x_1, x_2)\},$$

of order $r_P = \gcd\{w_i : x_i \neq 0\}$. If $r_P = 1$ then P is a smooth point; if $r_P > 1$ then P is an *orbifold point*.

Definition 4.14 (Weighted degree [23, §5.3]). For a Weil divisor $D = \sum_P m_P P$ on $X \subset \mathbb{P}(\mathbf{w})$, the *weighted degree* is

$$\deg_w(D) := \sum_P \frac{m_P}{r_P}. \quad (4.3)$$

A smooth point contributes 1 to the degree, while an orbifold point of order r_P contributes only $1/r_P$, consistent with intersection theory on the stack.

Define the *fractional-part correction*

$$\epsilon(D) := - \sum_{P \in \text{supp}(D)} \left\{ \frac{m_P}{r_P} \right\}, \quad (4.4)$$

where $\{x\} = x - \lfloor x \rfloor$ is the fractional part.

Remark 4.15. The term $\epsilon(D)$ is non-positive and vanishes whenever every m_P is a multiple of r_P . In that case the orbifold Riemann–Roch theorem reduces to its classical counterpart.

Theorem 4.16 (Weighted Riemann–Roch [23, Thm. 5.18]). *Let g be the genus of the normalisation $\tilde{X}$ and let K be a canonical divisor. For any divisor D on $X \subset \mathbb{P}(\mathbf{w})$,*

$$\ell(D) - \ell(K - D) = \deg_w(D) + 1 - g + \epsilon(D).$$

If $\deg(\lfloor D \rfloor) > 2g - 2$, then

$$\ell(D) = \deg_w(D) - g + 1 + \epsilon(D). \quad (4.5)$$

Proof. The weighted Riemann–Roch space $\mathcal{L}_w(D)$ is naturally isomorphic to the classical Riemann–Roch space $\mathcal{L}(\lfloor D \rfloor)$ on the coarse model of X . The classical degree of $\lfloor D \rfloor$ equals $\deg_w(D) + \epsilon(D)$. Applying the classical Riemann–Roch theorem to $\lfloor D \rfloor$ and substituting yields (4.5); see [23, Thm. 5.18]. $\square$

Example 4.17. Let X have genus $g = 1$ and let P be a stacky point with $r_P = 2$. For $D = 3P$, $\deg_w(D) = 3/2$ and $\epsilon(D) = -1/2$, so $\ell(D) = 3/2 - 1 + 1 - 1/2 = 1$. The integer outcome reflects the fact that poles of rational functions at a weight-2 point must have integer orders in the coarse model.

Proposition 4.18 (Monomial basis [23, Thm. 5.35]). *Let $\mathcal{X} : y^m = f(x)$ be embedded in $\mathbb{P}(m, n, 1)$ with $\gcd(m, n) = 1$ and $\deg f = n$. For $G = \delta P_\infty$,*

$$\mathcal{B} = \{x^a y^b : 0 \leq b < m, am + bn \leq \delta\}$$

is a basis of $\mathcal{L}_w(G)$. Each monomial $x^a y^b \in \mathcal{B}$ has pole order exactly $am + bn$ at P_∞ , and these orders are precisely the non-gaps of the Weierstrass semigroup $\langle m, n \rangle$ bounded by δ .

Proof. Since $\gcd(m, n) = 1$, every integer $s \in \langle m, n \rangle$ admits a unique representation $s = am + bn$ with $0 \leq b < m$. The monomials $x^a y^b$ have distinct valuations at P_∞ and are linearly independent; completeness follows from the dimension count given by the orbifold Riemann–Roch theorem. See [23, Thm. 5.35]. $\square$

4.4 Rational Points on Weighted Curves

A key arithmetic invariant for weighted AG codes is the number of $\mathbb{F}_q$ -rational points $|X(\mathbb{F}_q)|$, which determines the block length n of evaluation codes. In the weighted setting, point counting requires care because the weighted scaling action is non-uniform.

Definition 4.19 (Rational point [23, §5.1]). A point $[x_0 : x_1 : x_2] \in \mathbb{P}(\mathbf{w})$ is $\mathbb{F}_q$ -rational if it admits a representative with coordinates in $\mathbb{F}_q$.

Theorem 4.20 (Burnside formula [23, Thm. 5.22]).

$$|\mathbb{P}(\mathbf{w})(\mathbb{F}_q)| = \frac{1}{q-1} \sum_{\lambda \in \mathbb{F}_q^\times} (q^{N(\lambda)} - 1),$$

where $N(\lambda) = |\{i : \lambda^{w_i} = 1\}|$.

An equivalent combinatorial form is

$$|\mathbb{P}(\mathbf{w})(\mathbb{F}_q)| = \sum_{\emptyset \neq S \subseteq \{0,1,2\}} (q-1)^{|S|-1} \gcd(k_S, q-1),$$

with $k_S = \gcd\{w_i : i \in S\}$ [23, Lem. 2.4].

For the smooth locus one restricts to classes with $k_S = 1$:

$$|WP(\mathbf{w})(\mathbb{F}_q)| = \sum_{\substack{\emptyset \neq S \subseteq \{0,1,2\} \\ k_S=1}} (q-1)^{|S|-1} \gcd(k_S, q-1). \quad (4.6)$$

For evaluation codes, points in $\text{Sing}(\mathbb{P}(\mathbf{w}))$ and $\text{Sing}(X)$ are excluded; the divisor D is supported on the smooth locus.

4.4.1 Weil-type bound

For a quasi-smooth hypersurface $X \subset \mathbb{P}(\mathbf{w})$ of degree d over $\mathbb{F}_q$, a Weil-type bound incorporating orbifold corrections [23, Thm. 5.28] gives

$$\left| |X(\mathbb{F}_q)| - q - \cdots \right| \leq (d-1)\sqrt{q} + O(1),$$

with explicit correction terms determined by the local isotropy data of $\mathbb{P}(\mathbf{w})$. An Aubry-Perret pre-filter [23, Thm. 5.26] is used computationally to bound $|X(\mathbb{F}_q)|$ from above before the full enumeration.

CHAPTER FIVE

Coding Theory

This chapter develops the coding-theoretic framework underlying the weighted algebraic–geometric and quantum code constructions studied later in the thesis. Beginning with classical linear codes over finite fields, we develop the duality theory, stabiliser formalism, and algebraic–geometric code constructions required for the weighted and quantum graded AG-code families arising from superelliptic and weighted projective curves.

Section 5.1 recalls the fundamental parameters and duality theory of linear codes over finite fields. Section 5.2 constructs functional and differential AG-codes from the curve-theoretic machinery developed in Chapter 3 and establishes the Riemann–Roch parameter bounds underlying AG-code duality. Section 5.3 introduces the n -qudit Pauli group, the stabiliser formalism, and the Calderbank–Shor–Steane (CSS) construction, which integrates AG-codes into the CSS framework to produce quantum codes from algebraic curves. Section 5.4 extends the evaluation-code construction to weighted projective curves using weighted divisor theory and the weighted orbifold Riemann–Roch theorem. Finally, Section 5.5 develops the framework of quantum weighted algebraic–geometric codes together with self-orthogonality criteria and quantum Singleton-type bounds.

5.1 Linear Codes

Linear error-correcting codes are naturally modeled as finite-dimensional vector spaces over finite fields. Let $\mathbb{F}_q$ denote the finite field with $q = p^r$ elements, where p is prime.

5.1.1 Linear codes and basic parameters

Definition 5.1. A *linear code* of length n over $\mathbb{F}_q$ is a vector subspace

$$C \subseteq \mathbb{F}_q^n.$$

Its *dimension* is $k = \dim_{\mathbb{F}_q}(C)$, and its *minimum Hamming distance* is

$$d(C) = \min\{\text{wt}(\mathbf{c}) : \mathbf{c} \in C \setminus \{0\}\},$$

where $\text{wt}(\mathbf{c}) = |\{i : c_i \neq 0\}|$ denotes the Hamming weight of $\mathbf{c}$.

A linear code with parameters $[n, k, d]_q$ detects up to $d - 1$ errors and corrects up to $t = \lfloor (d - 1)/2 \rfloor$ errors.

A linear code may be described either by generators or by linear constraints.

A *generator matrix* of an $[n, k, d]_q$ code C is a matrix

$$G \in \mathbb{F}_q^{k \times n}$$

whose rows form an $\mathbb{F}_q$ -basis of C . Every codeword has the form $\mathbf{c} = uG$, where $u \in \mathbb{F}_q^k$.

The encoding process is represented by the linear map

$$E : \mathbb{F}_q^k \rightarrow \mathbb{F}_q^n, \quad E(u) = uG.$$

When G is in systematic form $G = [I_k \mid A]$, the first k coordinates of a codeword coincide with the information symbols, while the remaining coordinates encode parity information.

5.1.2 Duality and parity-check matrices

The orthogonal complement of a linear code plays a central role in decoding theory, weight enumeration, and quantum stabiliser constructions.

Definition 5.2. Let $C \subseteq \mathbb{F}_q^n$ be a linear code. The *dual code* of C is

$$C^\perp = \{\mathbf{y} \in \mathbb{F}_q^n : \langle \mathbf{x}, \mathbf{y} \rangle_E = 0 \text{ for all } \mathbf{x} \in C\},$$

where

$$\langle \mathbf{x}, \mathbf{y} \rangle_E = \sum_{i=1}^n x_i y_i$$

denotes the Euclidean inner product on $\mathbb{F}_q^n$.

One has $\dim C + \dim C^\perp = n$ and $(C^\perp)^\perp = C$.

A code is called:

- *self-orthogonal* if $C \subseteq C^\perp$,
- *self-dual* if $C = C^\perp$.

Self-orthogonality is the fundamental condition underlying CSS-type quantum stabiliser constructions.

A *parity-check matrix* of C is a matrix

$$H \in \mathbb{F}_q^{(n-k) \times n}$$

whose rows form a basis of $C^\perp$. Equivalently,

$$C = \ker(H) = \{\mathbf{x} \in \mathbb{F}_q^n : H\mathbf{x}^\top = 0\}.$$

5.1.3 Syndromes and error correction

The parity-check matrix provides a linear mechanism for detecting and correcting transmission errors.

Given a received vector $r \in \mathbb{F}_q^n$, its *syndrome* is defined by $s = Hr^\top$.

The syndrome measures the failure of the received vector to satisfy the defining linear constraints of the code.

If $r = c + e$, where $c \in C$ is the transmitted codeword and e is the error vector, then

$$Hr^\top = H(c + e)^\top = He^\top,$$

since $Hc^\top = 0$.

Thus the syndrome depends only on the error vector and determines the coset of r modulo C . Error correction therefore reduces to identifying the most likely error vector compatible with the observed syndrome.

5.1.4 Weight distributions and MacWilliams identities

The *weight enumerator* of a code C is the polynomial

$$W_C(x, y) = \sum_{c \in C} x^{n-\text{wt}(c)} y^{\text{wt}(c)} = \sum_{i=0}^n A_i x^{n-i} y^i,$$

where A_i denotes the number of codewords of weight i .

The weight enumerator encodes the distance distribution of the code. The MacWilliams identities show that the weight distributions of a code and its dual are related by an explicit linear transformation:

$$W_{C^\perp}(x, y) = \frac{1}{|C|} W_C(x + (q-1)y, x - y). \quad (5.1)$$

These identities play an important role in algebraic coding theory and association schemes.

5.1.5 Fundamental bounds

The parameters n , k , and d of a linear code satisfy several fundamental inequalities.

Proposition 5.3 (Singleton bound [23, Prop. 2.3]). *Every $[n, k, d]_q$ code satisfies*

$$d \leq n - k + 1.$$

Codes achieving equality are called maximum distance separable (MDS) codes.

Proposition 5.4 (Hamming bound [23, Prop. 2.4]). *An $[n, k, d]_q$ code correcting $t = \lfloor (d-1)/2 \rfloor$ errors satisfies*

$$q^k \sum_{i=0}^t \binom{n}{i} (q-1)^i \leq q^n.$$

Codes meeting this bound are called perfect codes.

Proposition 5.5 (Gilbert–Varshamov bound [23, Prop. 2.5]). *For every q, n, k , there exists an $[n, k, d]_q$ code satisfying*

$$q^k \sum_{i=0}^{d-2} \binom{n}{i} (q-1)^i \leq q^n.$$

Together these bounds illustrate the fundamental tradeoff between transmission rate and error-correcting capability.

5.1.6 Classical families of codes

The following examples illustrate several fundamental constructions in coding theory.

Example 5.6 (Repetition codes). The $[n, 1, n]_q$ repetition code consists of all constant vectors

$$(a, a, \dots, a), \quad a \in \mathbb{F}_q.$$

It detects up to $n-1$ errors and corrects up to $\lfloor (n-1)/2 \rfloor$ errors. Although highly redundant, repetition codes provide the simplest example of error-correcting redundancy.

Example 5.7 (Hamming codes). Binary Hamming codes form a family of perfect codes. For every integer $m \geq 2$, there exists a code with parameters $[2^m - 1, 2^m - 1 - m, 3]_2$.

The parity-check matrix consists of all nonzero binary column vectors of length m . The case $m = 3$ yields the classical $[7, 4, 3]_2$ Hamming code.

Example 5.8 (Cyclic and BCH codes). A linear code $C \subseteq \mathbb{F}_q^n$ is called *cyclic* if it is invariant under cyclic shifts of coordinates. Cyclic codes correspond to ideals in the quotient ring

$$\mathbb{F}_q[x]/(x^n - 1),$$

and are generated by a polynomial $g(x) \mid (x^n - 1)$.

Bose–Chaudhuri–Hocquenghem (BCH) codes form an important subclass of cyclic codes obtained by prescribing consecutive roots of $g(x)$ in an extension field. A primitive narrow-sense BCH code of length $n = q^m - 1$ with designed distance δ has minimum distance at least δ and admits efficient algebraic decoding.

Under suitable conditions on the defining parameters, BCH codes are self-orthogonal and therefore suitable for CSS-type quantum constructions.

Example 5.9 (Reed–Solomon codes). Reed–Solomon codes are MDS codes with parameters $[q - 1, k, q - k + 1]_q$.

The code $\text{RS}_k(q - 1, q)$ is constructed by evaluating all polynomials $f(x) \in \mathbb{F}_q[x]$ satisfying $\deg(f) < k$ at pairwise distinct nonzero field elements $\alpha_1, \dots, \alpha_{q-1} \in \mathbb{F}_q$.

Thus $C = \{(f(\alpha_1), \dots, f(\alpha_{q-1})) : \deg(f) < k\}$.

Reed–Solomon codes achieve equality in the Singleton bound and may be viewed as the simplest algebraic–geometric codes, arising from evaluation on the projective line.

5.2 Algebraic Geometry Codes

5.2.1 Curves, divisors, and Riemann–Roch spaces

We briefly recall the geometric framework underlying algebraic–geometric codes; see Chapter 3 and [26] for further details.

Let $\mathcal{X}/\mathbb{F}_q$ be a smooth, projective, geometrically irreducible curve of genus g . For a divisor $E = \sum_{P \in \mathcal{X}} n_P P$, the associated Riemann–Roch space is

$$L(E) = \{f \in \mathbb{F}_q(\mathcal{X})^\times : \operatorname{div}(f) + E \geq 0\} \cup \{0\}.$$

Its dimension $\ell(E) = \dim_{\mathbb{F}_q} L(E)$ is governed by the Riemann–Roch theorem.

Theorem 5.10 (Riemann–Roch). *For any divisor E and canonical divisor $K_{\mathcal{X}}$,*

$$\ell(E) - \ell(K_{\mathcal{X}} - E) = \deg(E) - g + 1.$$

In particular, if $\deg(E) > 2g - 2$, then

$$\ell(E) = \deg(E) - g + 1.$$

The number of rational points on $\mathcal{X}$ satisfies the Hasse–Weil bound

$$||\mathcal{X}(\mathbb{F}_q)| - (q + 1)| \leq 2g\sqrt{q}.$$

Curves with many rational points relative to genus produce strong families of algebraic–geometric codes. For asymptotic families of curves over $\mathbb{F}_q$, the Drinfeld–Vladüt bound states that

$$\limsup_{g \rightarrow \infty} \frac{|\mathcal{X}(\mathbb{F}_q)|}{g} \leq \sqrt{q} - 1.$$

5.2.2 Functional AG codes

Let $P_1, \dots, P_n \in \mathcal{X}(\mathbb{F}_q)$ be distinct rational points and set $D = P_1 + \dots + P_n$. Choose a divisor G satisfying $\operatorname{supp}(G) \cap \operatorname{supp}(D) = \emptyset$.

The evaluation map

$$\text{ev}_D : L(G) \rightarrow \mathbb{F}_q^n, \quad f \mapsto (f(P_1), \dots, f(P_n)),$$

defines the *functional algebraic-geometric code*

$$C_L(D, G) := \text{ev}_D(L(G)).$$

Theorem 5.11 ([23, Thm. 2.13]). *If $\deg(G) < n$ and $\text{supp}(G) \cap \text{supp}(D) = \emptyset$, then*

$$\dim C_L(D, G) = \ell(G) - \ell(G - D), \quad d \geq n - \deg(G).$$

If moreover $\deg(G) > 2g - 2$ and $\deg(G - D) < 0$, then

$$\dim C_L(D, G) = \deg(G) - g + 1.$$

The quantity $n - \deg(G)$ is called the *designed distance* or *Goppa bound*.

5.2.3 Differential AG codes and duality

Let $\Omega_{\mathcal{X}}$ denote the space of rational differentials on $\mathcal{X}$. For a divisor E , define

$$\Omega(E) = \{\omega \in \Omega_{\mathcal{X}} : \text{div}(\omega) \geq E\}.$$

The associated differential AG code is

$$C_{\Omega}(D, E) = \left\{ (\text{res}_{P_1}(\omega), \dots, \text{res}_{P_n}(\omega)) : \omega \in \Omega(E) \right\}.$$

Theorem 5.12 (Duality of AG codes [23, Thm. 2.14]).

$$C_L(D, G)^{\perp} = C_{\Omega}(D, K_{\mathcal{X}} - G + D).$$

This duality theorem is one of the foundational results of algebraic-geometric coding theory and provides the key mechanism underlying self-orthogonality criteria for quantum AG-code constructions.

5.2.4 Examples of AG codes

Example 5.13 (Reed–Solomon codes). Reed–Solomon codes arise naturally from the projective line $\mathbb{P}_{\mathbb{F}_q}^1$, which has genus $g = 0$.

Let $P_1, \dots, P_{q-1}$ denote the affine rational points and let P_∞ be the point at infinity. Setting $D = P_1 + \dots + P_{q-1}$ and $G = (k-1)P_\infty$, the Riemann–Roch space $L(G)$ consists precisely of polynomials of degree at most $k-1$.

The resulting code $C_L(D, G)$ is the classical Reed–Solomon code with parameters $[q-1, k, q-k+1]_q$.

Example 5.14 (Hermitian codes [23, Ex. 2.16]). Over $\mathbb{F}_{q^2}$, the Hermitian curve

$$\mathcal{X} : y^q + y = x^{q+1}$$

has genus $g = q(q-1)/2$ and exactly $q^3 + 1$ rational points.

One-point AG codes constructed from the point at infinity yield families of codes with parameters exceeding the Gilbert–Varshamov bound asymptotically. Hermitian curves also possess large automorphism groups, making them especially important in explicit AG-code constructions.

5.2.5 Asymptotic performance

A family of codes is called *asymptotically good* if both the rate $R = k/n$ and the relative distance $\delta = d/n$ remain positive as $n \rightarrow \infty$.

Using towers of curves approaching the Drinfeld–Vladuŭt bound, one obtains AG-code families satisfying

$$R + \delta \geq 1 - \frac{1}{\sqrt{q} - 1}.$$

Theorem 5.15 (Tsfasman–Vladuț–Zink bound [28]). *For q a perfect square, there exist families of AG codes over $\mathbb{F}_q$ satisfying*

$$R \geq 1 - \delta - \frac{1}{\sqrt{q} - 1}.$$

These families exceed the Gilbert–Varshamov bound whenever $q \geq 49$.

5.3 Quantum Stabiliser Codes

This section reviews the stabiliser formalism and the CSS construction following the framework developed in [23, Ch. 4]. Since these results are standard in quantum coding theory, we focus primarily on the structural ideas needed later for weighted algebraic–geometric and quantum weighted AG-code constructions. Additional details and proofs may be found in [23, §§4.1–4.4].

5.3.1 Quantum error model

Quantum information carried by n qudits of dimension $q = p^m$ is encoded in the Hilbert space

$$\mathcal{H} = (\mathbb{C}^q)^{\otimes n},$$

whose computational basis is $\{|x\rangle : x \in \mathbb{F}_q^n\}$.

Errors acting on $\mathcal{H}$ are modeled by the generalized Pauli group. For a single qudit, define the shift and phase operators

$$X(a)|b\rangle = |a + b\rangle, \quad Z(c)|b\rangle = \omega^{\text{Tr}(cb)}|b\rangle,$$

where $\omega = e^{2\pi i/p}$ and $\text{Tr} : \mathbb{F}_q \rightarrow \mathbb{F}_p$ denotes the field trace.

For vectors $a = (a_1, \dots, a_n)$ and $c = (c_1, \dots, c_n) \in \mathbb{F}_q^n$, set

$$X(a) = X(a_1) \otimes \cdots \otimes X(a_n), \quad Z(c) = Z(c_1) \otimes \cdots \otimes Z(c_n).$$

The n -qudit Pauli group is

$$\mathcal{P}_n = \{\omega^j X(a)Z(c) : j \in \mathbb{Z}_p, a, c \in \mathbb{F}_q^n\}.$$

The fundamental commutation relation is

$$Z(c)X(a) = \omega^{\langle c, a \rangle_s} X(a)Z(c), \quad (5.2)$$

where

$$\langle c, a \rangle_s = \text{Tr} \left(\sum_{i=1}^n c_i a_i \right).$$

Thus $X(a)$ and $Z(c)$ commute precisely when $\langle c, a \rangle_s = 0$.

The quotient of $\mathcal{P}_n$ by scalar multiples of the identity may be identified with $\mathbb{F}_q^{2n}$, equipped with the symplectic form

$$\langle (a|c), (a'|c') \rangle_s = \text{Tr}(ac' - a'c),$$

which governs commutation in the stabiliser formalism; see [23, §4.1].

The weight of an error operator $E = \omega^j X(a)Z(c)$ is defined by

$$\text{wt}(E) = |\{i : (a_i, c_i) \neq (0, 0)\}|,$$

measuring the number of qudits on which the operator acts nontrivially.

5.3.2 Stabiliser codes

The stabiliser formalism encodes quantum information as the common eigenspace of a commuting subgroup of the Pauli group.

Definition 5.16 ([23, Def. 4.1]). Let $S \leq \mathcal{P}_n$ be an abelian subgroup satisfying $-I \notin S$. The associated *stabiliser code* is

$$Q(S) = \{|\psi\rangle \in \mathcal{H} : s|\psi\rangle = |\psi\rangle \text{ for all } s \in S\}.$$

The subgroup S is called the *stabiliser* of the code. Each generator of S imposes a linear constraint on the ambient Hilbert space, and the code space consists precisely of the vectors fixed by every element of S .

The following result shows that the stabiliser code is a linear subspace of $\mathcal{H}$.

Lemma 5.17 ([23, Lem. 4.2]). *If $S \leq \mathcal{P}_n$ is abelian and $-I \notin S$, then $Q(S)$ is a linear subspace of $\mathcal{H}$.*

Associated to S is the averaging operator

$$P_S = \frac{1}{|S|} \sum_{s \in S} s,$$

called the *stabiliser projector*.

Proposition 5.18 ([23, Prop. 4.3]). *The operator P_S is the orthogonal projection onto $Q(S)$. In particular,*

$$P_S^2 = P_S, \quad P_S^\dagger = P_S.$$

The stabiliser projector extracts the component of a state lying inside the code space and plays a central role in syndrome measurement and decoding.

The dimension of a stabiliser code is determined by the number of independent stabiliser generators.

Theorem 5.19 (Dimension formula [23, Thm. 4.4]). *If S has r independent generators, then*

$$\dim Q(S) = q^{n-r}.$$

Equivalently, the stabiliser code encodes $k = n - r$ logical qudits.

The distance of a stabiliser code is defined as the minimum weight of a Pauli operator commuting with all elements of S but not itself belonging to S . Such

operators act nontrivially on the encoded information and represent undetectable logical errors.

Further details concerning stabiliser groups, syndrome extraction, and symplectic representations may be found in [23, §4.1].

5.3.3 The CSS construction

The Calderbank–Shor–Steane (CSS) construction provides a bridge between classical coding theory and quantum stabiliser codes. The key idea is to construct commuting X - and Z -type stabilisers from a pair of nested classical linear codes.

Let $C_1, C_2 \subseteq \mathbb{F}_q^n$ be linear codes satisfying

$$C_2^\perp \subseteq C_1. \quad (5.3)$$

The orthogonality condition ensures that the corresponding Pauli operators commute under the symplectic pairing.

Lemma 5.20 ([23, Lem. 4.5]). *If $C_2^\perp \subseteq C_1$, then the subgroup*

$$S = \langle Z(h) : h \in C_1^\perp, X(h') : h' \in C_2^\perp \rangle$$

is abelian.

In practice, the commutation condition is usually verified through parity-check matrices.

Proposition 5.21 (Matrix criterion [23, Prop. 4.6]). *Let H_1 and H_2 be parity-check matrices of C_1 and C_2 , respectively. Then*

$$C_2^\perp \subseteq C_1 \iff H_1 H_2^\top = 0.$$

The resulting stabiliser code is called the *CSS code* associated to the pair (C_1, C_2) .

Theorem 5.22 (CSS construction [23, Thm. 4.7]). *Let $C_1, C_2 \subseteq \mathbb{F}_q^n$ be linear codes satisfying $C_2^\perp \subseteq C_1$. If $\dim C_i = k_i$, then there exists a quantum stabiliser code with parameters*

$$[[n, k_1 + k_2 - n, d]]_q,$$

where

$$d = \min\left\{\text{wt}(C_1 \setminus C_2^\perp), \text{wt}(C_2 \setminus C_1^\perp)\right\}.$$

The theorem shows that the parameters of the quantum code are determined entirely by the algebraic relation between the two classical codes. The inclusion $C_2^\perp \subseteq C_1$ guarantees commutativity of the stabilisers, while the minimum distance is controlled by vectors lying outside the dual subcodes.

This construction is particularly important because it allows geometric and algebraic families of classical codes to produce quantum stabiliser codes systematically.

5.4 Weighted Algebraic Geometry Codes

The theory of weighted algebraic–geometric codes developed in this section follows the framework introduced in [23, Ch. 6], where algebraic–geometric coding theory was extended from smooth projective curves to quasi-smooth curves in weighted projective spaces. In contrast with the classical setting, weighted AG-codes incorporate graded coordinate rings, weighted divisors, orbifold singularities, and weighted Riemann–Roch spaces into the coding-theoretic construction. These weighted methods are especially well suited for superelliptic and low-degree models that admit simpler descriptions in weighted projective geometry than in ordinary projective space.

The principal advantage of the weighted setting is that many curves with large automorphism groups and explicit monomial bases admit natural weighted

embeddings with significantly lower defining degree. The resulting weighted Riemann–Roch spaces therefore produce computationally efficient evaluation codes while preserving the geometric structure needed for duality and quantum stabiliser constructions.

5.4.1 Construction and parameter bounds

Let $\mathcal{X} \subset \mathbb{P}(w_0, w_1, w_2)$ be a well-formed quasi-smooth curve defined over $\mathbb{F}_q$. Fix distinct rational points $P_1, \dots, P_n \in \mathcal{X}(\mathbb{F}_q)$ contained in the smooth locus and set $D = P_1 + \dots + P_n$.

Let G be a weighted divisor satisfying $\text{supp}(G) \cap \text{supp}(D) = \emptyset$. Following [23, §6.1], the associated *weighted algebraic–geometric code* is defined by

$$C_w(\mathcal{X}, D, G) = \{(f(P_1), \dots, f(P_n)) : f \in \mathcal{L}_w(G)\} \subseteq \mathbb{F}_q^n,$$

where $\mathcal{L}_w(G)$ denotes the weighted Riemann–Roch space introduced in Section 4.3.

The construction is formally analogous to the classical AG-code setting, but the dimension and distance are now governed by weighted divisor theory and the orbifold Riemann–Roch theorem.

Theorem 5.23 (Parameters of weighted AG codes [23, Thms. 6.2–6.3]). *If $\deg_w(G) < n$, then the evaluation map*

$$\text{ev}_D : \mathcal{L}_w(G) \rightarrow \mathbb{F}_q^n$$

is injective, and the code $C_w(\mathcal{X}, D, G)$ has parameters $[n, k, d]_q$ satisfying

$$k = \ell_w(G) \geq \deg_w(G) - g + 1 + \epsilon(G), \tag{5.4}$$

$$d \geq n - \deg_w(G). \tag{5.5}$$

If $\deg_w(G) > 2g - 2$, equality holds in (5.4).

Proof. Suppose $f \in \mathcal{L}_w(G)$ vanishes at all evaluation points $P_1, \dots, P_n$. Then the zero divisor of f has degree at least n , while the pole divisor has weighted degree at most $\deg_w(G) < n$, which is impossible. Hence the evaluation map is injective and $k = \ell_w(G)$.

The dimension estimate follows from the weighted Riemann–Roch theorem (Thm. 4.16). The distance bound follows because a nonzero section $f \in \mathcal{L}_w(G)$ can vanish at at most $\deg_w(G)$ evaluation points, since the weighted degree of its zero divisor equals the weighted degree of its pole divisor. $\square$

The correction term $\epsilon(G)$ reflects the orbifold contribution arising from the weighted structure. In particular, weighted singularities alter the classical Riemann–Roch count by introducing fractional contributions determined by the local isotropy data; see [23, §5.4].

Remark 5.24 (Semigroup gap advantage). The weighted setting admits an additional structural advantage absent from the classical theory. When the numerical semigroup $\langle w_0, w_1, w_2 \rangle$ contains gaps, low-degree monomials are missing from the graded coordinate ring. Consequently, low-weight codewords cannot arise from sections of small weighted degree, frequently improving the minimum distance relative to classical AG-codes of comparable dimension. This phenomenon, developed in [23, §6.1.2], is one of the principal coding-theoretic advantages of weighted projective models.

5.4.2 Duality and self-orthogonality

The residue theorem extends naturally to quasi-smooth weighted curves, which are normal projective Gorenstein curves. As a result, the duality theory of classical AG-codes extends to the weighted setting with only minor modifications.

Theorem 5.25 (Weighted AG-code duality [23, Thm. 6.15]).

$$C_w(\mathcal{X}, D, G)^\perp \cong C_w(\mathcal{X}, D, K_{\mathcal{X}} + D - G).$$

This duality theorem provides the geometric mechanism underlying self-orthogonality criteria for weighted AG-codes and forms the foundation for the quantum weighted constructions developed in the next section.

Corollary 5.26 (Euclidean self-orthogonality [23, Cor. 6.16]). *The code*

$C_w(\mathcal{X}, D, G)$ *satisfies*

$$C_w(\mathcal{X}, D, G) \subseteq C_w(\mathcal{X}, D, G)^\perp$$

if and only if

$$2G \leq K_{\mathcal{X}} + D.$$

The condition $2G \leq K_{\mathcal{X}} + D$ is the weighted analogue of the classical self-orthogonality criterion for AG-codes. Geometrically, it expresses the fact that the weighted divisor G lies sufficiently far below the canonical threshold determined by the weighted adjunction structure.

Proposition 5.27 (Hermitian self-orthogonality [23, Prop. 6.17]). *Over $\mathbb{F}_{q^2}$, the code $C_w(\mathcal{X}, D, G)$ satisfies*

$$C_w(\mathcal{X}, D, G) \subseteq C_w(\mathcal{X}, D, G)^{\perp H}$$

whenever

$$(q+1)G \leq K_{\mathcal{X}} + D.$$

Hermitian self-orthogonality is particularly important for quantum stabiliser constructions over quadratic extensions, where Hermitian duality naturally replaces Euclidean duality in the CSS framework.

5.4.3 Examples and explicit constructions

Weighted projective models frequently admit explicit monomial bases and highly symmetric defining equations, making them especially suitable for concrete code constructions.

Example 5.28 (Genus-1 curve in $\mathbb{P}(1, 1, 2)$). Consider the weighted curve

$$\mathcal{X} : z^2 = x^4 + y^4 \subset \mathbb{P}(1, 1, 2)$$

defined over $\mathbb{F}_7$.

Weighted adjunction (Eq. (4.1)) gives $K_{\mathcal{X}} \sim 0$ and genus $g = 1$. Direct enumeration yields $n = 8$ smooth rational points.

Let G denote the divisor corresponding to weighted degree 2. Then

$$\deg_w(G) = 4, \quad k = 4,$$

and

$$2 \deg_w(G) = 8 = \deg_w(D).$$

Hence the self-orthogonality criterion of Thm. 5.26 holds with equality, implying that $C_w(\mathcal{X}, D, G)$ is self-dual.

The resulting weighted AG-code has parameters $[8, 4, 4]_7$, providing an explicit example of a self-dual code arising from weighted projective geometry; see [23, §6.3].

Additional explicit constructions involving superelliptic curves, weighted Hermitian-type models, and orbifold towers are developed in [23, §§6.3–6.5].

5.4.4 Asymptotic behaviour

Weighted AG-codes are birationally equivalent to codes arising from smooth projective models and therefore inherit the asymptotic properties of classical AG-codes, including the Tsfasman–Vladut–Zink bound (Thm. 5.15).

The principal advantage of the weighted setting is computational rather than asymptotic. Weighted projective embeddings frequently admit smaller defining degrees, sparse monomial bases, and explicit graded coordinate structures that make generator matrix construction practical in genera where standard smooth embeddings become computationally infeasible.

Proposition 5.29 ([23, Prop. 6.22]). *Let $\{X_m\}$ be a tower of weighted curves over $\mathbb{F}_q$ with $g_m \rightarrow \infty$, and let*

$$A(q) = \limsup_{g \rightarrow \infty} \frac{N_q(g)}{g}$$

denote the Drinfeld–Vladut constant.

Then there exists a sequence of weighted AG-codes satisfying

$$R + \delta \geq 1 - \frac{1}{A(q)}.$$

Thus weighted AG-codes preserve the asymptotic strength of classical algebraic–geometric codes while simultaneously providing a substantially more flexible computational framework for explicit constructions.

5.5 Quantum Weighted Algebraic–Geometric Codes

The theory developed in this section establishes a quantum coding framework built from weighted algebraic–geometric codes on quasi-smooth curves in weighted projective spaces. The constructions, self-orthogonality criteria, orbifold refinements, and quantum parameter bounds presented here were introduced and systematically developed for the first time in [23, Ch. 7]. In particular, the

integration of weighted adjunction, orbifold Riemann–Roch theory, weighted AG-code duality, and CSS stabiliser constructions into a unified theory of quantum weighted algebraic–geometric (QWAG) codes originates in [23].

Unlike the classical setting, the weighted framework incorporates orbifold corrections, graded divisor structures, and weighted canonical geometry directly into the quantum coding-theoretic construction. These additional geometric features lead to refined parameter bounds and explicit families of quantum stabiliser codes arising naturally from superelliptic and weighted projective models.

5.5.1 Self-orthogonality from weighted adjunction

Let $\mathcal{X} = \{F = 0\} \subset \mathbb{P}(w_0, w_1, w_2)$ be a well-formed quasi-smooth curve of weighted degree d defined over $\mathbb{F}_q$. Write $H = c_1(\mathcal{O}_{\mathbb{P}(\mathbf{w})}(1))|_{\mathcal{X}}$ and set $a = w_0$, $b = w_1$, and $c = w_2$. The degree of the hyperplane class is

$$\deg H = \frac{d}{abc}, \tag{5.6}$$

and weighted adjunction (Eq. (4.1)) gives

$$\deg K_{\mathcal{X}} = (d - a - b - c) \deg H;$$

see [23, §7.1.2].

The key observation is that weighted adjunction converts geometric data into a numerical criterion for self-orthogonality. This provides the bridge between weighted algebraic geometry and quantum stabiliser constructions.

Theorem 5.30 (Self-orthogonality criterion for QWAG codes [23, Thm. 7.1]). *Let $G = tH|_{\mathcal{X}}$ and let $D = \sum_{i=1}^n P_i$ be a sum of distinct smooth rational points. If*

$$2t \deg H \leq (2g - 2) + n, \tag{5.7}$$

then

$$2G \leq K_{\mathcal{X}} + D,$$

and consequently

$$C_w(\mathcal{X}, D, G) \subseteq C_w(\mathcal{X}, D, G)^\perp.$$

Proof. Taking weighted degrees in the inequality $2G \leq K_{\mathcal{X}} + D$ gives

$$2 \deg_w(G) \leq \deg K_{\mathcal{X}} + n = (2g - 2) + n.$$

Since $\deg_w(G) = t \deg H$, condition (5.7) is precisely this inequality.

Self-orthogonality then follows from Thm. 5.26. □

The theorem shows that the self-orthogonality condition is governed entirely by weighted adjunction and divisor degree. In practice, this reduces the construction of quantum stabiliser codes to explicit numerical inequalities determined by the weighted geometry of the curve.

5.5.2 QWAG code parameters

The self-orthogonality criterion above allows weighted AG-codes to be embedded directly into the CSS framework developed in Sections 5.3.3 and 5.5.1.

Theorem 5.31 (QWAG code parameters [23, Cor. 7.5]). *Let $\mathcal{X}$, D , and $G = tH|_{\mathcal{X}}$ satisfy Thm. 5.30. Then the symmetric CSS construction applied to $C = C_w(\mathcal{X}, D, G)$ yields a quantum stabiliser code with parameters*

$$[[n, n - 2k, d_Q]]_q,$$

where

$$k = \ell_w(G)$$

and

$$d_Q \geq \min\{n - \deg_w(G), n - \deg_w(K_{\mathcal{X}} - G)\}.$$

Proof. Self-orthogonality follows from Thm. 5.30. Applying the self-orthogonal CSS construction (Thm. 5.22) gives

$$k_Q = n - 2k,$$

while the distance estimate follows from the Goppa bounds for $C_w(\mathcal{X}, D, G)$ and its dual code

$$C_w(\mathcal{X}, D, G)^\perp \cong C_w(\mathcal{X}, D, K_{\mathcal{X}} + D - G)$$

via weighted AG-code duality (Thm. 5.25). □

The theorem demonstrates that the quantum parameters are controlled simultaneously by:

- the weighted Riemann–Roch dimension,
- the weighted divisor degree,
- and the canonical geometry of the weighted curve.

This interaction between weighted geometry and quantum duality is one of the central structural features of QWAG codes.

Theorem 5.32 (Existence criterion [23, Thm. 7.7]). *Suppose $\deg_w(G) > 2g - 2$.*

Then the associated QWAG code has positive dimension $k_Q > 0$ if and only if

$$2g - 2 < \deg_w(G) \leq g - 1 + \frac{n}{2}. \tag{5.8}$$

Within this range,

$$k_Q = n - 2(\deg_w(G) - g + 1),$$

and

$$d_Q \geq n - \deg_w(G).$$

Proof. Since $\deg_w(G) > 2g - 2$, weighted Riemann–Roch gives

$$k = \deg_w(G) - g + 1.$$

The condition $k_Q = n - 2k > 0$ becomes

$$k < \frac{n}{2},$$

which is equivalent to

$$\deg_w(G) < g - 1 + \frac{n}{2}.$$

On the other hand, the self-orthogonality condition

$$2 \deg_w(G) \leq (2g - 2) + n$$

implies

$$\deg_w(G) \leq g - 1 + \frac{n}{2}.$$

Thus the admissible range is precisely (5.8). □

The interval (5.8) defines the geometric region in which weighted AG-codes simultaneously satisfy:

- weighted Riemann–Roch regularity,
- self-orthogonality,
- and nontrivial quantum dimension.

5.5.3 Explicit QWAG constructions

Weighted projective models frequently admit explicit canonical divisors and monomial bases, making them particularly suitable for concrete quantum code constructions.

Example 5.33 (Hyperelliptic QWAG code in $\mathbb{P}(1, 1, 2)$). Let

$$\mathcal{X} : y^2 = f_6(x, z) \subset \mathbb{P}(1, 1, 2)$$

be defined over $\mathbb{F}_q$, where $g = 4$.

From (5.6),

$$\deg H = \frac{6}{2} = 3.$$

Taking $G = tH|_{\mathcal{X}}$, condition (5.7) becomes

$$6t \leq 6 + n.$$

For $n \geq 12$, one may choose $t = 3$. Then

$$k = 9 - 4 + 1 = 6, \quad d_Q \geq n - 9.$$

At $n = 14$, this yields a quantum stabiliser code with parameters

$$[[14, 2, \geq 5]]_q.$$

Example 5.34 (Genus-3 QWAG code in $\mathbb{P}(1, 1, 4)$). Let

$$\mathcal{X} : y^2 = x^8 + z^8 + x^4 z^4 \subset \mathbb{P}(1, 1, 4)$$

be defined over $\mathbb{F}_{13}$.

Here $d = 8$, $\deg H = 2$, and

$$\deg K_{\mathcal{X}} = (8 - 6) \cdot 2 = 4,$$

so $g = 3$.

The Hasse–Weil bound permits up to 35 rational points; we take $n = 20$.
 Choosing $G = 5H|_{\mathcal{X}}$ gives

$$\deg_w(G) = 10.$$

The self-orthogonality condition becomes

$$20 \leq 4 + 20,$$

which holds.

Weighted Riemann–Roch yields

$$k = 10 - 3 + 1 = 8,$$

and the resulting CSS construction produces a quantum code with parameters

$$\llbracket 20, 4, \geq 6 \rrbracket_{13}.$$

Further examples involving superelliptic towers, weighted Hermitian curves, and orbifold constructions are developed in [23, §§7.3–7.5].

5.5.4 Refined quantum Singleton bounds

Orbifold geometry introduces additional corrections into the dimension theory of weighted Riemann–Roch spaces. Consequently, the classical quantum Singleton bound acquires an additional orbifold defect term.

Definition 5.35 (Orbifold defect). Let Σ denote the set of orbifold points of $\mathcal{X}$ with isotropy groups G_P . The *total orbifold defect* is defined by

$$\epsilon_\Sigma = \sum_{P \in \Sigma} \left(1 - \frac{1}{|G_P|} \right).$$

Theorem 5.36 (Refined orbifold Singleton bound [23, Thm. 7.12]). *Let $\mathcal{X}$ have coarse topological genus g_{top} and total orbifold defect ϵ_Σ .*

Then every QWAG code $[[n, k_Q, d]]_q$ constructed on $\mathcal{X}$ satisfies

$$d \leq \frac{n - k_Q}{2} + 1 - \left(g_{\text{top}} + \frac{\epsilon_\Sigma}{2} \right). \quad (5.9)$$

Proof sketch; see [23, Thm. 7.12]. Orbifold Riemann–Roch gives the effective genus

$$g_{\text{eff}} = g_{\text{top}} + \frac{\epsilon_\Sigma}{2}.$$

Combining this with the weighted Goppa bound

$$d \leq n - \deg_w(G)$$

and the relation

$$\deg_w(G) = k_C + g_{\text{eff}} - 1$$

obtained from weighted Riemann–Roch yields

$$d \leq n - k_C + 1 - g_{\text{eff}}.$$

Substituting $k_C = (n - k_Q)/2$ produces (5.9). □

The orbifold defect term measures the extent to which stacky singularities reduce the effective dimension of both primal and dual weighted Riemann–Roch spaces. In the smooth case, $\epsilon_\Sigma = 0$, and (5.9) reduces to the classical genus-corrected quantum Singleton bound.

The theory developed in this section demonstrates that weighted projective geometry provides a natural and highly structured framework for constructing quantum stabiliser codes. Through weighted adjunction, orbifold Riemann–Roch theory, and weighted AG-code duality, one obtains explicit families of quantum codes whose parameters are governed directly by the geometry of quasi-smooth weighted curves. The entire QWAG framework presented here was introduced and systematically developed for the first time in [23].

CHAPTER SIX

Mathematical Databases

The computations in this thesis begin with polynomial data, but the purpose of this chapter is not simply to describe an enumeration. A list of polynomials becomes useful only after one decides how the objects are represented, how they are identified, how repeated representatives are removed, and how the stored information can be checked. These choices affect the reliability of every later computation. For that reason, the database is treated here as part of the mathematical framework of the thesis, rather than as a separate file in which computed output is stored.

The central point is that the database is indexed by mathematical identity, not by an arbitrary row number. A record is determined first by a canonical coefficient tuple, and the remaining fields are arithmetic, invariant-theoretic, and Galois-theoretic data attached to that tuple. The purpose of this chapter is to explain how these canonical records are constructed, why this affects the choice of storage architecture, and how the resulting files are organized for the applications in the later chapters.

This chapter is organized as follows. We first explain what makes polynomial data different from ordinary application data. We then describe the passage from projective coefficient points to canonical database records. After that we discuss the storage architecture, the structure and validation of the septic records, and the cleaned data views used in the Galois-theoretic, machine-learning, and coding applications.

6.1 Mathematical Structure of Polynomial Data

The construction of the polynomial database can be viewed as an extract–transform–load procedure adapted to exact algebraic data. The extraction stage consists of enumerating projective coefficient points subject to a height bound. The transformation stage replaces each point by its canonical integral representative, applies exact arithmetic checks, and attaches the corresponding invariant and Galois-theoretic data. The loading stage stores the surviving records by their normalized coefficient tuples and exports the cleaned tables used in the later chapters.

6.1.1 Algebraic Data versus Application Data

The data considered in this chapter are exact algebraic objects. This separates the present database from many ordinary application databases, where a row may record an external event, transaction, or user entry. Here a row represents an irreducible polynomial over $\mathbb{Q}$ together with arithmetic data computed from that polynomial.

For a septic polynomial we write

$$f_{\mathbf{a}}(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0, \quad \mathbf{a} = (a_0, \dots, a_7).$$

After normalization, the tuple $\mathbf{a}$ is the identifier of the record. The remaining fields, such as height, discriminant, signature, invariant values, and Galois label, are data attached to this tuple.

Thus an error in the coefficient tuple is a mathematical error, not only a formatting error. It may change the polynomial and therefore all arithmetic data computed from it. Similarly, a repeated normalized tuple means that the same mathematical object has been counted twice.

This is the main difference from many application databases. In an application database, two similar rows may represent two different events. In the present database, two rows with the same normalized coefficient tuple represent the same object. Equality of keys is therefore equality of polynomials, not merely equality of stored text. The database design must preserve this distinction.

6.1.2 Design Requirements for a Polynomial Database

The database is designed around exact polynomial records, so its requirements are different from those of a database whose entries are approximate measurements or external observations.

Each polynomial must first have a canonical representative. Without a fixed normalization convention, the same projective point could occur in several forms, and the database would count different representatives of the same object as different rows. The primitive coefficient tuple is used to avoid this ambiguity.

The stored arithmetic must also remain exact. The coefficients, discriminants, and invariant values are integer or rational quantities. Replacing them by floating point approximations would change the nature of the data. In particular, the discriminant is not used only through its numerical size; its exact arithmetic properties are part of the information attached to the polynomial.

The construction must also prevent repeated canonical tuples. Once a tuple has been normalized, it should occur at most once. This is one reason that a key-value structure is useful during construction: the normalized tuple gives the natural lookup key, while the computed arithmetic data form the value.

The database must also support different views of the same records. A polynomial may be grouped by Galois label, converted into a feature vector, or

reduced modulo a prime for later finite-field computations. These are different uses of the same record, not independent datasets with unrelated identifiers.

Finally, the construction must be reproducible. The normalization convention, height bounds, filtering steps, and cleaning checks must be recorded clearly enough that the database can be checked again. This is what makes the database a mathematical object of the thesis, rather than only a file of computational output.

Table 6.1: Design requirements for the polynomial database.

Requirement	Purpose
Canonical representative	One tuple represents each projective coefficient point.
Exact arithmetic	Arithmetic quantities are stored as exact algebraic data.
Unique key	The normalized tuple prevents duplicate records.
Traceability	Computed fields remain attached to the same $f_{\mathbf{a}}$.
Derived views	Galois, feature, and reduction views come from the same records.
Reproducibility	Conventions and checks are recorded for later verification.

6.1.3 The Degree as a Parameter

Although the main database in this thesis consists of septic polynomials, the first stage of the construction can be carried out for polynomials of any fixed degree. For a fixed degree n , a polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

is represented by the coefficient tuple

$$(a_0, \dots, a_n).$$

The natural parameter space for such coefficient vectors is

$$\mathbb{P}^n(\mathbb{Q}).$$

After fixing a height bound, we enumerate the rational points in this projective space, replace them by primitive integral representatives, and keep only those representatives that define irreducible polynomials of degree n .

In this sense, the database architecture is degree-independent. The key is always the normalized coefficient tuple, and the first checks are always the same: valid coefficient length, nonzero leading coefficient, irreducibility, duplicate control, and exact storage of the computed arithmetic data.

The restriction to degree 7 appears when the records are enriched with arithmetic data. The possible transitive Galois groups, the resolvents used to assign exact labels, and the invariant columns $j_0, \dots, j_4$ are specific to binary septic forms. Thus the basic record construction applies in any fixed degree, while the additional arithmetic data attached to the records depend on the degree.

This distinction is important for future extensions. The same database design can be used in other degrees, but the arithmetic information attached to each record must be adapted to the degree. In this thesis we work in degree 7, because septic polynomials already realize several different transitive Galois groups while still allowing exact computations on a large database.

Thus the database architecture separates the degree-independent part of the construction from the degree-dependent arithmetic enrichment.

6.2 From Projective Coefficient Points to Database Records

We now specialize the preceding discussion to the septic case. A polynomial

$$f(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0$$

determines a projective coefficient point

$$[a_7 : \cdots : a_0] \in \mathbb{P}^7(\mathbb{Q}).$$

This projective point, rather than an arbitrary affine coefficient vector, is the natural starting object for the database. Since multiplying all coefficients by the same nonzero rational number gives the same projective point, one must choose a single representative before the polynomial can be stored as a database record.

For each admissible point we choose a primitive integral representative, together with a fixed sign convention. The resulting tuple is stored in the order

$$(a_0, a_1, \dots, a_n),$$

which is the order used by the computational routines. This is only a storage convention: the corresponding projective point is the same coefficient vector written in homogeneous coordinates. The stored tuple serves as the identifier of the record. The remaining fields, including the height, discriminant, signature, invariant values, and Galois label, are then attached to this fixed representative.

Not every rational point gives a row of the septic database. The polynomial must have degree 7 and must be irreducible over $\mathbb{Q}$. We also discard polynomials with

$$\Delta_f = 0,$$

so that only separable polynomials are retained. These checks are carried out using exact arithmetic in SageMath. Thus the stored rows are not raw output from the

search, but canonical records that have passed the arithmetic conditions needed in the later chapters.

6.2.1 Normalization and Height-Bounded Enumeration

The enumeration is carried out in projective coefficient space. Associated to

$$f(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0$$

is the binary septic form

$$F(X, Y) = \sum_{i=0}^7 a_i X^i Y^{7-i}.$$

Thus the coefficient vector determines a rational point

$$\mathbf{p} = [a_7 : a_6 : \cdots : a_0] \in \mathbb{P}^7(\mathbb{Q}).$$

The database is indexed by this projective coefficient point after choosing a canonical primitive representative.

Given a tuple $(b_0, \dots, b_7) \in \mathbb{Q}^8$, the normalization proceeds as follows:

1. clear denominators to obtain an integral tuple;
2. divide all coordinates by their greatest common divisor;
3. impose a fixed sign convention, here $a_7 > 0$.

The resulting tuple $(a_0, \dots, a_7) \in \mathbb{Z}^8$ satisfies

$$\gcd(a_0, \dots, a_7) = 1.$$

Since two primitive integral representatives of the same projective point differ only by multiplication by ± 1 , the sign convention removes the remaining ambiguity. Thus each admissible projective coefficient point determines exactly one stored tuple.

The naive height of this representative is

$$H(f) = \max_{0 \leq i \leq 7} |a_i|.$$

For a fixed height bound $h \geq 1$, let $\mathcal{P}_7^h$ denote the set of canonical primitive projective points corresponding to irreducible septic polynomials satisfying

$$H(f) \leq h, \quad \Delta_f \neq 0.$$

Proposition 6.1. *For every $h \geq 1$, the set $\mathcal{P}_7^h$ is finite.*

Proof. Every element of $\mathcal{P}_7^h$ has a canonical representative $(a_0, \dots, a_7)$ with

$$|a_i| \leq h \quad 0 \leq i \leq 7.$$

Thus all possible coefficient tuples lie in the finite set

$$[-h, h]^8 \cap \mathbb{Z}^8.$$

The conditions that the polynomial have degree 7, be irreducible, and have nonzero discriminant only remove elements from this finite set. Hence $\mathcal{P}_7^h$ is finite. $\square$

More generally, let $P_n(h)$ denote the number of primitive integer vectors

$$\mathbf{x} = (x_0, \dots, x_n) \in \mathbb{Z}^{n+1},$$

considered up to multiplication by ± 1 , such that

$$\max_i |x_i| = h, \quad \gcd(x_0, \dots, x_n) = 1.$$

Equivalently, $P_n(h)$ is the number of primitive projective points of exact height h in $\mathbb{P}^n$. The number of all integer vectors of exact height h , modulo sign, is

$$\frac{(2h+1)^{n+1} - (2h-1)^{n+1}}{2}.$$

Among these, the nonprimitive vectors are precisely those with a common divisor $d \geq 2$. Since the vector has exact height h , such a divisor must satisfy $d \mid h$, and division by d gives a primitive vector of exact height h/d . Therefore

$$P_n(h) = \frac{(2h+1)^{n+1} - (2h-1)^{n+1}}{2} - \sum_{\substack{d|h \\ d \geq 2}} P_n\left(\frac{h}{d}\right).$$

The first term counts projective points of exact height h , modulo sign, and the subtraction removes tuples whose coordinates have a common divisor.

For $n = 7$, the cumulative counts up to height 10 are shown in Table 6.2. The table records the size of the projective search space before irreducibility, discriminant, and Galois-theoretic checks are imposed. Its rapid growth is one reason that the database must be organized around canonical records rather than around repeated raw coefficient vectors.

Table 6.2: Primitive projective points in $\mathbb{P}^7(\mathbb{Q})$ by height.

Height h	Points with height $\leq h$	Points with exact height h
1	3,280	3,280
2	192,032	188,752
3	2,875,840	2,683,808
4	21,324,768	18,448,928
5	106,977,568	85,652,800
6	404,787,648	297,810,080
7	1,278,364,320	873,576,672
8	3,466,156,768	2,187,792,448
9	8,467,372,480	5,001,215,712
10	18,801,175,808	10,333,803,328

In practice, the bounded-height enumeration was implemented in SageMath by traversing rational points in projective space:

```
P = ProjectiveSpace(degree, QQ)
points = P.rational_points(h_max)
```

The parameter `degree` allows the same routine to be used in degree n . In this thesis we specialize to `degree` = 7. For each normalized point $p = (a_0, \dots, a_n)$, the corresponding polynomial is

$$f_p(x) = \sum_{j=0}^n a_j x^j.$$

The record is kept only when f_p has exact degree n , is irreducible over $\mathbb{Q}$, and satisfies the arithmetic checks above.

6.2.2 The Coefficient Tuple as a Database Key

After normalization, the tuple

$$\mathbf{a} = (a_0, \dots, a_7)$$

is used as the identifier of the record. If

$$f_{\mathbf{a}}(x) = a_7 x^7 + a_6 x^6 + \dots + a_1 x + a_0,$$

then the corresponding database record has the form

$$\mathcal{R}(f_{\mathbf{a}}) = (\mathbf{a}, H(f_{\mathbf{a}}), \Delta_{f_{\mathbf{a}}}, \text{sig}(f_{\mathbf{a}}), J(f_{\mathbf{a}}), \text{Gal}_{\mathbb{Q}}(f_{\mathbf{a}})).$$

Here $J(f_{\mathbf{a}})$ denotes the invariant data stored for the polynomial.

The important point is that the first entry of the record is not an arbitrary row label. It is the canonical representative of a projective coefficient point. Thus two records with the same normalized tuple represent the same polynomial object and should not appear as distinct entries. The remaining fields are computed data attached to this fixed representative.

This is the reason the choice of database structure matters. The storage system must preserve the coefficient tuple as the primary identifier, while still making it possible to search by derived quantities such as discriminant, invariants, or Galois label. The next section explains how this requirement leads to the storage architecture used in the implementation.

6.3 Storage Architecture

6.3.1 Key–Value Storage During Construction

The preceding section identifies the normalized coefficient tuple as the key of a polynomial record. The remaining question is which storage system is appropriate for the computations carried out in this thesis. This choice is not made by ranking database systems abstractly; it depends on the stage of the project and on the operations required at that stage.

The first stage is a construction stage rather than a public query stage. A single SageMath process enumerates projective coefficient points, normalizes each tuple, tests the corresponding polynomial, computes the required arithmetic data, and rejects tuples that have already appeared. Thus the dominant operation during construction is the repeated membership test

$$\mathbf{a} \in \mathcal{D},$$

where $\mathbf{a} = (a_0, \dots, a_7)$ is the normalized coefficient tuple and $\mathcal{D}$ is the collection of records already constructed.

For this construction stage, an in-memory dictionary is the most direct representation:

$$(a_0, \dots, a_7) \mapsto (H(f_{\mathbf{a}}), \Delta_{f_{\mathbf{a}}}, \text{sig}(f_{\mathbf{a}}), J(f_{\mathbf{a}}), \text{Gal}_{\mathbb{Q}}(f_{\mathbf{a}})).$$

The dictionary is used because the computation requires exact key-based lookup and duplicate control in a single-process batch construction. The key is the canonical coefficient tuple, while the value consists of the arithmetic, invariant-theoretic, and Galois-theoretic data attached to the corresponding polynomial. This choice preserves exact arithmetic inside SageMath, avoids premature conversion of integer, rational, and symbolic data into external formats, and makes duplicate detection depend on equality of normalized tuples. Thus the dictionary serves as the internal construction backend.

Once the records are constructed, they are exported to tabular files, which become the working format for cleaning, integrity checks, grouping by Galois label, feature extraction, and the later applications.

6.3.2 Relational, Document, and Cache Models

A relational database such as PostgreSQL would be natural for a public or long-term searchable version of the database. In that setting, one would want fixed schemas, uniqueness constraints, indexed queries, and linked tables. For example, one table could store the polynomial records, while separate tables could store reductions modulo primes, point counts on associated curves, or code parameters. Such a design is more appropriate for a persistent multi-user database than for the initial construction stage.

A document database such as MongoDB would serve a different purpose. It is well suited for nested or nonuniform records. For example, one polynomial may have reductions over many finite fields, another may have several code constructions attached to it, and another may store only rational arithmetic data. In the present construction, however, the central polynomial record has a fixed form, so this flexibility is not needed at the first stage.

Redis has a different role again. It should be understood as a possible cache for expensive intermediate computations, not as the archival database. A cache could avoid repeated evaluation of large invariant expressions or other intermediate quantities. Cached values, however, are auxiliary; they do not replace the verified record indexed by the normalized coefficient tuple.

Thus the backend choice follows the mathematical and computational role of the data. During construction, the normalized tuple is the object that must be looked up exactly, so an in-memory key–value structure is appropriate. For a future public searchable database, PostgreSQL would be more suitable. For highly nonuniform attached data, MongoDB could be useful. For repeated temporary computations, Redis could be added as a cache. The comparison is summarized in Table 6.3.

Table 6.3: Storage choices for the septic polynomial database.

Backend/format	Main role	Appropriate use
Python dictionary	Exact lookup by normalized coefficient tuple.	Construction stage and duplicate control.
Tabular files	Stable representation of verified records.	Cleaning, integrity checks, grouping, and feature extraction.
PostgreSQL	Relational schema, constraints, indexes, and linked tables.	Future public or multi-user searchable database.
MongoDB	Nested records with variable attached data.	Possible storage of reductions, point counts, and code outputs.
Redis	Temporary cache for repeated computations.	Auxiliary acceleration, not the archival record.

6.4 Schema and Integrity of the Septic Database

6.4.1 Schema of the Polynomial Records

After choosing the storage structure, we specify the schema of the records. Using the notation $f_{\mathbf{a}}$ introduced above, a typical row has the form

$$\mathcal{R}(f_{\mathbf{a}}) = (\mathbf{a}, H(f_{\mathbf{a}}), \Delta_{f_{\mathbf{a}}}, \text{sig}(f_{\mathbf{a}}), J(f_{\mathbf{a}}), \text{Gal}_{\mathbb{Q}}(f_{\mathbf{a}})).$$

Here $\mathbf{a} = (a_0, \dots, a_7)$ is the normalized coefficient tuple, $H(f_{\mathbf{a}})$ is the coefficient height, $\Delta_{f_{\mathbf{a}}}$ is the discriminant, $\text{sig}(f_{\mathbf{a}})$ is the real signature, $J(f_{\mathbf{a}})$ denotes the invariant data attached to the binary septic form, and $\text{Gal}_{\mathbb{Q}}(f_{\mathbf{a}})$ is the Galois label.

The schema separates three kinds of information. The first is identifying data: the coefficient tuple and the height. The second is arithmetic data: the discriminant, signature, and invariant values. The third is classification data: the Galois label and the corresponding group notation. This separation is useful because later computations use different parts of the same record. Some computations use only the coefficients, some group the records by Galois label, and the machine-learning experiments use the invariant columns as numerical features.

The enriched non- S_7 database contains the additional invariant columns

$$j_0, j_1, j_2, j_3, j_4.$$

These columns are stored explicitly because evaluating the invariant expressions is computationally expensive. Storing them once avoids recomputation and makes the later feature construction reproducible.

In this sense, the invariant columns are also feature-engineering choices. They are not generic numerical descriptors added after the computation; they come from the invariant theory of binary septic forms. Thus the numerical feature view

used later in the machine-learning chapter remains tied to the algebraic structure of the polynomial.

The full database and the enriched non- S_7 database have different roles. The full database records the large collection of irreducible septics, including the generic S_7 class. The enriched non- S_7 database is used when the non-generic classes and the invariant columns are needed.

Table 6.4: Main fields attached to a septic polynomial record.

Field	Type	Role
$\mathbf{a} = (a_0, \dots, a_7)$	Identifying	Canonical coefficient tuple.
$H(f_{\mathbf{a}})$	Identifying	Height used to record the search range.
$\Delta_{f_{\mathbf{a}}}$	Arithmetic	Discriminant of the polynomial.
$\text{sig}(f_{\mathbf{a}})$	Arithmetic	Real signature of the associated number field.
$J(f_{\mathbf{a}})$	Invariant	Invariant data attached to the binary septic form.
$j_0, \dots, j_4$	Invariant	Stored transvection invariants in the enriched non- S_7 database.
$\text{Gal}_{\mathbb{Q}}(f_{\mathbf{a}})$	Classification	Exact Galois label.
Group notation	Classification	Readable notation for the Galois group.

6.4.2 Cleaning and Integrity Checks

Before the databases are used in the later applications, the records are checked against the consistency conditions required of a septic polynomial database. Each coefficient entry must parse as an integer tuple of length 8, the leading coefficient must be nonzero, the tuple must be primitive, and the normalized

coefficient tuple must occur only once. We also require

$$\Delta_f \neq 0,$$

so that the polynomial is separable. Finally, the stored arithmetic and classification fields are checked as fields attached to the same normalized tuple.

These checks also record the provenance of the files used later. The height bound, normalization convention, filtering conditions, and cleaning steps are part of the construction data. Recording them is necessary because later chapters use derived views of the same records rather than independently constructed datasets.

The full septic database passed these checks without removing any rows and contains

$$1,686,353$$

distinct normalized coefficient tuples. In the enriched non- S_7 database, one malformed row was removed because it did not contain a valid coefficient tuple. After this correction, the cleaned enriched database contains

$$126,396$$

distinct normalized coefficient tuples, with no duplicate coefficient tuples.

Thus the databases used in the Galois-theoretic, machine-learning, and coding applications are indexed by distinct canonical coefficient tuples. The cleaning step does not introduce new mathematics; it verifies that the stored rows satisfy the conditions required before the later computations are performed.

6.5 Derived Views and Final Databases

6.5.1 Galois, Feature, and Reduction Views

The cleaned database can be used in several forms. It is useful to distinguish the main database from the views derived from it. A derived view is not a new mathematical object; it is a selected arrangement of the same records for a particular computation.

The first view is the Galois-label view. In this view, records are grouped by their stored Galois label. This makes it possible to count how many examples belong to each transitive group and to compare arithmetic quantities across groups. The methods used to compute the labels are treated in the Galois-theoretic chapters and are not repeated here.

The second view is the feature view. In this view, each polynomial record is converted into a numerical row. The coefficient tuple gives the basic coordinates, while the discriminant, signature, and invariant values provide additional features. This is the form used when the database is treated as a labelled dataset in the machine-learning chapter. The important point here is that these features are not external measurements; they are computed from the same polynomial record.

The third view is the reduction view. In this view, an integral polynomial is reduced modulo a prime q , provided that the reduction is admissible for the intended computation. The reduced polynomial over $\mathbb{F}_q$ is then used as input for the finite-field curve and code constructions considered later. The present chapter does not develop those constructions; it only records that the database must retain exact integer coefficients so that such reductions can be performed reliably.

Thus the role of the database is to keep one stable collection of records from which these different views can be produced. The Galois view uses the stored label, the feature view uses numerical columns attached to the polynomial, and the

reduction view uses the exact integer coefficients. Keeping all views tied to the same normalized tuple prevents the later computations from becoming separate unrelated datasets.

6.5.2 Final Cleaned Databases

We finish the chapter by recording the final cleaned databases used in the rest of the thesis. The purpose here is not to analyze Galois distributions, machine-learning performance, or code parameters. Those questions are treated in the later chapters. Here we only identify the cleaned datasets and the role each one plays.

The full cleaned database contains

$$1,686,353$$

distinct normalized irreducible septic polynomials. It includes all seven transitive Galois classes in degree 7 and is the main source for computations that require the complete collection, including the generic S_7 class.

The enriched non- S_7 database contains

$$126,396$$

polynomials after cleaning. It excludes the dominant S_7 class and includes the invariant columns

$$j_0, j_1, j_2, j_3, j_4.$$

This file is used when the non-generic classes are studied in more detail and when the invariant data are needed as stored features.

Table 6.5: Final cleaned databases used in the thesis.

Database		Rows after cleaning	Main role
Full septic database		1,686,353	Complete collection of irreducible septic records, including the generic S_7 class.
Enriched database	non- S_7	126,396	Non- S_7 records together with the invariant columns $j_0, \dots, j_4$.

These cleaned databases are the source for the computations in the later chapters. Subsequent tables, filters, groupings, and transformations are built from these records rather than from independent copies of the data.

This chapter has described the database as a finite collection of exact polynomial records, organized by canonical coefficient tuples and checked before use. The same architecture can be extended beyond degree 7. The degree-independent part consists of coefficient-tuple normalization, exact lookup, duplicate control, and integrity checking. What changes with the degree is the enrichment step: the relevant transitive Galois groups, the resolvents used for classification, and the invariant columns attached to the records. In larger degrees or at larger height bounds, the construction-stage backend could also be replaced by a more scalable key–value or relational system, while preserving the same canonical-key principle.

CHAPTER SEVEN

Applications to Galois Theory

7.1 From the Septic Database to Galois Groups

We now apply the septic database of Chapter 6 to the Galois-theoretic methods developed in Chapter 2. In particular, Chapter 2 recalled the passage from an irreducible polynomial to a transitive permutation group and introduced the resolvent method used to distinguish the possible groups. Here we use the Galois labels recorded in the database as the starting point for the analysis.

For a database entry, let

$$f(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0 \in \mathbb{Z}[x]$$

be the corresponding canonical primitive irreducible septic. After an ordering of the roots is chosen, its Galois group gives a transitive subgroup

$$G_f = \text{Gal}_{\mathbb{Q}}(f) \leq S_7,$$

well-defined up to conjugacy. Hence G_f is one of the seven transitive subgroups of S_7 .

The transitive permutation groups of degree 7 have been classified up to conjugacy; see [10]. We use the GAP numbering and notation [2]. The groups are listed in Table 7.1.

All seven groups occur as Galois groups over $\mathbb{Q}$; see [16, Chapter 3]. The first four groups in Table 7.1 are solvable, whereas $\text{PSL}(3, 2)$, A_7 , and S_7 are nonsolvable. Thus, in the septic database, small solvable groups and large nonsolvable groups appear within the same fixed degree.

Table 7.1: Transitive subgroups of S_7 up to conjugacy.

No.	Group	Order
1	C_7	7
2	D_7	14
3	$C_7 \rtimes C_3$	21
4	$C_7 \rtimes C_6$	42
5	$\text{PSL}(3, 2) \cong \text{PSL}(2, 7)$	168
6	A_7	2520
7	S_7	5040

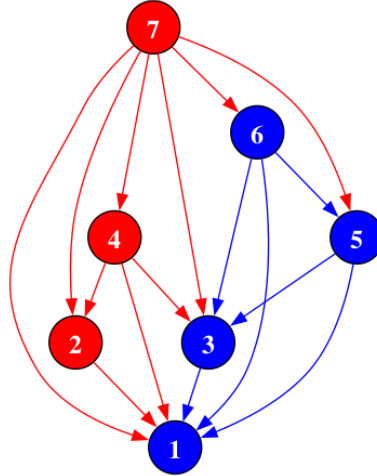


Figure 7.1: Containment relations among the transitive subgroups of S_7 of degree 7, with the numbering as in GAP.

The containment relations among the seven groups are shown in Figure 7.1. The numbering in the figure agrees with Table 7.1: node 1 corresponds to C_7 , and node 7 corresponds to S_7 . The arrows are read from a group to a subgroup.

Inside the Frobenius group $C_7 \rtimes C_6$ one has

$$C_7 \subset D_7 \subset C_7 \rtimes C_6$$

and

$$C_7 \subset C_7 \rtimes C_3 \subset C_7 \rtimes C_6.$$

There is also the exceptional chain

$$C_7 \subset C_7 \rtimes C_3 \subset \mathrm{PSL}(3, 2) \subset A_7 \subset S_7.$$

Thus D_7 and $C_7 \rtimes C_3$ are distinct intermediate groups above C_7 , and neither contains the other.

The rest of this chapter has two purposes. First, we recall the arithmetic data used to assign the exact Galois labels, especially the discriminant and resolvent factorizations. Second, we use the computed labels to study the distribution of Galois groups in the database and to exhibit explicit examples for the rare solvable cases.

7.2 Arithmetic Data Used in the Classification

Each database entry records a canonical primitive irreducible septic f , together with its height $H(f)$, discriminant Δ_f , signature, invariant vector, and Galois label, as described in Chapter 6. The invariant vector is obtained from classical transvections of the associated binary septic form; see [11]. In this chapter these data are used only insofar as they enter the Galois-theoretic classification.

The discriminant gives the first separation among the transitive subgroups of S_7 . For an irreducible septic polynomial f , one has

$$\mathrm{Gal}_{\mathbb{Q}}(f) \subseteq A_7 \iff \Delta_f \in (\mathbb{Q}^\times)^2.$$

Thus the square class of Δ_f separates the groups contained in A_7 from those containing odd permutations. This test does not determine the Galois group by itself. If Δ_f is not a square, then

$$G_f \not\subseteq A_7,$$

but further resolvent tests are still needed to distinguish among the remaining possibilities.

The invariant vector attached to the binary septic form is part of the database record. These invariants provide arithmetic features of the polynomial which are independent of the chosen representative of the binary form. They are not used here to define the Galois group. In this chapter the Galois label is obtained from exact algebraic tests: first the discriminant test, and then the resolvent factorizations described in Section 7.4. The use of the invariant vector as a feature set is taken up later in Chapter 8.

7.3 The Height-Bounded Septic Subset

Before turning to the full database analysis, we record the distribution obtained from the original height-bounded search with projective height $h \leq 4$. The construction of this height-bounded database was described in Chapter 6; here we use only the resulting Galois labels.

Table 7.2 shows the strong dominance of the generic group S_7 in the small-height search. The solvable groups occur very sparsely: only D_7 and $C_7 \rtimes C_6$ appear in this range, while no examples of $C_7 \rtimes C_3$ or C_7 occur for $h \leq 4$. This is

Table 7.2: Galois group counts in the height-bounded septic subset $h \leq 4$.

Galois group	Count
S_7	584,324
A_7	138
$\text{PSL}(3, 2)$	136
D_7	18
$C_7 \rtimes C_6$	4
$C_7 \rtimes C_3$	0
C_7	0

one reason that the height-bounded search is supplemented later by explicit constructions of polynomials in the rare solvable classes.

The Galois labels in Table 7.2 are not assigned from the height data. They are obtained from the exact discriminant and resolvent tests described in Section 7.4.

7.4 Resolvents of Septics

We now specialize the resolvent construction of Section 2.3 to irreducible septic polynomials. If $F(x_1, \dots, x_7)$ is a polynomial in the roots with stabilizer $H \subseteq S_7$, we denote by $R_F(f)$ the corresponding resolvent. By the resolvent principle recalled in Section 2.3, the factorization of $R_F(f)$ over $\mathbb{Q}$ records the orbit lengths of

$$G_f = \text{Gal}_{\mathbb{Q}}(f)$$

on the associated coset space.

For septic equations we use the classical resolvents of Foulkes [14]. They are attached to three root expressions F_1, F_2, F_3 , whose stabilizers in S_7 are respectively

$$A_7, \quad \text{PSL}(3, 2), \quad C_7 \rtimes C_6.$$

For brevity, we write

$$R_i(f) := R_{F_i}(f), \quad i = 1, 2, 3.$$

Thus

$$\deg R_1 = 2, \quad \deg R_2 = 30, \quad \deg R_3 = 120.$$

7.4.1 The quadratic resolvent

Let $\alpha_1, \dots, \alpha_7$ be the roots of f . Set

$$F_1(x_1, \dots, x_7) = \prod_{1 \leq i < j \leq 7} (x_i - x_j).$$

Then

$$F_1(\alpha_1, \dots, \alpha_7)^2 = \Delta_f,$$

where Δ_f is the discriminant of f . The expression F_1 is fixed by even permutations and changes sign under odd permutations. Hence its stabilizer in S_7 is A_7 .

The corresponding resolvent is

$$R_{F_1}(f)(X) = (X - F_1(\alpha_1, \dots, \alpha_7))(X + F_1(\alpha_1, \dots, \alpha_7)).$$

Therefore

$$R_1(f)(X) = R_{F_1}(f)(X) = X^2 - \Delta_f.$$

This recovers the discriminant test:

$$R_1(f) \text{ splits over } \mathbb{Q} \iff \Delta_f \in (\mathbb{Q}^\times)^2 \iff G_f \subseteq A_7.$$

If $R_1(f)$ is irreducible, then $G_f \not\subseteq A_7$. This conclusion does not determine G_f by itself. In that case the group may still be S_7 , $C_7 \rtimes C_6$, or D_7 , and the higher resolvents are needed.

7.4.2 The degree-30 resolvent

The second resolvent is attached to the exceptional subgroup

$$\mathrm{PSL}(3, 2) \cong \mathrm{PSL}(2, 7) \subset S_7.$$

Let

$$\begin{aligned} F_2(x_1, \dots, x_7) = & x_3x_1x_4 + x_4x_2x_5 + x_5x_3x_6 + x_6x_4x_7 \\ & + x_7x_5x_1 + x_1x_6x_2 + x_2x_7x_3. \end{aligned}$$

The stabilizer of F_2 in S_7 is isomorphic to $\mathrm{PSL}(3, 2)$. Thus

$$R_{F_2}(f)(X) = \prod_{\sigma \in S_7 / \mathrm{PSL}(3, 2)} (X - F_2^\sigma(\alpha_1, \dots, \alpha_7)).$$

Since

$$[S_7 : \mathrm{PSL}(3, 2)] = \frac{5040}{168} = 30,$$

we have $\deg R_{F_2} = 30$, and we write

$$R_2(f) := R_{F_2}(f).$$

The factorization of $R_2(f)$ refines the discriminant test. For the large nonsolvable groups one obtains the following patterns:

$$G_f = S_7 \implies R_2(f) \text{ is irreducible,}$$

$$G_f = A_7 \implies R_2(f) \text{ has factor degrees } 15, 15,$$

and

$$G_f = \mathrm{PSL}(3, 2) \implies R_2(f) \text{ has factor degrees } 1, 7, 8, 14.$$

Thus $R_2(f)$ separates the S_7 , A_7 , and $\mathrm{PSL}(3, 2)$ cases, and it also helps separate the branches leading to the solvable groups.

7.4.3 The degree-120 resolvent

The third resolvent is attached to the Frobenius group

$$F_{42} = C_7 \rtimes C_6.$$

Let

$$\begin{aligned} F_3(x_1, \dots, x_7) = & x_3x_1(x_4 + x_7) + x_2x_5(x_4 + x_3) + x_5x_6(x_3 + x_7) \\ & + x_4x_6(x_7 + x_3) + x_5x_1(x_7 + x_6) \\ & + x_1x_2(x_6 + x_4) + x_2x_7(x_3 + x_6). \end{aligned}$$

The stabilizer of F_3 in S_7 is isomorphic to $C_7 \rtimes C_6$. Hence

$$R_{F_3}(f)(X) = \prod_{\sigma \in S_7/(C_7 \rtimes C_6)} (X - F_3^\sigma(\alpha_1, \dots, \alpha_7)).$$

Since

$$[S_7 : C_7 \rtimes C_6] = \frac{5040}{42} = 120,$$

we have $\deg R_{F_3} = 120$, and we write

$$R_3(f) := R_{F_3}(f).$$

The role of $R_3(f)$ is to separate the solvable transitive groups which are not distinguished by the first two tests. In particular,

$$C_7, \quad D_7, \quad C_7 \rtimes C_3, \quad C_7 \rtimes C_6$$

have different factorization patterns for $R_3(f)$. The Galois group $\text{Gal}_{\mathbb{Q}}(f)$ is determined from the factorization patterns of the three resolvents

$$R_1(f), \quad R_2(f), \quad R_3(f).$$

Here the factorization pattern means the degrees of the irreducible factors over $\mathbb{Q}$.

Theorem 7.1. *Let $f \in \mathbb{Q}[x]$ be irreducible of degree 7, and let*

$$G_f = \text{Gal}_{\mathbb{Q}}(f) \subseteq S_7.$$

Then G_f is uniquely determined among the seven transitive subgroups of S_7 by the factorization patterns of the three resolvents

$$R_1(f), \quad R_2(f), \quad R_3(f).$$

More precisely:

1. $G_f = S_7$ if $R_1(f)$ and $R_2(f)$ are both irreducible.
2. $G_f = A_7$ if $R_1(f)$ factors as $(1, 1)$ and $R_2(f)$ factors as $(15, 15)$.
3. $G_f = \text{PSL}(3, 2)$ if $R_1(f)$ factors as $(1, 1)$ and $R_2(f)$ factors as $(1, 7, 8, 14)$.
4. $G_f = C_7 \rtimes C_6$ if $R_1(f)$ is irreducible, $R_2(f)$ factors as $(2, 14, 14)$, and $R_3(f)$ factors as $(1, 7, 14, 14, 14, 21, 21, 42)$.
5. $G_f = C_7 \rtimes C_3$ if $R_1(f)$ factors as $(1, 1)$, $R_2(f)$ factors as $(1, 1, 7, 7, 7, 7)$, and $R_3(f)$ factors as

$$1, 7, 7, 7, 7, 7, 21, 21, 21, 21.$$

6. $G_f = D_7$ if $R_1(f)$ is irreducible, $R_2(f)$ factors as $(2, 14, 14)$, and $R_3(f)$ factors as

$$1, 7^7, 14^5.$$

7. $G_f = C_7$ if $R_1(f)$ factors as $(1, 1)$, $R_2(f)$ factors as $(1, 7, 7, 7, 7)$, and $R_3(f)$ factors as

$$1, 7^{17}.$$

Proof. By the resolvent principle, the degrees of the irreducible factors of a resolvent $R_F(f)$ over $\mathbb{Q}$ are the orbit lengths of G_f acting on the corresponding coset space S_7/H . Moreover, a linear factor indicates that G_f is contained in a conjugate of the stabilizer subgroup H ; see [12, 21].

For the three resolvents used here, the stabilizers are

$$A_7, \quad \text{PSL}(3, 2), \quad C_7 \rtimes C_6.$$

Thus $R_1(f)$ detects whether $G_f \subseteq A_7$, while $R_2(f)$ separates the cases S_7 , A_7 , and $\text{PSL}(3, 2)$ and also separates the branches leading to the solvable subgroups. The remaining ambiguity among the solvable transitive subgroups is resolved by $R_3(f)$.

The orbit partitions for the seven transitive subgroups of S_7 give exactly the factorization patterns listed in the statement. Since these patterns are distinct, the three resolvents determine G_f uniquely. $\square$

The three resolvents $R_1(f), R_2(f), R_3(f)$, taken together, determine the Galois group of an irreducible septic. This suggests a natural question: can one find a single resolvent whose factorization still contains enough information to distinguish the possible septic Galois groups?

For degree seven there is a particularly simple construction. Instead of looking at the roots one at a time, we let S_7 act on triples of roots. The 3-element subsets of $\{1, \dots, 7\}$ form a set of size

$$\binom{7}{3} = 35.$$

This action gives a resolvent of degree 35. Its factorization records the orbit decomposition of G_f on triples of roots and separates several of the transitive groups directly.

7.4.4 The degree-35 triplet-sum resolvent

The degree-35 resolvent comes from the action of S_7 on the 3-element subsets of $\{1, \dots, 7\}$. It is an application of the resolvent construction recalled in Section 2.3; see also Soicher-McKay [25]. For numerical issues in computing resolvents, including the use of Tschirnhausen transformations when a resolvent is not squarefree, see Cohen [9].

Let

$$F_{35}(x_1, \dots, x_7) = x_1 + x_2 + x_3.$$

The stabilizer of F_{35} in S_7 is

$$H_{35} = S_{\{1,2,3\}} \times S_{\{4,5,6,7\}} \cong S_3 \times S_4.$$

Therefore

$$[S_7 : H_{35}] = \frac{7!}{3!4!} = \binom{7}{3} = 35.$$

The associated resolvent is

$$R_{F_{35}}(f)(X) = \prod_{\sigma \in S_7/H_{35}} (X - F_{35}^\sigma(\alpha_1, \dots, \alpha_7)).$$

Equivalently,

$$R_{F_{35}}(f)(X) = \prod_{1 \leq i < j < k \leq 7} (X - (\alpha_i + \alpha_j + \alpha_k)).$$

For brevity we write

$$R_{35}(f) := R_{F_{35}}(f).$$

The subscript 35 refers to the degree of this resolvent.

The coefficients of $R_{35}(f)$ are symmetric functions of the roots and therefore may be expressed in terms of the coefficients of f . In our symbolic computation we

use the power sums

$$p_m = \sum_{1 \leq i < j < k \leq 7} (\alpha_i + \alpha_j + \alpha_k)^m,$$

and then recover the elementary symmetric functions in the triplet sums by Newton's identities. Thus, if

$$R_{35}(f)(X) = X^{35} - e_1 X^{34} + e_2 X^{33} - \cdots - e_{35},$$

then the coefficients e_i are obtained from the p_m . For example, if s_i denotes the i -th elementary symmetric polynomial in the roots of f , then

$$p_1 = 15s_1 = -15a_6$$

and

$$p_2 = 15(s_1^2 - 2s_2) + 10s_2 = 15a_6^2 - 20a_5.$$

The remaining power sums are obtained in the same way. These power sums are expressed in terms of the s_i , and Newton's identities give the coefficients e_j .

$$e_1 = -15a_6,$$

$$e_2 = 105a_6^2 + 10a_5,$$

$$e_3 = -455a_6^3 - 140a_5a_6 - 2a_4,$$

$$e_4 = 1365a_6^4 + 910a_5a_6^2 + 32a_4a_6 + 45a_5^2 - 8a_3,$$

$$e_5 = -3003a_6^5 - 3640a_5a_6^3 - 234a_4a_6^2 - 585a_5^2a_6$$

$$+ 102a_3a_6 - 18a_4a_5 + 10a_2,$$

$$e_6 = 5005a_6^6 + 10010a_5a_6^4 + 1040a_4a_6^3 + 3510a_5^2a_6^2$$

$$- 600a_3a_6^2 + 270a_4a_5a_6 + 120a_5^3 - 140a_2a_6$$

$$- 60a_3a_5 - 3a_4^2 + 40a_1, \dots$$

The remaining power sums are obtained in the same way in appendix. The factor degrees of $R_{35}(f)$ over $\mathbb{Q}$ are the orbit lengths of G_f on the 3-element subsets of $\{1, \dots, 7\}$. The relevant partitions, computed from this action, are listed in Table 7.3.

Table 7.3: Orbit lengths on 3-element subsets for transitive subgroups of S_7 .

G_f	Factor degrees of $R_{35}(f)$
C_7	7, 7, 7, 7, 7
D_7	7, 7, 7, 14
$C_7 \rtimes C_3$	7, 7, 21
$C_7 \rtimes C_6$	14, 21
$\text{PSL}(3, 2)$	7, 28
A_7	35
S_7	35

Thus $R_{35}(f)$ separates several groups directly. The partitions

$$7, 7, 7, 7, 7, \quad 7, 7, 7, 14, \quad 7, 7, 21$$

distinguish C_7 , D_7 , and $C_7 \rtimes C_3$, respectively. The partition 7, 28 detects $\text{PSL}(3, 2)$.

The groups A_7 and S_7 are both transitive on 3-element subsets, so they are not distinguished by $R_{35}(f)$ alone. This remaining ambiguity is resolved by the discriminant test discussed above.

Proposition 7.2. *Let $f \in \mathbb{Q}[x]$ be irreducible of degree 7, and assume that the triplet sums*

$$\alpha_i + \alpha_j + \alpha_k, \quad 1 \leq i < j < k \leq 7,$$

are pairwise distinct. Then the factor degrees of $R_{35}(f)$ are the orbit lengths of G_f acting on the 3-element subsets of $\{1, \dots, 7\}$.

Proof. This is the resolvent principle of Section 2.3 applied to the action of S_7 on the cosets of

$$H_{35} = S_{\{1,2,3\}} \times S_{\{4,5,6,7\}} \cong S_3 \times S_4.$$

These cosets are naturally identified with the 3-element subsets of $\{1, \dots, 7\}$. Under the distinctness assumption, the roots of $R_{35}(f)$ are in one-to-one correspondence with these subsets. Therefore the irreducible factor degrees of $R_{35}(f)$ are exactly the orbit lengths of G_f on the 3-element subsets. $\square$

If the triplet sums are not distinct, one replaces f by a suitable Tschirnhausen transform, as in Section 2.3, before applying the same argument.

The resolvents in this section provide the exact algebraic labels used in the database. In the computations below, the Galois group of each irreducible septic is assigned from the factorization patterns of $R_1(f)$, $R_2(f)$, and $R_3(f)$. The degree-35 resolvent gives an additional check on the smaller groups through their action on triples of roots. We now turn from the classification method to the distribution of the resulting labels in the database.

7.5 Analysis of the Septic Database

We now record a short analysis of the septic database constructed in Chapter 6, using the Galois labels computed in Section 7.4. This section is not another method for determining the Galois group. The labels used below are the exact labels obtained from the discriminant and resolvent computations. The purpose is to record how the labelled polynomials are arranged with respect to height, discriminant size, and the invariant data attached to the corresponding binary septic.

The statistics in this section are descriptive. The database is not treated as a random sample from all irreducible septic polynomials over $\mathbb{Q}$. It contains height-bounded examples together with constructed examples in some of the sparse solvable classes. Thus the conclusions below describe the database used in this thesis.

7.5.1 Distribution of the non- S_7 groups

We focus on the cleaned non- S_7 part of the database. One malformed row was removed before the computations below were carried out. The cleaned non- S_7 database contains 126,396 polynomials.

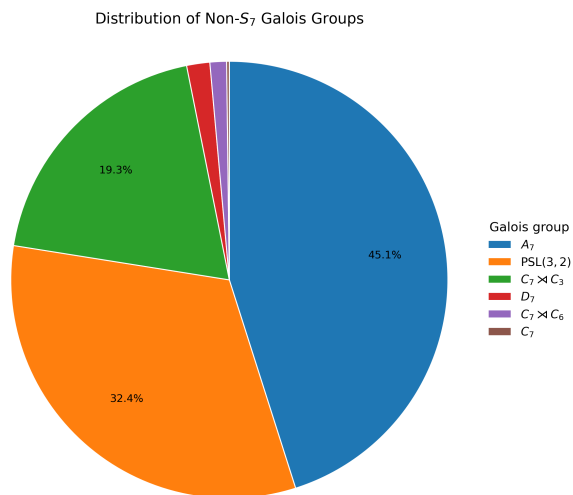


Figure 7.2: Distribution of the non- S_7 Galois groups in the septic database.

Figure Fig. 7.2 gives a visual summary of the class imbalance. The two largest classes are A_7 and $\text{PSL}(3, 2)$. The solvable groups C_7 , D_7 , $C_7 \rtimes C_3$, and $C_7 \rtimes C_6$ are much smaller. The exact counts are given in Table 7.4.

Table 7.4: Number of polynomials in each non- S_7 Galois class.

Galois group	Number of polynomials
A_7	56,997
$\text{PSL}(3, 2)$	40,977
$C_7 \rtimes C_3$	24,457
D_7	2,163
$C_7 \rtimes C_6$	1,550
C_7	252

Thus, even after removing the S_7 part, the database remains imbalanced. This will be important in the in Chapter 8. Ordinary accuracy alone is not enough to evaluate a classifier on this data, because a model can perform well on the large classes while still missing the rare solvable groups.

7.5.2 Height and discriminant size

We next compare the six non- S_7 groups using the height and discriminant data stored in the database. For a septic polynomial f , write

$$h(f) = \log_{10}(H(f) + 1), \quad \delta(f) = \log_{10}(1 + |\Delta(f)|).$$

Here $H(f)$ is the naive coefficient height. The logarithmic scale is used only to make the numerical ranges readable.

Table 7.5 shows a clear separation in the numerical ranges represented in the database. The groups $C_7 \rtimes C_3$ and C_7 have much larger median height than A_7 and $\text{PSL}(3, 2)$. This is not a theorem about all septic polynomials. It reflects the way the database was built: the rare solvable groups are not well represented by small-height enumeration alone and were therefore supplemented by explicit constructions.

The discriminant data show the same general pattern. The classes represented at larger height also tend to have larger discriminant size, as one

Table 7.5: Height and discriminant size by Galois group in the cleaned non- S_7 database.

Galois group	Count	Median $h(f)$	Mean $h(f)$	Median $\delta(f)$	Mean $\delta(f)$
$C_7 \rtimes C_3$	24,457	9.902	9.640	22.433	21.581
C_7	252	6.810	6.624	20.306	19.823
D_7	2,163	3.129	3.116	13.063	12.784
$C_7 \rtimes C_6$	1,550	2.659	2.746	14.778	15.270
$\text{PSL}(3, 2)$	40,977	1.813	1.917	12.076	12.367
A_7	56,997	1.633	1.631	12.787	12.964

expects from the fact that the discriminant is a polynomial expression in the coefficients. The point here is not to prove a new relation between height and discriminant, but to record the size ranges represented in the database.

Figures 7.3 and 7.4 give the same comparison visually. In each boxplot, the horizontal line inside the box marks the median.

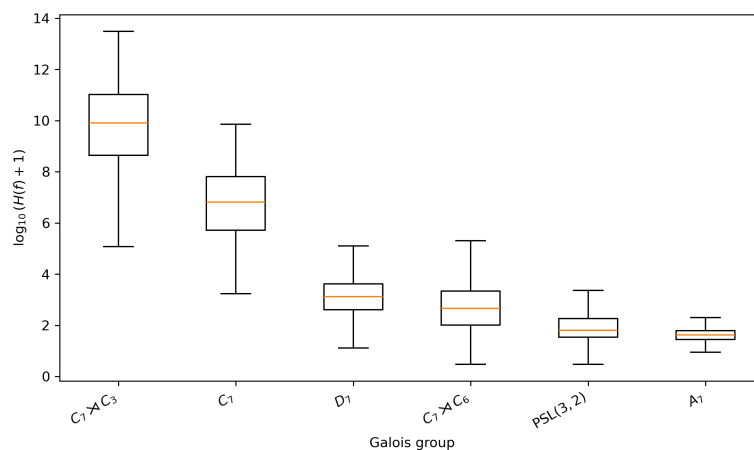


Figure 7.3: Boxplot of $h(f) = \log_{10}(H(f) + 1)$ by Galois group.

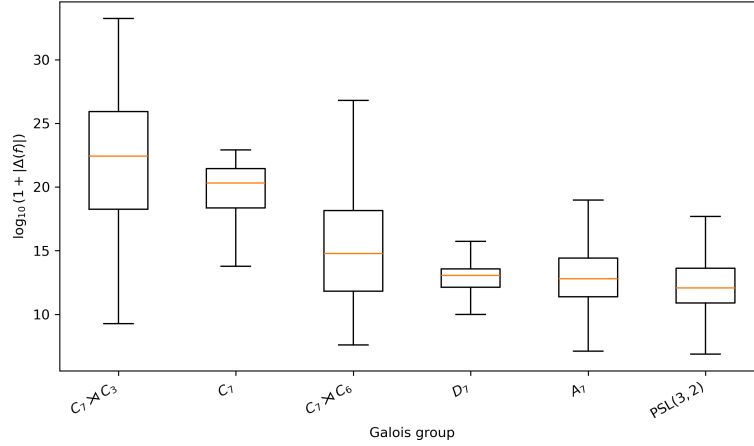


Figure 7.4: Boxplot of $\delta(f) = \log_{10}(1 + |\Delta(f)|)$ by Galois group.

7.5.3 Invariant classes

We now compare coefficient tuples with invariant data. This is the part of the analysis closest to the invariant-theoretic viewpoint for binary forms. The database stores canonical primitive coefficient tuples, so scalar multiples have already been removed. However, this does not mean that every remaining coefficient tuple represents a different invariant object. Distinct polynomial models may still have the same invariant values.

For a polynomial f in the cleaned non- S_7 database, set

$$J(f) = (j_0(f), j_1(f), j_2(f), j_3(f), j_4(f)).$$

Two polynomials are placed in the same invariant class if their five invariant values agree exactly. Thus an invariant class is a set of coefficient tuples with the same value of $J(f)$.

This is the same kind of reduction used in smaller degrees [12]. It is also consistent with the use of binary-form invariants and weighted heights in the

organization of polynomial data [11, 12]. The question here is whether a similar collapse appears in the septic database.

The Galois label is not defined from $J(f)$. The labels have already been computed by the exact discriminant and resolvent tests. The question here is whether the invariant classes are compatible with the exact Galois stratification in this database.

Table 7.6: Reduction from coefficient tuples to invariant classes in the cleaned non- S_7 database.

Galois group	Polynomials	Invariant classes	Reduction	Compression ratio
A_7	56,997	56,890	0.188%	1.002
$\text{PSL}(3, 2)$	40,977	40,866	0.271%	1.003
$C_7 \rtimes C_3$	24,457	3,094	87.349%	7.905
D_7	2,163	2,142	0.971%	1.010
$C_7 \rtimes C_6$	1,550	1,548	0.129%	1.001
C_7	252	159	36.905%	1.585

Table 7.6 compares the number of coefficient tuples with the number of distinct invariant classes. For A_7 , $\text{PSL}(3, 2)$, D_7 , and $C_7 \rtimes C_6$, almost every coefficient tuple has its own invariant vector. The behavior is different for $C_7 \rtimes C_3$ and C_7 . The group $C_7 \rtimes C_3$ contains 24,457 coefficient tuples but only 3,094 invariant classes. The cyclic group C_7 contains 252 coefficient tuples but only 159 invariant classes.

Thus the invariant vector reduces redundancy most strongly in the $C_7 \rtimes C_3$ class, and also noticeably in the C_7 class. This means that distinct coefficient representatives can determine the same invariant data.

Table 7.7: Repeated invariant classes in the cleaned non- S_7 database.

Galois group	Invariant classes	Repeated classes	Largest class	Polys. in repeated classes
A_7	56,890	68	4	0.307%
$\text{PSL}(3, 2)$	40,866	57	6	0.410%
$C_7 \rtimes C_3$	3,094	3,011	24	99.661%
D_7	2,142	9	4	1.387%
$C_7 \rtimes C_6$	1,548	2	2	0.258%
C_7	159	93	2	73.810%

Table 7.7 records how often an invariant class contains more than one coefficient tuple. Repetition is concentrated mainly in $C_7 \rtimes C_3$ and C_7 . For $C_7 \rtimes C_3$, almost every polynomial lies in a repeated invariant class, and the largest class has size 24. For C_7 , the largest class has size 2, but a large proportion of the cyclic examples belong to repeated classes.

No repeated invariant class in the cleaned non- S_7 database contains polynomials from two different Galois groups. Hence, for this dataset, equality of the five invariant values is compatible with the exact Galois label. We record this as an observation about the database:

$$J(f_1) = J(f_2) \implies \text{Gal}_{\mathbb{Q}}(f_1) = \text{Gal}_{\mathbb{Q}}(f_2)$$

for all pairs f_1, f_2 lying in the same repeated invariant class of the cleaned non- S_7 data.

This is not claimed as a theorem about all binary septic forms. It says only that, for the present data, the invariant vector $J(f)$ reduces redundancy without mixing the exact Galois labels.

Table 7.8 gives one concrete repeated invariant class. The coefficient tuples are distinct, so these are different polynomial models. They should not be

Table 7.8: Representative coefficient tuples in one repeated invariant class. The displayed polynomials have the same invariant vector J_1 , the same Galois group $C_7 \rtimes C_3$, and the same discriminant $\Delta(f) = 724,754,206,248,150,912$.

Class	Coefficient tuple $(a_0, \dots, a_7)$	$H(f)$
J_1	$(128, -448, -32480, 82320, 2370648, -3638516, -43555498, -16514581)$	43,555,498
J_1	$(128, 448, -32480, -82320, 2370648, 3638516, -43555498, 16514581)$	43,555,498
J_1	$(128, -448, -32480, 82320, 2370648, -3638516, -43555498, 61288427)$	61,288,427
J_1	$(128, 448, -32480, -82320, 2370648, 3638516, -43555498, -61288427)$	61,288,427
J_1	$(1, -14, -4060, 41160, 4741296, -29108128, -1393775936, -2113866368)$	2,113,866,368

interpreted as duplicate rows. For example, the first two rows are related by the change of variable $x \mapsto -x$, which changes the signs of the odd-degree coefficients and does not change the splitting field. More generally, invariant classes record agreement of the five invariant values, not equality of coefficient tuples.

In this particular class the discriminant is also the same. This should not be read as saying that the discriminant determines the invariant vector. The discriminant is only one invariant quantity, whereas $J(f)$ records five invariant values. The example shows instead that several different coefficient models can represent the same invariant data and the same exact Galois group.

7.5.4 Summary

The analysis gives three points used later in the thesis. First, after removing S_7 , the database is still imbalanced: A_7 and $\text{PSL}(3, 2)$ form the two largest classes, while C_7 remains very sparse. Second, in the cleaned non- S_7 database, the height and discriminant data show different numerical ranges for the sparse solvable classes and the larger nonsolvable classes. Third, invariant classes reduce redundancy, especially for $C_7 \rtimes C_3$ and C_7 , and the repeated classes do not mix exact Galois labels.

The invariants therefore do not replace the resolvent method. They provide a compact algebraic description of repeated coefficient models and explain why the invariant data are useful features in the later classification experiments of Chapter 8.

7.6 Polynomial Constructions

The preceding sections assigned Galois labels to the septic polynomials in the database by exact resolvent computations. The resulting distribution shows that the generic group S_7 dominates, while the solvable groups occur much less frequently. For this reason, the height-bounded search was supplemented by explicit constructions of septic polynomials with prescribed Galois group.

The general Galois-theoretic background for such constructions was recalled in Chapter 2. Here we use it only in a concrete form: the construction produces candidate polynomials over $\mathbb{Q}$, and each candidate is then checked by the same irreducibility and resolvent tests used for the rest of the database. Thus the construction supplies examples, but the final label is still assigned by the exact classification procedure of Section 7.4.

7.6.1 Constructing C_7 -septics

We first consider the cyclic group C_7 . As seen in the height-bounded part of the database, the cyclic case is rare at small height. The first example with Galois group C_7 occurs at height 28:

$$f(x) = x^7 + x^6 - 12x^5 - 7x^4 + 28x^3 + 14x^2 - 9x + 1.$$

This polynomial is obtained from a degree-7 subfield of a cyclotomic field.

Let $p \equiv 1 \pmod{7}$ be prime. Then

$$\text{Gal}(\mathbb{Q}(\zeta_p)/\mathbb{Q}) \cong (\mathbb{Z}/p\mathbb{Z})^\times$$

is cyclic of order $p - 1$. Since $7 \mid p - 1$, there is a unique subgroup

$$H \leq (\mathbb{Z}/p\mathbb{Z})^\times, \quad |H| = \frac{p-1}{7}.$$

The fixed field

$$L = \mathbb{Q}(\zeta_p)^H$$

has degree 7 over $\mathbb{Q}$, and

$$\text{Gal}(L/\mathbb{Q}) \cong C_7.$$

Thus, if $\alpha \in L$ generates L over $\mathbb{Q}$, then the minimal polynomial of α is an irreducible septic whose splitting field is L . Hence its Galois group over $\mathbb{Q}$ is cyclic of order 7.

In the computation we used Gaussian periods. After choosing H , we set

$$\alpha = \sum_{h \in H} \zeta_p^h.$$

This element is fixed by H , hence lies in $L = \mathbb{Q}(\zeta_p)^H$. We then compute the minimal polynomial of α , make it primitive, record its height, and verify its Galois group by the resolvent tests of Section 7.4.

The construction may be summarized as

$$\begin{aligned} p \equiv 1 \pmod{7} &\implies \mathbb{Q}(\zeta_p) \implies L = \mathbb{Q}(\zeta_p)^H, [L : \mathbb{Q}] = 7 \\ &\implies \text{minpoly}_{\mathbb{Q}}(\alpha). \end{aligned}$$

Example 7.3. Take $p = 29$. Then $p - 1 = 28$, and $(\mathbb{Z}/29\mathbb{Z})^\times$ is cyclic of order 28. Its subgroup H of order 4 fixes a degree-7 subfield of $\mathbb{Q}(\zeta_{29})$. For the Gaussian period

$$\alpha = \sum_{h \in H} \zeta_{29}^h,$$

the minimal polynomial over $\mathbb{Q}$ is

$$f(x) = x^7 + x^6 - 12x^5 - 7x^4 + 28x^3 + 14x^2 - 9x + 1.$$

This polynomial is irreducible over $\mathbb{Q}$. Since it generates the degree-7 subfield of the cyclic extension $\mathbb{Q}(\zeta_{29})/\mathbb{Q}$, its splitting field is cyclic of degree 7. Therefore

$$\text{Gal}_{\mathbb{Q}}(f) \cong C_7.$$

The resolvent factorization agrees with the C_7 case of Thm. 7.1. Thus this polynomial is labeled by the same method as the polynomials obtained from the height-bounded search.

Table 7.9: Representative cyclic septic polynomials obtained from degree-7 subfields of cyclotomic fields.

No.	Coefficients $(a_7, \dots, a_0)$	$H(f)$	p
1	$(1, 1, -12, -7, 28, 14, -9, 1)$	28	29
2	$(1, 1, -18, -35, 38, 104, 7, -49)$	104	43
3	$(1, 1, -30, 3, 254, -246, -245, 137)$	254	71
4	$(1, 1, -48, 37, 312, -12, -49, -1)$	312	113
5	$(1, 1, -54, -31, 558, -32, -1713, 1121)$	1,713	127
6	$(1, 1, -84, -217, 1348, 3988, -1433, -1163)$	3,988	197

Table 7.9 lists representative examples. In each case the polynomial was made primitive and its Galois group was verified by the same resolvent factorization tests used for the height-bounded part of the database. The full list of cyclic examples is retained in the database.

7.6.2 Constructing $C_7 \rtimes C_3$ -septics

We next consider the Frobenius group $C_7 \rtimes C_3$. This group is one of the sparse solvable groups appearing in the septic database. In the height-bounded search, the smallest example found was

$$f_{\min}(x) = x^7 - 8x^5 - 2x^4 + 16x^3 + 6x^2 - 6x - 2.$$

The construction used here is based on the degree-7 Chebyshev family of Bruen–Jensen–Yui [6].

Let $T_n(x)$ denote the Chebyshev polynomial of the first kind. For $n = 7$,

$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x.$$

Set

$$S = u^2 + 7v^2.$$

Then

$$S^{7/2}T_7(x/\sqrt{S}) = 64x^7 - 112Sx^5 + 56S^2x^3 - 7S^3x.$$

Only odd powers occur in T_7 , so the scaled expression has coefficients in $\mathbb{Q}[u, v]$.

Adding the constant term $-uS^3$ gives the two-parameter family

$$G_7(x; u, v) = 64x^7 - 112Sx^5 + 56S^2x^3 - 7S^3x - uS^3, \quad S = u^2 + 7v^2.$$

Lemma 7.4 (Integral scaling). *For any indeterminate S ,*

$$S^{7/2}T_7(x/\sqrt{S}) \in \mathbb{Q}[S][x].$$

Proof. The polynomial $T_7(x)$ contains only the powers x^7, x^5, x^3, x . A term $c_j x^j$ contributes $c_j S^{(7-j)/2} x^j$ after the scaling. For $j = 7, 5, 3, 1$, the exponent $(7-j)/2$ is a nonnegative integer. Hence the scaled expression lies in $\mathbb{Q}[S][x]$. $\square$

Theorem 7.5. *Outside a thin exceptional set of specializations, the polynomial*

$$G_7(x; u, v) = 64x^7 - 112Sx^5 + 56S^2x^3 - 7S^3x - uS^3, \quad S = u^2 + 7v^2,$$

has Galois closure with group $C_7 \rtimes C_3$.

Proof. This is the degree-7 case of the Chebyshev construction of Bruen–Jensen–Yui [6]. Lemma 7.4 shows that the displayed expression is a septic polynomial over $\mathbb{Q}(u, v)$. The specialization statement then follows from Hilbert irreducibility, as recalled in Chapter 2. □

For example, when $(u, v) = (1, 1)$, one has $S = 8$, and the family gives

$$G_7(x; 1, 1) = 64x^7 - 896x^5 + 3584x^3 - 3584x - 512.$$

Each specialization used in the database was checked separately for irreducibility over $\mathbb{Q}$, made primitive, and labeled by the resolvent factorization patterns of Section 7.4.

7.6.3 Contribution to the database

The constructions above were incorporated into the septic database to enlarge the sparse solvable classes. The height-bounded enumeration of Chapter 6 produces many examples with generic Galois group S_7 , whereas the smaller solvable groups occur much less frequently. The cyclic construction supplies examples in the C_7 class, and the Chebyshev family supplies examples in the $C_7 \rtimes C_3$ class. This use of explicit families is consistent with the constructive approach to inverse Galois theory recalled in Chapter 2; see also [20, 16].

The constructed polynomials were not added to the database merely because of their method of construction. Each polynomial was first made primitive and recorded with its height. Irreducibility over $\mathbb{Q}$ was then checked. After that, the

resolvents from Section 7.4 were factored over $\mathbb{Q}$, and the Galois label was assigned from the resulting factorization pattern. Thus the constructed examples and the height-bounded examples are labeled by the same exact procedure.

For C_7 , the cyclotomic construction gives septic fields from the degree-7 subfields of $\mathbb{Q}(\zeta_p)$, where $p \equiv 1 \pmod{7}$. The examples in Table 7.9 are representative of this construction. For $C_7 \rtimes C_3$, the Chebyshev construction gives a two-parameter family whose generic Galois group is $C_7 \rtimes C_3$. After specialization, the candidates were again checked for irreducibility and then verified by the resolvent tests.

These constructed examples are used later as part of the enlarged labeled dataset. In Chapter 8, the same database is used for the machine-learning classification problem, but the labels themselves remain the exact Galois labels computed in the present chapter. Thus the role of the constructions is not to replace the resolvent method, but to supply additional polynomials in rare classes where the height-bounded search alone is too sparse.

The focus on degree 7 is also practical. Septic polynomials already exhibit cyclic, solvable, exceptional, alternating, and symmetric Galois groups, while the exact resolvent computations are still feasible on a large database. General algorithms for computing Galois groups are not restricted to degree 7 [9], but the number of transitive subgroups, the size of the resolvents, and the cost of the computations increase rapidly with the degree. Similarly, the invariant theory becomes less explicit in higher degree; for binary forms, concrete systems of invariants are available in small degrees, including the cases up to degree 10 [11]. For this reason, a natural next step is to study selected families in degrees 8, 9, and 10, rather than trying first to build a full analogue of the septic database in much higher degree.

CHAPTER EIGHT

Machine Learning for Galois Group Classification

The preceding chapters constructed the septic database and assigned its Galois labels by exact algebraic methods. The database itself was described in Chapter 6, and the discriminant and resolvent tests used to compute the Galois labels were described in Chapter 7. In this chapter the same labelled database is used as a supervised classification problem.

A row of the database consists of a canonical primitive irreducible septic

$$f(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0 \in \mathbb{Z}[x],$$

together with its coefficient tuple, discriminant, invariant data, and exact Galois label. The labels are fixed before training. They are not produced by the machine-learning model.

The main question in this chapter is the following:

Can a machine-learning model recover the Galois group from the coefficient tuple of the polynomial?

Equivalently, if

$$\mathbf{a}(f) = (a_0, a_1, \dots, a_7) \in \mathbb{Z}^8,$$

we ask how much information about

$$y(f) = \text{Gal}_{\mathbb{Q}}(f)$$

is visible from this vector and from the additional arithmetic features stored in the database.

This is not meant to replace the resolvent method. The resolvents give the exact Galois group. The purpose here is to use the labelled database as a testing

ground and to measure which algebraic features make the Galois label more visible to a numerical model.

The experiment is organized in stages. First we train using only the coefficient vector. This gives the baseline. We then add numerical summaries of the discriminant, and finally the five transvection invariants $j_0, \dots, j_4$. By comparing these feature sets, we separate the question of model performance from the more mathematical question of which algebraic data carry information about the Galois group.

8.1 The Classification Problem

We use two related datasets. The first is the full septic database, containing all seven transitive groups listed in Table 7.1. The second is the cleaned non- S_7 database studied in Section 7.5. The full dataset is used for the baseline comparison between coefficients and discriminant data. The non- S_7 dataset is used for the feature comparison involving the transvection invariants.

The full dataset contains 1,686,353 irreducible septic polynomials. The class distribution is shown in Table 8.1.

Table 8.1: Galois group distribution in the full dataset used in Chapter 8.

Group	Count
S_7	1,559,957
A_7	56,997
$\text{PSL}(3, 2)$	40,977
$C_7 \rtimes C_3$	24,457
D_7	2,163
$C_7 \rtimes C_6$	1,550
C_7	252

The distribution is highly imbalanced. The generic group S_7 accounts for more than 92% of the full dataset, while the solvable groups occur much less frequently. For this reason ordinary accuracy is not the main statistic. A classifier that mostly predicts S_7 can have high ordinary accuracy but still perform poorly on rare groups.

We therefore use balanced accuracy. If the labels are $G_1, \dots, G_r$, then

$$\text{BalAcc} = \frac{1}{r} \sum_{i=1}^r \frac{\#\{f : y(f) = G_i \text{ and } \hat{y}(f) = G_i\}}{\#\{f : y(f) = G_i\}}.$$

Thus each Galois group contributes equally, independent of its frequency in the database.

8.2 Feature Sets

The baseline feature is the coefficient vector

$$\mathbf{a}(f) = (a_0, a_1, \dots, a_7) \in \mathbb{Z}^8.$$

This vector determines the polynomial model, but it gives eight numerical input coordinates to the classifier.

The discriminant is a single arithmetic quantity, but its raw value can be very large. Instead of using Δ_f itself as a numerical feature, we use the two summaries stored in the database:

$$\text{sgn}(\Delta_f), \quad \log_{10}(1 + |\Delta_f|).$$

This numerical representation should not be confused with the exact square-discriminant test from Chapter 7. The exact criterion remains

$$\text{Gal}_{\mathbb{Q}}(f) \subseteq A_7 \iff \Delta_f \in (\mathbb{Q}^\times)^2.$$

Here the discriminant summaries are used only as numerical inputs for the model.

For the invariant features, we use the five transvection invariants

$$j_0, j_1, j_2, j_3, j_4.$$

These values can be very large, especially j_4 . We therefore use the signed logarithmic quantities

$$J_i(f) = \text{sgn}(j_i(f)) \log_{10}(1 + |j_i(f)|), \quad 0 \leq i \leq 4.$$

This transformation keeps both the sign and the order of magnitude of each invariant, while placing the coordinates on a scale suitable for numerical training.

The feature sets used in the experiments are

$$\mathcal{F}_1(f) = \mathbf{a}(f),$$

$$\mathcal{F}_2(f) = (\mathbf{a}(f), \text{sgn}(\Delta_f), \log_{10}(1 + |\Delta_f|)),$$

$$\mathcal{F}_3(f) = (\mathbf{a}(f), J_0(f), J_1(f), J_2(f), J_3(f), J_4(f)),$$

and

$$\mathcal{F}_4(f) = (\mathbf{a}(f), \text{sgn}(\Delta_f), \log_{10}(1 + |\Delta_f|), J_0(f), J_1(f), J_2(f), J_3(f), J_4(f)).$$

Their numbers of numerical input features are respectively

$$8, \quad 10, \quad 13, \quad 15.$$

8.3 Model and Evaluation

For all experiments we use the same histogram-based gradient boosting classifier. This choice is suitable for the present data because the features are tabular and numerical, and the relation between the coefficient tuple, the discriminant, the invariants, and the Galois group is not expected to be linear.

Table 8.2: Feature sets used in the machine-learning experiments.

Feature set	Number of numerical features
Coefficient vector $\mathbf{a}(f)$	8
Coefficient vector + discriminant summaries	10
Coefficient vector + five invariant summaries	13
Coefficient vector + discriminant summaries + five invariant summaries	15

We do not compare several learning algorithms in this chapter. The classifier is fixed, and the feature set is varied. This keeps the experiment focused on the algebraic data: when balanced accuracy improves, the improvement comes from the additional features supplied to the same model.

The data are divided into training and test sets using a stratified 60/40 split. Stratification preserves the Galois group proportions in both parts. On the training set we use five-fold stratified cross-validation. The final reported value is the balanced accuracy on the held-out test set.

To reduce the effect of class imbalance, the training uses sample weights. For a class G , the weight is proportional to

$$\left(\frac{\text{median class size}}{\#G} \right)^{1/2}.$$

This gives more weight to small classes without applying the full inverse-class frequency correction.

8.3.1 Full Database Baseline

The first experiment uses the full database. Since the full file does not contain the five invariant columns, we compare only two feature sets: the coefficient vector and the coefficient vector together with the two discriminant summaries.

Table 8.3: Full database baseline using the same gradient-boosting classifier.

Feature set	No. features	CV balanced accuracy	Test balanced accuracy
Coefficients	8	0.7526 ± 0.0095	0.7726
Coefficients + discriminant	10	0.8103 ± 0.0093	0.8236

Using only

$$(a_0, \dots, a_7),$$

the model reaches test balanced accuracy 0.7726 on the full database. Thus the coefficient vector alone already contains substantial information about the Galois label.

Adding the discriminant summaries improves the test balanced accuracy from

$$0.7726 \quad \text{to} \quad 0.8236.$$

This is consistent with the algebraic role of the discriminant. In the exact classification of Chapter 7, the discriminant gives the first separation according to whether the Galois group lies inside A_7 . The model is not given the exact square-class test, but it is given numerical information derived from the discriminant, and this information is useful.

This experiment is only a baseline, since the full dataset is dominated by S_7 . We therefore next remove S_7 and compare the feature sets on the remaining six groups.

8.4 The Non- S_7 Feature Comparison

We now restrict to the non- S_7 database. The remaining labels are

$$A_7, \quad \text{PSL}(3, 2), \quad C_7 \rtimes C_3, \quad D_7, \quad C_7 \rtimes C_6, \quad C_7.$$

This gives a more sensitive test because the classifier is no longer dominated by the generic group. The class distribution is shown in Table 8.4; it is the same cleaned non- S_7 dataset analyzed in Chapter 7.

Table 8.4: Galois group distribution in the non- S_7 invariant database.

Group	Count
A_7	56,997
$\text{PSL}(3, 2)$	40,977
$C_7 \rtimes C_3$	24,457
D_7	2,163
$C_7 \rtimes C_6$	1,550
C_7	252

In this experiment we compare all four feature sets from Table 8.2. The results are shown in Table 8.5.

Table 8.5: Feature comparison on the non- S_7 dataset using all five transvection invariants $j_0, \dots, j_4$.

Feature set	No. features	CV balanced accuracy	Test balanced accuracy
Coefficients	8	0.7679 ± 0.0131	0.7835
Coefficients + discriminant	10	0.8343 ± 0.0121	0.8449
Coefficients + invariants	13	0.7748 ± 0.0156	0.7877
Coefficients + discriminant + invariants	15	0.8387 ± 0.0120	0.8516

The coefficient-only model again performs substantially better than random guessing. On the non- S_7 dataset, it reaches test balanced accuracy

$$0.7835.$$

Thus even after removing the generic group S_7 , the coefficient tuple still carries strong information about the Galois group.

Adding the discriminant summaries gives the largest improvement:

$$0.7835 \longrightarrow 0.8449.$$

This agrees with the role of the discriminant in Chapter 7.

The invariant summaries alone give a smaller improvement:

$$0.7835 \longrightarrow 0.7877.$$

Thus the invariant vector contains some information about the Galois label, but it does not replace the discriminant information.

The best result is obtained from the combined feature set:

$$0.8516.$$

This improves on the coefficient-discriminant model:

$$0.8449 \longrightarrow 0.8516.$$

The five invariants therefore add information, but their contribution is secondary to the discriminant in this experiment.

8.5 Performance by Galois Group

The class-wise results show where the model still has difficulty. The most persistent confusion is between A_7 and $\text{PSL}(3, 2)$. This is not surprising: both

groups are contained in A_7 , so the discriminant cannot separate them. In the exact classification, the distinction between these two groups is made by the higher resolvents described in Chapter 7.

The group $C_7 \rtimes C_3$ is classified very accurately. This agrees with the database analysis of Chapter 7, where this group appears separately from the larger nonsolvable classes in the height, discriminant, and invariant data. The cyclic group C_7 is also recognized well, although its test support is small and the corresponding scores should be interpreted with care.

The groups D_7 and $C_7 \rtimes C_6$ are more difficult than $C_7 \rtimes C_3$. Their performance improves when the discriminant and invariant summaries are used together, which suggests that these additional features help the model separate some of the smaller solvable classes.

8.5.1 Interpretation

The experiments support three conclusions. First, the coefficient vector already contains substantial information about the Galois group. This is expected in one sense, since the polynomial is determined by its coefficients. However, the map

$$(a_0, \dots, a_7) \longmapsto \text{Gal}_{\mathbb{Q}}(f)$$

is highly nonlinear. The classifier is not computing resolvents explicitly; rather, it detects patterns in coefficient space that correlate with the exact Galois labels.

Second, the discriminant summaries give the main improvement. This matches the role of the discriminant in the exact classification, where it gives the first separation according to whether the Galois group is contained in A_7 .

Third, the five transvection invariants give a further improvement when used together with the discriminant. The improvement is smaller, but it shows that the

invariant vector retains additional information about the arithmetic form of the polynomial.

For the experiments above, the feature sets are ordered by test balanced accuracy as

$$\begin{aligned} \text{coefficients} &< \text{coefficients} + \text{invariants} \\ &< \text{coefficients} + \text{discriminant} \\ &< \text{coefficients} + \text{discriminant} + \text{invariants}. \end{aligned}$$

This order is consistent with the algebraic roles of the features. The coefficients define the polynomial, the discriminant gives a classical Galois-theoretic separation, and the transvection invariants refine the arithmetic profile of the associated binary septic form.

The machine-learning model should therefore be interpreted as a feature study, not as a replacement for the resolvent method. The labels remain the exact Galois labels computed in Chapter 7. The model measures how much of those labels is visible from the numerical features stored in the database.

8.6 A Neurosymbolic Network for Galois Group Classification

The previous experiments show that the best performance is obtained when the coefficient vector is combined with the discriminant and the transvection invariants. We now use this observation to organize the classifier in a more structured way. Following the invariant-based and neurosymbolic viewpoint of [22, 18, 12], we separate the computation into two parts: an algebraic preprocessing layer and a learned classification layer. The purpose is not to replace the exact Galois group computation, but to build a fast predictor from arithmetic data attached to the coefficients.

At prediction time the model does not compute resolvent polynomials and does not call a Galois group routine. The resolvents remain the exact method used to produce the labels in the database; see [14, 9, 18]. Here the input is the coefficient tuple. From it we compute the discriminant and the invariant values, and then use these quantities to predict the Galois label.

8.6.1 Symbolic invariant layer

Let

$$f(x) = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0 \in \mathbb{Z}[x]$$

be an irreducible septic. The input to the model is

$$\mathbf{a}(f) = (a_0, a_1, \dots, a_7).$$

From this tuple we compute

$$\mathcal{I}(f) = (\Delta_f, j_0(f), j_1(f), j_2(f), j_3(f), j_4(f)).$$

This is the symbolic part of the network. It is fixed and contains no learned parameters. The use of transvection invariants follows the classical invariant-theoretic approach to binary forms and is consistent with the coefficient-based framework used in the construction of the septic database [11, 18].

Since these values can be large, we do not pass the raw integers directly to the classifier. For the invariants we use

$$J_i(f) = \text{sgn}(j_i(f)) \log_{10}(1 + |j_i(f)|), \quad 0 \leq i \leq 4.$$

For the discriminant we use

$$\text{sgn}(\Delta_f), \quad \log_{10}(1 + |\Delta_f|),$$

and also the square-discriminant indicator

$$\mathbf{1}_{\Delta_f \in (\mathbb{Q}^\times)^2}.$$

This last coordinate is included because the square class of the discriminant detects whether the Galois group is contained in A_7 .

The full numerical vector used by the classifier is

$$\begin{aligned} \Phi(f) = (a_0, \dots, a_7, H(f), \|\mathbf{a}(f)\|_2, \text{sgn}(\Delta_f), \log_{10}(1 + |\Delta_f|), \\ \mathbf{1}_{\Delta_f \in (\mathbb{Q}^\times)^2}, J_0(f), \dots, J_4(f)). \end{aligned}$$

Thus the input has 18 numerical coordinates.

Table 8.6: Input coordinates for the neurosymbolic network.

Block	Number of coordinates
Coefficient vector $(a_0, \dots, a_7)$	8
Height and Euclidean coefficient norm	2
Discriminant sign, logarithmic size, and square indicator	3
Signed logarithms of $j_0, \dots, j_4$	5
Total	18

The network can be summarized as

$$(a_0, \dots, a_7) \mapsto \mathcal{I}(f) \mapsto \Phi(f) \mapsto \widehat{\text{Gal}_{\mathbb{Q}}(f)}.$$

Only the last arrow is learned. The first two arrows are algebraic computations from the coefficients.

For efficiency, the invariant layer is cached. When a coefficient tuple is given, the program first checks whether the tuple is already present in the invariant

database. If it is present, the stored values of

$$\Delta_f, j_0(f), \dots, j_4(f)$$

are used. If it is not present, the formulas are evaluated once and the new row is added to the cache. This matters computationally because j_4 is much more expensive to evaluate than the lower invariants.

8.6.2 Training on the non- S_7 data

The network in this section is trained on the non- S_7 invariant database. This choice is intentional. In the full database the S_7 class is dominant, and a random irreducible septic is very likely to have Galois group S_7 . Thus testing only on random new septics mostly checks whether the generic case is recognized. That is not the main difficulty here.

The purpose is to test the non-generic classes

$$A_7, \quad \text{PSL}(3, 2), \quad C_7 \rtimes C_6, \quad C_7 \rtimes C_3, \quad D_7, \quad C_7.$$

These are the non- S_7 transitive subgroups of S_7 occurring in the database. For the classification of transitive subgroups in small degree, see [10, 7]. The network should therefore be read as a classifier for the non- S_7 part of the database, not as a complete seven-class Galois group algorithm.

8.6.3 Test-set performance

The same histogram-based gradient boosting classifier was used for the learned layer. We used a stratified 60/40 train-test split and class weights to reduce the effect of imbalance among the six groups.

The test accuracy was

$$0.8129,$$

Table 8.7: Performance of the neurosymbolic network on the held-out non- S_7 test set.

Group	Precision	Recall	F_1 -score	Support
A_7	0.78	0.83	0.80	22,799
$\text{PSL}(3, 2)$	0.74	0.67	0.70	16,391
$C_7 \rtimes C_6$	0.87	0.79	0.83	620
$C_7 \rtimes C_3$	1.00	1.00	1.00	9,783
D_7	0.86	0.91	0.89	865
C_7	0.97	0.95	0.96	101

the balanced accuracy was

$$0.8590,$$

and the macro F_1 -score was

$$0.8630.$$

Balanced accuracy is the relevant statistic here because the six classes remain unevenly represented.

The strongest performance is on the solvable classes. The group $C_7 \rtimes C_3$ is recognized with recall 1.00. The groups D_7 and C_7 are also recognized well. The main remaining confusion is between A_7 and $\text{PSL}(3, 2)$. This is consistent with the algebraic feature set: both groups lie inside A_7 , so the square-discriminant coordinate cannot separate them.

Once the invariant vector is available, prediction is fast. The held-out test set contained 50,559 examples, and the classifier evaluated them in 0.2303 seconds, about

$$2.20 \times 10^5$$

predictions per second. This timing does not include the one-time computation of invariants for a new polynomial.

8.6.4 Checks on constructed non- S_7 examples

To test the model beyond the random train-test split, we used additional non- S_7 examples from the constructions of Chapter 7. These polynomials were not part of the train-test split used for Table 8.7. For each polynomial, the exact Galois group was checked independently in Sage/GAP before comparison with the network prediction [27, 2]. The network was then applied using only the coefficient tuple and the symbolic invariant layer.

Some invariant vectors were already present in the cache, while others were computed during this test. This affects only the running time; the prediction procedure is the same in both cases.

The first set consists of $C_7 \rtimes C_3$ -septics constructed as in Section 7.6.2. Since the exact label for every polynomial in Table 8.8 is $C_7 \rtimes C_3$, the table records the prediction, the probability assigned to each possible non- S_7 label, and whether the prediction is correct.

Table 8.8: Predictions for constructed $C_7 \rtimes C_3$ septics.

Ex.	Polynomial	Prediction	$P(A_7)$	$P(\text{PSL})$	$P(C_7 \rtimes C_6)$	$P(C_7 \rtimes C_3)$	$P(D_7)$	$P(C_7)$	Correct
1	$64x^7 - 1232x^5 + 6776x^3 - 9317x - 2662$	$C_7 \rtimes C_3$	0.0008	0.0023	0.0001	0.9965	0.0001	0.0002	yes
2	$x^7 - 28x^5 + 224x^3 - 448x - 192$	$C_7 \rtimes C_3$	0.4098	0.1283	0.0007	0.4568	0.0007	0.0036	yes
3	$64x^7 - 3248x^5 + 47096x^3 - 170723x - 24389$	$C_7 \rtimes C_3$	0.0007	0.0004	0.0001	0.9985	0.0001	0.0002	yes
4	$64x^7 - 2576x^5 + 29624x^3 - 85169x - 48668$	$C_7 \rtimes C_3$	0.0005	0.0006	0.0001	0.9987	0.0001	0.0001	yes
5	$64x^7 - 7504x^5 + 251384x^3 - 2105341x - 601526$	$C_7 \rtimes C_3$	0.0060	0.0037	0.0007	0.9868	0.0007	0.0020	yes
6	$64x^7 - 5936x^5 + 157304x^3 - 1042139x - 744385$	$C_7 \rtimes C_3$	0.0008	0.0004	0.0001	0.9985	0.0001	0.0001	yes

All six examples in Table 8.8 were classified correctly. Five of the six predictions assign probability at least 0.9868 to the correct group. The least decisive case is the second polynomial,

$$x^7 - 28x^5 + 224x^3 - 448x - 192.$$

The network still predicts $C_7 \rtimes C_3$, but with probability 0.4568, while A_7 receives probability 0.4098. This example shows why the probability should be read as a confidence score, not as a proof of the Galois group.

The second set consists of cyclic septics constructed as in Section 7.6.1. Since the exact label for every polynomial in Table 8.9 is C_7 , the table again records the full probability vector and whether the prediction is correct.

Table 8.9: Predictions for constructed C_7 septics.

Ex.	Polynomial	Prediction	$P(A_7)$	$P(\text{PSL})$	$P(C_7 \rtimes C_6)$	$P(C_7 \rtimes C_3)$	$P(D_7)$	$P(C_7)$	Correct
1	$x^7 + x^6 - 432x^5 - 247x^4 + 59784x^3 + 15272x^2 - 2647129x + 374699$	C_7	0.0001	0.0002	0.0000	0.0014	0.0000	0.9982	yes
2	$x^7 + x^6 - 450x^5 + 4247x^4 - 6450x^3 - 22774x^2 + 25691x - 5653$	$C_7 \rtimes C_3$	0.0333	0.2108	0.0058	0.5366	0.0058	0.2077	no
3	$x^7 + x^6 - 468x^5 + 2543x^4 + 23004x^3 - 169598x^2 + 130839x + 450131$	C_7	0.000009	0.000017	0.000002	0.000018	0.000002	0.999951	yes

Two of the three cyclic examples were classified correctly. The first and third examples were predicted as C_7 with probabilities 0.9982 and 0.999951, respectively. The second example was predicted as $C_7 \rtimes C_3$, although its exact label is C_7 . In that case the network assigned probability 0.5366 to $C_7 \rtimes C_3$, 0.2108 to $\text{PSL}(3, 2)$, and 0.2077 to C_7 .

The error is between closely related groups:

$$C_7 \subset C_7 \rtimes C_3.$$

Thus the model placed the example in the cyclic/Frobenius part of the subgroup lattice, but did not separate the two groups correctly in this case. This is consistent with the small size of the C_7 class in the training data.

These constructed checks are more informative than random tests for the present model. Random height-bounded irreducible septics are usually S_7 , while the network in this section was trained only on the non- S_7 invariant database. The examples above therefore test the part of the problem for which this classifier is designed: distinguishing among the rare non-generic Galois groups.

8.6.5 Conclusion

The neurosymbolic network gives a fast invariant-based prediction, but it is not an exact Galois group algorithm. Its role is complementary to the resolvent method. The algebraic layer computes coefficient-derived invariants, and the learned layer uses their distribution across the labelled database to predict a Galois label.

The experiments show that this approach is useful for the non- S_7 classes, especially for $C_7 \rtimes C_3$ and C_7 , but exact verification still belongs to the resolvent method or to trusted Galois group software.

CHAPTER NINE

Applications to Quantum Error-Correcting Codes

We now apply the geometric and coding-theoretic constructions of Chapters 3–5 to the septic database of Chapter 6. The geometric input is the superelliptic curve attached to a septic polynomial after reduction modulo a finite field, and the coding-theoretic input is the one-point AG/CSS construction recalled in Chapter 5. The purpose of this chapter is to carry out this construction explicitly for a large family of curves arising from the database.

Let $f(x) \in \mathbb{Z}[x]$ be an irreducible septic polynomial from the database. For a prime q of good reduction, we reduce f modulo q and consider

$$X_{f,5}/\mathbb{F}_q : \quad y^5 = \bar{f}(x).$$

The rational points of $X_{f,5}$ are used as evaluation points for one-point AG codes, and compatible pairs of such codes give CSS quantum codes.

Throughout this chapter we restrict to

$$m = 5, \quad \deg f = 7.$$

Thus $\gcd(5, 7) = 1$. The corresponding smooth projective model has genus $g = 12$, a unique point at infinity, and pole semigroup $\langle 5, 7 \rangle$ at that point, as in the superelliptic theory recalled in Chapter 3. The computations below are carried out over

$$\mathbb{F}_{11}, \quad \mathbb{F}_{31}, \quad \mathbb{F}_{61},$$

so that $5 \mid (q - 1)$ and the base field contains the fifth roots of unity.

9.1 From the database to a quantum code

We describe the construction in four steps.

9.1.1 Reduction

Let

$$f(x) = a_7x^7 + \cdots + a_0 \in \mathbb{Z}[x]$$

be an irreducible septic from the database of Chapter 6. For a prime q satisfying

$$q \nmid a_7\Delta_f,$$

the reduced polynomial $\bar{f}(x) \in \mathbb{F}_q[x]$ has degree 7 and is separable.

We use

$$q \in \{11, 31, 61\}.$$

All three fields contain a primitive fifth root of unity, since $5 \mid (q - 1)$. This ensures that the automorphism

$$(x, y) \longmapsto (x, \zeta_5 y)$$

is defined over $\mathbb{F}_q$.

9.1.2 The superelliptic curve

We attach to $\bar{f}$ the superelliptic curve

$$X/\mathbb{F}_q : \quad y^5 = \bar{f}(x).$$

Since $\gcd(5, 7) = 1$, the smooth projective model has a unique point P_∞ above infinity, and by Theorem 3.4 its genus is

$$g = \frac{(5-1)(7-1)}{2} = 12.$$

The Weierstrass semigroup at P_∞ is

$$H(P_\infty) = \langle 5, 7 \rangle,$$

with gap set

$$\{1, 2, 3, 4, 6, 8, 9, 11, 13, 16, 18, 23\}$$

and conductor 24.

The cyclic automorphism

$$\sigma : (x, y) \longmapsto (x, \zeta_5 y)$$

has order 5. It partitions the affine rational points into fixed points, which occur when $\bar{f}(a) = 0$, and free orbits of size 5. Hence, if

$$r_f = \#\{a \in \mathbb{F}_q : \bar{f}(a) = 0\},$$

and N denotes the number of affine rational points of X , then

$$N \equiv r_f \pmod{5}.$$

9.1.3 The weighted model and the monomial basis

We work in the weighted projective plane $\mathbb{P}(5, 7, 5)$, with coordinates $[X : Y : Z]$ satisfying

$$\deg X = \deg Z = 5, \quad \deg Y = 7.$$

The homogenized equation

$$Y^5 = F_7(X, Z)$$

is weighted homogeneous of degree 35. Indeed, the left side has weighted degree $5 \cdot 7 = 35$, and each monomial $X^i Z^{7-i}$ on the right has weighted degree $5 \cdot 7 = 35$. Thus the grading is dictated by the equation.

By Proposition 4.18, the one-point Riemann–Roch space is

$$L(rP_\infty) = \text{span}_{\mathbb{F}_q} \left\{ x^i y^j : i \geq 0, 0 \leq j < 5, 5i + 7j \leq r \right\}.$$

For $r > 2g - 2 = 22$, the Riemann–Roch theorem gives

$$\ell(rP_\infty) = r - g + 1 = r - 11.$$

9.1.4 The evaluation codes and the CSS construction

Let $P_1, \dots, P_N$ be the affine rational points of X , and set

$$D = P_1 + \dots + P_N.$$

For divisor degrees

$$r_1 > r_2 > 2g - 2,$$

we define

$$C_i = C_L(D, r_i P_\infty), \quad i = 1, 2.$$

Since $r_2 P_\infty \leq r_1 P_\infty$, one has

$$C_2 \subset C_1.$$

By the AG-code parameter theorem recalled in Chapter 5,

$$k_i = \dim C_i = r_i - g + 1, \quad d(C_i) \geq N - r_i.$$

The duality theorem for one-point AG codes gives the CSS compatibility condition

$$C_2^\perp \subseteq C_1$$

whenever

$$r_1 + r_2 \leq N + 2g - 2.$$

Applying the CSS construction to the pair $C_2^\perp \subseteq C_1$ gives a quantum code

$$Q = \llbracket N, k_1 - k_2, d_Q \rrbracket_q,$$

with designed distance

$$d_Q \geq \min\{N - r_1, r_2 - (2g - 2)\}.$$

9.2 One-point CSS codes on the saturated line

We now specialize the AG–CSS construction to the one-point divisors used in the computations. Let

$$C_i = C_L(D, r_i P_\infty), \quad i = 1, 2,$$

with

$$r_1 > r_2 > 2g - 2.$$

For these divisors, Riemann–Roch gives

$$k_i = \dim C_i = r_i - g + 1,$$

and the Goppa bound gives

$$d(C_i) \geq N - r_i.$$

The CSS construction applied to the pair $C_2^\perp \subseteq C_1$ gives

$$k_Q = k_1 - k_2 = r_1 - r_2,$$

with designed distance bounded below by

$$d_Q \geq \min\{N - r_1, r_2 - (2g - 2)\}.$$

These are the one-point AG-code and CSS formulas recalled in Chapter 5.

We choose r_1 and r_2 on the line

$$r_1 + r_2 = N + 2g - 2.$$

This is the boundary case of the CSS compatibility condition

$$r_1 + r_2 \leq N + 2g - 2.$$

Writing

$$r_1 = N - s, \quad r_2 = 2g - 2 + s,$$

the two terms in the designed distance bound become equal:

$$N - r_1 = s, \quad r_2 - (2g - 2) = s.$$

Therefore

$$d_Q \geq s.$$

Moreover,

$$k_Q = r_1 - r_2 = N - 2g + 2 - 2s.$$

Thus the divisors on this line give designed CSS parameters

$$\llbracket N, N - 2g + 2 - 2s, \geq s \rrbracket_q.$$

The largest possible designed distance on this line is obtained by taking the largest integer s for which $r_2 < r_1$. Since

$$r_2 < r_1 \iff s < \frac{N - 2g + 2}{2},$$

this largest integer is

$$s_{\max} = \left\lfloor \frac{N - 2g + 1}{2} \right\rfloor.$$

For this choice the quantum dimension is very small:

$$k_Q = N - 2g + 2 - 2 \left\lfloor \frac{N - 2g + 1}{2} \right\rfloor \in \{1, 2\}.$$

More precisely, $k_Q = 1$ when N is odd, and $k_Q = 2$ when N is even.

For the examples in this chapter, we usually do not use this extremal choice. Although it maximizes the designed distance, it leaves only one or two logical qudits. Instead, we use a balanced choice of s , chosen so that both k_Q and the designed distance are nontrivial. Since

$$k_Q d_Q \geq (N - 2g + 2 - 2s)s,$$

the product-balanced value is near

$$s = \frac{N - 2g + 2}{4}.$$

In the computations we choose an admissible integer near this value.

Finally, for every admissible s on the line $r_1 + r_2 = N + 2g - 2$, the designed parameters satisfy

$$k_Q + 2s = N - 2g + 2.$$

Equivalently,

$$(N + 2) - (k_Q + 2s) = 2g.$$

Thus the designed Singleton defect is controlled by the genus of the curve. In the present chapter $g = 12$, so this quantity is 24.

9.3 Explicit examples

We illustrate the construction on three curves, one from each of three Galois classes.

9.3.1 An S_7 curve over $\mathbb{F}_{11}$

The database entry has Galois group S_7 and

$$\bar{f}(x) = x^7 + 10x^6 + 8x^5 + 2x^4 + 5x^3 + 8x^2 + 8x + 1 \in \mathbb{F}_{11}[x].$$

The curve $y^5 = \bar{f}(x)$ has genus $g = 12$ and $N = 46$ affine rational points. Since $\bar{f}$ has one root in $\mathbb{F}_{11}$, we have

$$N \equiv 1 \pmod{5}.$$

The product-balanced selection gives $s = 6$, hence

$$r_1 = 40, \quad r_2 = 28.$$

By Riemann–Roch,

$$\ell(40P_\infty) = 29, \quad \ell(28P_\infty) = 17.$$

Evaluating the monomial bases at the 46 affine rational points gives

$$C_1 = [46, 29, \geq 6]_{11}, \quad C_2 = [46, 17, \geq 18]_{11}.$$

The CSS construction therefore gives

$$Q = \llbracket 46, 12, \geq 6 \rrbracket_{11}.$$

For this example, an exact search in the relevant CSS logical set finds a word of Hamming weight 6. Hence the designed lower bound is sharp and

$$Q = \llbracket 46, 12, 6 \rrbracket_{11}.$$

The Singleton defect is

$$\Delta = (46 + 2) - (12 + 2 \cdot 6) = 24 = 2g.$$

9.3.2 A C_7 curve over $\mathbb{F}_{11}$

From the C_7 class, take

$$\bar{f}(x) = x^7 + 10x^6 + x^5 + 9x^4 + 8x^3 + 10x^2 + 3x + 1 \in \mathbb{F}_{11}[x].$$

The corresponding curve has $N = 35$ affine rational points. Here $\bar{f}$ has no roots in $\mathbb{F}_{11}$, so

$$N \equiv 0 \pmod{5}.$$

Equivalently, all affine rational points occur in free orbits under the automorphism $\sigma : (x, y) \mapsto (x, \zeta_5 y)$.

With

$$s = 3, \quad r_1 = 32, \quad r_2 = 25,$$

the Riemann–Roch dimensions are

$$\ell(32P_\infty) = 21, \quad \ell(25P_\infty) = 14.$$

Thus

$$C_1 = [35, 21, \geq 3]_{11}, \quad C_2 = [35, 14, \geq 10]_{11},$$

and the associated CSS code has designed parameters

$$Q = \llbracket 35, 7, \geq 3 \rrbracket_{11}.$$

The designed Singleton defect is

$$(35 + 2) - (7 + 2 \cdot 3) = 24 = 2g.$$

Compared to the S_7 example above, the smaller point count $N = 35$ gives a smaller range of admissible CSS parameters.

9.3.3 An S_7 curve over $\mathbb{F}_{61}$

Over $\mathbb{F}_{61}$, take

$$\bar{f}(x) = x^7 + 60x^6 + 60x^5 + 3x^4 + 53x^3 + 52x^2 + 56x + 60.$$

The curve $y^5 = \bar{f}(x)$ has $N = 145$ affine rational points, the largest point count found in the database for this family. The balanced selection with $s = 31$ gives

$$r_1 = 114, \quad r_2 = 53.$$

Therefore

$$\ell(114P_\infty) = 103, \quad \ell(53P_\infty) = 42,$$

and the resulting codes have designed parameters

$$C_1 = [145, 103, \geq 31]_{61}, \quad C_2 = [145, 42, \geq 92]_{61}.$$

The CSS code is

$$Q = \llbracket 145, 61, \geq 31 \rrbracket_{61}.$$

The designed Singleton defect is

$$(145 + 2) - (61 + 2 \cdot 31) = 24 = 2g.$$

9.4 Computational verification

We now record the computational check used for the examples in this chapter. The purpose is not to introduce a new construction, but to apply the one-point AG/CSS construction of Chapter 5 to the septic database of Chapter 6.

For each irreducible septic polynomial in the database, and for each

$$q \in \{11, 31, 61\},$$

we reduced the polynomial modulo q and considered the superelliptic curve

$$X_{f,5}/\mathbb{F}_q : \quad y^5 = \bar{f}(x).$$

Only rows satisfying the hypotheses of the construction were retained: $\bar{f}$ had to be squarefree of degree 7, the weighted model had to be admissible, the evaluation set had to be nonempty, and the chosen pair of one-point AG codes had to satisfy the CSS compatibility condition.

The numbers of retained CSS rows were

q	retained CSS rows
11	871,090
31	1,144,311
61	1,168,425

Thus the computation produced

$$3,183,826$$

retained CSS rows in total.

For every retained row, the curve is of the fixed type considered throughout this chapter. Hence $g = 12$, the point P_∞ is unique, and the Riemann–Roch bases used for evaluation are the monomial bases described in Section 9.1.3.

For each retained row, the stored code data consist of

$$N, \quad r_1, \quad r_2, \quad C_1, \quad C_2, \quad Q = \llbracket N, k_Q, \geq d_Q \rrbracket_q.$$

Here $N = |X_{f,5}(\mathbb{F}_q)|$ is the affine rational point count and hence the length of the evaluation code. The integers r_1 and r_2 determine the one-point divisors

$$r_1 P_\infty, \quad r_2 P_\infty.$$

The dimensions and designed distances are then obtained from the Riemann–Roch and Goppa bounds, and the quantum parameters come from the CSS construction.

The main variable in the computation is therefore the rational point count N . Once N , $g = 12$, r_1 , and r_2 are fixed, the designed code parameters follow from the formulas in Sections 9.1.4 and 9.2. In this sense, the computation serves as a consistency check and as a source of explicit examples, not as a replacement for the theoretical construction.

9.5 The Galois group and the code parameters

We now relate the Galois data of the original polynomial to the code parameters obtained after reduction modulo q . The Galois group $\text{Gal}_{\mathbb{Q}}(f)$ is attached to the rational polynomial f , while the code parameters are attached to the reduced curve

$$X_{f,5}/\mathbb{F}_q : \quad y^5 = \bar{f}(x).$$

Thus the Galois group does not determine a single code parameter triple. Rather, it organizes the data into arithmetic Galois classes. Within each class, the main geometric quantity affecting the code is the rational point count

$$N = |X_{f,5}(\mathbb{F}_q)|.$$

Since $5 \mid (q - 1)$ for $q = 11, 31, 61$, every nonzero element of $\mathbb{F}_q$ has either no fifth root or exactly five fifth roots. Hence the affine point count is

$$N = r_f + 5 \cdot \#\{a \in \mathbb{F}_q : \bar{f}(a) \neq 0 \text{ and } \bar{f}(a) \in (\mathbb{F}_q^\times)^5\},$$

where

$$r_f = \#\{a \in \mathbb{F}_q : \bar{f}(a) = 0\}.$$

In particular,

$$N \equiv r_f \pmod{5}.$$

The value of r_f is the number of linear factors of $\bar{f}$ over $\mathbb{F}_q$. Equivalently, it is the number of fixed points of the corresponding Frobenius element acting on the seven roots of f . This gives the connection between the Galois-theoretic data of Chapter 7 and the point count used in the code construction.

Table 9.1 records, for each Galois group and each field, the best code found in the scan. Here “best” means largest value of the designed product

$$k_Q d_{\text{des}},$$

among the retained product-balanced CSS rows, where d_{des} is the designed distance lower bound.

Table 9.1: Best designed CSS codes by Galois group and field for the $m = 5$, $g = 12$ family in $\mathbb{P}(5, 7, 5)$, using the product-balanced selection.

$\text{Gal}(f)$	$q = 11$	$q = 31$	$q = 61$
S_7	$\llbracket 46, 12, \geq 6 \rrbracket$	$\llbracket 95, 37, \geq 18 \rrbracket$	$\llbracket 145, 61, \geq 31 \rrbracket$
A_7	$\llbracket 45, 11, \geq 6 \rrbracket$	$\llbracket 80, 28, \geq 15 \rrbracket$	$\llbracket 131, 55, \geq 27 \rrbracket$
$\text{PSL}(3, 2)$	$\llbracket 45, 11, \geq 6 \rrbracket$	$\llbracket 91, 35, \geq 17 \rrbracket$	$\llbracket 130, 54, \geq 27 \rrbracket$
$C_7 \rtimes C_3$	$\llbracket 31, 5, \geq 2 \rrbracket$	$\llbracket 76, 26, \geq 14 \rrbracket$	$\llbracket 121, 49, \geq 25 \rrbracket$
D_7	$\llbracket 36, 6, \geq 4 \rrbracket$	$\llbracket 80, 28, \geq 15 \rrbracket$	$\llbracket 111, 45, \geq 22 \rrbracket$
$C_7 \rtimes C_6$	$\llbracket 31, 5, \geq 2 \rrbracket$	$\llbracket 66, 22, \geq 11 \rrbracket$	$\llbracket 111, 45, \geq 22 \rrbracket$
C_7	$\llbracket 35, 7, \geq 3 \rrbracket$	$\llbracket 60, 18, \geq 10 \rrbracket$	$\llbracket 105, 41, \geq 21 \rrbracket$

The table should be read as a summary of the best examples found in the database, not as a classification theorem. The visible pattern is that the largest code parameters occur for rows with larger point count N . This is consistent with

the construction: after $g = 12$ is fixed, larger N permits larger admissible choices of r_1 and r_2 , and hence larger values of k_Q and d_{des} under the product-balanced selection. The Galois group enters through the splitting behaviour of $\bar{f}$, which affects the possible values of N inside each class.

9.5.1 The congruence of the point count modulo 5

The congruence

$$N \equiv r_f \pmod{5}, \quad r_f = \#\{a \in \mathbb{F}_q : \bar{f}(a) = 0\},$$

comes from the cyclic automorphism

$$\sigma : (x, y) \mapsto (x, \zeta_5 y).$$

The fixed affine points of σ are exactly the points with $y = 0$, and these are counted by r_f . All other affine rational points occur in orbits of size 5.

Table 9.2 records the distribution of $N \bmod 5$ over $\mathbb{F}_{61}$. The table is included to show how the Galois classes influence the possible point counts through the number of linear factors of $\bar{f}$.

Table 9.2: Distribution of $N \bmod 5$ by Galois group over $\mathbb{F}_{61}$.

$\text{Gal}(f)$	$N \equiv 0$	$N \equiv 1$	$N \equiv 2$	$N \equiv 3$	$N \equiv 4$	Total
S_7	459,623	382,029	154,478	41,330	7,203	1,044,663
A_7	22,043	20,307	10,280	1,718	1,070	55,418
$\text{PSL}(3, 2)$	12,761	23,334	103	3,940	0	40,138
$C_7 \rtimes C_3$	24	24,369	2	0	0	24,395
D_7	968	1,008	88	0	0	2,064
$C_7 \rtimes C_6$	52	1,451	1	0	0	1,504
C_7	212	0	31	0	0	243

The most visible feature is the concentration of the $C_7 \rtimes C_3$ class: over $\mathbb{F}_{61}$, almost all retained rows satisfy

$$N \equiv 1 \pmod{5}.$$

Equivalently, for most of these reductions, $\bar{f}$ has exactly one linear factor over $\mathbb{F}_{61}$. This is consistent with the permutation action of $C_7 \rtimes C_3$ on the seven roots, where elements with one fixed point occur naturally. We use this only as evidence of the arithmetic organization of the database, not as a classification theorem.

The construction of this chapter connects the septic database of Chapter 6 to quantum codes through the superelliptic curve theory of Chapter 3, the weighted projective model of Chapter 4, and the CSS construction of Chapter 5. For each retained polynomial, the reduction modulo q gives a curve

$$X_{f,5}/\mathbb{F}_q : \quad y^5 = \bar{f}(x),$$

and the rational point count $N = |X_{f,5}(\mathbb{F}_q)|$ determines the length of the evaluation codes. The one-point divisors $r_1 P_\infty$ and $r_2 P_\infty$ then determine the AG and CSS parameters.

The strongest designed code found in the scan is

$$[[145, 61, \geq 31]]_{61},$$

arising from an S_7 -type polynomial over $\mathbb{F}_{61}$ with $N = 145$ affine rational points.

The Galois group does not determine the code parameters by itself, but it organizes the reductions into arithmetic classes. Through the factorization of $\bar{f}(x)$ over $\mathbb{F}_q$, it influences the possible values of N , and hence the range of code parameters obtained from the construction.

9.6 Open problems

The computations in this thesis suggest several directions for further work. We state them as concrete problems which remain within the framework developed above.

1. **Higher-degree Galois databases.** This thesis focused on irreducible septic polynomials because degree 7 already contains cyclic, dihedral, Frobenius, exceptional, alternating, and symmetric Galois groups, while the exact resolvent computations remain feasible on a large database. A natural next step is to construct analogous labelled databases in degrees 8, 9, and 10.

The main difficulty is that the number of transitive subgroups and the size of the relevant resolvents grow quickly with the degree. Thus one should not begin with a full analogue of the septic database in very high degree. A more realistic problem is to build selected height-bounded and constructed families in degrees 8, 9, and 10, compute exact Galois labels using trusted algebraic methods, and compare the resulting distributions with the degree-7 case. This would test how much of the septic behaviour is special to degree 7, and how much persists in nearby degrees. General algorithms for computing Galois groups are available beyond degree 7, but their cost increases rapidly [9].

2. **Invariant features beyond binary septics.** In Chapter 8, the feature set used the transvection invariants $j_0, \dots, j_4$ of binary septic forms. These invariants gave a modest but visible improvement when combined with the discriminant. For higher-degree polynomials, one needs an analogue of this invariant layer.

The problem is to determine which computable invariant-theoretic quantities are most useful for Galois classification in each degree. One possibility is to

use classical invariants of binary forms of degree n , where explicit systems are known in small degrees [11]. Another possibility is to use degree-independent arithmetic features, such as discriminant data, factorization statistics modulo primes, signatures, and selected resolvent factorization patterns. The goal is not to replace the exact Galois computation, but to build a fast filtering method which identifies promising rare examples for exact verification.

3. **Exact minimum distances of the quantum codes.** Most code parameters in Chapter 9 are designed parameters obtained from the Goppa bound and the CSS construction. For example, a code written as

$$[[N, k, \geq d]]_q$$

has designed distance at least d , but its exact minimum distance may be larger.

A natural computational problem is to determine the exact minimum distances for the best codes in Table 9.1. This requires searching the relevant logical codewords in the CSS construction, not just computing Riemann–Roch dimensions. The first cases to test are the moderate-length codes over $\mathbb{F}_{11}$ and $\mathbb{F}_{31}$, where exact distance computations are more feasible. This would show which of the designed bounds are sharp and which codes improve beyond the lower bound.

4. **Point-count distributions inside Galois classes.** The Galois group of $f \in \mathbb{Q}[x]$ does not determine the code parameters after reduction modulo q , but it organizes the possible splitting types of $\bar{f}(x)$. Since the code length is the affine point count

$$N = |X_{f,5}(\mathbb{F}_q)|,$$

understanding the distribution of N inside each Galois class is a natural arithmetic problem.

For $q \equiv 1 \pmod{5}$, the congruence

$$N \equiv r_f \pmod{5}$$

relates the point count to the number r_f of linear factors of $\bar{f}$ over $\mathbb{F}_q$.

Equivalently, r_f is the number of fixed points of the Frobenius element acting on the seven roots. A precise problem is to compare the observed point-count distributions with the fixed-point statistics of conjugacy classes in each transitive subgroup of S_7 . This would make the connection between the Galois data of Chapter 7 and the code parameters of Chapter 9 more explicit.

5. **Equivalence of weighted models.** In Chapter 9, the curves were represented in the weighted projective model

$$Y^5 = F_7(X, Z) \subset \mathbb{P}(5, 7, 5).$$

This model is adapted to the pole orders of x and y at the point at infinity. However, the same curve may admit other weighted presentations, for example models closer to $\mathbb{P}(5, 7, 1)$.

The problem is to determine when two weighted presentations of the same superelliptic curve give monomially equivalent evaluation codes, and when they only give codes with the same numerical parameters. This requires comparing the graded coordinate rings, the chosen evaluation sets, and the monomial bases of the Riemann–Roch spaces. This question belongs naturally to the weighted Riemann–Roch and weighted AG-code framework developed in Chapters 4 and 5.

6. **Other superelliptic exponents.** This thesis used the family

$$y^5 = \bar{f}(x), \quad \deg f = 7.$$

The same database can be used to study curves

$$y^m = \bar{f}(x), \quad \gcd(m, 7) = 1.$$

The first natural comparisons are $m = 2$ and $m = 3$, because they give smaller genus than the $m = 5$ case. The genus formula gives

$$g = \frac{(m-1)(7-1)}{2} = 3(m-1),$$

and the pole semigroup at infinity is

$$H(P_\infty) = \langle m, 7 \rangle.$$

This would separate two effects which are combined in Chapter 9: the arithmetic of the septic polynomial f , and the geometry of the cyclic cover $y^m = \bar{f}(x)$. Comparing several values of m over the same database would show how the exponent changes the point counts, the Riemann–Roch bases, and the resulting CSS parameters.

7. **Automorphisms and decoding structure.** Every curve

$$y^5 = \bar{f}(x)$$

used in Chapter 9 has the automorphism

$$(x, y) \mapsto (x, \zeta_5 y).$$

This automorphism partitions the affine rational points into fixed points and orbits of size 5, and it explains the congruence $N \equiv r_f \pmod{5}$. A further problem is to use this symmetry more directly in the code construction.

One direction is to study how the automorphism group of the curve acts on the evaluation code and on the associated CSS code. This may lead to more structured generator matrices, faster equivalence testing, or decoding methods which exploit orbit decompositions. In the best cases, the automorphism group could provide a way to reduce the cost of exact distance computations.

APPENDIX A
COEFFICIENT FORMULAS

The following expressions give the coefficients $e_1, \dots, e_{35}$ in terms of the parameters $a_0, a_1, \dots, a_6$.

$$e_1 = -15a_6 \quad (\text{A.1})$$

$$e_2 = 105a_6^2 + 10a_5 \quad (\text{A.2})$$

$$e_3 = -455a_6^3 - 140a_5a_6 - 2a_4 \quad (\text{A.3})$$

$$e_4 = 1365a_6^4 + 910a_5a_6^2 + 32a_4a_6 + 45a_5^2 - 8a_3 \quad (\text{A.4})$$

$$e_5 = -3003a_6^5 - 3640a_5a_6^3 - 234a_4a_6^2 - 585a_5^2a_6 + 102a_3a_6 - 18a_4a_5 + 10a_2 \quad (\text{A.5})$$

$$e_6 = 5005a_6^6 + 10010a_5a_6^4 + 1040a_4a_6^3 + 3510a_5^2a_6^2 - 600a_3a_6^2 + 270a_4a_5a_6 + 120a_5^3 - 140a_2a_6 - 60a_3a_5 - 3a_4^2 + 40a_1 \quad (\text{A.6})$$

$$e_7 = -6435a_6^7 - 20020a_5a_6^5 - 3146a_4a_6^4 - 12870a_5^2a_6^3 + 2156a_3a_6^3 - 1836a_4a_5a_6^2 - 1440a_5^3a_6 + 904a_2a_6^2 + 702a_3a_5a_6 + 27a_4^2a_6 - 72a_4a_5^2 - 454a_1a_6 + 72a_2a_5 + 18a_3a_4 - 302a_0 \quad (\text{A.7})$$

$$e_8 = 6435a_6^8 + 30030a_5a_6^6 + 6864a_4a_6^5 + 32175a_5^2a_6^4 - 5280a_3a_6^4 + 7524a_4a_5a_6^3 + 7920a_5^3a_6^2 - 3570a_2a_6^3 - 3759a_3a_5a_6^2 - 84a_4^2a_6^2 + 1008a_4a_5^2a_6 + 210a_5^4 + 2370a_1a_6^2 - 951a_2a_5a_6 - 237a_3a_4a_6 - 192a_3a_5^2 - 24a_4^2a_5 + 3624a_0a_6 + 222a_1a_5 + 33a_2a_4 + 6a_3^2 \quad (\text{A.8})$$

$$\begin{aligned}
e_9 = & -5005a_6^9 - 34320a_5a_6^7 - 11154a_4a_6^6 - 57915a_5^2a_6^5 \\
& + 9306a_3a_6^5 - 20790a_4a_5a_6^4 - 26400a_5^3a_6^3 + 9636a_2a_6^4 \\
& + 12177a_3a_5a_6^3 - 6336a_4a_5^2a_6^2 - 2310a_5^4a_6 - 7536a_1a_6^3 \\
& + 5739a_2a_5a_6^2 + 1413a_3a_4a_6^2 + 2040a_3a_5^2a_6 + 192a_4^2a_5a_6 \\
& - 168a_4a_5^3 - 20310a_0a_6^2 - 2234a_1a_5a_6 - 307a_2a_4a_6 \\
& + 224a_2a_5^2 - 48a_3^2a_6 + 120a_3a_4a_5 + 8a_4^3 \\
& - 1534a_0a_5 - 162a_1a_4 - 50a_2a_3
\end{aligned} \tag{A.9}$$

$$\begin{aligned}
e_{10} = & 3003a_6^{10} + 30030a_5a_6^8 + 13728a_4a_6^7 + 77220a_5^2a_6^6 \\
& - 12144a_3a_6^6 + 40986a_4a_5a_6^5 + 59400a_5^3a_6^4 - 18810a_2a_6^5 \\
& - 26565a_3a_5a_6^4 + 825a_4^2a_6^4 + 23760a_4a_5^2a_6^3 + 11550a_5^4a_6^2 \\
& + 16290a_1a_6^4 - 20973a_2a_5a_6^3 - 5055a_3a_4a_6^3 - 9816a_3a_5^2a_6^2 \\
& - 480a_4^2a_5a_6^2 + 2184a_4a_5^3a_6 + 252a_5^5 + 70524a_0a_6^3 \\
& + 10194a_1a_5a_6^2 + 1191a_2a_4a_6^2 - 2796a_2a_5^2a_6 + 135a_3^2a_6^2 \\
& - 1452a_3a_4a_5a_6 - 336a_3a_5^3 - 96a_4^3a_6 - 84a_4^2a_5^2 \\
& + 17196a_0a_5a_6 + 1716a_1a_4a_6 + 480a_1a_5^2 + 618a_2a_3a_6 \\
& + 228a_2a_4a_5 + 12a_3a_4^2 - 144a_0a_4 + 192a_1a_3 \\
& - 135a_2^2
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
e_{11} = & -1365a_6^{11} - 20020a_5a_6^9 - 12870a_4a_6^8 - 77220a_5^2a_6^7 \\
& + 11880a_3a_6^7 - 59400a_4a_5a_6^6 - 95040a_5^3a_6^5 + 27390a_2a_6^6 \\
& + 41085a_3a_5a_6^5 - 3069a_4^2a_6^5 - 59400a_4a_5^2a_6^4 - 34650a_5^4a_6^3 \\
& - 25290a_1a_6^5 + 51825a_2a_5a_6^4 + 12105a_3a_4a_6^4 + 28200a_3a_5^2a_6^3 \\
& - 480a_4^2a_5a_6^3 - 12600a_4a_5^3a_6^2 - 2520a_5^5a_6 - 169632a_0a_6^4 \\
& - 27828a_1a_5a_6^3 - 2265a_2a_4a_6^3 + 15738a_2a_5^2a_6^2 - 27a_3^2a_6^3 \\
& + 7866a_3a_4a_5a_6^2 + 3192a_3a_5^3a_6 + 504a_4^3a_6^2 + 588a_4^2a_5^2a_6 \\
& - 252a_4a_5^4 - 89934a_0a_5a_6^2 - 8292a_1a_4a_6^2 - 4064a_1a_5^2a_6 \\
& - 3426a_2a_3a_6^2 - 1912a_2a_4a_5a_6 + 392a_2a_5^3 + 120a_3^2a_5a_6 \\
& - 48a_3a_4^2a_6 + 336a_3a_4a_5^2 + 56a_4^3a_5 + 1704a_0a_4a_6 \\
& - 3262a_0a_5^2 - 2292a_1a_3a_6 - 858a_1a_4a_5 + 1263a_2^2a_6 \\
& - 230a_2a_3a_5 - 108a_2a_4^2 - 30a_3^2a_4 - 282a_0a_3 \\
& + 642a_1a_2
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
e_{12} = & 455a_6^{12} + 10010a_5a_6^{10} + 9152a_4a_6^9 + 57915a_5^2a_6^8 \\
& - 8712a_3a_6^8 + 64152a_4a_5a_6^7 + 110880a_5^3a_6^6 - 30228a_2a_6^7 \\
& - 46134a_3a_5a_6^6 + 6336a_4^2a_6^6 + 104544a_4a_5^2a_6^5 + 69300a_5^4a_6^4 \\
& + 29028a_1a_6^6 - 91494a_2a_5a_6^5 - 20466a_3a_4a_6^5 - 53640a_3a_5^2a_6^4 \\
& + 6120a_4^2a_5a_6^4 + 42840a_4a_5^3a_6^3 + 11340a_5^5a_6^2 + 299118a_0a_6^5 \\
& + 50431a_1a_5a_6^4 + 1088a_2a_4a_6^4 - 52951a_2a_5^2a_6^3 - 885a_3^2a_6^4 \\
& - 25239a_3a_4a_5a_6^3 - 13524a_3a_5^3a_6^2 - 1516a_4^3a_6^3 - 1092a_4^2a_5^2a_6^2 \\
& + 3024a_4a_5^4a_6 + 210a_5^6 + 290729a_0a_5a_6^3 + 24183a_1a_4a_6^3 \\
& + 14992a_1a_5^2a_6^2 + 11308a_2a_3a_6^3 + 6503a_2a_4a_5a_6^2 - 4648a_2a_5^3a_6 \\
& - 1200a_3^2a_5a_6^2 - 189a_3a_4^2a_6^2 - 3696a_3a_4a_5^2a_6 - 336a_3a_5^4 \\
& - 616a_4^3a_5a_6 - 168a_4^2a_5^3 - 8709a_0a_4a_6^2 + 34457a_0a_5^2a_6 \\
& + 12456a_1a_3a_6^2 + 8011a_1a_4a_5a_6 + 456a_1a_5^3 - 5343a_2^2a_6^2 \\
& + 2725a_2a_3a_5a_6 + 1154a_2a_4^2a_6 + 680a_2a_4a_5^2 + 285a_3^2a_4a_6 \\
& - 120a_3^2a_5^2 + 60a_3a_4^2a_5 + 2a_4^4 + 1710a_0a_3a_6 \\
& - 1267a_0a_4a_5 - 6078a_1a_2a_6 + 1194a_1a_3a_5 + 219a_1a_4^2 \\
& - 663a_2^2a_5 - 185a_2a_3a_4 + 60a_3^3 + 2778a_0a_2 \\
& - 1614a_1^2
\end{aligned} \tag{A.12}$$

$$\begin{aligned}
e_{13} = & -105a_6^{13} - 3640a_5a_6^{11} - 4862a_4a_6^{10} - 32175a_5^2a_6^9 \\
& + 4730a_3a_6^9 - 51678a_4a_5a_6^8 - 95040a_5^3a_6^7 + 25410a_2a_6^8 \\
& + 37818a_3a_5a_6^7 - 8712a_4^2a_6^7 - 133056a_4a_5^2a_6^6 - 97020a_5^4a_6^5 \\
& - 24960a_1a_6^7 + 118782a_2a_5a_6^6 + 25074a_3a_4a_6^6 + 70704a_3a_5^2a_6^5 \\
& - 18432a_4^2a_5a_6^5 - 95760a_4a_5^3a_6^4 - 30240a_5^5a_6^3 - 398814a_0a_6^6 \\
& - 63513a_1a_5a_6^5 + 5112a_2a_4a_6^5 + 118809a_2a_5^2a_6^4 + 2943a_3^2a_6^5 \\
& + 53361a_3a_4a_5a_6^4 + 33516a_3a_5^3a_6^3 + 2844a_4^3a_6^4 - 2772a_4^2a_5^2a_6^3 \\
& - 15876a_4a_5^4a_6^2 - 1890a_5^6a_6 - 648159a_0a_5a_6^4 - 47445a_1a_4a_6^4 \\
& - 30950a_1a_5^2a_6^3 - 24816a_2a_3a_6^4 - 9973a_2a_4a_5a_6^3 + 24374a_2a_5^3a_6^2 \\
& + 5418a_3^2a_5a_6^3 + 1875a_3a_4^2a_6^3 + 17934a_3a_4a_5^2a_6^2 + 2772a_3a_5^4a_6 \\
& + 2912a_4^3a_5a_6^2 + 1008a_4^2a_5^3a_6 - 252a_4a_5^5 + 25095a_0a_4a_6^3 \\
& - 169777a_0a_5^2a_6^2 - 40896a_1a_3a_6^3 - 33613a_1a_4a_5a_6^2 - 2592a_1a_5^3a_6 \\
& + 13473a_2^2a_6^3 - 14075a_2a_3a_5a_6^2 - 5474a_2a_4^2a_6^2 - 5084a_2a_4a_5^2a_6 \\
& + 420a_2a_5^4 - 1149a_3^2a_4a_6^2 + 1416a_3^2a_5^2a_6 - 120a_3a_4^2a_5a_6 \\
& + 504a_3a_4a_5^3 + 10a_4^4a_6 + 168a_4^3a_5^2 - 2544a_0a_3a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 14017a_0a_4a_5a_6 - 3930a_0a_5^3 + 26316a_1a_2a_6^2 - 12842a_1a_3a_5a_6 \\
& - 2229a_1a_4^2a_6 - 1878a_1a_4a_5^2 + 5517a_2^2a_5a_6 + 1267a_2a_3a_4a_6 \\
& - 394a_2a_3a_5^2 - 600a_2a_4^2a_5 - 570a_3^3a_6 - 90a_3^2a_4a_5 \\
& - 52a_3a_4^3 - 26610a_0a_2a_6 - 424a_0a_3a_5 - 900a_0a_4^2 \\
& + 16296a_1^2a_6 + 2064a_1a_2a_5 + 348a_1a_3a_4 + 600a_2^2a_4 \\
& - 50a_2a_3^2 - 2706a_0a_1
\end{aligned} \tag{A.13}$$

$$\begin{aligned}
e_{14} = & 15a_6^{14} + 910a_5a_6^{12} + 1872a_4a_6^{11} + 12870a_5^2a_6^{10} \\
& - 1848a_3a_6^{10} + 30690a_4a_5a_6^9 + 59400a_5^3a_6^8 - 16192a_2a_6^9 \\
& - 22374a_3a_5a_6^8 + 8415a_4^2a_6^8 + 123552a_4a_5^2a_6^7 + 97020a_5^4a_6^6 \\
& + 16092a_1a_6^8 - 114858a_2a_5a_6^7 - 22446a_3a_4a_6^7 - 65520a_3a_5^2a_6^6 \\
& + 32256a_4^2a_5a_6^6 + 148176a_4a_5^3a_6^5 + 52920a_5^5a_6^4 + 408384a_0a_6^7 \\
& + 56304a_1a_5a_6^6 - 14778a_2a_4a_6^6 - 187524a_2a_5^2a_6^5 - 5274a_3^2a_6^6 \\
& - 78084a_3a_4a_5a_6^5 - 53424a_3a_5^3a_6^4 - 3312a_4^3a_6^5 + 18648a_4^2a_5^2a_6^4 \\
& + 48384a_4a_5^4a_6^3 + 7560a_5^6a_6^2 + 1051818a_0a_5a_6^5 + 66036a_1a_4a_5^5 \\
& + 37091a_1a_5^2a_6^4 + 38220a_2a_3a_6^5 - 278a_2a_4a_5a_6^4 - 75047a_2a_5^3a_6^3 \\
& - 14565a_3^2a_5a_6^4 - 6396a_3a_4^2a_6^4 - 50631a_3a_4a_5^2a_6^3 - 9912a_3a_5^4a_6^2 \\
& - 7700a_4^3a_5a_6^3 - 1176a_4^2a_5^3a_6^2 + 2772a_4a_5^5a_6 + 120a_5^7 \\
& - 43716a_0a_4a_6^4 + 515287a_0a_5^2a_6^3 + 90681a_1a_3a_6^4 + 83517a_1a_4a_5a_6^3 \\
& + 3960a_1a_5^3a_6^2 - 22446a_2^2a_6^4 + 42245a_2a_3a_5a_6^3 + 15168a_2a_4^2a_6^3 \\
& + 14859a_2a_4a_5^2a_6^2 - 4788a_2a_5^4a_6 + 2469a_3^2a_4a_6^3 - 7404a_3^2a_5^2a_6^2 \\
& - 1653a_3a_4^2a_5a_6^2 - 4956a_3a_4a_5^3a_6 - 168a_3a_5^5 - 186a_4^4a_6^2 \\
& - 1680a_4^3a_5^2a_6 - 210a_4^2a_5^4 - 8277a_0a_3a_6^3 - 66519a_0a_4a_5a_6^2 \\
& + 39825a_0a_5^3a_6 - 68949a_1a_2a_6^3 + 62532a_1a_3a_5a_6^2 + 10182a_1a_4^2a_6^2 \\
& + 14931a_1a_4a_5^2a_6 + 48a_1a_5^4 - 20472a_2^2a_5a_6^2 - 3195a_2a_3a_4a_6^2 \\
& + 4629a_2a_3a_5^2a_6 + 5862a_2a_4^2a_5a_6 + 1140a_2a_4a_5^3 + 2475a_3^3a_6^2 \\
& + 645a_3^2a_4a_5a_6 - 384a_3^2a_5^3 + 468a_3a_4^3a_6 + 108a_3a_4^2a_5^2 \\
& + 12a_4^4a_5 + 118521a_0a_2a_6^2 - 2307a_0a_3a_5a_6 + 6192a_0a_4^2a_6 \\
& - 3879a_0a_4a_5^2 - 76560a_1^2a_6^2 - 16868a_1a_2a_5a_6 - 2121a_1a_3a_4a_6 \\
& + 2970a_1a_3a_5^2 + 1143a_1a_4^2a_5 - 5470a_2^2a_4a_6 - 1369a_2^2a_5^2 \\
& + 390a_2a_3^2a_6 - 825a_2a_3a_4a_5 - 28a_2a_4^3 + 300a_3^3a_5 \\
& + 30a_3^2a_4^2 + 24420a_0a_1a_6 + 8021a_0a_2a_5 + 4014a_0a_3a_4 \\
& - 4686a_1^2a_5 - 1485a_1a_2a_4 - 960a_1a_3^2 + 415a_2^2a_3 \\
& + 3828a_0^2
\end{aligned} \tag{A.14}$$

$$\begin{aligned}
e_{15} = & -a_6^{15} - 140a_5a_6^{13} - 494a_4a_6^{12} - 3510a_5^2a_6^{11} \\
& + 492a_3a_6^{11} - 13068a_4a_5a_6^{10} - 26400a_5^3a_6^9 + 7700a_2a_6^{10} \\
& + 9240a_3a_5a_6^9 - 5775a_4^2a_6^9 - 83160a_4a_5^2a_6^8 - 69300a_5^4a_6^7 \\
& - 7690a_1a_6^9 + 82746a_2a_5a_6^8 + 14580a_3a_4a_6^8 + 42192a_3a_5^2a_6^7 \\
& - 37440a_4^2a_5a_6^7 - 162288a_4a_5^3a_6^6 - 63504a_5^5a_6^5 - 322614a_0a_6^8 \\
& - 34452a_1a_5a_6^7 + 21366a_2a_4a_6^7 + 213696a_2a_5^2a_6^6 + 6210a_3^2a_6^7 \\
& + 80640a_3a_4a_5a_6^6 + 56448a_3a_5^3a_6^5 + 2016a_4^3a_6^6 - 45864a_4^2a_5^2a_6^5 \\
& - 95256a_4a_5^4a_6^4 - 17640a_5^6a_6^3 - 1276968a_0a_5a_6^6 - 66948a_1a_4a_6^6 \\
& - 20608a_1a_5^2a_6^5 - 42420a_2a_3a_6^6 + 31276a_2a_4a_5a_6^5 + 151312a_2a_5^3a_6^4 \\
& + 25872a_3^2a_5a_6^5 + 12684a_3a_4^2a_6^5 + 92064a_3a_4a_5^2a_6^4 + 19824a_3a_5^4a_6^3 \\
& + 12208a_4^3a_5a_6^4 - 6720a_4^2a_5^3a_6^3 - 13104a_4a_5^5a_6^2 - 960a_5^7a_6 \\
& + 42954a_0a_4a_6^5 - 1070554a_0a_5^2a_6^4 - 143655a_1a_3a_6^5 - 136131a_1a_4a_5a_6^4 \\
& + 7542a_1a_5^3a_6^3 + 25822a_2^2a_6^5 - 82307a_2a_3a_5a_6^4 - 27090a_2a_4^2a_6^4 \\
& - 17063a_2a_4a_5^2a_6^3 + 23394a_2a_5^4a_6^2 - 2676a_3^2a_4a_6^4 + 22698a_3^2a_5^2a_6^3 \\
& + 10281a_3a_4^2a_5a_6^3 + 21042a_3a_4a_5^3a_6^2 + 1092a_3a_5^5a_6 + 874a_4^4a_6^3 \\
& + 7056a_4^3a_5^2a_6^2 + 1050a_4^2a_5^4a_6 - 168a_4a_5^6 + 45651a_0a_3a_6^4 \\
& + 178019a_0a_4a_5a_6^3 - 187648a_0a_5^3a_6^2 + 121841a_1a_2a_6^4 - 182692a_1a_3a_5a_6^3 \\
& - 27625a_1a_4^2a_6^3 - 51742a_1a_4a_5^2a_6^2 + 1604a_1a_5^4a_6 + 44441a_2^2a_5a_6^3 \\
& + 2039a_2a_3a_4a_6^3 - 22358a_2a_3a_5^2a_6^2 - 24995a_2a_4^2a_5a_6^2 - 7496a_2a_4a_5^3a_6 \\
& + 280a_2a_5^5 - 6478a_3^3a_6^3 - 1500a_3^2a_4a_5a_6^2 + 3576a_3^2a_5^3a_6 \\
& - 1767a_3a_4^3a_6^2 + 144a_3a_4^2a_5^2a_6 + 420a_3a_4a_5^4 + 72a_4^4a_5a_6 \\
& + 280a_4^3a_5^3 - 324905a_0a_2a_6^3 + 36323a_0a_3a_5a_6^2 - 16547a_0a_4^2a_6^2 \\
& + 39373a_0a_4a_5^2a_6 - 3282a_0a_5^4 + 221718a_1^3a_6^3 + 61712a_1a_2a_5a_6^2 \\
& + 4001a_1a_3a_4a_6^2 - 28704a_1a_3a_5^2a_6 - 10133a_1a_4^2a_5a_6 - 2194a_1a_4a_5^3 \\
& + 22567a_2^2a_4a_6^2 + 10018a_2^2a_5^2a_6 - 1649a_2a_3^2a_6^2 + 4709a_2a_3a_4a_5a_6 \\
& - 270a_2a_3a_5^3 - 86a_2a_4^3a_6 - 1388a_2a_4^2a_5^2 - 2490a_3^3a_5a_6 \\
& - 381a_3^2a_4^2a_6 - 6a_3^2a_4a_5^2 - 240a_3a_4^3a_5 - 12a_4^5 \\
& - 101790a_0a_1a_6^2 - 73373a_0a_2a_5a_6 - 33662a_0a_3a_4a_6 + 2104a_0a_3a_5^2 \\
& - 3965a_0a_4^2a_5 + 44832a_1^2a_5a_6 + 12893a_1a_2a_4a_6 + 2358a_1a_2a_5^2 \\
& + 8378a_1a_3^2a_6 + 928a_1a_3a_4a_5 - 55a_1a_4^3 - 2700a_2^2a_3a_6 \\
& + 2281a_2^2a_4a_5 - 340a_2a_3^2a_5 + 619a_2a_3a_4^2 - 90a_3^3a_4 \\
& - 34452a_0^2a_6 - 7416a_0a_1a_5 + 4624a_0a_2a_4 - 3890a_0a_3^2 \\
& - 2238a_1^2a_4 + 1908a_1a_2a_3 - 1229a_2^3
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
e_{16} = & 10a_5a_6^{14} + 80a_4a_6^{13} + 585a_5^2a_6^{12} - 80a_3a_6^{12} \\
& + 3780a_4a_5a_6^{11} + 7920a_5^3a_6^{10} - 2650a_2a_6^{11} - 2475a_3a_5a_6^{10} \\
& + 2772a_4^2a_6^{10} + 39600a_4a_5^2a_6^9 + 34650a_5^4a_6^8 + 2650a_1a_6^{10} \\
& - 43875a_2a_5a_6^9 - 6705a_3a_4a_6^9 - 18000a_3a_5^2a_6^8 + 29880a_4^2a_5a_6^8 \\
& + 126000a_4a_5^3a_6^7 + 52920a_5^5a_6^6 + 195548a_0a_6^9 + 13392a_1a_5a_6^8 \\
& - 20007a_2a_4a_6^8 - 177282a_2a_5^2a_6^7 - 5076a_3^2a_6^8 - 58626a_3a_4a_5a_6^7 \\
& - 38808a_3a_5^3a_6^6 + 216a_4^3a_6^7 + 67032a_4^2a_5^2a_6^6 + 127008a_4a_5^4a_6^5 \\
& + 26460a_5^6a_6^4 + 1172886a_0a_5a_6^7 + 49890a_1a_4a_6^7 - 8764a_1a_5^2a_6^6 \\
& + 34224a_2a_3a_6^7 - 66374a_2a_4a_5a_6^6 - 210308a_2a_5^3a_6^5 - 31836a_3^2a_5a_6^6 \\
& - 16422a_3a_4^2a_6^6 - 112308a_3a_4a_5^2a_6^5 - 23520a_3a_5^4a_6^4 - 10976a_4^3a_5a_6^5 \\
& + 30576a_4^2a_5^3a_6^4 + 35280a_4a_5^5a_6^3 + 3360a_5^7a_6^2 - 9774a_0a_4a_6^6 \\
& + 1599628a_0a_5^2a_6^5 + 167553a_1a_3a_6^6 + 151972a_1a_4a_5a_6^5 - 41940a_1a_5^3a_6^4 \\
& - 20733a_2^2a_6^6 + 109586a_2a_3a_5a_6^5 + 32270a_2a_4^2a_6^5 - 12820a_2a_4a_5^2a_6^4 \\
& - 65415a_2a_5^4a_6^3 + 114a_3^2a_4a_6^5 - 45378a_3^2a_5^2a_6^4 - 28173a_3a_4^2a_5a_6^4 \\
& - 50547a_3a_4a_5^3a_6^3 - 2730a_3a_5^5a_6^2 - 2155a_4^4a_6^4 - 16044a_4^3a_5^2a_6^3 \\
& - 420a_4^2a_5^4a_6^2 + 1680a_4a_5^6a_6 + 45a_5^8 - 103125a_0a_3a_6^5 \\
& - 292372a_0a_4a_5a_6^4 + 539340a_0a_5^3a_6^3 - 153111a_1a_2a_6^5 + 357422a_1a_3a_5a_6^4 \\
& + 49602a_1a_4^2a_6^4 + 101292a_1a_4a_5^2a_6^3 - 13500a_1a_5^4a_6^2 - 62052a_2^2a_5a_6^4 \\
& + 7706a_2a_3a_4a_6^4 + 60154a_2a_3a_5^2a_6^3 + 60987a_2a_4^2a_5a_6^3 + 18315a_2a_4a_5^3a_6^2 \\
& - 3150a_2a_5^5a_6 + 11340a_3^3a_6^4 - 438a_3^2a_4a_5a_6^3 - 14976a_3^2a_5^3a_6^2 \\
& + 3541a_3a_4^3a_6^3 - 5049a_3a_4^2a_5^2a_6^2 - 3570a_3a_4a_5^4a_6 - 1044a_4^4a_5a_6^2 \\
& - 2520a_4^3a_5^3a_6 - 168a_4^2a_5^5 + 611373a_0a_2a_6^4 - 160634a_0a_3a_5a_6^3 \\
& + 16770a_0a_4^2a_6^3 - 168324a_0a_4a_5^2a_6^2 + 32265a_0a_5^4a_6 - 441771a_1^2a_6^4 \\
& - 132644a_1a_2a_5a_6^3 + 3402a_1a_3a_4a_6^3 + 124605a_1a_3a_5^2a_6^2 + 39711a_1a_4^2a_5a_6^2 \\
& + 13899a_1a_4a_5^3a_6 - 300a_1a_5^5 - 55672a_2^2a_4a_6^3 - 32113a_2^2a_5^2a_6^2 \\
& + 5030a_2a_3^2a_6^3 - 8520a_2a_3a_4a_5a_6^2 + 3573a_2a_3a_5^3a_6 + 1721a_2a_4^3a_6^2 \\
& + 12318a_2a_4^2a_5^2a_6 + 1170a_2a_4a_5^4 + 9345a_3^3a_5a_6^2 + 1899a_3^2a_4^2a_6^2 \\
& - 531a_3^2a_4a_5^2a_6 - 540a_3^2a_5^4 + 1884a_3a_4^3a_5a_6 + 60a_3a_4^2a_5^3 \\
& + 96a_4^5a_6 + 30a_4^4a_5^2 + 258960a_0a_1a_6^3 + 309656a_0a_2a_5a_6^2 \\
& + 126930a_0a_3a_4a_6^2 - 30807a_0a_3a_5^2a_6 + 22155a_0a_4^2a_5a_6 - 5787a_0a_4a_5^3 \\
& - 198742a_1^2a_5a_6^2 - 50912a_1a_2a_4a_6^2 - 14777a_1a_2a_5^2a_6 - 33426a_1a_3^2a_6^2 \\
& - 3048a_1a_3a_4a_5a_6 + 3786a_1a_3a_5^3 + 805a_1a_4^3a_6 + 2499a_1a_4^2a_5^2 \\
& + 7372a_2^2a_3a_6^2 - 18762a_2^2a_4a_5a_6 - 1563a_2^2a_5^3 + 2655a_2a_3^2a_5a_6 \\
& - 4902a_2a_3a_4^2a_6 - 1401a_2a_3a_4a_5^2 - 156a_2a_4^3a_5 + 825a_3^3a_4a_6
\end{aligned}$$

$$\begin{aligned}
& + 540a_3^3a_5^2 + 180a_3^2a_4^2a_5 + 12a_3a_4^4 + 142680a_0^2a_6^2 \\
& + 63980a_0a_1a_5a_6 - 36040a_0a_2a_4a_6 + 9953a_0a_2a_5^2 + 34920a_0a_3^2a_6 \\
& + 11910a_0a_3a_4a_5 + 2345a_0a_4^3 + 19572a_1^2a_4a_6 - 5781a_1^2a_5^2 \\
& - 18304a_1a_2a_3a_6 - 4974a_1a_2a_4a_5 - 2454a_1a_3^2a_5 - 1170a_1a_3a_4^2 \\
& + 9828a_2^3a_6 + 1158a_2^2a_3a_5 - 21a_2^2a_4^2 - 45a_2a_3^2a_4 \\
& - 15a_3^4 + 11600a_0^2a_5 - 5220a_0a_1a_4 - 3680a_0a_2a_3 \\
& + 624a_1^2a_3 + 2472a_1a_2^2
\end{aligned} \tag{A.16}$$

$$\begin{aligned}
e_{17} = & -6a_4a_6^{14} - 45a_5^2a_6^{13} + 6a_3a_6^{13} - 666a_4a_5a_6^{12} \\
& - 1440a_5^3a_6^{11} + 624a_2a_6^{12} + 357a_3a_5a_6^{11} - 888a_4^2a_6^{11} \\
& - 12672a_4a_5^2a_6^{10} - 11550a_5^4a_6^9 - 624a_1a_6^{11} + 16647a_2a_5a_6^{10} \\
& + 2073a_3a_4a_6^{10} + 4440a_3a_5^2a_6^9 - 16320a_4^2a_5a_6^9 - 68040a_4a_5^3a_6^8 \\
& - 30240a_5^5a_6^7 - 89466a_0a_6^{10} - 2256a_1a_5a_6^9 + 12837a_2a_4a_6^9 \\
& + 106326a_2a_5^2a_6^8 + 2910a_3^2a_6^9 + 29214a_3a_4a_5a_6^8 + 15624a_3a_5^3a_6^7 \\
& - 1584a_4^3a_6^8 - 64008a_4^2a_5^2a_6^7 - 116424a_4a_5^4a_6^6 - 26460a_5^6a_6^5 \\
& - 813936a_0a_5a_6^8 - 27168a_1a_4a_6^8 + 27380a_1a_5^2a_6^7 - 19938a_2a_3a_6^8 \\
& + 77626a_2a_4a_5a_6^7 + 205996a_2a_5^3a_6^6 + 27636a_3^2a_5a_6^7 + 14394a_3a_4^2a_6^7 \\
& + 92316a_3a_4a_5^2a_6^6 + 15288a_3a_5^4a_6^5 + 3136a_4^3a_5a_6^6 - 61152a_4^2a_5^3a_6^5 \\
& - 59976a_4a_5^5a_6^4 - 6720a_5^7a_6^3 - 35934a_0a_4a_6^7 - 1757868a_0a_5^2a_6^6 \\
& - 145911a_1a_3a_6^7 - 117124a_1a_4a_5a_6^6 + 83412a_1a_5^3a_6^5 + 11363a_2^2a_6^7 \\
& - 101766a_2a_3a_5a_6^6 - 25298a_2a_4^2a_6^6 + 72828a_2a_4a_5^2a_6^5 + 116865a_2a_5^4a_6^4 \\
& + 4074a_3^2a_4a_6^6 + 62118a_3^2a_5^2a_6^5 + 45675a_3a_4^2a_5a_6^5 + 75117a_3a_4a_5^3a_6^4 \\
& + 2646a_3a_5^5a_6^3 + 3213a_4^4a_6^5 + 20580a_4^3a_5^2a_6^4 - 8820a_4^2a_5^4a_6^3 \\
& - 7056a_4a_5^6a_6^2 - 315a_5^8a_6 + 143673a_0a_3a_6^6 + 290532a_0a_4a_5a_6^5 \\
& - 1044500a_0a_5^3a_6^4 + 140423a_1a_2a_6^6 - 494304a_1a_3a_5a_6^5 - 62088a_1a_4^2a_6^5 \\
& - 119636a_1a_4a_5^2a_6^4 + 46410a_1a_5^4a_6^3 + 57358a_2^2a_5a_6^5 - 23250a_2a_3a_4a_6^5 \\
& - 100960a_2a_3a_5^2a_6^4 - 93283a_2a_4^2a_5a_6^4 - 13461a_2a_4a_5^3a_6^3 + 14364a_2a_5^5a_6^2 \\
& - 13914a_3^3a_6^5 + 9852a_3^2a_4a_5a_6^4 + 37002a_3^2a_5^3a_6^3 - 3705a_3a_4^3a_6^4 \\
& + 21987a_3a_4^2a_5^2a_6^3 + 12600a_3a_4a_5^4a_6^2 - 168a_3a_5^6a_6 + 4200a_4^4a_5a_6^3 \\
& + 9240a_4^3a_5^3a_6^2 + 672a_4^2a_5^5a_6 - 72a_4a_5^7 - 833831a_0a_2a_6^5 \\
& + 390892a_0a_3a_5a_6^4 + 14900a_0a_4^2a_6^4 + 398668a_0a_4a_5^2a_6^3 - 145405a_0a_5^4a_6^2 \\
& + 639719a_1^2a_6^5 + 184086a_1a_2a_5a_6^4 - 31256a_1a_3a_4a_6^4 - 321607a_1a_3a_5^2a_6^3 \\
& - 90685a_1a_4^2a_5a_6^3 - 34957a_1a_4a_5^3a_6^2 + 3836a_1a_5^5a_6 + 91394a_2^2a_4a_6^4 \\
& + 58691a_2^2a_5^2a_6^3 - 11452a_2a_3^2a_6^4 - 3288a_2a_3a_4a_5a_6^3 - 16355a_2a_3a_5^3a_6^2
\end{aligned}$$

$$\begin{aligned}
& -7347a_2a_4^3a_6^3 - 46634a_2a_4^2a_5^2a_6^2 - 6650a_2a_4a_5^4a_6 + 112a_2a_6^5 \\
& - 20839a_3^3a_5a_6^3 - 5061a_3^2a_4^2a_6^3 + 4971a_3^2a_4a_5^2a_6^2 + 4200a_3^2a_5^4a_6 \\
& - 5928a_3a_4^3a_5a_6^2 + 840a_3a_4^2a_5^3a_6 + 168a_3a_4a_5^5 - 300a_4^5a_6^2 \\
& + 210a_4^4a_5^2a_6 + 280a_4^3a_5^4 - 447430a_0a_1a_6^4 - 794886a_0a_2a_5a_6^3 \\
& - 282728a_0a_3a_4a_6^3 + 167455a_0a_3a_5^2a_6^2 - 40363a_0a_4^2a_5a_6^2 + 53125a_0a_4a_5^3a_6 \\
& - 2302a_0a_5^5 + 539702a_1^2a_5a_6^3 + 120830a_1a_2a_4a_6^3 + 36519a_1a_2a_5^2a_6^2 \\
& + 80516a_1a_3^2a_6^3 - 7106a_1a_3a_4a_5a_6^2 - 32790a_1a_3a_5^3a_6 - 4233a_1a_4^3a_6^2 \\
& - 18905a_1a_4^2a_5^2a_6 - 1490a_1a_4a_5^4 - 10428a_2^2a_3a_6^3 + 69054a_2^2a_4a_5a_6^2 \\
& + 9979a_2^2a_5^3a_6 - 10753a_2a_3^2a_5a_6^2 + 16674a_2a_3a_4^2a_6^2 + 6155a_2a_3a_4a_5^2a_6 \\
& + 10a_2a_3a_5^4 - 340a_2a_4^3a_5a_6 - 1720a_2a_4^2a_5^3 - 3471a_3^3a_4a_6^2 \\
& - 3786a_3^3a_5^2a_6 - 1722a_3^2a_4^2a_5a_6 + 270a_3^2a_4a_5^3 - 228a_3a_4^4a_6 \\
& - 420a_3a_4^3a_5^2 - 60a_5^5a_5 - 363460a_0^2a_6^3 - 250960a_0a_1a_5a_6^2 \\
& + 124954a_0a_2a_4a_6^2 - 90053a_0a_2a_5^2a_6 - 143586a_0a_3^2a_6^2 - 87316a_0a_3a_4a_5a_6 \\
& + 6576a_0a_3a_5^3 - 17767a_0a_4^3a_6 - 6832a_0a_4^2a_5^2 - 77812a_1^2a_4a_6^2 \\
& + 52881a_1^2a_5^2a_6 + 80916a_1a_2a_3a_6^2 + 37864a_1a_2a_4a_5a_6 + 1124a_1a_2a_5^3 \\
& + 19214a_1a_3^2a_5a_6 + 8416a_1a_3a_4^2a_6 + 756a_1a_3a_4a_5^2 - 420a_1a_3^3a_5 \\
& - 36236a_2^3a_6^2 - 6154a_2^2a_3a_5a_6 + 1079a_2^2a_4^2a_6 + 3398a_2^2a_4a_5^2 \\
& + 1283a_2a_3^2a_4a_6 - 726a_2a_3^2a_5^2 + 2068a_2a_3a_4^2a_5 + 192a_2a_4^4 \\
& + 15a_3^4a_6 - 420a_3^3a_4a_5 + 42a_3^2a_4^3 - 83350a_0^2a_5a_6 \\
& + 37328a_0a_1a_4a_6 - 12780a_0a_1a_5^2 + 30984a_0a_2a_3a_6 + 17722a_0a_2a_4a_5 \\
& - 12198a_0a_3^2a_5 - 2324a_0a_3a_4^2 - 7530a_1^2a_3a_6 - 5574a_1^2a_4a_5 \\
& - 19216a_1a_2^2a_6 + 4156a_1a_2a_3a_5 + 642a_1a_2a_4^2 + 1022a_1a_3^2a_4 \\
& - 2410a_2^3a_5 - 1910a_2^2a_3a_4 + 250a_2a_3^3 - 15550a_0^2a_4 \\
& + 16836a_0a_1a_3 - 6800a_0a_2^2 + 102a_1^2a_2
\end{aligned} \tag{A.17}$$

$$\begin{aligned}
e_{18} = & 54a_4a_5a_6^{13} + 120a_3^3a_6^{12} - 90a_2a_6^{13} - 9a_3a_5a_6^{12} \\
& + 171a_4^2a_6^{12} + 2448a_4a_5^2a_6^{11} + 2310a_5^4a_6^{10} + 90a_1a_6^{12} \\
& - 4281a_2a_5a_6^{11} - 387a_3a_4a_6^{11} - 312a_3a_5^2a_6^{10} + 5856a_4^2a_5a_6^{10} \\
& + 24360a_4a_5^3a_6^9 + 11340a_5^5a_6^8 + 29940a_0a_6^{11} - 634a_1a_5a_6^{10} \\
& - 5645a_2a_4a_6^{10} - 44984a_2a_5^2a_6^9 - 1149a_3^2a_6^{10} - 9336a_3a_4a_5a_6^9 \\
& - 2016a_3a_5^3a_6^8 + 1456a_4^3a_6^9 + 40572a_4^2a_5^2a_6^8 + 72576a_4a_5^4a_6^7 \\
& + 17640a_5^6a_6^6 + 420964a_0a_5a_6^9 + 10560a_1a_4a_6^9 - 25630a_1a_5^2a_6^8 \\
& + 8186a_2a_3a_6^9 - 58652a_2a_4a_5a_6^8 - 142394a_2a_5^3a_6^7 - 16878a_3^2a_5a_6^8 \\
& - 8520a_3a_4^2a_6^8 - 49434a_3a_4a_5^2a_6^7 - 2352a_3a_5^4a_6^6 + 4648a_4^3a_5a_6^7
\end{aligned}$$

$$\begin{aligned}
& + 72912a_4^2a_5^3a_6^6 + 67032a_4a_5^5a_6^5 + 8400a_5^7a_6^4 + 59028a_0a_4a_6^8 \\
& + 1428056a_0a_5^2a_6^7 + 95007a_1a_3a_6^8 + 60456a_1a_4a_5a_6^7 - 98532a_1a_5^3a_6^6 \\
& - 3902a_2^2a_6^8 + 65722a_2a_3a_5a_6^7 + 11826a_2a_4^2a_6^7 - 117446a_2a_4a_5^2a_6^6 \\
& - 140091a_2a_5^4a_6^5 - 6438a_3^2a_4a_6^7 - 59346a_3^2a_5^2a_6^6 - 47355a_3a_4^2a_5a_6^6 \\
& - 70119a_3a_4a_5^3a_6^5 + 1470a_3a_5^5a_6^4 - 2975a_4^4a_6^6 - 12348a_4^3a_5^2a_6^5 \\
& + 29400a_4^2a_5^4a_6^4 + 16464a_4a_5^6a_6^3 + 945a_5^8a_6^2 - 135555a_0a_3a_6^7 \\
& - 137152a_0a_4a_5a_6^6 + 1424216a_0a_5^3a_6^5 - 94789a_1a_2a_6^7 + 496454a_1a_3a_5a_6^6 \\
& + 55484a_1a_4^2a_6^6 + 79904a_1a_4a_5^2a_6^5 - 91235a_1a_5^4a_6^4 - 34252a_2^2a_5a_6^6 \\
& + 32246a_2a_3a_4a_6^6 + 110530a_2a_3a_5^2a_6^5 + 91111a_2a_4^2a_5a_6^5 - 28501a_2a_4a_5^3a_6^4 \\
& - 36085a_2a_5^5a_6^3 + 12182a_3^3a_6^6 - 24576a_3^2a_4a_5a_6^5 - 59475a_3^2a_5^3a_6^4 \\
& + 957a_3a_4^3a_6^5 - 47385a_3a_4^2a_5^2a_6^4 - 23625a_3a_4a_5^4a_6^3 + 1176a_3a_5^6a_6^2 \\
& - 8730a_4^4a_5a_6^4 - 17500a_4^3a_5^3a_6^3 + 336a_4^2a_5^5a_6^2 + 648a_4a_5^7a_6 \\
& + 10a_5^9 + 848551a_0a_2a_6^6 - 610300a_0a_3a_5a_6^5 - 68420a_0a_2^2a_6^5 \\
& - 566912a_0a_4a_5^2a_6^4 + 391250a_0a_5^4a_6^3 - 692814a_1^2a_6^6 - 169246a_1a_2a_5a_6^5 \\
& + 72242a_1a_3a_4a_6^5 + 549848a_1a_3a_5^2a_6^4 + 133646a_1a_4^2a_5a_6^4 + 38598a_1a_4a_5^3a_6^3 \\
& - 17860a_1a_5^5a_6^2 - 105104a_2^2a_4a_6^5 - 66228a_2^2a_5^2a_6^4 + 18988a_2a_3^2a_6^5 \\
& + 39234a_2a_3a_4a_5a_6^4 + 38044a_2a_3a_5^3a_6^3 + 16347a_2a_4^3a_6^4 + 98247a_2a_4^2a_5^2a_6^3 \\
& + 13045a_2a_4a_5^4a_6^2 - 1316a_2a_5^6a_6 + 30458a_3^3a_5a_6^4 + 7953a_3^2a_4^2a_6^4 \\
& - 19548a_3^2a_4a_5^2a_6^3 - 14550a_3^2a_5^4a_6^2 + 8985a_3a_4^3a_5a_6^3 - 7425a_3a_4^2a_5^3a_6^2 \\
& - 1092a_3a_4a_5^5a_6 + 48a_3a_5^7 + 420a_4^5a_6^3 - 2400a_4^4a_5^2a_6^2 \\
& - 2240a_4^3a_5^4a_6 - 84a_4^2a_5^6 + 552666a_0a_1a_6^5 + 1380400a_0a_2a_5a_6^4 \\
& + 410284a_0a_3a_4a_6^4 - 495024a_0a_3a_5^2a_6^3 - 8506a_0a_4^2a_5a_6^3 - 199410a_0a_4a_5^3a_6^2 \\
& + 21089a_0a_5^5a_6 - 999884a_1^2a_5a_6^4 - 191730a_1a_2a_4a_6^4 - 36604a_1a_2a_5^2a_6^3 \\
& - 130162a_1a_3^2a_6^4 + 60926a_1a_3a_4a_5a_6^3 + 126180a_1a_3a_5^3a_6^2 + 11924a_1a_4^3a_6^3 \\
& + 61737a_1a_4^2a_5^2a_6^4 + 6435a_1a_4a_5^4a_6 - 288a_1a_5^6 + 6646a_2^2a_3a_6^4 \\
& - 149960a_2^2a_4a_5a_6^3 - 27329a_2^2a_5^3a_6^2 + 29046a_2a_3^2a_5a_6^3 - 31570a_2a_3a_4^2a_6^3 \\
& - 4980a_2a_3a_4a_5^2a_6^2 + 865a_2a_3a_5^4a_6 + 6909a_2a_4^3a_5a_6^2 + 13770a_2a_4^2a_5^3a_6 \\
& + 748a_2a_4a_5^5 + 8780a_3^3a_4a_6^3 + 11787a_3^3a_5^2a_6^2 + 6660a_3^2a_4^2a_5a_6^2 \\
& - 2835a_3^2a_4a_5^3a_6 - 384a_3^2a_5^5 + 1215a_3a_4^4a_6^2 + 2760a_3a_4^3a_5^2a_6 \\
& - 60a_3a_4^2a_5^4 + 420a_4^5a_5a_6 + 40a_4^4a_5^3 + 641649a_0^2a_6^4 \\
& + 589102a_0a_1a_5a_6^3 - 254712a_0a_2a_4a_6^3 + 364000a_0a_2a_5^2a_6^2 + 357814a_0a_2^2a_6^3 \\
& + 280164a_0a_3a_4a_5a_6^2 - 63327a_0a_3a_5^3a_6 + 59448a_0a_4^3a_6^2 + 28821a_0a_4^2a_5^2a_6 \\
& - 4663a_0a_4a_5^4 + 185474a_1^2a_4a_6^3 - 221969a_1^2a_5^2a_6^2 - 217322a_1a_2a_3a_6^3 \\
& - 129046a_1a_2a_4a_5a_6^2 - 1902a_1a_2a_5^3a_6 - 68196a_1a_3^2a_5a_6^2 - 26892a_1a_3a_4^2a_6^2 \\
& + 2145a_1a_3a_4a_5^2a_6 + 2602a_1a_3a_5^4 + 4383a_1a_4^3a_5a_6 + 2967a_1a_4^2a_5^3
\end{aligned}$$

$$\begin{aligned}
& + 81674a_2^3a_6^3 + 12767a_2^2a_3a_5a_6^2 - 7014a_2^2a_4a_6^2 - 25216a_2^2a_4a_5^2a_6 \\
& - 1099a_2^2a_5^4 - 7122a_2a_3^2a_4a_6^2 + 5508a_2a_3^2a_5^2a_6 - 14097a_2a_3a_4^2a_5a_6 \\
& - 1041a_2a_3a_4a_5^3 - 1326a_2a_4^4a_6 - 352a_2a_4^3a_5^2 + 300a_3^4a_6^2 \\
& + 3270a_3^3a_4a_5a_6 + 364a_3^3a_5^3 - 135a_3^2a_4^3a_6 + 402a_3^2a_4^2a_5^2 \\
& + 60a_3a_4^4a_5 + 2a_4^6 + 276778a_0^2a_5a_6^2 - 117106a_0a_1a_4a_6^2 \\
& + 101056a_0a_1a_5^2a_6 - 116746a_0a_2a_3a_6^2 - 116228a_0a_2a_4a_5a_6 + 8105a_0a_2a_5^3 \\
& + 98298a_0a_3^2a_5a_6 + 20178a_0a_3a_4^2a_6 + 12774a_0a_3a_4a_5^2 + 7739a_0a_4^3a_5 \\
& + 36484a_1^2a_3a_6^2 + 46344a_1^2a_4a_5a_6 - 4488a_1^2a_5^3 + 68022a_1a_2^2a_6^2 \\
& - 40504a_1a_2a_3a_5a_6 - 5506a_1a_2a_4^2a_6 - 7507a_1a_2a_4a_5^2 - 7782a_1a_3^2a_4a_6 \\
& - 1724a_1a_3^2a_5^2 - 3230a_1a_3a_4^2a_5 - 141a_1a_4^4 + 17591a_2^3a_5a_6 \\
& + 12275a_2^2a_3a_4a_6 + 1241a_2^2a_3a_5^2 + 307a_2^2a_4^2a_5 - 1970a_2a_3^3a_6 \\
& - 670a_2a_3^2a_4a_5 - 201a_2a_3a_4^3 + 150a_3^4a_5 - 60a_3^3a_4^2 \\
& + 107228a_0^2a_4a_6 + 7489a_0^2a_5^2 - 137996a_0a_1a_3a_6 - 6450a_0a_1a_4a_5 \\
& + 51288a_0a_2^2a_6 - 11738a_0a_2a_3a_5 - 8354a_0a_2a_4^2 + 1164a_0a_3^2a_4 \\
& + 2976a_1^2a_2a_6 + 4152a_1^2a_3a_5 - 3357a_1^2a_4^2 + 3694a_1a_2^2a_5 \\
& + 6414a_1a_2a_3a_4 - 1760a_1a_3^3 - 77a_2^3a_4 + 635a_2^2a_3^2 \\
& + 22276a_0^2a_3 + 4092a_0a_1a_2 - 5072a_1^3
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
e_{19} = & 6a_2a_6^{14} - 3a_3a_5a_6^{13} - 15a_4^2a_6^{13} - 216a_4a_5^2a_6^{12} \\
& - 210a_5^4a_6^{11} - 6a_1a_6^{13} + 669a_2a_5a_6^{12} + 33a_3a_4a_6^{12} \\
& - 120a_3a_5^2a_6^{11} - 1248a_4^2a_5a_6^{11} - 5208a_4a_5^3a_6^{10} - 2520a_5^5a_6^9 \\
& - 6924a_0a_6^{12} + 480a_1a_5a_6^{11} + 1635a_2a_4a_6^{11} + 12750a_2a_5^2a_6^{10} \\
& + 297a_3^2a_6^{11} + 1614a_3a_4a_5a_6^{10} - 1176a_3a_5^3a_6^9 - 696a_4^3a_6^{10} \\
& - 16548a_4^2a_5^2a_6^9 - 29484a_4a_5^4a_6^8 - 7560a_5^6a_6^7 - 157554a_0a_5a_6^{10} \\
& - 2784a_1a_4a_6^{10} + 13904a_1a_5^2a_6^9 - 2250a_2a_3a_6^{10} + 29428a_2a_4a_5a_6^9 \\
& + 68152a_2a_5^3a_6^8 + 7080a_3^2a_5a_6^9 + 3276a_3a_4^2a_6^9 + 15456a_3a_4a_5^2a_6^8 \\
& - 4368a_3a_5^4a_6^7 - 6440a_4^3a_5a_6^8 - 55104a_4^2a_5^3a_6^7 - 49392a_4a_5^5a_6^6 \\
& - 6720a_5^7a_6^5 - 49104a_0a_4a_6^9 - 849572a_0a_5^2a_6^8 - 45741a_1a_3a_6^9 \\
& - 18602a_1a_4a_5a_6^8 + 76320a_1a_5^3a_6^7 + 576a_2^2a_6^9 - 28660a_2a_3a_5a_6^8 \\
& - 1798a_2a_4^2a_6^8 + 108926a_2a_4a_5^2a_6^7 + 114513a_2a_4^4a_6^6 + 5406a_3^2a_4a_6^8 \\
& + 39474a_3^2a_5^2a_6^7 + 31713a_3a_4^2a_5a_6^7 + 38661a_3a_4a_5^3a_6^6 - 6762a_3a_5^5a_6^5 \\
& + 1601a_4^4a_6^7 - 2940a_4^3a_5^2a_6^6 - 47040a_4^2a_5^4a_6^5 - 23520a_4a_5^6a_6^4 \\
& - 1575a_5^8a_6^3 + 88917a_0a_3a_6^8 - 45424a_0a_4a_5a_6^7 - 1390428a_0a_5^3a_6^6 \\
& + 46779a_1a_2a_6^8 - 365488a_1a_3a_5a_6^7 - 35532a_1a_4^2a_6^7 - 14904a_1a_4a_5^2a_6^6
\end{aligned}$$

$$\begin{aligned}
& + 114600a_1a_5^4a_6^5 + 11526a_2^2a_5a_6^7 - 27370a_2a_3a_4a_6^7 - 79016a_2a_3a_5^2a_6^6 \\
& - 53649a_2a_4^2a_5a_6^6 + 85263a_2a_4a_5^3a_6^5 + 55986a_2a_5^5a_6^4 - 7590a_3^3a_6^7 \\
& + 33132a_3^2a_4a_5a_6^6 + 64656a_3^2a_5^3a_6^5 + 2497a_3a_4^3a_6^6 + 59967a_3a_4^2a_5^2a_6^5 \\
& + 24150a_3a_4a_5^4a_6^4 - 3528a_3a_5^6a_6^3 + 10548a_4^4a_5a_6^5 + 16800a_4^3a_5^3a_6^4 \\
& - 6720a_4^2a_5^5a_6^3 - 2376a_4a_5^7a_6^2 - 60a_5^9a_6 - 653421a_0a_2a_6^7 \\
& + 649036a_0a_3a_5a_6^6 + 98736a_0a_4^2a_6^6 + 469752a_0a_4a_5^2a_6^5 - 691075a_0a_5^4a_6^4 \\
& + 568260a_1^2a_6^7 + 99166a_1a_2a_5a_6^6 - 98088a_1a_3a_4a_6^6 - 655528a_1a_3a_5^2a_6^5 \\
& - 132500a_1a_4^2a_5a_6^5 + 1568a_1a_4a_5^3a_6^4 + 44174a_1a_5^5a_6^3 + 86786a_2^2a_4a_6^6 \\
& + 45818a_2^2a_5^2a_6^5 - 22540a_2a_3^2a_6^6 - 74606a_2a_3a_4a_5a_6^5 - 50510a_2a_3a_5^3a_6^4 \\
& - 22265a_2a_4^3a_6^5 - 124667a_2a_4^2a_5^2a_6^4 - 3545a_2a_4a_5^4a_6^3 + 5614a_2a_5^6a_6^2 \\
& - 30292a_3^3a_5a_6^5 - 7317a_3^2a_4^2a_6^5 + 43527a_3^2a_4a_5^2a_6^4 + 29340a_3^2a_5^4a_6^3 \\
& - 4725a_3a_4^3a_5a_6^4 + 23025a_3a_4^2a_5^3a_6^3 + 2478a_3a_4a_5^5a_6^2 - 360a_3a_5^7a_6 \\
& - 90a_4^5a_6^4 + 8100a_4^4a_5^2a_6^3 + 7000a_4^3a_5^4a_6^2 + 252a_4^2a_5^6a_6 \\
& - 18a_4a_5^8 - 500178a_0a_1a_6^6 - 1709686a_0a_2a_5a_6^5 - 402282a_0a_3a_4a_5^5 \\
& + 917188a_0a_3a_5^2a_6^4 + 158606a_0a_4^2a_5a_6^4 + 400910a_0a_4a_5^3a_6^3 - 86940a_0a_5^5a_6^2 \\
& + 1331492a_1^2a_5a_6^5 + 213844a_1a_2a_4a_6^5 - 18244a_1a_2a_5^2a_6^4 + 148338a_1a_3^2a_6^5 \\
& - 165100a_1a_3a_4a_5a_6^4 - 285318a_1a_3a_5^3a_6^3 - 20900a_1a_4^3a_6^4 - 113596a_1a_4^2a_5^2a_6^3 \\
& - 5670a_1a_4a_5^4a_6^2 + 2736a_1a_5^6a_6 + 2002a_2^2a_3a_6^5 + 213266a_2^2a_4a_5a_6^4 \\
& + 41342a_2^2a_5^3a_6^3 - 55066a_2a_3^2a_5a_6^4 + 35624a_2a_3a_4^2a_6^4 - 21236a_2a_3a_4a_5^2a_6^3 \\
& - 4050a_2a_3a_5^4a_6^2 - 25853a_2a_4^3a_5a_6^3 - 45555a_2a_4^2a_5^3a_6^2 - 3592a_2a_4a_5^5a_6 \\
& + 24a_2a_5^7 - 14684a_3^3a_4a_6^4 - 21210a_3^3a_5^2a_6^3 - 13362a_3^2a_4^2a_5a_6^3 \\
& + 12780a_3^2a_4a_5^3a_6^2 + 2472a_3^2a_5^5a_6 - 3105a_3a_4^4a_6^3 - 6615a_3a_4^3a_5^2a_6^2 \\
& + 1200a_3a_4^2a_5^4a_6 - 1080a_4^5a_5a_6^2 + 320a_4^4a_5^3a_6 + 168a_4^3a_5^5 \\
& - 836421a_0^2a_6^5 - 917608a_0a_1a_5a_6^4 + 339818a_0a_2a_4a_6^4 - 868772a_0a_2a_5^2a_6^3 \\
& - 602010a_0a_3^2a_6^4 - 507998a_0a_3a_4a_5a_6^3 + 261737a_0a_3a_5^3a_6^2 - 114886a_0a_4^3a_6^3 \\
& - 20990a_0a_4^2a_5^2a_6^2 + 38073a_0a_4a_5^4a_6 - 1354a_0a_5^6 - 293652a_1^2a_4a_6^4 \\
& + 564811a_1^2a_5^2a_6^3 + 394214a_1a_2a_3a_6^4 + 258604a_1a_2a_4a_5a_6^3 - 16992a_1a_2a_5^3a_6^2 \\
& + 144300a_1a_3^2a_5a_6^3 + 49938a_1a_3a_4^2a_6^3 - 33835a_1a_3a_4a_5^2a_6^2 - 20212a_1a_3a_5^4a_6 \\
& - 17965a_1a_4^3a_5a_6^2 - 18737a_1a_4^2a_5^3a_6 - 622a_1a_4a_5^5 - 125694a_2^3a_6^4 \\
& - 11033a_2^2a_3a_5a_6^3 + 21376a_2^2a_4^2a_6^3 + 82473a_2^2a_4a_5^2a_6^2 + 6134a_2^2a_5^4a_6 \\
& + 18392a_2a_3^2a_4a_6^3 - 21155a_2a_3^2a_5^2a_6^2 + 39555a_2a_3a_4^2a_5a_6^2 + 2581a_2a_3a_4a_5^3a_6 \\
& + 110a_2a_3a_5^5 + 3736a_2a_4^4a_6^2 - 538a_2a_4^3a_5^2a_6 - 1220a_2a_4^2a_5^4 \\
& - 1402a_3^4a_6^3 - 11562a_3^3a_4a_5a_6^2 - 1914a_3^3a_5^3a_6 - 321a_3^2a_4^3a_6^2 \\
& - 2847a_3^2a_4^2a_5^2a_6 + 390a_3^2a_4a_5^4 - 900a_3a_4^4a_5a_6 - 320a_3a_4^3a_5^3 \\
& - 42a_4^6a_6 - 120a_4^5a_5^2 - 574894a_0^2a_5a_6^3 + 210762a_0a_1a_4a_6^3
\end{aligned}$$

$$\begin{aligned}
& -354600a_0a_1a_5^2a_6^2 + 260434a_0a_2a_3a_6^3 + 335358a_0a_2a_4a_5a_6^2 - 71521a_0a_2a_5^3a_6 \\
& -362256a_0a_3^2a_5a_6^2 - 78852a_0a_3a_4^2a_6^2 - 78326a_0a_3a_4a_5^2a_6 + 7148a_0a_3a_5^4 \\
& -50639a_0a_4^3a_5a_6 - 5729a_0a_4^2a_5^3 - 100290a_1^2a_3a_6^3 - 174420a_1^2a_4a_5a_6^2 \\
& + 38116a_1^2a_5^3a_6 - 144894a_1a_2^2a_6^3 + 175920a_1a_2a_3a_5a_6^2 + 21354a_1a_2a_4^2a_6^2 \\
& + 49647a_1a_2a_4a_5^2a_6 + 238a_1a_2a_5^4 + 27624a_1a_3^2a_4a_6^2 + 12006a_1a_3^2a_5^2a_6 \\
& + 19432a_1a_3a_4^2a_5a_6 + 184a_1a_3a_4a_5^3 + 651a_1a_4^4a_6 - 1011a_1a_4^3a_5^2 \\
& - 59359a_2^3a_5a_6^2 - 34653a_2^2a_3a_4a_6^2 - 5314a_2^2a_3a_5^2a_6 + 787a_2^2a_4^2a_5a_6 \\
& + 2527a_2^2a_4a_5^3 + 7354a_2a_3^3a_6^2 + 6902a_2a_3^2a_4a_5a_6 - 632a_2a_3^2a_5^3 \\
& + 2461a_2a_3a_4^3a_6 + 2455a_2a_3a_4^2a_5^2 + 696a_2a_4^4a_5 - 1320a_3^4a_5a_6 \\
& + 336a_3^3a_4^2a_6 - 654a_3^3a_4a_5^2 - 6a_3^2a_4^3a_5 + 48a_3a_4^5 \\
& - 331236a_0^2a_4a_6^2 - 32227a_0^2a_5^2a_6 + 517932a_0a_1a_3a_6^2 + 22858a_0a_1a_4a_5a_6 \\
& - 13294a_0a_1a_5^3 - 177684a_0a_2^2a_6^2 + 85862a_0a_2a_3a_5a_6 + 57014a_0a_2a_4^2a_6 \\
& + 22776a_0a_2a_4a_5^2 - 7296a_0a_3^2a_4a_6 - 13560a_0a_3^2a_5^2 - 6304a_0a_3a_4^2a_5 \\
& - 750a_0a_4^4 - 25062a_1^2a_2a_6^2 - 32890a_1^2a_3a_5a_6 + 24615a_1^2a_4^2a_6 \\
& - 4926a_1^2a_4a_5^2 - 24736a_1a_2^2a_5a_6 - 46718a_1a_2a_3a_4a_6 + 2544a_1a_2a_3a_5^2 \\
& + 2208a_1a_2a_4^2a_5 + 12152a_1a_3^3a_6 + 1962a_1a_3^2a_4a_5 + 846a_1a_3a_4^3 \\
& + 989a_2^3a_4a_6 - 1799a_2^3a_5^2 - 4795a_2^2a_3^2a_6 - 2998a_2^2a_3a_4a_5 \\
& - 906a_2^2a_4^3 + 330a_2a_3^3a_5 - 404a_2a_3^2a_4^2 + 120a_3^4a_4 \\
& - 160926a_0^2a_3a_6 - 31270a_0^2a_4a_5 - 27312a_0a_1a_2a_6 + 31756a_0a_1a_3a_5 \\
& + 10446a_0a_1a_4^2 - 12248a_0a_2^2a_5 + 1516a_0a_2a_3a_4 - 1070a_0a_3^3 \\
& + 37350a_1^3a_6 + 384a_1^2a_2a_5 - 2886a_1^2a_3a_4 - 906a_1a_2^2a_4 \\
& + 1922a_1a_2a_3^2 - 296a_2^3a_3 - 3198a_0^2a_2 - 2778a_0a_1^2
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
e_{20} = & -48a_2a_5a_6^{13} + 24a_3a_5^2a_6^{12} + 120a_4^2a_5a_6^{12} + 504a_4a_5^3a_6^{11} \\
& + 252a_5^5a_6^{10} + 990a_0a_6^{13} - 117a_1a_5a_6^{12} - 282a_2a_4a_6^{12} \\
& - 2175a_2a_5^2a_6^{11} - 45a_3^2a_6^{12} - 63a_3a_4a_5a_6^{11} + 588a_3a_5^3a_6^{10} \\
& + 180a_4^3a_6^{11} + 3948a_4^2a_5^2a_6^{10} + 7056a_4a_5^4a_6^9 + 1890a_5^6a_6^8 \\
& + 40377a_0a_5a_6^{11} + 447a_1a_4a_6^{11} - 4652a_1a_5^2a_6^{10} + 372a_2a_3a_6^{11} \\
& - 9529a_2a_4a_5a_6^{10} - 21532a_2a_5^3a_6^9 - 1932a_3^2a_5a_6^{10} - 741a_3a_4^2a_6^{10} \\
& - 1764a_3a_4a_5^2a_6^9 + 3696a_3a_5^4a_6^8 + 3752a_4^3a_5a_6^9 + 26040a_4^2a_5^3a_6^8 \\
& + 23184a_4a_5^5a_6^7 + 3360a_5^7a_6^6 + 25515a_0a_4a_6^{10} + 360499a_0a_5^2a_6^9 \\
& + 15843a_1a_3a_6^{10} + 1721a_1a_4a_5a_6^9 - 39492a_1a_5^3a_6^8 + 128a_2^2a_6^{10} \\
& + 7817a_2a_3a_5a_6^9 - 1400a_2a_4^2a_6^9 - 63852a_2a_4a_5^2a_6^8 - 63189a_2a_5^4a_6^7 \\
& - 2799a_3^2a_4a_6^9 - 17838a_3^2a_5^2a_6^8 - 13131a_3a_4^2a_5a_6^8 - 9177a_3a_4a_5^3a_6^7
\end{aligned}$$

$$\begin{aligned}
& + 7938a_3a_5^5a_6^6 - 339a_4^4a_6^8 + 11004a_4^3a_5^2a_6^7 + 44100a_4^2a_5^4a_6^6 \\
& + 21168a_4a_5^6a_6^5 + 1575a_5^8a_6^4 - 40221a_0a_3a_6^9 + 121971a_0a_4a_5a_6^8 \\
& + 969898a_0a_3^3a_6^7 - 16471a_1a_2a_6^9 + 196032a_1a_3a_5a_6^8 + 16035a_1a_4^2a_6^8 \\
& - 23498a_1a_4a_5^2a_6^7 - 96009a_1a_5^4a_6^6 - 611a_2^2a_5a_6^8 + 14991a_2a_3a_4a_6^8 \\
& + 34988a_2a_3a_5^2a_6^7 + 13745a_2a_4^2a_5a_6^7 - 104619a_2a_4a_5^3a_6^6 - 56175a_2a_5^5a_6^5 \\
& + 3288a_3^3a_6^8 - 27930a_3^2a_4a_5a_6^7 - 47841a_3^2a_5^3a_6^6 - 3513a_3a_4^3a_6^7 \\
& - 45999a_3a_4^2a_5^2a_6^6 - 10395a_3a_4a_5^4a_6^5 + 5880a_3a_5^6a_6^4 - 7302a_4^4a_5a_6^6 \\
& - 3780a_4^3a_5^3a_6^5 + 16800a_4^2a_5^5a_6^4 + 4680a_4a_5^7a_6^3 + 150a_5^9a_6^2 \\
& + 381739a_0a_2a_6^8 - 479885a_0a_3a_5a_6^7 - 81094a_0a_4^2a_6^7 - 159962a_0a_4a_5^2a_6^6 \\
& + 833973a_0a_5^4a_6^5 - 353026a_1^2a_6^8 - 29559a_1a_2a_5a_6^7 + 89170a_1a_3a_4a_6^7 \\
& + 557362a_1a_3a_5^2a_6^6 + 88665a_1a_4^2a_5a_6^6 - 60519a_1a_4a_5^3a_6^5 - 66660a_1a_5^5a_6^4 \\
& - 51874a_2^2a_4a_6^7 - 16712a_2^2a_5^2a_6^6 + 18986a_2a_3^2a_6^7 + 75880a_2a_3a_4a_5a_6^6 \\
& + 38028a_2a_3a_5^3a_6^5 + 19499a_2a_4^3a_6^6 + 94026a_2a_4^2a_5^2a_6^5 - 27915a_2a_4a_5^4a_6^4 \\
& - 12453a_2a_5^6a_6^3 + 20539a_3^3a_5a_6^6 + 3183a_3^2a_4^2a_6^6 - 60642a_3^2a_4a_5^2a_6^5 \\
& - 37710a_3^2a_5^4a_6^4 - 5280a_3a_4^3a_5a_6^5 - 36900a_3a_4^2a_5^3a_6^4 - 1365a_3a_4a_5^5a_6^3 \\
& + 1164a_3a_5^7a_6^2 - 552a_4^5a_6^5 - 13725a_4^4a_5^2a_6^4 - 10500a_4^3a_5^4a_6^3 \\
& + 420a_4^2a_5^6a_6^2 + 144a_4a_5^8a_6 + a_5^{10} + 334184a_0a_1a_6^7 \\
& + 1553076a_0a_2a_5a_6^6 + 265789a_0a_3a_4a_6^6 - 1135628a_0a_3a_5^2a_6^5 - 304813a_0a_4^2a_5a_6^5 \\
& - 457725a_0a_4a_5^3a_6^4 + 209039a_0a_5^5a_6^3 - 1306868a_1^2a_5a_6^6 - 171281a_1a_2a_4a_6^6 \\
& + 101602a_1a_2a_5^2a_6^5 - 121642a_1a_3^2a_6^6 + 260014a_1a_3a_4a_5a_6^5 + 421494a_1a_3a_5^3a_6^4 \\
& + 24465a_1a_4^3a_6^5 + 127821a_1a_4^2a_5^2a_6^4 - 19315a_1a_4a_5^4a_6^3 - 10176a_1a_5^6a_6^2 \\
& - 7714a_2^2a_3a_6^6 - 208414a_2^2a_4a_5a_6^5 - 36583a_2^2a_5^3a_6^4 + 73529a_2a_3^2a_5a_6^5 \\
& - 22908a_2a_3a_4^2a_6^5 + 63786a_2a_3a_4a_5^2a_6^4 + 6295a_2a_3a_5^4a_6^3 + 48744a_2a_4^3a_5a_6^4 \\
& + 80620a_2a_4^2a_5^3a_6^3 + 5301a_2a_4a_5^5a_6^2 - 336a_2a_5^7a_6 + 16839a_3^3a_4a_6^5 \\
& + 23883a_3^3a_5^2a_6^4 + 14031a_3^2a_4^2a_5a_6^4 - 32505a_3^2a_4a_5^3a_6^3 - 6960a_3^2a_5^5a_6^2 \\
& + 4380a_3a_4^4a_6^4 + 5460a_3a_4^3a_5^2a_6^3 - 5655a_3a_4^2a_5^4a_6^2 + 168a_3a_4a_5^6a_6 \\
& + 24a_3a_5^8 + 1020a_4^5a_5a_6^3 - 2880a_4^4a_5^3a_6^2 - 1176a_4^3a_5^5a_6 \\
& - 24a_4^2a_5^7 + 833513a_0^2a_6^6 + 994417a_0a_1a_5a_6^5 - 311770a_0a_2a_4a_6^5 \\
& + 1367146a_0a_2a_5^2a_6^4 + 720474a_0a_3^2a_6^5 + 552986a_0a_3a_4a_5a_6^4 - 619134a_0a_3a_5^3a_6^3 \\
& + 140185a_0a_4^3a_6^4 - 104946a_0a_4^2a_5^2a_6^3 - 120891a_0a_4a_5^4a_6^2 + 10771a_0a_5^6a_6 \\
& + 322099a_1^2a_4a_6^5 - 969006a_1^2a_5^2a_6^4 - 508302a_1a_2a_3a_6^5 - 335688a_1a_2a_4a_5a_6^4 \\
& + 83720a_1a_2a_5^3a_6^3 - 201138a_1a_3^2a_5a_6^4 - 58570a_1a_3a_4^2a_6^4 + 122766a_1a_3a_4a_5^2a_6^3 \\
& + 68649a_1a_3a_5^4a_6^2 + 40170a_1a_4^3a_5a_6^3 + 49164a_1a_4^2a_5^3a_6^2 + 1081a_1a_4a_5^5a_6 \\
& - 120a_1a_5^7 + 139550a_2^3a_6^5 - 1909a_2^2a_3a_5a_6^4 - 38709a_2^2a_4^2a_6^4 \\
& - 156348a_2^2a_4a_5^2a_6^3 - 14408a_2^2a_5^4a_6^2 - 26681a_2a_3^2a_4a_6^4 + 51399a_2a_3^2a_5^2a_6^3
\end{aligned}$$

$$\begin{aligned}
& -57555a_2a_3a_4^2a_5a_6^3 + 5013a_2a_3a_4a_5^3a_6^2 - 457a_2a_3a_5^5a_6 - 5310a_2a_4^4a_6^3 \\
& + 11016a_2a_4^3a_5^2a_6^2 + 8750a_2a_4^2a_5^4a_6 + 288a_2a_4a_5^6 + 3135a_3^4a_6^4 \\
& + 24225a_3^3a_4a_5a_6^3 + 4113a_3^3a_5^3a_6^2 + 2355a_3^2a_4^3a_6^3 + 7929a_3^2a_4^2a_5^2a_6^2 \\
& - 3105a_3^2a_4a_5^4a_6 - 120a_3^2a_5^6 + 3915a_3a_4^4a_5a_6^2 + 1560a_3a_4^3a_5^3a_6 \\
& - 108a_3a_4^2a_5^5 + 204a_4^6a_6^2 + 720a_4^5a_5^2a_6 + 30a_4^4a_5^4 \\
& + 856777a_0^2a_5a_6^4 - 237281a_0a_1a_4a_6^4 + 726120a_0a_1a_5^2a_6^3 - 383862a_0a_2a_3a_6^4 \\
& - 564884a_0a_2a_4a_5a_6^3 + 266286a_0a_2a_5^3a_6^2 + 805246a_0a_3^2a_5a_6^3 + 181226a_0a_3a_4^2a_6^3 \\
& + 195060a_0a_3a_4a_5^2a_6^2 - 55655a_0a_3a_5^4a_6 + 144074a_0a_4^3a_5a_6^2 + 15002a_0a_4^2a_5^3a_6 \\
& - 2081a_0a_4a_5^5 + 179040a_1^2a_3a_6^4 + 390492a_1^2a_4a_5a_6^3 - 147630a_1^2a_5^3a_6^2 \\
& + 206536a_1a_2^2a_6^4 - 450536a_1a_2a_3a_5a_6^3 - 49298a_1a_2a_4^2a_6^3 - 142974a_1a_2a_4a_5^2a_6^2 \\
& + 4025a_1a_2a_5^4a_6 - 60278a_1a_3^2a_4a_6^3 - 37232a_1a_3^2a_5^2a_6^2 - 51362a_1a_3a_4^2a_5a_6^2 \\
& + 5074a_1a_3a_4a_5^3a_6 + 910a_1a_3a_5^5 - 1035a_1a_4^4a_6^2 + 8402a_1a_4^3a_5^2a_6 \\
& + 2073a_1a_4^2a_5^4 + 122665a_2^3a_5a_6^3 + 56179a_2^2a_3a_4a_6^3 + 8269a_2^2a_3a_5^2a_6^2 \\
& - 11386a_2^2a_4^2a_5a_6^2 - 17148a_2^2a_4a_5^3a_6 - 501a_2^2a_5^5 - 16668a_2a_3^3a_6^3 \\
& - 25375a_2a_3^2a_4a_5a_6^2 + 4877a_2a_3^2a_5^3a_6 - 10179a_2a_3a_4^3a_6^2 - 13605a_2a_3a_4^2a_5^2a_6 \\
& - 179a_2a_3a_4a_5^4 - 4172a_2a_4^4a_5a_6 - 408a_2a_4^3a_5^3 + 4920a_3^4a_5a_6^2 \\
& - 606a_3^3a_4^2a_6^2 + 4143a_3^3a_4a_5^2a_6 - 44a_3^3a_5^4 + 627a_3^2a_4^3a_5a_6 \\
& + 408a_3^2a_4^2a_5^3 - 222a_3a_4^5a_6 + 120a_3a_4^4a_5^2 + 8a_4^6a_5 \\
& + 604492a_0^2a_4a_6^3 + 48558a_0^2a_5^2a_6^2 - 1181502a_0a_1a_3a_6^3 + 7386a_0a_1a_4a_5a_6^2 \\
& + 91013a_0a_1a_5^3a_6 + 376442a_0a_2^2a_6^3 - 274724a_0a_2a_3a_5a_6^2 - 171436a_0a_2a_4^2a_6^2 \\
& - 119678a_0a_2a_4a_5^2a_6 + 5057a_0a_2a_5^4 + 24930a_0a_3^2a_4a_6^2 + 98137a_0a_3^2a_5^2a_6 \\
& + 49003a_0a_3a_4^2a_5a_6 + 6262a_0a_3a_4a_5^3 + 5084a_0a_4^4a_6 + 9662a_0a_4^3a_5^2 \\
& + 89376a_1^2a_2a_6^3 + 119029a_1^2a_3a_5a_6^2 - 80659a_1^2a_4^2a_6^2 + 41243a_1^2a_4a_5^2a_6 \\
& - 2499a_1^2a_5^4 + 72993a_1a_2^2a_5a_6^2 + 152210a_1a_2a_3a_4a_6^2 - 31942a_1a_2a_3a_5^2a_6 \\
& - 17417a_1a_2a_4^2a_5a_6 - 6932a_1a_2a_4a_5^3 - 38976a_1a_3^3a_6^2 - 12327a_1a_3^2a_4a_5a_6 \\
& + 238a_1a_3^2a_5^3 - 4916a_1a_3a_4^3a_6 - 3172a_1a_3a_4^2a_5^2 - 363a_1a_4^4a_5 \\
& - 4347a_2^3a_4a_6^2 + 12418a_2^3a_5^2a_6 + 15343a_2^2a_3^2a_6^2 + 15924a_2^2a_3a_4a_5a_6 \\
& + 656a_2^2a_3a_5^3 + 5386a_2^2a_4^3a_6 + 998a_2^2a_4^2a_5^2 - 2845a_2a_3^3a_5a_6 \\
& + 692a_2a_3^2a_4^2a_6 - 1643a_2a_3^2a_4a_5^2 - 607a_2a_3a_4^3a_5 - 42a_2a_4^5 \\
& - 585a_3^4a_4a_6 + 495a_3^4a_5^2 - 60a_3^3a_4^2a_5 - 78a_3^2a_4^4 \\
& + 529368a_0^2a_3a_6^2 + 181254a_0^2a_4a_5a_6 - 2177a_0^2a_5^3 + 79638a_0a_1a_2a_6^2 \\
& - 224474a_0a_1a_3a_5a_6 - 67810a_0a_1a_4^2a_6 + 1075a_0a_1a_4a_5^2 + 77379a_0a_2^2a_5a_6 \\
& - 20894a_0a_2a_3a_4a_6 - 15508a_0a_2a_3a_5^2 - 19562a_0a_2a_4^2a_5 + 4278a_0a_3^3a_6 \\
& + 2089a_0a_3^2a_4a_5 + 103a_0a_3a_4^3 - 125502a_1^3a_6^2 - 91a_1^2a_2a_5a_6 \\
& + 14985a_1^2a_3a_4a_6 + 5178a_1^2a_3a_5^2 - 7404a_1^2a_4^2a_5 + 7295a_1a_2^2a_4a_6
\end{aligned}$$

$$\begin{aligned}
& + 4328a_1a_2^2a_5^2 - 6958a_1a_2a_3^2a_6 + 14062a_1a_2a_3a_4a_5 + 591a_1a_2a_4^3 \\
& - 3506a_1a_3^3a_5 - 1009a_1a_3^2a_4^2 + 2468a_2^3a_3a_6 - 1846a_2^3a_4a_5 \\
& + 1177a_2^2a_3^2a_5 + 1151a_2^2a_3a_4^2 + 805a_2a_3^3a_4 - 204a_3^5 \\
& + 27096a_0^2a_2a_6 + 38668a_0^2a_3a_5 + 7751a_0^2a_4^2 + 24216a_0a_1^2a_6 \\
& + 7498a_0a_1a_2a_5 - 10560a_0a_1a_3a_4 + 6251a_0a_2^2a_4 + 7138a_0a_2a_3^2 \\
& - 9998a_1^3a_5 + 6939a_1^2a_2a_4 + 2922a_1^2a_3^2 - 10396a_1a_2^2a_3 \\
& + 1477a_2^4 - 18084a_0^2a_1
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
e_{21} = & -66a_0a_6^{14} + 11a_1a_5a_6^{13} + 22a_2a_4a_6^{13} + 169a_2a_5^2a_6^{12} \\
& + 3a_3^2a_6^{13} - 15a_3a_4a_5a_6^{12} - 84a_3a_5^3a_6^{11} - 20a_4^3a_6^{12} \\
& - 420a_4^2a_5^2a_6^{11} - 756a_4a_5^4a_6^{10} - 210a_5^6a_6^9 - 6347a_0a_5a_6^{12} \\
& - 33a_1a_4a_6^{12} + 898a_1a_5^2a_6^{11} - 28a_2a_3a_6^{12} + 1811a_2a_4a_5a_6^{11} \\
& + 4046a_2a_5^3a_6^{10} + 306a_2^3a_5a_6^{11} + 75a_3a_4^2a_6^{11} - 378a_3a_4a_5^2a_6^{10} \\
& - 1260a_3a_5^4a_6^9 - 1120a_4^3a_5a_6^{10} - 7056a_4^2a_5^3a_6^9 - 6300a_4a_5^5a_6^8 \\
& - 960a_5^7a_6^7 - 8385a_0a_4a_6^{11} - 103491a_0a_5^2a_6^{10} - 3741a_1a_3a_6^{11} \\
& + 1001a_1a_4a_5a_6^{10} + 13260a_1a_5^3a_6^9 - 80a_2^2a_6^{11} - 1071a_2a_3a_5a_6^{10} \\
& + 988a_2a_4^2a_6^{10} + 23480a_2a_4a_5^2a_6^9 + 22575a_2a_5^4a_6^8 + 891a_3^2a_4a_6^{10} \\
& + 5178a_3^2a_5^2a_6^9 + 2913a_3a_4^2a_5a_6^9 - 1953a_3a_4a_5^3a_6^8 - 4830a_3a_5^5a_6^7 \\
& - 115a_4^4a_6^9 - 8316a_4^3a_5^2a_6^8 - 24780a_4^2a_5^4a_6^7 - 11760a_4a_5^6a_6^6 \\
& - 945a_5^8a_6^5 + 12027a_0a_3a_6^{10} - 91773a_0a_4a_5a_6^9 - 473290a_0a_5^3a_6^8 \\
& + 3929a_1a_2a_6^{10} - 74782a_1a_3a_5a_6^9 - 4867a_1a_4^2a_6^9 + 24338a_1a_4a_5^2a_6^8 \\
& + 53890a_1a_5^4a_6^7 - 1243a_2^2a_5a_6^9 - 5201a_2a_3a_4a_6^9 - 7814a_2a_3a_5^2a_6^8 \\
& + 4063a_2a_4^2a_5a_6^8 + 74225a_2a_4a_5^3a_6^7 + 36680a_2a_5^5a_6^6 - 940a_3^3a_6^9 \\
& + 15078a_3^2a_4a_5a_6^8 + 23586a_3^2a_5^3a_6^7 + 2169a_3a_4^3a_6^8 + 20169a_3a_4^2a_5^2a_6^7 \\
& - 3780a_3a_4a_5^4a_6^6 - 5880a_3a_5^6a_6^5 + 2304a_4^4a_5a_6^7 - 8680a_4^3a_5^3a_6^6 \\
& - 20832a_4^2a_5^5a_6^5 - 5400a_4a_5^7a_6^4 - 200a_5^9a_6^3 - 167693a_0a_2a_6^9 \\
& + 245189a_0a_3a_5a_6^8 + 40120a_0a_4^2a_6^8 - 90746a_0a_4a_5^2a_6^7 - 695310a_0a_5^4a_6^6 \\
& + 164046a_1^2a_6^9 - 3721a_1a_2a_5a_6^8 - 56272a_1a_3a_4a_6^8 - 338570a_1a_3a_5^2a_6^7 \\
& - 38331a_1a_4^2a_5a_6^7 + 84601a_1a_4a_5^3a_6^6 + 65070a_1a_5^5a_6^5 + 22342a_2^2a_4a_6^8 \\
& + 128a_2^2a_5^2a_6^7 - 11210a_2a_3^2a_6^8 - 46460a_2a_3a_4a_5a_6^7 - 12678a_2a_3a_5^3a_6^6 \\
& - 10921a_2a_4^3a_6^7 - 34978a_2a_4^2a_5^2a_6^6 + 54255a_2a_4a_5^4a_6^5 + 16275a_2a_5^6a_6^4 \\
& - 9185a_3^3a_5a_6^7 + 591a_3^2a_4^2a_6^7 + 54681a_3^2a_4a_5^2a_6^6 + 31740a_3^2a_5^4a_6^5 \\
& + 10776a_3a_4^3a_5a_6^6 + 32640a_3a_4^2a_5^3a_6^5 - 3885a_3a_4a_5^5a_6^4 - 2100a_3a_5^7a_6^3 \\
& + 834a_4^5a_6^6 + 12645a_4^4a_5^2a_6^5 + 6300a_4^3a_5^4a_6^4 - 2940a_4^2a_5^6a_6^3
\end{aligned}$$

$$\begin{aligned}
& -450a_4a_5^8a_6^2 - 5a_5^{10}a_6 - 163668a_0a_1a_6^8 - 1047710a_0a_2a_5a_6^7 \\
& -112481a_0a_3a_4a_6^7 + 964148a_0a_3a_5^2a_6^6 + 308739a_0a_4^2a_5a_6^6 + 261159a_0a_4a_5^3a_6^5 \\
& -321052a_0a_5^5a_6^4 + 952490a_1^2a_5a_6^7 + 98629a_1a_2a_4a_6^7 - 143072a_1a_2a_5^2a_6^6 \\
& + 71780a_1a_3^2a_6^7 - 269844a_1a_3a_4a_5a_6^6 - 426504a_1a_3a_5^3a_6^5 - 19735a_1a_4^3a_6^6 \\
& - 87087a_1a_4^2a_5^2a_6^5 + 61910a_1a_4a_5^4a_6^4 + 20382a_1a_5^6a_6^3 + 7076a_2^2a_3a_6^7 \\
& + 142936a_2^2a_4a_5a_6^6 + 17624a_2^2a_5^3a_6^5 - 68771a_2a_3^2a_5a_6^6 + 5610a_2a_3a_4^2a_6^6 \\
& - 80160a_2a_3a_4a_5^2a_6^5 - 1280a_2a_3a_5^4a_6^4 - 54216a_2a_3^3a_5a_6^5 - 80780a_2a_4^2a_5^3a_6^4 \\
& + 1545a_2a_4a_5^5a_6^3 + 1338a_2a_5^7a_6^2 - 13313a_3^3a_4a_6^6 - 16776a_3^3a_5^2a_6^5 \\
& - 4878a_3^2a_4^2a_5a_6^5 + 51270a_3^2a_4a_5^3a_6^4 + 11118a_3^2a_5^5a_6^3 - 3402a_3a_4^4a_6^5 \\
& + 4560a_3a_4^3a_5^2a_6^4 + 11745a_3a_4^2a_5^4a_6^3 - 1134a_3a_4a_5^6a_6^2 - 138a_3a_5^8a_6 \\
& + 570a_4^5a_5a_6^4 + 7920a_4^4a_5^3a_6^3 + 3024a_4^3a_5^5a_6^2 + 48a_4^2a_5^7a_6 \\
& - 2a_4a_5^9 - 644307a_0^2a_6^7 - 764043a_0a_1a_5a_6^6 + 199408a_0a_2a_4a_6^6 \\
& - 1501632a_0a_2a_5^2a_6^5 - 629048a_0a_3^2a_6^6 - 335796a_0a_3a_4a_5a_6^5 + 932940a_0a_3a_5^3a_6^4 \\
& - 109859a_0a_4^3a_6^5 + 311718a_0a_4^2a_5^2a_6^4 + 192005a_0a_4a_5^4a_6^3 - 38259a_0a_5^6a_6^2 \\
& - 246247a_1^2a_4a_6^6 + 1177694a_1^2a_5^2a_6^5 + 477518a_1a_2a_3a_6^6 + 291722a_1a_2a_4a_5a_6^5 \\
& - 179490a_1a_2a_5^3a_6^4 + 192194a_1a_3^2a_5a_6^5 + 42884a_1a_3a_4^2a_6^5 - 239412a_1a_3a_4a_5^2a_6^4 \\
& - 134875a_1a_3a_5^4a_6^3 - 55622a_1a_4^3a_5a_6^4 - 67980a_1a_4^2a_5^3a_6^3 + 5433a_1a_4a_5^5a_6^2 \\
& + 976a_1a_5^7a_6 - 114936a_2^3a_6^6 + 13299a_2^2a_3a_5a_6^5 + 45749a_2^2a_4a_6^5 \\
& + 190099a_2^2a_4a_5^2a_6^4 + 18245a_2^2a_5^4a_6^3 + 22519a_2a_3^2a_4a_6^5 - 83100a_2a_3^2a_5^2a_6^4 \\
& + 42471a_2a_3a_4^2a_5a_6^4 - 27025a_2a_3a_4a_5^3a_6^3 + 1599a_2a_3a_5^5a_6^2 + 3392a_2a_4^4a_6^4 \\
& - 35200a_2a_4^3a_5^2a_6^3 - 24750a_2a_4^2a_5^4a_6^2 - 1132a_2a_4a_5^6a_6 + 2a_2a_5^8 \\
& - 4347a_3^4a_6^5 - 32769a_3^3a_4a_5a_6^4 - 4135a_3^3a_5^3a_6^3 - 5142a_3^2a_4^3a_6^4 \\
& - 9810a_3^2a_4^2a_5^2a_6^3 + 10665a_3^2a_4a_5^4a_6^2 + 600a_3^2a_5^6a_6 - 7845a_3a_4^4a_5a_6^3 \\
& - 1800a_3a_4^3a_5^3a_6^2 + 792a_3a_4^2a_5^5a_6 - 24a_3a_4a_5^7 - 440a_4^6a_6^3 \\
& - 1440a_4^5a_5^2a_6^2 + 270a_4^4a_5^4a_6 + 56a_4^3a_5^6 - 989507a_0^2a_5a_6^5 \\
& + 167843a_0a_1a_4a_6^5 - 959930a_0a_1a_5^2a_6^4 + 395270a_0a_2a_3a_6^5 + 622958a_0a_2a_4a_5a_6^4 \\
& - 564410a_0a_2a_5^3a_6^3 - 1197854a_0a_3^2a_5a_6^4 - 269264a_0a_3a_4^2a_6^4 - 227076a_0a_3a_4a_5^2a_6^3 \\
& + 190305a_0a_3a_5^4a_6^2 - 231034a_0a_4^3a_5a_6^3 + 20460a_0a_4^2a_5^3a_6^2 + 14739a_0a_4a_5^5a_6 \\
& - 542a_0a_5^7 - 221348a_1^2a_3a_6^5 - 572422a_1^2a_4a_5a_6^4 + 344890a_1^2a_5^3a_6^3 \\
& - 206982a_1a_2^2a_6^5 + 758308a_1a_2a_3a_5a_6^4 + 75054a_1a_2a_4^2a_6^4 + 231554a_1a_2a_4a_5^2a_6^3 \\
& - 29775a_1a_2a_5^4a_6^2 + 89414a_1a_3^2a_4a_6^4 + 66690a_1a_3^2a_5^2a_6^3 + 77250a_1a_3a_4^2a_5a_6^3 \\
& - 33030a_1a_3a_4a_5^3a_6^2 - 6422a_1a_3a_5^5a_6 + 210a_1a_4^4a_6^3 - 27700a_1a_4^3a_5^2a_6^2 \\
& - 10735a_1a_4^2a_5^4a_6 - 178a_1a_4a_5^6 - 173079a_2^3a_5a_6^4 - 57277a_2^2a_3a_4a_6^4 \\
& - 4340a_2^2a_3a_5^2a_6^3 + 36615a_2^2a_4^2a_5a_6^3 + 50055a_2^2a_4a_5^3a_6^2 + 2483a_2^2a_5^5a_6 \\
& + 24870a_2a_3^3a_6^4 + 46479a_2a_3^2a_4a_5a_6^3 - 18105a_2a_3^2a_5^3a_6^2 + 21271a_2a_3a_4^3a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 27975a_2a_3a_4^2a_5^2a_6^2 - 1135a_2a_3a_4a_5^4a_6 + 50a_2a_3a_5^6 + 9745a_2a_4^4a_5a_6^2 \\
& - 440a_2a_4^3a_5^3a_6 - 488a_2a_4^2a_5^5 - 10590a_3^4a_5a_6^3 - 201a_3^3a_4^2a_6^3 \\
& - 11715a_3^3a_4a_5^2a_6^2 + 610a_3^3a_5^4a_6 - 3900a_3^2a_4^3a_5a_6^2 - 1980a_3^2a_4^2a_5^3a_6 \\
& + 210a_3^2a_4a_5^5 + 255a_3a_4^5a_6^2 - 1320a_3a_4^4a_5^2a_6 - 60a_3a_4^3a_5^4 \\
& - 160a_4^6a_5a_6 - 120a_4^5a_5^3 - 724660a_0^2a_4a_6^4 - 24910a_0^2a_5^2a_6^3 \\
& + 1831186a_0a_1a_3a_6^4 - 178714a_0a_1a_4a_5a_6^3 - 269665a_0a_1a_5^3a_6^2 - 546700a_0a_2^2a_6^4 \\
& + 509528a_0a_2a_3a_5a_6^3 + 301828a_0a_2a_4^2a_6^3 + 273166a_0a_2a_4a_5^2a_6^2 - 39865a_0a_2a_5^4a_6 \\
& - 62426a_0a_3^2a_4a_6^3 - 326051a_0a_3^2a_5^2a_6^2 - 170309a_0a_3a_4^2a_5a_6^2 - 28728a_0a_3a_4a_5^3a_6 \\
& + 3448a_0a_3a_5^5 - 15893a_0a_4^4a_6^2 - 52710a_0a_4^3a_5^2a_6 - 2332a_0a_4^2a_5^4 \\
& - 190544a_1^2a_2a_6^4 - 261529a_1^2a_3a_5a_6^3 + 154107a_1^2a_4^2a_6^3 - 155047a_1^2a_4a_5^2a_6^2 \\
& + 18565a_1^2a_5^4a_6 - 123479a_1a_2^2a_5a_6^3 - 291658a_1a_2a_3a_4a_6^3 + 148104a_1a_2a_3a_5^2a_6^2 \\
& + 60879a_1a_2a_4^2a_5a_6^2 + 39834a_1a_2a_4a_5^3a_6 + 72a_1a_2a_5^5 + 77010a_1a_3^3a_6^3 \\
& + 37033a_1a_3^2a_4a_5a_6^2 - 1582a_1a_3^2a_5^3a_6 + 13016a_1a_3a_4^3a_6^2 + 15434a_1a_3a_4^2a_5^2a_6 \\
& + 68a_1a_3a_4a_5^4 + 999a_1a_4^4a_5a_6 - 1072a_1a_4^3a_5^3 + 10109a_2^3a_4a_6^3 \\
& - 39775a_2^3a_5^2a_6^2 - 27531a_2^2a_3^2a_6^3 - 35286a_2^2a_3a_4a_5a_6^2 - 2376a_2^2a_3a_5^3a_6 \\
& - 13941a_2^2a_4^3a_6^2 - 2337a_2^2a_4^2a_5^2a_6 + 1004a_2^2a_4a_5^4 + 10811a_2a_3^3a_5a_6^2 \\
& + 3937a_2a_3^2a_4^2a_6^2 + 11197a_2a_3^2a_4a_5^2a_6 - 190a_2a_3^2a_5^4 + 6247a_2a_3a_4^3a_5a_6 \\
& + 1136a_2a_3a_4^2a_5^3 + 516a_2a_4^5a_6 + 952a_2a_4^4a_5^2 + 1221a_3^4a_4a_6^2 \\
& - 3129a_3^4a_5^2a_6 + 24a_3^3a_4^2a_5a_6 - 344a_3^3a_4a_5^3 + 519a_3^2a_4^4a_6 \\
& - 252a_3^2a_4^3a_5^2 + 120a_3a_4^5a_5 + 8a_4^7 - 1047146a_0^2a_3a_6^3 \\
& - 453846a_0^2a_4a_5a_6^2 + 33735a_0^2a_5^3a_6 - 132766a_0a_1a_2a_6^3 + 731162a_0a_1a_3a_5a_6^2 \\
& + 195928a_0a_1a_4^2a_6^2 - 46301a_0a_1a_4a_5^2a_6 - 6510a_0a_1a_5^4 - 227793a_0a_2^2a_5a_6^2 \\
& + 88966a_0a_2a_3a_4a_6^2 + 93508a_0a_2a_3a_5^2a_6 + 104752a_0a_2a_4^2a_5a_6 + 12902a_0a_2a_4a_5^3 \\
& - 2826a_0a_3^3a_6^2 - 7315a_0a_3^2a_4a_5a_6 - 5300a_0a_3^2a_5^3 + 1057a_0a_3a_4^3a_6 \\
& - 7700a_0a_3a_4^2a_5^2 - 1751a_0a_4^4a_5 + 254162a_1^3a_6^3 - 13149a_1^2a_2a_5a_6^2 \\
& - 30811a_1^2a_3a_4a_6^2 - 33488a_1^2a_3a_5^2a_6 + 53454a_1^2a_4^2a_5a_6 - 2198a_1^2a_4a_5^3 \\
& - 26745a_1a_2^2a_4a_6^2 - 23126a_1a_2^2a_5^2a_6 + 2964a_1a_2a_3^2a_6^2 - 92266a_1a_2a_3a_4a_5a_6 \\
& + 248a_1a_2a_3a_5^3 - 3443a_1a_2a_4^3a_6 + 2800a_1a_2a_4^2a_5^2 + 21774a_1a_3^3a_5a_6 \\
& + 4727a_1a_3^2a_4^2a_6 + 984a_1a_3^2a_4a_5^2 + 1742a_1a_3a_4^3a_5 + 165a_1a_4^5 \\
& - 8382a_2^3a_3a_6^2 + 11724a_2^3a_4a_5a_6 - 796a_2^3a_5^3 - 7129a_2^2a_3^2a_5a_6 \\
& - 7160a_2^2a_3a_4^2a_6 - 1244a_2^2a_3a_4a_5^2 - 1945a_2^2a_4^3a_5 - 4523a_2a_3^3a_4a_6 \\
& - 124a_2a_3^3a_5^2 + 254a_2a_3^2a_4^2a_5 - 513a_2a_3a_4^4 + 1140a_3^5a_6 \\
& + 118a_3^3a_4^3 - 110646a_0^2a_2a_6^2 - 249926a_0^2a_3a_5a_6 - 47125a_0^2a_4^2a_6 \\
& - 27636a_0^2a_4a_5^2 - 95304a_0a_1^2a_6^2 - 32186a_0a_1a_2a_5a_6 + 59280a_0a_1a_3a_4a_6 \\
& + 13756a_0a_1a_3a_5^2 + 27598a_0a_1a_4^2a_5 - 27193a_0a_2^2a_4a_6 - 7718a_0a_2^2a_5^2
\end{aligned}$$

$$\begin{aligned}
& -41902a_0a_2a_3^2a_6 + 12420a_0a_2a_3a_4a_5 + 2810a_0a_2a_4^3 - 4540a_0a_3^3a_5 \\
& -2558a_0a_3^2a_4^2 + 67582a_1^3a_5a_6 - 37961a_1^2a_2a_4a_6 + 390a_1^2a_2a_5^2 \\
& -21518a_1^2a_3^2a_6 - 2704a_1^2a_3a_4a_5 - 3852a_1^2a_4^3 + 64314a_1a_2^2a_3a_6 \\
& -5344a_1a_2^2a_4a_5 + 2284a_1a_2a_3^2a_5 + 2526a_1a_2a_3a_4^2 + 608a_1a_3^3a_4 \\
& -10335a_2^4a_6 + 280a_2^3a_3a_5 + 1057a_2^3a_4^2 - 2030a_2^2a_3^2a_4 \\
& + 400a_2a_3^4 + 131790a_0^2a_1a_6 + 11024a_0^2a_2a_5 - 4730a_0^2a_3a_4 \\
& -3174a_0a_1^2a_5 - 24292a_0a_1a_2a_4 + 17822a_0a_1a_3^2 - 10590a_0a_2^2a_3 \\
& -3998a_1^3a_4 + 438a_1^2a_2a_3 + 2940a_1a_2^3 - 36250a_0^3
\end{aligned} \tag{A.21}$$

$$\begin{aligned}
e_{22} = & 462a_0a_5a_6^{13} - 77a_1a_5^2a_6^{12} - 154a_2a_4a_5a_6^{12} - 343a_2a_5^3a_6^{11} \\
& -21a_3^2a_5a_6^{12} + 105a_3a_4a_5^2a_6^{11} + 168a_3a_5^4a_6^{10} + 140a_4^3a_5a_6^{11} \\
& + 840a_4^2a_5^3a_6^{10} + 756a_4a_5^5a_6^9 + 120a_5^7a_6^8 + 1608a_0a_4a_6^{12} \\
& + 18041a_0a_5^2a_6^{11} + 540a_1a_3a_6^{12} - 397a_1a_4a_5a_6^{11} - 2628a_1a_5^3a_6^{10} \\
& + 15a_2^2a_6^{12} + a_2a_3a_5a_6^{11} - 266a_2a_4^2a_6^{11} - 4973a_2a_4a_5^2a_6^{10} \\
& - 4725a_2a_5^4a_6^9 - 159a_2^2a_4a_6^{11} - 858a_2^2a_5^2a_6^{10} - 180a_3a_4^2a_5a_6^{10} \\
& + 1743a_3a_4a_5^3a_6^9 + 1554a_3a_5^5a_6^8 + 85a_4^4a_6^{10} + 2940a_4^3a_5^2a_6^9 \\
& + 7770a_4^2a_5^4a_6^8 + 3696a_4a_5^6a_6^7 + 315a_5^8a_6^6 - 2148a_0a_3a_6^{11} \\
& + 37615a_0a_4a_5a_6^{10} + 153855a_0a_5^3a_6^9 - 570a_1a_2a_6^{11} + 19270a_1a_3a_5a_6^{10} \\
& + 894a_1a_4^2a_6^{10} - 11259a_1a_4a_5^2a_6^9 - 19605a_1a_5^4a_6^8 + 540a_2^2a_5a_6^{10} \\
& + 1045a_2a_3a_4a_6^{10} - 175a_2a_3a_5^2a_6^9 - 4497a_2a_4^2a_5a_6^9 - 31659a_2a_4a_5^3a_6^8 \\
& - 15099a_2a_5^5a_6^7 + 159a_3^3a_6^{10} - 5037a_3^2a_4a_5a_6^9 - 7317a_3^2a_5^3a_6^8 \\
& - 685a_3a_4^3a_6^9 - 3807a_3a_4^2a_5^2a_6^8 + 6825a_3a_4a_5^4a_6^7 + 3528a_3a_5^6a_6^6 \\
& + 270a_4^4a_5a_6^8 + 9660a_4^3a_5^3a_6^7 + 14448a_4^2a_5^5a_6^6 + 3672a_4a_5^7a_6^5 \\
& + 150a_5^9a_6^4 + 54048a_0a_2a_6^{10} - 83647a_0a_3a_5a_6^9 - 10680a_0a_4^2a_6^9 \\
& + 138567a_0a_4a_5^2a_6^8 + 395520a_0a_5^4a_6^7 - 55410a_1^2a_6^{10} + 7744a_1a_2a_5a_6^9 \\
& + 24531a_1a_3a_4a_6^9 + 144142a_1a_3a_5^2a_6^8 + 9065a_1a_4^2a_5a_6^8 - 62148a_1a_4a_5^3a_6^7 \\
& - 41676a_1a_5^5a_6^6 - 6814a_2^2a_4a_6^9 + 2749a_2^2a_5^2a_6^8 + 4522a_2a_3^2a_6^9 \\
& + 16823a_2a_3a_4a_5a_6^8 - 2760a_2a_3a_5^3a_6^7 + 3657a_2a_4^3a_6^8 - 2301a_2a_4^2a_5^2a_6^7 \\
& - 49425a_2a_4a_5^4a_6^6 - 13062a_2a_5^6a_6^5 + 2464a_3^3a_5a_6^8 - 1539a_3^2a_4^2a_6^8 \\
& - 31716a_3^2a_4a_5^2a_6^7 - 17250a_3^2a_5^4a_6^6 - 7767a_3a_4^3a_5a_6^7 - 14265a_3a_4^2a_5^3a_6^6 \\
& + 8442a_3a_4a_5^5a_6^5 + 2280a_3a_5^7a_6^4 - 540a_4^5a_6^7 - 5610a_4^4a_5^2a_6^6 \\
& + 2520a_4^3a_5^4a_6^5 + 5460a_4^2a_5^6a_6^4 + 720a_4a_5^8a_6^3 + 10a_5^{10}a_6^2 \\
& + 57342a_0a_1a_6^9 + 524177a_0a_2a_5a_6^8 + 25512a_0a_3a_4a_6^8 - 560689a_0a_3a_5^2a_6^7 \\
& - 185708a_0a_4^2a_5a_6^7 + 8412a_0a_4a_5^3a_6^6 + 326016a_0a_5^5a_6^5 - 511429a_1^2a_5a_6^8
\end{aligned}$$

$$\begin{aligned}
& -40043a_1a_2a_4a_6^8 + 115847a_1a_2a_5^2a_6^7 - 29862a_1a_3^2a_6^8 + 191914a_1a_3a_4a_5a_6^7 \\
& + 300087a_1a_3a_5^3a_6^6 + 10978a_1a_4^3a_6^7 + 29637a_1a_4^2a_5^2a_6^6 - 83880a_1a_4a_5^4a_6^5 \\
& - 24699a_1a_5^6a_6^4 - 3569a_2^2a_3a_6^8 - 69120a_2^2a_4a_5a_6^7 - 2523a_2^2a_5^3a_6^6 \\
& + 44512a_2a_3^2a_5a_6^7 + 2854a_2a_3a_4^2a_6^7 + 54624a_2a_3a_4a_5^2a_6^6 - 7935a_2a_3a_5^4a_6^5 \\
& + 36753a_2a_4^3a_5a_6^6 + 41820a_2a_4^2a_5^3a_6^5 - 14052a_2a_4a_5^5a_6^4 - 2541a_2a_5^7a_6^3 \\
& + 7118a_3^3a_4a_6^7 + 6555a_3^3a_5^2a_6^6 - 5436a_3^2a_4^2a_5a_6^6 - 51705a_3^2a_4a_5^3a_6^5 \\
& - 10875a_3^2a_5^5a_6^4 + 1143a_3a_4^4a_6^6 - 13470a_3a_4^3a_5^2a_6^5 - 11730a_3a_4^2a_5^4a_6^4 \\
& + 3171a_3a_4a_5^6a_6^3 + 336a_3a_5^8a_6^2 - 2190a_4^5a_5a_6^5 - 10380a_4^4a_5^3a_6^4 \\
& - 3276a_4^3a_5^5a_6^3 + 168a_4^2a_5^7a_6^2 + 14a_4a_5^9a_6 + 385455a_0^2a_6^8 \\
& + 415100a_0a_1a_5a_6^7 - 85900a_0a_2a_4a_6^7 + 1187434a_0a_2a_5^2a_6^6 + 404019a_0a_2^2a_6^7 \\
& + 53513a_0a_3a_4a_5a_6^6 - 936129a_0a_3a_5^3a_6^5 + 52394a_0a_4^3a_6^6 - 405609a_0a_4^2a_5^2a_6^5 \\
& - 146724a_0a_4a_5^4a_6^4 + 77979a_0a_5^6a_6^3 + 127008a_1^2a_4a_6^7 - 1034558a_1^2a_5^2a_6^6 \\
& - 329293a_1a_2a_3a_6^7 - 167495a_1a_2a_4a_5a_6^6 + 228933a_1a_2a_5^3a_6^5 - 126438a_1a_2^3a_5a_6^6 \\
& - 15903a_1a_3a_4^2a_6^6 + 295764a_1a_3a_4a_5^2a_6^5 + 170001a_1a_3a_5^4a_6^4 + 50361a_1a_4^3a_5a_6^5 \\
& + 48891a_1a_4^2a_5^3a_6^4 - 22887a_1a_4a_5^5a_6^3 - 3048a_1a_5^7a_6^2 + 70938a_2^3a_6^7 \\
& - 12864a_2^2a_3a_5a_6^6 - 36821a_2^2a_4^2a_6^6 - 155148a_2^2a_4a_5^2a_6^5 - 13182a_2^2a_5^4a_6^4 \\
& - 9746a_2a_3^2a_4a_6^6 + 90507a_2a_3^2a_5^2a_6^5 - 7461a_2a_3a_4^2a_5a_6^5 + 42357a_2a_3a_4a_5^3a_6^4 \\
& - 4707a_2a_3a_5^5a_6^3 + 552a_2a_4^4a_6^5 + 54294a_2a_4^3a_5^2a_6^4 + 35310a_2a_4^2a_5^4a_6^3 \\
& + 1113a_2a_4a_5^6a_6^2 - 48a_2a_5^8a_6 + 4030a_3^4a_6^6 + 29187a_3^3a_4a_5a_6^5 \\
& + 657a_3^3a_5^3a_6^4 + 5769a_3^2a_4^3a_6^5 + 1566a_3^2a_4^2a_5^2a_6^4 - 20595a_3^2a_4a_5^4a_6^3 \\
& - 1260a_3^2a_5^6a_6^2 + 7845a_3a_4^4a_5a_6^4 - 2910a_3a_4^3a_5^3a_6^3 - 2079a_3a_4^2a_5^5a_6^2 \\
& + 204a_3a_4a_5^7a_6 + 4a_3a_5^9 + 471a_4^6a_6^4 + 600a_4^5a_5^2a_6^3 \\
& - 1890a_4^4a_5^4a_6^2 - 336a_4^3a_5^6a_6 - 3a_4^2a_5^8 + 912569a_0^2a_5a_6^6 \\
& - 66036a_0a_1a_4a_6^6 + 854579a_0a_1a_5^2a_6^5 - 292961a_0a_2a_3a_6^6 - 477020a_0a_2a_4a_5a_6^5 \\
& + 770681a_0a_2a_5^3a_6^4 + 1250328a_0a_3^2a_5a_6^5 + 268125a_0a_3a_4^2a_6^5 + 50256a_0a_3a_4a_5^2a_6^4 \\
& - 374262a_0a_3a_5^4a_6^3 + 223505a_0a_4^3a_5a_6^4 - 136080a_0a_4^2a_5^3a_6^3 - 36603a_0a_4a_5^5a_6^2 \\
& + 3627a_0a_5^7a_6 + 194991a_1^2a_3a_6^6 + 567509a_1^2a_4a_5a_6^5 - 539102a_1^2a_5^3a_6^4 \\
& + 148857a_1a_2^2a_6^6 - 883424a_1a_2a_3a_5a_6^5 - 78823a_1a_2a_4^2a_6^5 - 224052a_1a_2a_4a_5^2a_6^4 \\
& + 84663a_1a_2a_5^4a_6^3 - 93840a_1a_3^2a_4a_6^5 - 74384a_1a_3^2a_5^2a_6^4 - 69023a_1a_3a_4^2a_5a_6^4 \\
& + 89487a_1a_3a_4a_5^3a_6^3 + 19252a_1a_3a_5^5a_6^2 + 1635a_1a_4^4a_6^4 + 49448a_1a_4^3a_5^2a_6^3 \\
& + 21776a_1a_4^2a_5^4a_6^2 - 159a_1a_4a_5^6a_6 - 24a_1a_5^8 + 175606a_2^3a_5a_6^5 \\
& + 37712a_2^2a_3a_4a_6^5 - 1703a_2^2a_3a_5^2a_6^4 - 61363a_2^2a_4^2a_5a_6^4 - 82524a_2^2a_4a_5^3a_6^3 \\
& - 5130a_2^2a_5^5a_6^2 - 25196a_2a_3^3a_6^5 - 45451a_2a_3^2a_4a_5a_6^4 + 39372a_2a_3^2a_5^3a_6^3 \\
& - 25569a_2a_3a_4^3a_6^4 - 22440a_2a_3a_4^2a_5^2a_6^3 + 7277a_2a_3a_4a_5^4a_6^2 - 345a_2a_3a_5^6a_6 \\
& - 10463a_2a_4^4a_5a_6^3 + 8794a_2a_4^3a_5^3a_6^2 + 3114a_2a_4^2a_5^5a_6 + 60a_2a_4a_5^7
\end{aligned}$$

$$\begin{aligned}
& + 14817a_3^4a_5a_6^4 + 2529a_3^3a_4^2a_6^4 + 19062a_3^3a_4a_5^2a_6^3 - 2423a_3^3a_5^4a_6^2 \\
& + 10068a_3^2a_4^3a_5a_6^3 + 3096a_3^2a_4^2a_5^3a_6^2 - 1305a_3^2a_4a_5^5a_6 + 339a_3a_4^5a_6^3 \\
& + 4350a_3a_4^4a_5^2a_6^2 - 60a_3a_4^3a_5^4a_6 - 60a_3a_4^2a_5^6 + 656a_4^6a_5a_6^2 \\
& + 600a_4^5a_3^3a_6 + 12a_4^4a_5^5 + 604008a_0^2a_4a_6^5 + 17613a_0^2a_5^2a_6^4 \\
& - 2038959a_0a_1a_3a_6^5 + 445321a_0a_1a_4a_5a_6^4 + 452498a_0a_1a_5^3a_6^3 + 576186a_0a_2a_6^5 \\
& - 612696a_0a_2a_3a_5a_6^4 - 350473a_0a_2a_4^2a_6^4 - 366972a_0a_2a_4a_5^2a_6^3 + 127334a_0a_2a_5^4a_6^2 \\
& + 116748a_0a_2^3a_4a_6^4 + 650228a_0a_2^2a_5^2a_6^3 + 340358a_0a_3a_4^2a_5a_6^3 + 34752a_0a_3a_4a_5^3a_6^2 \\
& - 22969a_0a_3a_5^5a_6 + 30220a_0a_4^4a_6^3 + 123288a_0a_4^3a_5^2a_6^2 + 418a_0a_4^2a_5^4a_6 \\
& - 525a_0a_4a_6^6 + 272823a_1^2a_2a_6^5 + 387914a_1^2a_3a_5a_6^4 - 188110a_1^2a_4^2a_6^4 \\
& + 338586a_1^2a_4a_5^2a_6^3 - 63998a_1^2a_5^4a_6^2 + 128962a_1a_2^2a_5a_6^4 + 362356a_1a_2a_3a_4a_6^4 \\
& - 371134a_1a_2a_3a_5^2a_6^3 - 123329a_1a_2a_4^2a_5a_6^3 - 95631a_1a_2a_4a_5^3a_6^2 + 2648a_1a_2a_5^5a_6 \\
& - 104464a_1a_3^3a_6^4 - 70206a_1a_3^2a_4a_5a_6^3 + 5332a_1a_3^2a_5^3a_6^2 - 21337a_1a_3a_4^3a_6^3 \\
& - 33377a_1a_3a_4^2a_5^2a_6^2 + 2137a_1a_3a_4a_5^4a_6 + 126a_1a_3a_5^6 + 88a_1a_4^4a_5a_6^2 \\
& + 7414a_1a_4^3a_5^3a_6 + 861a_1a_4^2a_5^5 - 14348a_2^3a_4a_6^4 + 78077a_2^3a_5^2a_6^3 \\
& + 30510a_2^2a_3^2a_6^4 + 41009a_2^2a_3a_4a_5a_6^3 + 3583a_2^2a_3a_5^3a_6^2 + 20452a_2^2a_4^3a_6^3 \\
& - 3652a_2^2a_4^2a_5^2a_6^2 - 6494a_2^2a_4a_5^4a_6 - 147a_2^2a_5^6 - 22802a_2a_3^3a_5a_6^3 \\
& - 16586a_2a_3^2a_4^2a_6^3 - 28804a_2a_3^2a_4a_5^2a_6^2 + 1894a_2a_3^2a_5^4a_6 - 20800a_2a_3a_4^3a_5a_6^2 \\
& - 4034a_2a_3a_4^2a_5^3a_6 + 189a_2a_3a_4a_5^5 - 1927a_2a_4^5a_6^2 - 4864a_2a_4^4a_5^2a_6 \\
& - 252a_2a_4^3a_5^4 - 1254a_3^4a_4a_6^3 + 8715a_3^4a_5^2a_6^2 + 1242a_3^3a_4^2a_5a_6^2 \\
& + 1460a_3^3a_4a_5^3a_6 - 156a_3^3a_5^5 - 1227a_3^2a_4^4a_6^2 + 2034a_3^2a_4^3a_5^2a_6 \\
& + 162a_3^2a_4^2a_5^4 - 372a_3a_4^5a_5a_6 + 120a_3a_4^4a_5^3 - 32a_4^7a_6 \\
& + 12a_4^6a_5^2 + 1383324a_0^2a_3a_6^4 + 627024a_0^2a_4a_5a_6^3 - 143448a_0^2a_5^3a_6^2 \\
& + 138699a_0a_1a_2a_6^4 - 1454832a_0a_1a_3a_5a_6^3 - 333100a_0a_1a_4^2a_6^3 + 244440a_0a_1a_4a_5^2a_6^2 \\
& + 37007a_0a_1a_5^4a_6 + 419582a_0a_2^2a_5a_6^3 - 191748a_0a_2a_3a_4a_6^3 - 241148a_0a_2a_3a_5^2a_6^2 \\
& - 238390a_0a_2a_4^2a_5a_6^2 - 50386a_0a_2a_4a_5^3a_6 + 2319a_0a_2a_5^5 - 15446a_0a_3^3a_6^3 \\
& + 18924a_0a_3^2a_4a_5a_6^2 + 37121a_0a_3^2a_5^3a_6 - 4530a_0a_3a_4^3a_6^2 + 53099a_0a_3a_4^2a_5^2a_6 \\
& + 1802a_0a_3a_4a_5^4 + 8501a_0a_4^4a_5a_6 + 5854a_0a_4^3a_5^3 - 345018a_1^3a_6^4 \\
& + 64242a_1^2a_2a_5a_6^3 + 28808a_1^2a_3a_4a_6^3 + 103391a_1^2a_3a_5^2a_6^2 - 164591a_1^2a_4^2a_5a_6^2 \\
& + 21643a_1^2a_4a_5^3a_6 - 966a_1^2a_5^5 + 58452a_1a_2^2a_4a_6^3 + 50531a_1a_2^2a_5^2a_6^2 \\
& + 28146a_1a_2a_3^2a_6^3 + 267384a_1a_2a_3a_4a_5a_6^2 - 13282a_1a_2a_3a_5^3a_6 + 8375a_1a_2a_4^3a_6^2 \\
& - 22118a_1a_2a_4^2a_5^2a_6 - 4003a_1a_2a_4a_5^4 - 63070a_1a_3^3a_5a_6^2 - 9390a_1a_3^2a_4^2a_6^2 \\
& - 4283a_1a_3^2a_4a_5^2a_6 + 664a_1a_3^2a_5^4 - 7345a_1a_3a_4^3a_5a_6 - 1292a_1a_3a_4^2a_5^3 \\
& - 711a_1a_4^5a_6 - 378a_1a_4^4a_5^2 + 15680a_2^3a_3a_6^3 - 33743a_2^3a_4a_5a_6^2 \\
& + 5477a_2^3a_5^3a_6 + 17435a_2^2a_3^2a_5a_6^2 + 19313a_2^2a_3a_4^2a_6^2 + 4761a_2^2a_3a_4a_5^2a_6 \\
& + 129a_2^2a_3a_5^4 + 10338a_2^2a_4^3a_5a_6 + 1078a_2^2a_4^2a_5^3 + 9994a_2a_3^3a_4a_6^2
\end{aligned}$$

$$\begin{aligned}
& -305a_2a_3^3a_5^2a_6 - 5679a_2a_3^2a_4^2a_5a_6 - 1476a_2a_3^2a_4a_5^3 + 1808a_2a_3a_4^4a_6 \\
& - 770a_2a_3a_4^3a_5^2 - 84a_2a_4^5a_5 - 2895a_3^5a_6^2 + 495a_3^4a_4a_5a_6 \\
& + 436a_3^4a_5^3 - 753a_3^3a_4^3a_6 + 168a_3^3a_4^2a_5^2 - 168a_3^2a_4^4a_5 \\
& - 28a_3a_4^6 + 281550a_0^2a_2a_6^3 + 733735a_0^2a_3a_5a_6^2 + 130748a_0^2a_4^2a_6^2 \\
& + 137634a_0^2a_4a_5^2a_6 - 4032a_0^2a_5^4 + 224628a_0a_1^2a_6^3 + 38498a_0a_1a_2a_5a_6^2 \\
& - 148660a_0a_1a_3a_4a_6^2 - 83083a_0a_1a_3a_5^2a_6 - 150122a_0a_1a_4^2a_5a_6 + 4487a_0a_1a_4a_5^3 \\
& + 39652a_0a_2^2a_4a_6^2 + 40069a_0a_2^2a_5^2a_6 + 108484a_0a_2a_3^2a_6^2 - 86388a_0a_2a_3a_4a_5a_6 \\
& - 10754a_0a_2a_3a_5^3 - 15344a_0a_2a_4^3a_6 - 15514a_0a_2a_4^2a_5^2 + 19601a_0a_3^3a_5a_6 \\
& + 7821a_0a_3^2a_4^2a_6 - 997a_0a_3^2a_4a_5^2 + 415a_0a_3a_4^3a_5 + 495a_0a_4^5 \\
& - 206488a_1^3a_5a_6^2 + 89570a_1^2a_2a_4a_6^2 - 3687a_1^2a_2a_5^2a_6 + 70721a_1^2a_3^2a_6^2 \\
& + 546a_1^2a_3a_4a_5a_6 + 2802a_1^2a_3a_5^3 + 19800a_1^2a_4^3a_6 - 7488a_1^2a_4^2a_5^2 \\
& - 180387a_1a_2^2a_3a_6^2 + 36569a_1a_2^2a_4a_5a_6 + 3094a_1a_2^2a_5^3 - 4100a_1a_2a_3^2a_5a_6 \\
& - 12311a_1a_2a_3a_4^2a_6 + 12708a_1a_2a_3a_4a_5^2 + 2124a_1a_2a_4^3a_5 - 2145a_1a_3^3a_4a_6 \\
& - 2342a_1a_3^3a_5^2 - 2037a_1a_3^2a_4^2a_5 - 612a_1a_3a_4^4 + 32831a_2^4a_6^2 \\
& - 1739a_2^3a_3a_5a_6 - 5956a_2^3a_4^2a_6 - 2219a_2^3a_4a_5^2 + 13215a_2^2a_3^2a_4a_6 \\
& + 1092a_2^2a_3^2a_5^2 + 474a_2^2a_3a_4^2a_5 + 159a_2^2a_4^4 - 1970a_2a_3^4a_6 \\
& + 1095a_2a_3^3a_4a_5 + 1145a_2a_3^2a_4^3 - 240a_3^5a_5 - 195a_3^4a_4^2 \\
& - 432354a_0^2a_1a_6^2 - 66785a_0^2a_2a_5a_6 + 23081a_0^2a_3a_4a_6 + 25796a_0^2a_3a_5^2 \\
& + 9409a_0^2a_4^2a_5 + 25048a_0a_1^2a_5a_6 + 149494a_0a_1a_2a_4a_6 + 8229a_0a_1a_2a_5^2 \\
& - 102028a_0a_1a_3^2a_6 - 14490a_0a_1a_3a_4a_5 - 4608a_0a_1a_4^3 + 63833a_0a_2^2a_3a_6 \\
& + 5443a_0a_2^2a_4a_5 + 10361a_0a_2a_3^2a_5 - 349a_0a_2a_3a_4^2 + 4478a_0a_3^3a_4 \\
& + 23540a_1^3a_4a_6 - 7530a_1^3a_5^2 - 1502a_1^2a_2a_3a_6 + 9276a_1^2a_2a_4a_5 \\
& + 3366a_1^2a_3^2a_5 + 6948a_1^2a_3a_4^2 - 18023a_1a_2^3a_6 - 11898a_1a_2^2a_3a_5 \\
& - 2649a_1a_2^2a_4^2 - 3933a_1a_2a_3^2a_4 - 200a_1a_3^4 + 1747a_2^4a_5 \\
& + 649a_2^3a_3a_4 - 435a_2^2a_3^3 + 217500a_0^3a_6 - 22454a_0^2a_1a_5 \\
& + 9879a_0^2a_2a_4 + 932a_0^2a_3^2 + 576a_0a_1^2a_4 - 12064a_0a_1a_2a_3 \\
& + 2453a_0a_2^3 + 640a_1^3a_3 + 57a_1^2a_2^2
\end{aligned} \tag{A.22}$$

$$\begin{aligned}
e_{23} = & -138a_0a_4a_6^{13} - 1444a_0a_5^2a_6^{12} - 36a_1a_3a_6^{13} + 47a_1a_4a_5a_6^{12} \\
& + 234a_1a_5^3a_6^{11} - a_2^2a_6^{13} + 13a_2a_3a_5a_6^{12} + 28a_2a_4^2a_6^{12} \\
& + 465a_2a_4a_5^2a_6^{11} + 441a_2a_5^4a_6^{10} + 12a_3^2a_4a_6^{12} + 60a_3^2a_5^2a_6^{11} \\
& - 30a_3a_4^2a_5a_6^{11} - 315a_3a_4a_5^3a_6^{10} - 210a_3a_5^5a_6^9 - 15a_4^4a_6^{11} \\
& - 420a_4^3a_5^2a_6^{10} - 1050a_4^2a_5^4a_6^9 - 504a_4a_5^6a_6^8 - 45a_5^8a_6^7 \\
& + 174a_0a_3a_6^{12} - 8475a_0a_4a_5a_6^{11} - 29996a_0a_5^3a_6^{10} + 38a_1a_2a_6^{12}
\end{aligned}$$

$$\begin{aligned}
& -3012a_1a_3a_5a_6^{11} - 75a_1a_4a_6^{11} + 2734a_1a_4a_5^2a_6^{10} + 4200a_1a_5^4a_6^9 \\
& - 95a_2^2a_5a_6^{11} - 93a_2a_3a_4a_6^{11} + 494a_2a_3a_5^2a_6^{10} + 1424a_2a_4^2a_5a_6^{10} \\
& + 7581a_2a_4a_5^3a_6^9 + 3570a_2a_5^5a_6^8 - 12a_3^3a_6^{11} + 936a_3^2a_4a_5a_6^{10} \\
& + 1260a_3^2a_5^3a_6^9 + 90a_3a_4^3a_6^{10} - 315a_3a_4^2a_5^2a_6^9 - 3150a_3a_4a_5^4a_6^8 \\
& - 1176a_3a_5^6a_6^7 - 420a_4^4a_5a_6^9 - 4200a_4^3a_5^3a_6^8 - 5376a_4^2a_5^5a_6^7 \\
& - 1368a_4a_5^7a_6^6 - 60a_5^9a_6^5 - 12146a_0a_2a_6^{11} + 17599a_0a_3a_5a_6^{10} \\
& + 575a_0a_4^2a_6^{10} - 74213a_0a_4a_5^2a_6^9 - 147065a_0a_5^4a_6^8 + 12878a_1^2a_6^{11} \\
& - 3388a_1a_2a_5a_6^{10} - 7085a_1a_3a_4a_6^{10} - 40968a_1a_3a_5^2a_6^9 - 221a_1a_4^2a_5a_6^9 \\
& + 27112a_1a_4a_5^3a_6^8 + 17050a_1a_5^5a_6^7 + 1419a_2^2a_4a_6^{10} - 1172a_2^2a_5^2a_6^9 \\
& - 1183a_2a_3^2a_6^{10} - 3079a_2a_3a_4a_5a_6^9 + 4256a_2a_3a_5^3a_6^8 - 577a_2a_4^3a_6^9 \\
& + 8093a_2a_4^2a_5^2a_6^8 + 25205a_2a_4a_5^4a_6^7 + 6356a_2a_5^6a_6^6 - 288a_3^3a_5a_6^9 \\
& + 822a_3^2a_4^2a_6^9 + 11283a_3^2a_4a_5^2a_6^8 + 5700a_3^2a_5^4a_6^7 + 2595a_3a_4^3a_5a_6^8 \\
& + 825a_3a_4^2a_5^3a_6^7 - 7308a_3a_4a_5^5a_6^6 - 1488a_3a_5^7a_6^5 + 150a_4^5a_6^8 \\
& + 150a_4^4a_5^2a_6^7 - 6160a_4^3a_5^4a_6^6 - 4956a_4^2a_5^6a_6^5 - 630a_4a_5^8a_6^4 \\
& - 10a_5^{10}a_6^3 - 13648a_0a_1a_6^{10} - 190889a_0a_2a_5a_6^9 - 350a_0a_3a_4a_6^9 \\
& + 217060a_0a_3a_5^2a_6^8 + 62945a_0a_4^2a_5a_6^8 - 123688a_0a_4a_5^3a_6^7 - 219040a_0a_5^5a_6^6 \\
& + 197172a_1^2a_5a_6^9 + 10917a_1a_2a_4a_6^9 - 59770a_1a_2a_5^2a_6^8 + 8336a_1a_3^2a_6^9 \\
& - 93308a_1a_3a_4a_5a_6^8 - 145098a_1a_3a_5^3a_6^7 - 4059a_1a_4^3a_6^8 + 2630a_1a_4^2a_5^2a_6^7 \\
& + 65970a_1a_4a_5^4a_6^6 + 18864a_1a_5^6a_6^5 + 1062a_2^2a_3a_6^9 + 23389a_2^2a_4a_5a_6^8 \\
& - 1710a_2^2a_5^3a_6^7 - 19432a_2a_3^2a_5a_6^8 - 2787a_2a_3a_4^2a_6^8 - 19376a_2a_3a_4a_5^2a_6^7 \\
& + 11110a_2a_3a_5^4a_6^6 - 14501a_2a_4^3a_5a_6^7 - 4095a_2a_4^2a_5^3a_6^6 + 18570a_2a_4a_5^5a_6^5 \\
& + 2664a_2a_5^7a_6^4 - 2448a_3^3a_4a_6^8 - 618a_3^3a_5^2a_6^7 + 7662a_3^2a_4^2a_5a_6^7 \\
& + 33000a_3^2a_4a_5^3a_6^6 + 6504a_3^2a_5^5a_6^5 + 195a_3a_4^4a_6^7 + 11325a_3a_4^3a_5^2a_6^6 \\
& + 3930a_3a_4^2a_5^4a_6^5 - 4704a_3a_4a_5^6a_6^4 - 444a_3a_5^8a_6^3 + 1980a_4^5a_5a_6^6 \\
& + 6480a_4^4a_5^3a_6^5 + 504a_4^3a_5^5a_6^4 - 672a_4^2a_5^7a_6^3 - 36a_4a_5^9a_6^2 \\
& - 175315a_0^2a_6^9 - 155058a_0a_1a_5a_6^8 + 20796a_0a_2a_4a_6^8 - 684230a_0a_2a_5^2a_6^7 \\
& - 189771a_0a_3^2a_6^8 + 80477a_0a_3a_4a_5a_6^7 + 631212a_0a_3a_5^3a_6^6 - 11760a_0a_4^3a_6^7 \\
& + 297682a_0a_4^2a_5^2a_6^6 + 9624a_0a_4a_5^4a_6^5 - 98974a_0a_5^6a_6^4 - 39339a_1^2a_4a_6^8 \\
& + 656304a_1^2a_5^2a_6^7 + 165435a_1a_2a_3a_6^8 + 58121a_1a_2a_4a_5a_6^7 - 189028a_1a_2a_5^3a_6^6 \\
& + 55486a_1a_3^2a_5a_6^7 - 2337a_1a_3a_4^2a_6^7 - 244563a_1a_3a_4a_5^2a_6^6 - 143121a_1a_3a_5^4a_6^5 \\
& - 29955a_1a_4^3a_5a_6^6 - 8601a_1a_4^2a_5^3a_6^5 + 38315a_1a_4a_5^5a_6^4 + 5010a_1a_5^7a_6^3 \\
& - 32625a_2^3a_6^8 + 5840a_2^2a_3a_5a_6^7 + 20509a_2^2a_4^2a_6^7 + 86767a_2^2a_4a_5^2a_6^6 \\
& + 5267a_2^2a_5^4a_6^5 + 132a_2a_3^2a_4a_6^7 - 66204a_2a_3^2a_5^2a_6^6 - 12445a_2a_3a_4^2a_5a_6^6 \\
& - 31017a_2a_3a_4a_5^3a_6^5 + 8419a_2a_3a_5^5a_6^4 - 2562a_2a_4^4a_6^6 - 46824a_2a_4^3a_5^2a_6^5 \\
& - 25640a_2a_4^2a_5^4a_6^4 + 1227a_2a_4a_5^6a_6^3 + 174a_2a_5^8a_6^2 - 2544a_3^4a_6^7
\end{aligned}$$

$$\begin{aligned}
& -16701a_3^3a_4a_5a_6^6 + 3153a_3^3a_5^3a_6^5 - 3477a_3^2a_4^3a_6^6 + 10539a_3^2a_4^2a_5^2a_6^5 \\
& + 24180a_3^2a_4a_5^4a_6^4 + 1398a_3^2a_5^6a_6^3 - 3105a_3a_4^4a_5a_6^5 + 9330a_3a_4^3a_5^3a_6^4 \\
& + 2115a_3a_4^2a_5^5a_6^3 - 666a_3a_4a_5^7a_6^2 - 18a_3a_5^9a_6 - 207a_4^6a_5^5 \\
& + 1740a_4^5a_5^4a_6^4 + 4050a_4^4a_5^4a_6^3 + 672a_4^3a_5^6a_6^2 + 3a_4^2a_5^8a_6 \\
& - 664650a_0^2a_5a_6^7 + 3336a_0a_1a_4a_6^7 - 516086a_0a_1a_5^2a_6^6 + 158109a_0a_2a_3a_6^7 \\
& + 257736a_0a_2a_4a_5a_6^6 - 724398a_0a_2a_5^3a_6^5 - 933982a_0a_3^2a_5a_6^6 - 179352a_0a_3a_4^2a_6^6 \\
& + 205149a_0a_3a_4a_5^2a_6^5 + 464399a_0a_3a_5^4a_6^4 - 124953a_0a_4^3a_5a_6^5 + 242767a_0a_4^2a_5^3a_6^4 \\
& + 36671a_0a_4a_5^5a_6^3 - 10792a_0a_5^7a_6^2 - 122898a_1^2a_3a_6^7 - 377012a_1^2a_4a_5a_6^6 \\
& + 588334a_1^2a_5^3a_6^5 - 76827a_1a_2^2a_6^7 + 728752a_1a_2a_3a_5a_6^6 + 57902a_1a_2a_4^2a_6^6 \\
& + 120055a_1a_2a_4a_5^2a_6^5 - 134101a_1a_2a_5^4a_6^4 + 70146a_1a_3^2a_4a_6^6 + 50772a_1a_3^2a_5^2a_6^5 \\
& + 29868a_1a_3a_4^2a_5a_6^5 - 140740a_1a_3a_4a_5^3a_6^4 - 32640a_1a_3a_5^5a_6^3 - 2889a_1a_4^4a_6^5 \\
& - 53559a_1a_4^3a_5^2a_6^4 - 20089a_1a_4^2a_5^4a_6^3 + 3182a_1a_4a_5^6a_6^2 + 178a_1a_5^8a_6 \\
& - 130988a_2^3a_5a_6^6 - 15788a_2^2a_3a_4a_6^6 + 1322a_2^2a_3a_5^2a_6^5 + 63071a_2^2a_4^2a_5a_6^5 \\
& + 85025a_2^2a_4a_5^3a_6^4 + 5761a_2^2a_5^5a_6^3 + 17434a_2a_3^3a_6^6 + 19809a_2a_3^2a_4a_5a_6^5 \\
& - 53207a_2a_3^2a_5^3a_6^4 + 18225a_2a_3a_4^3a_6^5 - 4695a_2a_3a_4^2a_5^2a_6^4 - 13433a_2a_3a_4a_5^4a_6^3 \\
& + 1270a_2a_3a_5^6a_6^2 + 3159a_2a_4^4a_5a_6^4 - 23106a_2a_4^3a_5^3a_6^3 - 7289a_2a_4^2a_5^5a_6^2 \\
& - 184a_2a_4a_5^7a_6 - 14136a_3^4a_5a_6^5 - 4449a_3^3a_4^2a_6^5 - 18801a_3^3a_4a_5^2a_6^4 \\
& + 4902a_3^3a_5^4a_6^3 - 13374a_3^2a_4^3a_5a_6^4 + 576a_3^2a_4^2a_5^3a_6^3 + 3396a_3^2a_4a_5^5a_6^2 \\
& - 24a_3^2a_5^7a_6 - 1125a_3a_4^5a_6^4 - 5970a_3a_4^4a_5^2a_6^3 + 1515a_3a_4^3a_5^4a_6^2 \\
& + 240a_3a_4^2a_5^6a_6 - 6a_3a_4a_5^8 - 1104a_4^6a_5a_6^3 - 840a_4^5a_5^3a_6^2 \\
& + 120a_4^4a_5^5a_6 + 8a_4^3a_5^7 - 367473a_0^2a_4a_6^6 - 98748a_0^2a_5^2a_6^5 \\
& + 1678527a_0a_1a_3a_6^6 - 594537a_0a_1a_4a_5a_6^5 - 471266a_0a_1a_5^3a_6^4 - 453116a_0a_2^2a_6^6 \\
& + 506630a_0a_2a_3a_5a_6^5 + 288337a_0a_2a_4^2a_6^5 + 338171a_0a_2a_4a_5^2a_6^4 - 227123a_0a_2a_5^3a_6^3 \\
& - 156630a_0a_3^2a_4a_6^5 - 856024a_0a_3^2a_5^2a_6^4 - 422677a_0a_3a_4^2a_5a_6^4 + 51125a_0a_3a_4a_5^3a_6^3 \\
& + 67293a_0a_3a_5^5a_6^2 - 38206a_0a_4^4a_6^4 - 157376a_0a_4^3a_5^2a_6^3 + 27741a_0a_4^2a_5^4a_6^2 \\
& + 3079a_0a_4a_5^6a_6 - 120a_0a_5^8 - 276070a_1^2a_2a_6^6 - 405494a_1^2a_3a_5a_6^5 \\
& + 151379a_1^2a_4a_6^5 - 466302a_1^2a_4a_5^2a_6^4 + 134836a_1^2a_5^4a_6^3 - 80584a_1a_2^2a_5a_6^5 \\
& - 301902a_1a_2a_3a_4a_6^5 + 579526a_1a_2a_3a_5^2a_6^4 + 159328a_1a_2a_4^2a_5a_6^4 + 119542a_1a_2a_4a_5^3a_6^3 \\
& - 15232a_1a_2a_5^5a_6^2 + 102228a_1a_3^3a_6^5 + 91538a_1a_3^2a_4a_5a_6^4 - 12016a_1a_3^2a_5^3a_6^3 \\
& + 24502a_1a_3a_4^3a_6^4 + 40948a_1a_3a_4^2a_5^2a_6^3 - 11349a_1a_3a_4a_5^4a_6^2 - 840a_1a_3a_5^6a_6 \\
& - 3742a_1a_4^4a_5a_6^3 - 20022a_1a_4^3a_5^3a_6^2 - 3647a_1a_4^2a_5^5a_6 - 38a_1a_4a_5^7 \\
& + 12934a_2^3a_4a_6^5 - 104220a_2^3a_5^2a_6^4 - 21384a_2^2a_3^2a_6^5 - 24867a_2^2a_3a_4a_5a_6^4 \\
& - 4255a_2^2a_3a_5^3a_6^3 - 18516a_2^2a_4^3a_6^4 + 20352a_2^2a_4^2a_5^2a_6^3 + 17173a_2^2a_4a_5^4a_6^2 \\
& + 658a_2^2a_5^6a_6 + 29044a_2a_3^3a_5a_6^4 + 27896a_2a_3^2a_4^2a_6^4 + 34826a_2a_3^2a_4a_5^2a_6^3 \\
& - 7095a_2a_3^2a_5^4a_6^2 + 33040a_2a_3a_4^3a_5a_6^3 + 1578a_2a_3a_4^2a_5^3a_6^2 - 1369a_2a_3a_4a_5^5a_6
\end{aligned}$$

$$\begin{aligned}
& + 6a_2a_3a_5^7 + 3457a_2a_4^5a_6^3 + 9042a_2a_4^4a_5^2a_6^2 - 202a_2a_4^3a_5^4a_6 \\
& - 100a_2a_4^2a_5^6 + 189a_3^4a_4a_6^4 - 14265a_3^4a_5^2a_6^3 - 4938a_3^3a_4^2a_5a_6^3 \\
& - 2436a_3^3a_4a_5^3a_6^2 + 834a_3^3a_5^5a_6 + 1083a_3^2a_4^4a_6^3 - 6642a_3^2a_4^3a_5^2a_6^2 \\
& - 363a_3^2a_4^2a_5^4a_6 + 30a_3^2a_4a_5^6 - 126a_3a_4^5a_5a_6^2 - 840a_3a_4^4a_5^3a_6 \\
& + 48a_3a_4^3a_5^5 + 24a_4^7a_6^2 - 228a_4^6a_5^2a_6 - 60a_4^5a_5^4 \\
& - 1279863a_0^2a_3a_6^5 - 500499a_0^2a_4a_5a_6^4 + 292876a_0^2a_5^3a_6^3 - 92949a_0a_1a_2a_6^5 \\
& + 1976640a_0a_1a_3a_5a_6^4 + 370641a_0a_1a_4^2a_6^4 - 597558a_0a_1a_4a_5^2a_6^3 - 85414a_0a_1a_5^4a_6^2 \\
& - 542414a_0a_2^2a_5a_6^4 + 247282a_0a_2a_3a_4a_6^4 + 352324a_0a_2a_3a_5^2a_6^3 + 307707a_0a_2a_4^2a_5a_6^3 \\
& + 86215a_0a_2a_4a_5^3a_6^2 - 15223a_0a_2a_5^5a_6 + 45144a_0a_3^3a_6^4 - 59116a_0a_3^2a_4a_5a_6^3 \\
& - 119534a_0a_3^2a_5^3a_6^2 + 3355a_0a_3a_4^3a_6^3 - 161627a_0a_3a_4^2a_5^2a_6^2 - 4794a_0a_3a_4a_5^4a_6 \\
& + 648a_0a_3a_6^6 - 20372a_0a_4^4a_5a_6^2 - 24966a_0a_4^3a_5^3a_6 - 367a_0a_4^2a_5^5 \\
& + 330172a_1^3a_6^5 - 155540a_1^2a_2a_5a_6^4 - 5584a_1^2a_3a_4a_6^4 - 201538a_1^2a_3a_5^2a_6^3 \\
& + 281620a_1^2a_4^2a_5a_6^3 - 86716a_1^2a_4a_5^3a_6^2 + 6112a_1^2a_5^5a_6 - 83928a_1a_2^2a_4a_6^4 \\
& - 52072a_1a_2^2a_5^2a_6^3 - 74856a_1a_2a_3^2a_6^4 - 447974a_1a_2a_3a_4a_5a_6^3 + 69646a_1a_2a_3a_5^3a_6^2 \\
& - 10684a_1a_2a_4^3a_6^3 + 72218a_1a_2a_4^2a_5^2a_6^2 + 19963a_1a_2a_4a_5^4a_6 + 26a_1a_2a_5^6 \\
& + 112590a_1a_3^3a_5a_6^3 + 10952a_1a_3^2a_4^2a_6^3 + 9784a_1a_3^2a_4a_5^2a_6^2 - 3914a_1a_3^2a_5^4a_6 \\
& + 14528a_1a_3a_4^3a_5a_6^2 + 5032a_1a_3a_4^2a_5^3a_6 + 48a_1a_3a_4a_5^5 + 1290a_1a_4^5a_6^2 \\
& + 430a_1a_4^4a_5^2a_6 - 525a_1a_4^3a_5^4 - 17942a_2^3a_3a_6^4 + 57751a_2^3a_4a_5a_6^3 \\
& - 17303a_2^3a_5^3a_6^2 - 21710a_2^2a_3^2a_5a_6^3 - 29293a_2^2a_3a_4^2a_6^3 - 5023a_2^2a_3a_4a_5^2a_6^2 \\
& - 368a_2^2a_3a_5^4a_6 - 23440a_2^2a_4^3a_5a_6^2 - 3206a_2^2a_4^2a_5^3a_6 + 215a_2^2a_4a_5^5 \\
& - 10490a_2a_3^3a_4a_6^3 + 2798a_2a_3^3a_5^2a_6^2 + 24250a_2a_3^2a_4^2a_5a_6^2 + 7028a_2a_3^2a_4a_5^3a_6 \\
& + 12a_2a_3^2a_5^5 - 1710a_2a_3a_4^4a_6^2 + 5762a_2a_3a_4^3a_5^2a_6 + 73a_2a_3a_4^2a_5^4 \\
& + 1136a_2a_4^5a_5a_6 + 600a_2a_4^4a_5^3 + 4422a_3^5a_6^3 - 2226a_3^4a_4a_5a_6^2 \\
& - 2088a_3^4a_5^3a_6 + 2130a_3^3a_4^3a_6^2 - 1152a_3^3a_4^2a_5^2a_6 + 42a_3^3a_4a_5^4 \\
& + 780a_3^2a_4^4a_5a_6 - 300a_3^2a_4^3a_5^3 + 192a_3a_4^6a_6 + 72a_3a_4^5a_5^2 \\
& + 24a_4^7a_5 - 487103a_0^2a_2a_6^4 - 1282187a_0^2a_3a_5a_6^3 - 209584a_0^2a_4^2a_6^3 \\
& - 267948a_0^2a_4a_5^2a_6^2 + 32659a_0^2a_5^4a_6 - 355322a_0a_1^2a_6^4 + 39908a_0a_1a_2a_5a_6^3 \\
& + 216494a_0a_1a_3a_4a_6^3 + 229559a_0a_1a_3a_5^2a_6^2 + 361024a_0a_1a_4^2a_5a_6^2 - 54279a_0a_1a_4a_5^3a_6 \\
& - 1380a_0a_1a_5^5 - 1930a_0a_2^2a_4a_6^3 - 103856a_0a_2^2a_5^2a_6^2 - 163300a_0a_2a_3^2a_6^3 \\
& + 239878a_0a_2a_3a_4a_5a_6^2 + 53922a_0a_2a_3a_5^3a_6 + 32097a_0a_2a_4^3a_6^2 + 56029a_0a_2a_4^2a_5^2a_6 \\
& + 3776a_0a_2a_4a_5^4 - 28623a_0a_3^3a_5a_6^2 - 249a_0a_3^2a_4^2a_6^2 + 10811a_0a_3^2a_4a_5^2a_6 \\
& + 414a_0a_3^2a_5^4 + 1986a_0a_3a_4^3a_5a_6 - 5136a_0a_3a_4^2a_5^3 - 1379a_0a_4^5a_6 \\
& - 1650a_0a_4^4a_5^2 + 375834a_1^3a_5a_6^3 - 114870a_1^2a_2a_4a_6^3 + 10760a_1^2a_2a_5^2a_6^2 \\
& - 138353a_1^2a_3^2a_6^3 + 44630a_1^2a_3a_4a_5a_6^2 - 15830a_1^2a_3a_5^3a_6 - 42737a_1^2a_4^3a_6^2 \\
& + 50545a_1^2a_4^2a_5^2a_6 - 746a_1^2a_4a_5^4 + 302029a_1a_2^2a_3a_6^3 - 112725a_1a_2^2a_4a_5a_6^2
\end{aligned}$$

$$\begin{aligned}
& -14042a_1a_2^2a_5^3a_6 - 16198a_1a_2a_3^2a_5a_6^2 + 23663a_1a_2a_3a_4^2a_6^2 - 72468a_1a_2a_3a_4a_5^2a_6 \\
& - 44a_1a_2a_3a_5^4 - 9677a_1a_2a_4^3a_5a_6 + 1576a_1a_2a_4^2a_5^3 + 1977a_1a_3^3a_4a_6^2 \\
& + 13556a_1a_3^3a_5^2a_6 + 6959a_1a_3^2a_4^2a_5a_6 + 244a_1a_3^2a_4a_5^3 + 2177a_1a_3a_4^4a_6 \\
& + 908a_1a_3a_4^3a_5^2 + 390a_1a_4^5a_5 - 62677a_2^4a_6^3 + 6487a_2^3a_3a_5a_6^2 \\
& + 15488a_2^3a_4^2a_6^2 + 12319a_2^3a_4a_5^2a_6 - 203a_2^3a_5^4 - 37718a_2^2a_3^2a_4a_6^2 \\
& - 5121a_2^2a_3^2a_5^2a_6 - 3838a_2^2a_3a_4^2a_5a_6 - 108a_2^2a_3a_4a_5^3 - 1123a_2^2a_4^4a_6 \\
& - 1488a_2^2a_4^3a_5^2 + 4761a_2a_3^4a_6^2 - 3883a_2a_3^3a_4a_5a_6 - 236a_2a_3^3a_5^3 \\
& - 5063a_2a_3^2a_4^3a_6 + 1136a_2a_3^2a_4^2a_5^2 - 614a_2a_3a_4^4a_5 - 100a_2a_4^6 \\
& + 960a_3^5a_5a_6 + 834a_3^4a_4^2a_6 - 336a_3^4a_4a_5^2 + 232a_3^3a_4^3a_5 \\
& + 6a_3^2a_4^5 + 847150a_0^2a_1a_6^3 + 160689a_0^2a_2a_5a_6^2 - 63309a_0^2a_3a_4a_6^2 \\
& - 154605a_0^2a_3a_5^2a_6 - 74112a_0^2a_4^2a_5a_6 - 14273a_0^2a_4a_5^3 - 84296a_0a_1^2a_5a_6^2 \\
& - 408632a_0a_1a_2a_4a_6^2 - 23377a_0a_1a_2a_5^2a_6 + 265254a_0a_1a_3^2a_6^2 + 80308a_0a_1a_3a_4a_5a_6 \\
& + 234a_0a_1a_3a_5^3 + 23222a_0a_1a_4^3a_6 + 23257a_0a_1a_4^2a_5^2 - 169589a_0a_2^2a_3a_6^2 \\
& - 1713a_0a_2^2a_4a_5a_6 - 2966a_0a_2^2a_5^3 - 52425a_0a_2a_3^2a_5a_6 + 13763a_0a_2a_3a_4^2a_6 \\
& + 11856a_0a_2a_3a_4a_5^2 + 7927a_0a_2a_4^3a_5 - 28218a_0a_3^3a_4a_6 - 5446a_0a_3^3a_5^2 \\
& - 701a_0a_3^2a_4^2a_5 - 2413a_0a_3a_4^4 - 61586a_1^3a_4a_6^2 + 46062a_1^3a_5^2a_6 \\
& + 2336a_1^2a_2a_3a_6^2 - 52026a_1^2a_2a_4a_5a_6 + 16a_1^2a_2a_5^3 - 20828a_1^2a_3^2a_5a_6 \\
& - 40672a_1^2a_3a_4^2a_6 + 250a_1^2a_3a_4a_5^2 - 5736a_1^2a_4^3a_5 + 50877a_1a_2^3a_6^2 \\
& + 65806a_1a_2^2a_3a_5a_6 + 14528a_1a_2^2a_4^2a_6 - 3894a_1a_2^2a_4a_5^2 + 24833a_1a_2a_3^2a_4a_6 \\
& + 284a_1a_2a_3^2a_5^2 + 1990a_1a_2a_3a_4^2a_5 - 402a_1a_2a_4^4 + 528a_1a_4^4a_6 \\
& + 208a_1a_3^3a_4a_5 + 1531a_1a_3^2a_4^3 - 10991a_2^4a_5a_6 - 4601a_2^3a_3a_4a_6 \\
& + 24a_2^3a_3a_5^2 + 800a_2^3a_4^2a_5 + 1120a_2^2a_3^3a_6 - 1505a_2^2a_3^2a_4a_5 \\
& + 616a_2^2a_3a_4^3 + 400a_2a_3^4a_5 - 1331a_2a_3^3a_4^2 + 288a_3^5a_4 \\
& - 591588a_0^3a_6^2 + 141070a_0^2a_1a_5a_6 - 35059a_0^2a_2a_4a_6 + 20677a_0^2a_2a_5^2 \\
& + 9294a_0^2a_3a_6^2 + 2832a_0^2a_3a_4a_5 + 9367a_0^2a_4^3 - 8156a_0a_1^2a_4a_6 \\
& - 3174a_0a_1^2a_5^2 + 54116a_0a_1a_2a_3a_6 - 37190a_0a_1a_2a_4a_5 + 14100a_0a_1a_3^2a_5 \\
& - 394a_0a_1a_3a_4^2 - 19381a_0a_2^3a_6 - 8174a_0a_2^2a_3a_5 - 9321a_0a_2^2a_4^2 \\
& - 3252a_0a_2a_3^2a_4 + 2040a_0a_3^4 - 4248a_1^3a_3a_6 - 5118a_1^3a_4a_5 \\
& - 1733a_1^2a_2^2a_6 + 1274a_1^2a_2a_3a_5 + 6522a_1^2a_2a_4^2 - 680a_1^2a_3^2a_4 \\
& + 2822a_1a_2^3a_5 - 3788a_1a_2^2a_3a_4 + 688a_1a_2a_3^3 + 632a_2^4a_4 \\
& + 861a_2^3a_3^2 - 33378a_0^3a_5 - 2330a_0^2a_1a_4 - 33802a_0^2a_2a_3 \\
& + 9822a_0a_1^2a_3 + 20000a_0a_1a_2^2 - 1682a_1^3a_2
\end{aligned} \tag{A.23}$$

$$e_{24} = 828a_0a_4a_5a_6^{12} + 2658a_0a_5^3a_6^{11} + 216a_1a_3a_5a_6^{12} - 282a_1a_4a_5^2a_6^{11}$$

$$\begin{aligned}
& -403a_1a_5^4a_6^{10} + 6a_2^2a_5a_6^{12} - 78a_2a_3a_5^2a_6^{11} - 168a_2a_4^2a_5a_6^{11} \\
& - 788a_2a_4a_5^3a_6^{10} - 371a_2a_5^5a_6^9 - 72a_3^2a_4a_5a_6^{11} - 87a_3^2a_5^3a_6^{10} \\
& + 180a_3a_4^2a_5^2a_6^{10} + 525a_3a_4a_5^4a_6^9 + 168a_3a_5^6a_6^8 + 90a_4^4a_5a_6^{10} \\
& + 700a_4^3a_5^3a_6^9 + 840a_4^2a_5^5a_6^8 + 216a_4a_5^7a_6^7 + 10a_5^9a_6^6 \\
& + 1710a_0a_2a_6^{12} - 1899a_0a_3a_5a_6^{11} + 432a_0a_4^2a_6^{11} + 19902a_0a_4a_5^2a_6^{10} \\
& + 32320a_0a_5^4a_6^9 - 1845a_1^2a_6^{12} + 717a_1a_2a_5a_6^{11} + 1224a_1a_3a_4a_6^{11} \\
& + 6993a_1a_3a_5^2a_6^{10} - 438a_1a_4^2a_5a_6^{10} - 6678a_1a_4a_5^3a_6^9 - 4064a_1a_5^5a_6^8 \\
& - 186a_2^2a_4a_6^{11} + 201a_2^2a_5^2a_6^{10} + 180a_2a_3^2a_6^{11} + 96a_2a_3a_4a_5a_6^{10} \\
& - 1525a_2a_3a_5^3a_6^9 - 12a_2a_3^3a_6^{10} - 3138a_2a_4^2a_5^2a_6^9 - 6955a_2a_4a_5^4a_6^8 \\
& - 1729a_2a_5^6a_6^7 - 18a_3^3a_5a_6^{10} - 198a_3^2a_4^2a_6^{10} - 2187a_3^2a_4a_5^2a_6^9 \\
& - 990a_3^2a_5^4a_6^8 - 270a_3a_4^3a_5a_6^9 + 1650a_3a_4^2a_5^3a_6^8 + 3087a_3a_4a_5^5a_6^7 \\
& + 540a_3a_5^7a_6^6 + 825a_4^4a_5^2a_6^8 + 3500a_4^3a_5^4a_6^7 + 2268a_4^2a_5^6a_6^6 \\
& + 288a_4a_5^8a_6^5 + 5a_5^{10}a_6^4 + 1980a_0a_1a_6^{11} + 48276a_0a_2a_5a_6^{10} \\
& - 1233a_0a_3a_4a_6^{10} - 52167a_0a_3a_5^2a_6^9 - 8172a_0a_4^2a_5a_6^9 + 87162a_0a_4a_5^3a_6^8 \\
& + 94079a_0a_5^5a_6^7 - 51768a_1^2a_5a_6^{10} - 1797a_1a_2a_4a_6^{10} + 19545a_1a_2a_5^2a_6^9 \\
& - 1404a_1a_3^2a_6^{10} + 29778a_1a_3a_4a_5a_6^9 + 46110a_1a_3a_5^3a_6^8 + 900a_1a_4^3a_6^9 \\
& - 7152a_1a_4^2a_5^2a_6^8 - 31475a_1a_4a_5^4a_6^7 - 8970a_1a_5^6a_6^6 - 174a_2^2a_3a_6^{10} \\
& - 5412a_2^2a_4a_5a_6^9 + 894a_2^2a_5^3a_6^8 + 5430a_2a_3^2a_5a_6^9 + 894a_2a_3a_4^2a_6^9 \\
& + 2055a_2a_3a_4a_5^2a_6^8 - 6875a_2a_3a_5^4a_6^7 + 2706a_2a_4^3a_5a_6^8 - 7120a_2a_4^2a_5^3a_6^7 \\
& - 11875a_2a_4a_5^5a_6^6 - 1590a_2a_5^7a_6^5 + 486a_3^3a_4a_6^9 - 591a_3^3a_5^2a_6^8 \\
& - 4041a_3^2a_4^2a_5a_6^8 - 12615a_3^2a_4a_5^3a_6^7 - 2202a_3^2a_5^5a_6^6 - 270a_3a_4^4a_6^8 \\
& - 3900a_3a_4^3a_5^2a_6^7 + 2445a_3a_4^2a_5^4a_6^6 + 3906a_3a_4a_5^6a_6^5 + 336a_3a_5^8a_6^4 \\
& - 720a_4^5a_5a_6^7 - 1000a_4^4a_5^3a_6^6 + 2016a_4^3a_5^5a_6^5 + 888a_4^2a_5^7a_6^4 \\
& + 44a_4a_5^9a_6^3 + 58560a_0^2a_6^{10} + 36958a_0a_1a_5a_6^9 + 464a_0a_2a_4a_6^9 \\
& + 284704a_0a_2a_5^2a_6^8 + 63771a_0a_3^2a_6^9 - 68775a_0a_3a_4a_5a_6^8 - 279882a_0a_3a_5^3a_6^7 \\
& - 1100a_0a_4^3a_6^8 - 121632a_0a_4^2a_5^2a_6^7 + 79481a_0a_4a_5^4a_6^6 + 79949a_0a_5^6a_6^5 \\
& + 3862a_1^2a_4a_6^9 - 294225a_1^2a_5^2a_6^8 - 59049a_1a_2a_3a_6^9 - 7823a_1a_2a_4a_5a_6^8 \\
& + 102872a_1a_2a_5^3a_6^7 - 14740a_1a_3^2a_5a_6^8 + 6179a_1a_3a_4^2a_6^8 + 135954a_1a_3a_4a_5^2a_6^7 \\
& + 80536a_1a_3a_5^4a_6^6 + 10968a_1a_4^3a_5a_6^7 - 15604a_1a_4^2a_5^3a_6^6 - 35941a_1a_4a_5^5a_6^5 \\
& - 4824a_1a_5^7a_6^4 + 10914a_2^3a_6^9 - 1030a_2^2a_3a_5a_6^8 - 7885a_2^2a_4^2a_6^8 \\
& - 33536a_2^2a_4a_5^2a_6^7 - 1107a_2^2a_5^4a_6^6 + 2146a_2a_3^2a_4a_6^8 + 31866a_2a_3^2a_5^2a_6^7 \\
& + 10299a_2a_3a_4^2a_5a_6^7 + 8547a_2a_3a_4a_5^3a_6^6 - 8675a_2a_3a_5^5a_6^5 + 1866a_2a_4^4a_6^7 \\
& + 22504a_2a_4^3a_5^2a_6^6 + 6490a_2a_4^2a_5^4a_6^5 - 3558a_2a_4a_5^6a_6^4 - 267a_2a_5^8a_6^3 \\
& + 1077a_4^3a_6^8 + 5640a_3^3a_4a_5a_6^7 - 3803a_3^3a_5^3a_6^6 + 909a_3^2a_4^3a_6^7 \\
& - 13479a_3^2a_4^2a_5^2a_6^6 - 17295a_3^2a_4a_5^4a_6^5 - 810a_3^2a_5^6a_6^4 - 795a_3a_4^4a_5a_6^6
\end{aligned}$$

$$\begin{aligned}
& -8910a_3a_4^3a_5^3a_6^5 + 279a_3a_4^2a_5^5a_6^4 + 1119a_3a_4a_5^7a_6^3 + 33a_3a_5^9a_6^2 \\
& -38a_4^6a_6^6 - 2640a_4^5a_5^2a_6^5 - 3735a_4^4a_5^4a_6^4 - 420a_4^3a_5^6a_6^3 \\
& + 24a_4^2a_5^8a_6^2 + 368690a_0^2a_5a_6^8 + 11194a_0a_1a_4a_6^8 + 205032a_0a_1a_5^2a_6^7 \\
& - 61607a_0a_2a_3a_6^8 - 92308a_0a_2a_4a_5a_6^7 + 485574a_0a_2a_5^3a_6^6 + 499834a_0a_3^2a_5a_6^7 \\
& + 78208a_0a_3a_4^2a_6^7 - 295800a_0a_3a_4a_5^2a_6^6 - 374464a_0a_3a_5^4a_6^5 + 28688a_0a_4^3a_5a_6^6 \\
& - 222428a_0a_4^2a_5^3a_6^5 + 1249a_0a_4a_5^5a_6^4 + 18062a_0a_5^7a_6^3 + 54392a_1^2a_3a_6^8 \\
& + 157356a_1^2a_4a_5a_6^7 - 452342a_1^2a_5^3a_6^6 + 27879a_1a_2^2a_6^8 - 426468a_1a_2a_3a_5a_6^7 \\
& - 29392a_1a_2a_4^2a_6^7 - 15966a_1a_2a_4a_5^2a_6^6 + 131996a_1a_2a_5^4a_6^5 - 36610a_1a_3^2a_4a_6^7 \\
& - 17453a_1a_3^2a_5^2a_6^6 + 6826a_1a_3a_4^2a_5a_6^6 + 142088a_1a_3a_4a_5^3a_6^5 + 34596a_1a_3a_5^5a_6^4 \\
& + 2619a_1a_4^4a_6^6 + 36048a_1a_4^3a_5^2a_6^5 + 3265a_1a_4^2a_5^4a_6^4 - 8158a_1a_4a_5^6a_6^3 \\
& - 470a_1a_5^8a_6^2 + 71860a_2^3a_5a_6^7 + 3952a_2^2a_3a_4a_6^7 + 2823a_2^2a_3a_5^2a_6^6 \\
& - 42298a_2^2a_4^2a_5a_6^6 - 57596a_2^2a_4a_5^3a_6^5 - 3971a_2^2a_5^5a_6^4 - 8110a_2a_3^3a_6^7 \\
& + 3650a_2a_3^2a_4a_5a_6^6 + 45646a_2a_3^2a_5^3a_6^5 - 7293a_2a_3a_4^3a_6^6 + 21975a_2a_3a_4^2a_5^2a_6^5 \\
& + 9755a_2a_3a_4a_5^4a_6^4 - 2600a_2a_3a_5^6a_6^3 + 4122a_2a_4^4a_5a_6^5 + 27670a_2a_4^3a_5^3a_6^4 \\
& + 7843a_2a_4^2a_5^5a_6^3 + 89a_2a_4a_5^7a_6^2 - 3a_2a_5^9a_6 + 9267a_3^4a_5a_6^6 \\
& + 3951a_3^3a_4^2a_6^6 + 10512a_3^3a_4a_5^2a_6^5 - 5970a_3^3a_5^4a_6^4 + 9183a_3^2a_4^3a_5a_6^5 \\
& - 7950a_3^2a_4^2a_5^3a_6^4 - 4776a_3^2a_4a_5^5a_6^3 + 96a_3^2a_5^7a_6^2 + 1092a_3a_4^5a_6^5 \\
& + 2670a_3a_4^4a_5^2a_6^4 - 4035a_3a_4^3a_5^4a_6^3 - 243a_3a_4^2a_5^6a_6^2 + 39a_3a_4a_5^8a_6 \\
& + 780a_4^6a_5a_6^4 - 240a_4^5a_5^3a_6^3 - 636a_4^4a_5^5a_6^2 - 40a_4^3a_5^7a_6 \\
& + 175036a_0^2a_4a_6^7 + 200465a_0^2a_5^2a_6^6 - 1032739a_0a_1a_3a_6^7 + 505654a_0a_1a_4a_5a_6^6 \\
& + 311296a_0a_1a_5^3a_6^5 + 268142a_0a_2^2a_6^7 - 297551a_0a_2a_3a_5a_6^6 - 176702a_0a_2a_4^2a_6^6 \\
& - 234124a_0a_2a_4a_5^2a_6^5 + 259677a_0a_2a_5^4a_6^4 + 147379a_0a_3^2a_4a_6^6 + 773937a_0a_3^2a_5^2a_6^5 \\
& + 329259a_0a_3a_4^2a_5a_6^5 - 207971a_0a_3a_4a_5^3a_6^4 - 111710a_0a_3a_5^5a_6^3 + 32562a_0a_4^4a_6^5 \\
& + 110428a_0a_4^3a_5^2a_6^4 - 74132a_0a_4^2a_5^4a_6^3 - 4638a_0a_4a_5^6a_6^2 + 675a_0a_5^8a_6 \\
& + 201319a_1^2a_2a_6^7 + 301193a_1^2a_3a_5a_6^6 - 80151a_1^2a_4^2a_6^6 + 412756a_1^2a_4a_5^2a_6^5 \\
& - 189864a_1^2a_5^4a_6^4 + 21855a_1a_2^2a_5a_6^6 + 166893a_1a_2a_3a_4a_6^6 - 600913a_1a_2a_3a_5^2a_6^5 \\
& - 135755a_1a_2a_4^2a_5a_6^5 - 72617a_1a_2a_4a_5^3a_6^4 + 35688a_1a_2a_5^5a_6^3 - 73495a_1a_3^3a_6^6 \\
& - 82914a_1a_3^2a_4a_5a_6^5 + 19367a_1a_3^2a_5^3a_6^4 - 20238a_1a_3a_4^3a_6^5 - 26421a_1a_3a_4^2a_5^2a_6^4 \\
& + 26194a_1a_3a_4a_5^4a_6^3 + 2223a_1a_3a_5^6a_6^2 + 7066a_1a_4^4a_5a_6^4 + 28910a_1a_4^3a_5^3a_6^3 \\
& + 5583a_1a_4^2a_5^5a_6^2 - 79a_1a_4a_5^7a_6 - 2a_1a_5^9 - 7058a_2^3a_4a_6^6 \\
& + 98834a_2^3a_5^2a_6^5 + 9085a_2^2a_3^2a_6^6 + 5296a_2^2a_3a_4a_5a_6^5 + 6649a_2^2a_3a_5^3a_6^4 \\
& + 10502a_2^2a_4^3a_6^5 - 33437a_2^2a_4^2a_5^2a_6^4 - 24668a_2^2a_4a_5^4a_6^3 - 1229a_2^2a_5^6a_6^2 \\
& - 22959a_2a_3^3a_5a_6^5 - 26052a_2a_3^2a_4^2a_6^5 - 16362a_2a_3^2a_4a_5^2a_6^4 + 13376a_2a_3^2a_5^4a_6^3 \\
& - 27319a_2a_3a_4^3a_5a_6^4 + 10780a_2a_3a_4^2a_5^3a_6^3 + 3042a_2a_3a_4a_5^5a_6^2 - 81a_2a_3a_5^7a_6 \\
& - 3382a_2a_4^5a_6^4 - 6525a_2a_4^4a_5^2a_6^3 + 3609a_2a_4^3a_5^4a_6^2 + 562a_2a_4^2a_5^6a_6
\end{aligned}$$

$$\begin{aligned}
& + 5a_2a_4a_5^8 + 1215a_3^4a_4a_5^5 + 15207a_3^4a_5^2a_6^4 + 8823a_3^3a_4^2a_5a_6^4 \\
& + 1470a_3^3a_4a_5^3a_6^3 - 1893a_3^3a_5^5a_6^2 + 291a_3^2a_4^4a_6^4 + 10860a_3^2a_4^3a_5^2a_6^3 \\
& - 459a_3^2a_4^2a_5^4a_6^2 - 105a_3^2a_4a_5^6a_6 + 6a_3^2a_5^8 + 1683a_3a_4^5a_5a_6^3 \\
& + 1710a_3a_4^4a_5^3a_6^2 - 372a_3a_4^3a_5^5a_6 - 12a_3a_4^2a_5^7 + 60a_4^7a_6^3 \\
& + 756a_4^6a_5^2a_6^2 + 240a_4^5a_5^4a_6 + 2a_4^4a_5^6 + 845015a_0^2a_3a_6^6 \\
& + 212034a_0^2a_4a_5a_6^5 - 329154a_0^2a_5^3a_6^4 + 38531a_0a_1a_2a_6^6 - 1938064a_0a_1a_3a_5a_6^5 \\
& - 283070a_0a_1a_4^2a_6^5 + 860396a_0a_1a_4a_5^2a_6^4 + 101106a_0a_1a_5^4a_6^3 + 516026a_0a_2^2a_5a_6^5 \\
& - 206498a_0a_2a_3a_4a_6^5 - 327423a_0a_2a_3a_5^2a_6^4 - 261086a_0a_2a_4^2a_5a_6^4 - 98892a_0a_2a_4a_5^3a_6^3 \\
& + 39924a_0a_2a_5^5a_6^2 - 60741a_0a_3^3a_6^5 + 132858a_0a_3^2a_4a_5a_6^4 + 224836a_0a_3^2a_5^3a_6^3 \\
& + 9003a_0a_3a_4^3a_6^4 + 273138a_0a_3a_4^2a_5^2a_6^3 - 10446a_0a_3a_4a_5^4a_6^2 - 4005a_0a_3a_5^6a_6 \\
& + 33289a_0a_4^4a_5a_6^3 + 45290a_0a_4^3a_5^3a_6^2 - 2649a_0a_4^2a_5^5a_6 - 81a_0a_4a_5^7 \\
& - 227812a_1^3a_6^6 + 233705a_1^2a_2a_5a_6^5 - 12145a_1^2a_3a_4a_6^5 + 270446a_1^2a_3a_5^2a_6^4 \\
& - 291942a_1^2a_4^2a_5a_6^4 + 184314a_1^2a_4a_5^3a_6^3 - 18686a_1^2a_5^5a_6^2 + 82444a_1a_2^2a_4a_6^5 \\
& + 10004a_1a_2^2a_5^2a_6^4 + 96945a_1a_2a_3^2a_6^5 + 473970a_1a_2a_3a_4a_5a_6^4 - 167512a_1a_2a_3a_5^3a_6^3 \\
& + 6737a_1a_2a_4^3a_6^4 - 129372a_1a_2a_4^2a_5^2a_6^3 - 39368a_1a_2a_4a_5^4a_6^2 + 849a_1a_2a_5^6a_6 \\
& - 137252a_1a_3^3a_5a_6^4 - 10125a_1a_3^2a_4^2a_6^4 - 16656a_1a_3^2a_4a_5^2a_6^3 + 10470a_1a_3^2a_5^4a_6^2 \\
& - 20326a_1a_3a_4^3a_5a_6^3 - 10068a_1a_3a_4^2a_5^3a_6^2 + 132a_1a_3a_4a_5^5a_6 - 2a_1a_3a_5^7 \\
& - 1369a_1a_4^5a_6^3 + 1592a_1a_4^4a_5^2a_6^2 + 3201a_1a_4^3a_5^4a_6 + 201a_1a_4^2a_6^6 \\
& + 12886a_2^3a_3a_6^5 - 64270a_2^3a_4a_5a_6^4 + 33004a_2^3a_5^3a_6^3 + 13065a_2^2a_3^3a_5a_6^4 \\
& + 27187a_2^2a_3a_4^2a_6^4 - 3004a_2^2a_3a_4a_5^2a_6^3 + 834a_2^2a_3a_5^4a_6^2 + 29464a_2^2a_4^3a_5a_6^3 \\
& + 1798a_2^2a_4^2a_5^3a_6^2 - 1426a_2^2a_4a_5^5a_6 - 25a_2^2a_5^7 + 3767a_2a_3^3a_4a_6^4 \\
& - 5395a_2a_3^3a_5^2a_6^3 - 46752a_2a_3^2a_4^2a_5a_6^3 - 11778a_2a_3^2a_4a_5^3a_6^2 + 249a_2a_3^2a_5^5a_6 \\
& - 1248a_2a_3a_4^4a_6^3 - 13472a_2a_3a_4^3a_5^2a_6^2 + 936a_2a_3a_4^2a_5^4a_6 + 101a_2a_3a_4a_5^6 \\
& - 3671a_2a_4^5a_5a_6^2 - 2556a_2a_4^4a_5^3a_6 - 76a_2a_4^3a_5^5 - 4485a_3^5a_6^4 \\
& + 4845a_3^4a_4a_5a_6^3 + 4428a_3^4a_5^3a_6^2 - 3172a_3^3a_4^3a_6^3 + 3420a_3^3a_4^2a_5^2a_6^2 \\
& - 537a_3^3a_4a_5^4a_6 - 44a_3^3a_6^6 - 888a_3^2a_4^4a_5a_6^2 + 1566a_3^2a_4^3a_5^3a_6 \\
& - 12a_3^2a_4^2a_5^5 - 423a_3a_4^6a_6^2 + 60a_3a_4^4a_5^4 - 72a_4^7a_5a_6 \\
& + 8a_4^6a_5^3 + 596436a_0^2a_2a_6^5 + 1465855a_0^2a_3a_5a_6^4 + 203105a_0^2a_4^2a_6^4 \\
& + 225192a_0^2a_4a_5^2a_6^3 - 102216a_0^2a_5^4a_6^2 + 399738a_0a_1^2a_6^5 - 177327a_0a_1a_2a_5a_6^4 \\
& - 200481a_0a_1a_3a_4a_6^4 - 393677a_0a_1a_3a_5^2a_6^3 - 506652a_0a_1a_4^2a_5a_6^3 + 200658a_0a_1a_4a_5^3a_6^2 \\
& + 5094a_0a_1a_5^5a_6 - 69008a_0a_2^2a_4a_6^4 + 180474a_0a_2^2a_5^2a_6^3 + 158579a_0a_2a_3^2a_6^4 \\
& - 354724a_0a_2a_3a_4a_5a_6^3 - 113205a_0a_2a_3a_5^3a_6^2 - 30372a_0a_2a_4^3a_6^3 - 73272a_0a_2a_4^2a_5^2a_6^2 \\
& - 10116a_0a_2a_4a_5^4a_6 + 819a_0a_2a_5^6 + 4169a_0a_3^3a_5a_6^3 - 30342a_0a_3^2a_4^2a_6^3 \\
& - 22857a_0a_3^2a_4a_5^2a_6^2 + 1482a_0a_3^2a_5^4a_6 - 6502a_0a_3a_4^3a_5a_6^2 + 32382a_0a_3a_4^2a_5^3a_6 \\
& + 450a_0a_3a_4a_5^5 + 319a_0a_4^5a_6^2 + 4363a_0a_4^4a_5^2a_6 + 1997a_0a_4^3a_5^4
\end{aligned}$$

$$\begin{aligned}
& -452971a_1^3a_5a_6^4 + 79702a_1^2a_2a_4a_6^4 - 6966a_1^2a_2a_5^2a_6^3 + 180883a_1^2a_3^2a_6^4 \\
& - 143265a_1^2a_3a_4a_5a_6^3 + 45210a_1^2a_3a_5^2a_6^2 + 49728a_1^2a_4^3a_6^3 - 135846a_1^2a_4^2a_5^2a_6^2 \\
& + 9506a_1^2a_4a_5^4a_6 - 307a_1^2a_5^6 - 334576a_1a_2^2a_3a_6^4 + 204062a_1a_2^2a_4a_5a_6^3 \\
& + 23420a_1a_2^2a_5^3a_6^2 + 71181a_1a_2a_3^2a_5a_6^3 - 18896a_1a_2a_3a_4^2a_6^3 + 181569a_1a_2a_3a_4a_5^2a_6^2 \\
& - 4396a_1a_2a_3a_5^4a_6 + 17172a_1a_2a_4^3a_5a_6^2 - 13902a_1a_2a_4^2a_5^3a_6 - 1294a_1a_2a_4a_5^5 \\
& + 2148a_1a_3^3a_4a_6^3 - 36553a_1a_3^3a_5^2a_6^2 - 8253a_1a_3^2a_4^2a_5a_6^2 - 174a_1a_3^2a_4a_5^3a_6 \\
& + 230a_1a_3^2a_5^5 - 2680a_1a_3a_4^4a_6^2 - 1796a_1a_3a_4^3a_5^2a_6 - 106a_1a_3a_4^2a_5^4 \\
& - 1215a_1a_4^5a_5a_6 - 254a_1a_4^4a_5^3 + 80102a_2^4a_6^4 - 16195a_2^3a_3a_5a_6^3 \\
& - 24890a_2^3a_4^2a_6^3 - 31160a_2^3a_4a_5^2a_6^2 + 1616a_2^3a_5^4a_6 + 62025a_2^2a_3^2a_4a_6^3 \\
& + 9084a_2^2a_3^2a_5^2a_6^2 + 9819a_2^2a_3a_4^2a_5a_6^2 - 536a_2^2a_3a_4a_5^3a_6 - 50a_2^2a_3a_5^5 \\
& + 3249a_2^2a_4^4a_6^2 + 7208a_2^2a_4^3a_5^2a_6 + 503a_2^2a_4^2a_5^4 - 7724a_2a_3^4a_6^3 \\
& + 3217a_2a_3^3a_4a_5a_6^2 + 838a_2a_3^3a_5^3a_6 + 8524a_2a_3^2a_4^3a_6^2 - 8340a_2a_3^2a_4^2a_5^2a_6 \\
& - 467a_2a_3^2a_4a_5^4 + 907a_2a_3a_4^4a_5a_6 - 550a_2a_3a_4^3a_5^3 + 326a_2a_4^6a_6 \\
& - 40a_2a_4^5a_5^2 - 1740a_3^5a_5a_6^2 - 1842a_3^4a_4^2a_6^2 + 1830a_3^4a_4a_5^2a_6 \\
& + 63a_3^4a_5^4 - 1028a_3^3a_4^3a_5a_6 + 264a_3^3a_4^2a_5^3 - 201a_3^2a_4^5a_6 \\
& - 84a_3^2a_4^4a_5^2 - 60a_3a_4^6a_5 - 3a_4^8 - 1107126a_0^2a_1a_6^4 \\
& - 187550a_0^2a_2a_5a_6^3 + 130798a_0^2a_3a_4a_6^3 + 418621a_0^2a_3a_5^2a_6^2 + 222482a_0^2a_4^2a_5a_6^2 \\
& + 56544a_0^2a_4a_5^3a_6 - 914a_0^2a_5^5 + 169169a_0a_1^2a_5a_6^3 + 655408a_0a_1a_2a_4a_6^3 \\
& - 4179a_0a_1a_2a_5^2a_6^2 - 410776a_0a_1a_3^2a_6^3 - 186988a_0a_1a_3a_4a_5a_6^2 + 6296a_0a_1a_3a_5^3a_6 \\
& - 51384a_0a_1a_4^3a_6^2 - 106952a_0a_1a_4^2a_5^2a_6 + 3170a_0a_1a_4a_5^4 + 262884a_0a_2^2a_3a_6^3 \\
& - 60108a_0a_2^2a_4a_5a_6^2 + 14320a_0a_2^2a_5^3a_6 + 113806a_0a_2a_3^2a_5a_6^2 - 60102a_0a_2a_3a_4^2a_6^2 \\
& - 63434a_0a_2a_3a_4a_5^2a_6 - 5736a_0a_2a_3a_5^4 - 29084a_0a_2a_4^3a_5a_6 - 4852a_0a_2a_4^2a_5^3 \\
& + 73149a_0a_3^3a_4a_6^2 + 19643a_0a_3^3a_5^2a_6 - 8556a_0a_3^2a_4^2a_5a_6 - 1614a_0a_3^2a_4a_5^3 \\
& + 9334a_0a_3a_4^4a_6 - 954a_0a_3a_4^3a_5^2 + 1307a_0a_4^5a_5 + 95071a_1^3a_4a_6^3 \\
& - 124280a_1^3a_5^2a_6^2 - 5048a_1^2a_2a_3a_6^3 + 116337a_1^2a_2a_4a_5a_6^2 - 4050a_1^2a_2a_5^3a_6 \\
& + 56984a_1^2a_3^2a_5a_6^2 + 101091a_1^2a_3a_4^2a_6^2 - 18910a_1^2a_3a_4a_5^2a_6 + 1556a_1^2a_3a_5^4 \\
& + 23544a_1^2a_4^3a_5a_6 - 4908a_1^2a_4^2a_5^3 - 86800a_1a_2^3a_6^3 - 161068a_1a_2^2a_3a_5a_6^2 \\
& - 34481a_1a_2^2a_4^2a_6^2 + 29720a_1a_2^2a_4a_5^2a_6 + 1288a_1a_2^2a_5^4 - 67167a_1a_2a_3^2a_4a_6^2 \\
& + 4069a_1a_2a_3^2a_5^2a_6 - 9182a_1a_2a_3a_4^2a_5a_6 + 4828a_1a_2a_3a_4a_5^3 + 2150a_1a_2a_4^4a_6 \\
& + 3050a_1a_2a_4^3a_5^2 + 408a_1a_3^4a_6^2 + 924a_1a_3^3a_4a_5a_6 - 708a_1a_3^3a_5^3 \\
& - 6376a_1a_3^2a_4^3a_6 - 1194a_1a_3^2a_4^2a_5^2 - 938a_1a_3a_4^4a_5 - 103a_1a_4^6 \\
& + 31186a_2^4a_5a_6^2 + 14155a_2^3a_3a_4a_6^2 - 1359a_2^3a_3a_5^2a_6 - 3756a_2^3a_4^2a_5a_6 \\
& - 1316a_2^3a_4a_5^3 + 332a_2^2a_3^3a_6^2 + 9713a_2^2a_3^2a_4a_5a_6 + 706a_2^2a_3^2a_5^3 \\
& - 1990a_2^2a_3a_4^3a_6 + 418a_2^2a_3a_4^2a_5^2 - 233a_2^2a_4^4a_5 - 2000a_2a_3^4a_5a_6 \\
& + 6708a_2a_3^3a_4^2a_6 - 38a_2a_3^3a_4a_5^2 + 1188a_2a_3^2a_4^3a_5 + 381a_2a_3a_4^5
\end{aligned}$$

$$\begin{aligned}
& -1128a_3^5a_4a_6 + 120a_3^5a_5^2 - 300a_3^4a_4^2a_5 + 4a_3^3a_4^4 \\
& + 973784a_0^3a_6^3 - 403204a_0^2a_1a_5a_6^2 + 37522a_0^2a_2a_4a_6^2 - 102288a_0^2a_2a_5^2a_6 \\
& - 69795a_0^2a_3^2a_6^2 - 3640a_0^2a_3a_4a_5a_6 + 1675a_0^2a_3a_5^3 - 37716a_0^2a_4^3a_6 \\
& + 11132a_0^2a_4^2a_5^2 + 27403a_0^2a_1^2a_4a_6^2 + 12625a_0^2a_1^2a_5^2a_6 - 101730a_0a_1a_2a_3a_6^2 \\
& + 194908a_0a_1a_2a_4a_5a_6 + 4423a_0a_1a_2a_5^3 - 79678a_0a_1a_3^2a_5a_6 - 4740a_0a_1a_3a_4^2a_6 \\
& - 13823a_0a_1a_3a_4a_5^2 - 3704a_0a_1a_4^3a_5 + 65442a_0a_2^3a_6^2 + 33187a_0a_2^2a_3a_5a_6 \\
& + 38616a_0a_2^2a_4^2a_6 - 462a_0a_2^2a_4a_5^2 + 19686a_0a_2a_3^2a_4a_6 + 12985a_0a_2a_3^2a_5^2 \\
& - 5624a_0a_2a_3a_4^2a_5 - 2594a_0a_2a_4^4 - 5676a_0a_3^4a_6 + 3920a_0a_3^3a_4a_5 \\
& + 3664a_0a_3^2a_4^3 + 9092a_1^3a_3a_6^2 + 26139a_1^3a_4a_5a_6 - 3424a_1^3a_5^3 \\
& + 7548a_1^2a_2^2a_6^2 + 3967a_1^2a_2a_3a_5a_6 - 22874a_1^2a_2a_4^2a_6 + 8779a_1^2a_2a_4a_5^2 \\
& + 6369a_1^2a_3^2a_4a_6 - 1003a_1^2a_3^2a_5^2 + 11888a_1^2a_3a_4^2a_5 - 108a_1^2a_4^4 \\
& - 17401a_1a_2^3a_5a_6 + 14562a_1a_2^2a_3a_4a_6 - 6610a_1a_2^2a_3a_5^2 - 7607a_1a_2^2a_4^2a_5 \\
& - 6380a_1a_2a_3^3a_6 - 1604a_1a_2a_3^2a_4a_5 + 612a_1a_2a_3a_4^3 + 16a_1a_3^4a_5 \\
& - 1332a_1a_3^3a_4^2 - 3930a_2^4a_4a_6 + 1127a_2^4a_5^2 - 3064a_2^3a_3^2a_6 \\
& + 1669a_2^3a_3a_4a_5 + 402a_2^3a_4^3 - 772a_2^2a_3^3a_5 - 1636a_2^2a_3^2a_4^2 \\
& + 840a_2a_3^4a_4 - 208a_3^6 + 141284a_0^3a_5a_6 + 22972a_0^2a_1a_4a_6 \\
& + 2302a_0^2a_1a_5^2 + 182606a_0^2a_2a_3a_6 - 6592a_0^2a_2a_4a_5 + 15788a_0^2a_3^2a_5 \\
& - 18300a_0^2a_3a_4^2 - 40378a_0a_1^2a_3a_6 + 4257a_0a_1^2a_4a_5 - 110766a_0a_1a_2^2a_6 \\
& - 10510a_0a_1a_2a_3a_5 + 1268a_0a_1a_2a_4^2 + 10020a_0a_1a_3^2a_4 + 510a_0a_2^3a_5 \\
& + 15677a_0a_2^2a_3a_4 - 11236a_0a_2a_3^3 + 13042a_1^3a_2a_6 + 4482a_1^3a_3a_5 \\
& - 2091a_1^3a_4^2 - 1795a_1^2a_2^2a_5 - 15849a_1^2a_2a_3a_4 + 1452a_1^2a_3^3 \\
& + 5377a_1a_2^3a_4 + 4220a_1a_2^2a_3^2 - 1499a_2^4a_3 + 42524a_0^3a_4 \\
& - 51124a_0^2a_1a_3 + 6813a_0^2a_2^2 + 10434a_0a_1^2a_2 - 5147a_1^4
\end{aligned} \tag{A.24}$$

$$\begin{aligned}
e_{25} = & -114a_0a_2a_6^{13} + 57a_0a_3a_5a_6^{12} - 84a_0a_4a_6^{12} - 2226a_0a_4a_5^2a_6^{11} \\
& - 3191a_0a_5^4a_6^{10} + 123a_1^2a_6^{13} - 63a_1a_2a_5a_6^{12} - 96a_1a_3a_4a_6^{12} \\
& - 543a_1a_3a_5^2a_6^{11} + 78a_1a_4^2a_5a_6^{11} + 718a_1a_4a_5^3a_6^{10} + 430a_1a_5^5a_6^9 \\
& + 12a_2^2a_4a_6^{12} - 11a_2^2a_5^2a_6^{11} - 12a_2a_3^2a_6^{12} + 36a_2a_3a_4a_5a_6^{11} \\
& + 197a_2a_3a_5^3a_6^{10} + 12a_2a_4^3a_6^{11} + 422a_2a_4^2a_5^2a_6^{10} + 815a_2a_4a_5^4a_6^9 \\
& + 203a_2a_5^6a_6^8 + 6a_3^3a_5a_6^{11} + 18a_3^2a_4^2a_6^{11} + 168a_3^2a_4a_5^2a_6^{10} \\
& + 60a_3^2a_5^4a_6^9 - 30a_3a_4^3a_5a_6^{10} - 450a_3a_4^2a_5^3a_6^9 - 525a_3a_4a_5^5a_6^8 \\
& - 84a_3a_5^7a_6^7 - 6a_4^5a_6^{10} - 225a_4^4a_5^2a_6^9 - 700a_4^3a_5^4a_6^8 \\
& - 420a_4^2a_5^6a_6^7 - 54a_4a_5^8a_6^6 - a_5^{10}a_6^5 - 132a_0a_1a_6^{12} \\
& - 7632a_0a_2a_5a_6^{11} + 207a_0a_3a_4a_6^{11} + 6645a_0a_3a_5^2a_6^{10} - 1338a_0a_4^2a_5a_6^{10}
\end{aligned}$$

$$\begin{aligned}
& -27698a_0a_4a_5^3a_6^9 - 23490a_0a_5^5a_6^8 + 8304a_1^2a_5a_6^{11} + 135a_1a_2a_4a_6^{11} \\
& - 3719a_1a_2a_5^2a_6^{10} + 108a_1a_3^2a_6^{11} - 5634a_1a_3a_4a_5a_6^{10} - 8682a_1a_3a_5^3a_6^9 \\
& - 90a_1a_4^3a_6^{10} + 2806a_1a_4^2a_5^2a_6^9 + 8520a_1a_4a_5^4a_6^8 + 2442a_1a_5^6a_6^7 \\
& + 12a_2^2a_3a_6^{11} + 802a_2^2a_4a_5a_6^{10} - 131a_2^2a_5^3a_6^9 - 870a_2a_3^2a_5a_6^{10} \\
& - 108a_2a_3a_4^2a_6^{10} + 713a_2a_3a_4a_5^2a_6^9 + 2130a_2a_3a_5^4a_6^8 + 14a_2a_4^3a_5a_6^9 \\
& + 3630a_2a_4^2a_5^3a_6^8 + 3877a_2a_4a_5^5a_6^7 + 510a_2a_5^7a_6^6 - 42a_3^3a_4a_6^{10} \\
& + 246a_3^3a_5^2a_6^9 + 996a_3^2a_4^2a_5a_6^9 + 2520a_3^2a_4a_5^3a_6^8 + 330a_3^2a_5^5a_6^7 \\
& + 60a_3a_4^4a_6^9 + 150a_3a_4^3a_5^2a_6^8 - 2475a_3a_4^2a_5^4a_6^7 - 1722a_3a_4a_5^6a_6^6 \\
& - 138a_3a_5^8a_6^5 + 30a_4^5a_5a_6^8 - 800a_4^4a_5^3a_6^7 - 1680a_4^3a_5^5a_6^6 \\
& - 528a_4^2a_5^7a_6^5 - 26a_4a_5^9a_6^4 - 13536a_0^2a_6^{11} - 4530a_0a_1a_5a_6^{10} \\
& - 2304a_0a_2a_4a_6^{10} - 82072a_0a_2a_5^2a_6^9 - 14625a_0a_3^2a_6^{10} + 24777a_0a_3a_4a_5a_6^9 \\
& + 76594a_0a_3a_5^3a_6^8 + 1140a_0a_4^3a_6^9 + 20594a_0a_4^2a_5^2a_6^8 - 68487a_0a_4a_5^4a_6^7 \\
& - 40187a_0a_5^6a_6^6 + 1974a_1^2a_4a_6^{10} + 88633a_1^2a_5^2a_6^9 + 14211a_1a_2a_3a_6^{10} \\
& - 2255a_1a_2a_4a_5a_6^9 - 35880a_1a_2a_5^3a_6^8 + 1668a_1a_3^2a_5a_6^9 - 3297a_1a_3a_4^2a_6^9 \\
& - 48890a_1a_3a_4a_5^2a_6^8 - 29132a_1a_3a_5^4a_6^7 - 1892a_1a_4^3a_5a_6^8 + 14588a_1a_4^2a_5^3a_6^7 \\
& + 20023a_1a_4a_5^5a_6^6 + 2772a_1a_5^7a_6^5 - 2520a_2^3a_6^{10} - 154a_2^2a_3a_5a_6^9 \\
& + 2041a_2^2a_4^2a_6^9 + 8931a_2^2a_4a_5^2a_6^8 + 244a_2^2a_5^4a_6^7 - 1098a_2a_3^2a_4a_6^9 \\
& - 9601a_2a_3^2a_5^2a_6^8 - 3135a_2a_3a_4^2a_5a_6^8 + 2201a_2a_3a_4a_5^3a_6^7 + 5113a_2a_3a_5^5a_6^6 \\
& - 628a_2a_4^4a_6^8 - 5108a_2a_4^3a_5^2a_6^7 + 2930a_2a_4^2a_5^4a_6^6 + 3114a_2a_4a_5^6a_6^5 \\
& + 207a_2a_5^8a_6^4 - 291a_3^4a_6^9 - 858a_3^3a_4a_5a_6^8 + 2061a_3^3a_5^3a_6^7 \\
& + 78a_3^2a_4^3a_6^8 + 7428a_3^2a_4^2a_5^2a_6^7 + 7065a_3^2a_4a_5^4a_6^6 + 162a_3^2a_5^6a_6^5 \\
& + 1065a_3a_4^4a_5a_6^7 + 3050a_3a_4^3a_5^3a_6^6 - 2349a_3a_4^2a_5^5a_6^5 - 1041a_3a_4a_5^7a_6^4 \\
& - 31a_3a_5^9a_6^3 + 66a_4^6a_6^7 + 1320a_4^5a_5^2a_6^6 + 1125a_4^4a_5^4a_6^5 \\
& - 252a_4^3a_5^6a_6^4 - 60a_4^2a_5^8a_6^3 - 148368a_0^2a_5a_6^9 - 6162a_0a_1a_4a_6^9 \\
& - 47914a_0a_1a_5^2a_6^8 + 16653a_0a_2a_3a_6^9 + 15280a_0a_2a_4a_5a_6^8 - 233626a_0a_2a_5^3a_6^7 \\
& - 188442a_0a_3^2a_5a_6^8 - 20520a_0a_3a_4^2a_6^8 + 197960a_0a_3a_4a_5^2a_6^7 + 194080a_0a_3a_5^4a_6^6 \\
& + 7760a_0a_4^3a_5a_6^7 + 108464a_0a_4^2a_5^3a_6^6 - 37599a_0a_4a_5^5a_6^5 - 18154a_0a_5^7a_6^4 \\
& - 16098a_1^2a_3a_6^9 - 32512a_1^2a_4a_5a_6^8 + 240890a_1^2a_5^3a_6^7 - 6771a_1a_2^2a_6^9 \\
& + 173702a_1a_2a_3a_5a_6^8 + 9856a_1a_2a_4^2a_6^8 - 23198a_1a_2a_4a_5^2a_6^7 - 83310a_1a_2a_5^4a_6^6 \\
& + 12672a_1a_3^2a_4a_6^8 - 1224a_1a_3^2a_5^2a_6^7 - 17770a_1a_3a_4^2a_5a_6^7 - 93862a_1a_3a_4a_5^3a_6^6 \\
& - 23502a_1a_3a_5^5a_6^5 - 1440a_1a_4^4a_6^7 - 13656a_1a_4^3a_5^2a_6^6 + 11331a_1a_4^2a_5^4a_6^5 \\
& + 10094a_1a_4a_5^6a_6^4 + 622a_1a_5^8a_6^3 - 28370a_2^3a_5a_6^8 - 518a_2^2a_3a_4a_6^8 \\
& - 3816a_2^2a_3a_5^2a_6^7 + 18997a_2^2a_4^2a_5a_6^7 + 26569a_2^2a_4a_5^3a_6^6 + 1945a_2^2a_5^5a_6^5 \\
& + 2424a_2a_3^3a_6^8 - 8828a_2a_3^2a_4a_5a_6^7 - 24691a_2a_3^2a_5^3a_6^6 + 1241a_2a_3a_4^3a_6^7 \\
& - 15895a_2a_3a_4^2a_5^2a_6^6 - 213a_2a_3a_4a_5^4a_6^5 + 2998a_2a_3a_5^6a_6^4 - 4749a_2a_4^4a_5a_6^6
\end{aligned}$$

$$\begin{aligned}
& -17026a_2a_4^3a_5^3a_6^5 - 3455a_2a_4^2a_5^5a_6^4 + 257a_2a_4a_5^7a_6^3 + 9a_2a_5^9a_6^2 \\
& - 4080a_3^4a_5a_6^7 - 1956a_3^3a_4^2a_6^7 - 2229a_3^3a_4a_5^2a_6^6 + 4572a_3^3a_5^4a_6^5 \\
& - 2436a_3^2a_4^3a_5a_6^6 + 10656a_3^2a_4^2a_5^3a_6^5 + 3774a_3^2a_4a_5^5a_6^4 - 162a_3^2a_5^7a_6^3 \\
& - 423a_3a_4^5a_6^6 + 1530a_3a_4^4a_5^2a_6^5 + 4215a_3a_4^3a_5^4a_6^4 - 279a_3a_4^2a_5^6a_6^3 \\
& - 93a_3a_4a_5^8a_6^2 - 48a_4^6a_5a_6^5 + 1500a_4^5a_5^3a_6^4 + 984a_4^4a_5^5a_6^3 \\
& + 56a_4^3a_5^7a_6^2 - 71646a_0^2a_4a_6^8 - 205113a_0^2a_5^2a_6^7 + 472545a_0a_1a_3a_6^8 \\
& - 288014a_0a_1a_4a_5a_6^7 - 124032a_0a_1a_5^3a_6^6 - 118548a_0a_2^2a_6^8 + 125015a_0a_2a_3a_5a_6^7 \\
& + 82650a_0a_2a_4^2a_6^7 + 119996a_0a_2a_4a_5^2a_6^6 - 205341a_0a_2a_5^4a_6^5 - 96243a_0a_3^2a_4a_6^7 \\
& - 486300a_0a_3^2a_5^2a_6^6 - 151625a_0a_3a_4^2a_5a_6^6 + 283779a_0a_3a_4a_5^3a_6^5 + 113436a_0a_3a_5^5a_6^4 \\
& - 18051a_0a_4^4a_6^6 - 29952a_0a_4^3a_5^2a_6^5 + 87450a_0a_4^2a_5^4a_6^4 - 1486a_0a_4a_5^6a_6^3 \\
& - 1673a_0a_5^8a_6^2 - 105579a_1^2a_2a_6^8 - 156095a_1^2a_3a_5a_6^7 + 27291a_1^2a_4^2a_6^7 \\
& - 227716a_1^2a_4a_5^2a_6^6 + 182762a_1^2a_5^4a_6^5 + 7905a_1a_2^2a_5a_6^7 - 56741a_1a_2a_3a_4a_6^7 \\
& + 423481a_1a_2a_3a_5^2a_6^6 + 75525a_1a_2a_4^2a_5a_6^6 + 293a_1a_2a_4a_5^3a_6^5 - 46134a_1a_2a_5^5a_6^4 \\
& + 38625a_1a_3^3a_6^7 + 50244a_1a_3^2a_4a_5a_6^6 - 22191a_1a_3^2a_5^3a_6^5 + 11346a_1a_3a_4^3a_6^6 \\
& - 135a_1a_3a_4^2a_5^2a_6^5 - 35520a_1a_3a_4a_5^4a_6^4 - 3237a_1a_3a_5^6a_6^3 - 7200a_1a_4^4a_5a_6^5 \\
& - 24530a_1a_4^3a_5^3a_6^4 - 2725a_1a_4^2a_5^5a_6^3 + 673a_1a_4a_5^7a_6^2 + 14a_1a_5^9a_6 \\
& + 1694a_2^3a_4a_6^7 - 67245a_2^3a_5^2a_6^6 - 1883a_2^2a_3^2a_6^7 + 1982a_2^2a_3a_4a_5a_6^6 \\
& - 8769a_2^2a_3a_5^3a_6^5 - 3515a_2^2a_4^3a_6^6 + 29894a_2^2a_4^2a_5^2a_6^5 + 21680a_2^2a_4a_5^4a_6^4 \\
& + 1301a_2^2a_5^6a_6^3 + 10989a_2a_3^3a_5a_6^6 + 14405a_2a_3^2a_4^2a_6^6 - 6348a_2a_3^2a_4a_5^2a_6^5 \\
& - 14190a_2a_3^2a_5^4a_6^4 + 10503a_2a_3a_4^3a_5a_6^5 - 19080a_2a_3a_4^2a_5^3a_6^4 - 2306a_2a_3a_4a_5^5a_6^3 \\
& + 287a_2a_3a_5^7a_6^2 + 1800a_2a_4^5a_6^5 - 765a_2a_4^4a_5^2a_6^4 - 7355a_2a_4^3a_5^4a_6^3 \\
& - 1030a_2a_4^2a_5^6a_6^2 - 11a_2a_4a_5^8a_6 - 1686a_3^4a_4a_6^6 - 10869a_3^4a_5^2a_6^5 \\
& - 8463a_3^3a_4^2a_5a_6^5 + 1350a_3^3a_4a_5^3a_6^4 + 2427a_3^3a_5^5a_6^3 - 1260a_3^2a_4^4a_6^5 \\
& - 8790a_3^2a_4^3a_5^2a_6^4 + 2595a_3^2a_4^2a_5^4a_6^3 + 105a_3^2a_4a_5^6a_6^2 - 24a_3^2a_5^8a_6 \\
& - 2265a_3a_4^5a_5a_6^4 - 690a_3a_4^4a_5^3a_6^3 + 1008a_3a_4^3a_5^5a_6^2 + 24a_3a_4^2a_5^7a_6 \\
& - 120a_4^7a_6^4 - 900a_4^6a_5^2a_6^3 - 180a_4^5a_5^4a_6^2 + 22a_4^4a_5^6a_6 \\
& - 397641a_0^2a_3a_6^7 - 28908a_0^2a_4a_5a_6^6 + 198874a_0^2a_5^3a_6^5 - 8277a_0a_1a_2a_6^7 \\
& + 1406330a_0a_1a_3a_5a_6^6 + 150204a_0a_1a_4^2a_6^6 - 803620a_0a_1a_4a_5^2a_6^5 - 60128a_0a_1a_5^4a_6^4 \\
& - 365574a_0a_2^2a_5a_6^6 + 116868a_0a_2a_3a_4a_6^6 + 207491a_0a_2a_3a_5^2a_6^5 + 168906a_0a_2a_4^2a_5a_6^5 \\
& + 98582a_0a_2a_4a_5^3a_6^4 - 58502a_0a_2a_5^5a_6^3 + 49497a_0a_3^3a_6^6 - 182028a_0a_3^2a_4a_5a_6^5 \\
& - 268203a_0a_3^2a_5^3a_6^4 - 21537a_0a_3a_4^3a_6^5 - 268728a_0a_3a_4^2a_5^2a_6^4 + 61164a_0a_3a_4a_5^4a_6^3 \\
& + 10553a_0a_3a_5^6a_6^2 - 38376a_0a_4^4a_5a_6^4 - 41854a_0a_4^3a_5^3a_6^3 + 12697a_0a_4^2a_5^5a_6^2 \\
& + 359a_0a_4a_5^7a_6 - 12a_0a_5^9 + 113640a_1^3a_6^7 - 234111a_1^2a_2a_5a_6^6 \\
& + 9501a_1^2a_3a_4a_6^6 - 253518a_1^2a_3a_5^2a_6^5 + 186586a_1^2a_4^2a_5a_6^5 - 228058a_1^2a_4a_5^3a_6^4 \\
& + 35642a_1^2a_5^5a_6^3 - 55986a_1a_2^2a_4a_6^6 + 39449a_1a_2^2a_5^2a_6^5 - 76949a_1a_2a_3^2a_6^6
\end{aligned}$$

$$\begin{aligned}
& -321754a_1a_2a_3a_4a_5a_6^5 + 234588a_1a_2a_3a_5^3a_6^4 - 151a_1a_2a_4^3a_6^5 + 140158a_1a_2a_4^2a_5^2a_6^4 \\
& + 35826a_1a_2a_4a_5^4a_6^3 - 4143a_1a_2a_5^6a_6^2 + 119010a_1a_3^3a_5a_6^5 + 9672a_1a_3^2a_4^2a_6^5 \\
& + 20697a_1a_3^2a_4a_5^2a_6^4 - 16728a_1a_3^2a_5^4a_6^3 + 22656a_1a_3a_4^3a_5a_6^4 + 12192a_1a_3a_4^2a_5^3a_6^3 \\
& - 886a_1a_3a_4a_5^5a_6^2 + 22a_1a_3a_5^7a_6 + 990a_1a_4^5a_6^4 - 4848a_1a_4^4a_5^2a_6^3 \\
& - 7381a_1a_4^3a_5^4a_6^2 - 707a_1a_4^2a_5^6a_6 - 4a_1a_4a_5^8 - 5546a_2^3a_3a_6^6 \\
& + 47344a_2^3a_4a_5a_6^5 - 41825a_2^3a_5^3a_6^4 - 621a_2^2a_3^2a_5a_6^5 - 15850a_2^2a_3a_4^2a_6^5 \\
& + 10239a_2^2a_3a_4a_5^2a_6^4 - 2352a_2^2a_3a_5^4a_6^3 - 22539a_2^2a_4^3a_5a_6^4 + 4108a_2^2a_4^2a_5^3a_6^3 \\
& + 3456a_2^2a_4a_5^5a_6^2 + 101a_2^2a_5^7a_6 + 2931a_2a_3^3a_4a_6^5 + 3930a_2a_3^3a_5^2a_6^4 \\
& + 49215a_2a_3^2a_4^2a_5a_6^4 + 6666a_2a_3^2a_4a_5^3a_6^3 - 995a_2a_3^2a_5^5a_6^2 + 3795a_2a_3a_4^4a_6^4 \\
& + 12852a_2a_3a_4^3a_5^2a_6^3 - 4860a_2a_3a_4^2a_5^4a_6^2 - 399a_2a_3a_4a_5^6a_6 + 5241a_2a_4^5a_5a_6^3 \\
& + 3556a_2a_4^4a_5^3a_6^2 - 52a_2a_4^3a_5^5a_6 - 8a_2a_4^2a_5^7 + 3114a_3^5a_6^5 \\
& - 6726a_3^4a_4a_5a_6^4 - 5532a_3^4a_5^3a_6^3 + 2415a_3^3a_4^3a_6^4 - 5934a_3^3a_4^2a_5^2a_6^3 \\
& + 1731a_3^3a_4a_5^4a_6^2 + 162a_3^3a_5^6a_6 - 978a_3^2a_4^4a_5a_6^3 - 3336a_3^2a_4^3a_5^3a_6^2 \\
& + 150a_3^2a_4^2a_5^5a_6 - 6a_3^2a_4a_5^7 + 345a_3a_4^6a_6^3 - 864a_3a_4^5a_5^2a_6^2 \\
& - 180a_3a_4^4a_5^4a_6 + 20a_3a_4^3a_5^6 - 144a_4^6a_5^3a_6 - 12a_4^5a_5^5 \\
& - 528396a_0^2a_2a_6^6 - 1135983a_0^2a_3a_5a_6^5 - 111603a_0^2a_4^2a_6^5 - 10252a_0^2a_4a_5^2a_6^4 \\
& + 164698a_0^2a_5^4a_6^3 - 329694a_0a_1^2a_6^6 + 251603a_0a_1a_2a_5a_6^5 + 124929a_0a_1a_3a_4a_6^5 \\
& + 481897a_0a_1a_3a_5^2a_6^4 + 459372a_0a_1a_4^2a_5a_6^4 - 375954a_0a_1a_4a_5^3a_6^3 - 3418a_0a_1a_5^5a_6^2 \\
& + 105952a_0a_2^2a_4a_6^5 - 225643a_0a_2^2a_5^2a_6^4 - 103569a_0a_2a_3^2a_6^5 + 308160a_0a_2a_3a_4a_5a_6^4 \\
& + 132891a_0a_2a_3a_5^3a_6^3 + 6592a_0a_2a_4^3a_6^4 + 37822a_0a_2a_4^2a_5^2a_6^3 + 14386a_0a_2a_4a_5^4a_6^2 \\
& - 4279a_0a_2a_5^6a_6 + 39381a_0a_3^3a_5a_6^4 + 57633a_0a_3^2a_4^2a_6^4 + 1107a_0a_3^2a_4a_5^2a_6^3 \\
& - 13088a_0a_3^2a_5^4a_6^2 - 4024a_0a_3a_4^3a_5a_6^3 - 86174a_0a_3a_4^2a_5^3a_6^2 - 548a_0a_3a_4a_5^5a_6 \\
& + 2779a_0a_4^5a_6^3 - 6449a_0a_4^4a_5^2a_6^2 - 5915a_0a_4^3a_5^4a_6 + 16a_0a_4^2a_5^6 \\
& + 379855a_1^3a_5a_6^5 - 19000a_1^2a_2a_4a_6^5 - 26278a_1^2a_2a_5^2a_6^4 - 167853a_1^2a_3^2a_6^5 \\
& + 212773a_1^2a_3a_4a_5a_6^4 - 84362a_1^2a_3a_5^3a_6^3 - 32936a_1^2a_4^3a_6^4 + 190596a_1^2a_4^2a_5^2a_6^3 \\
& - 37054a_1^2a_4a_5^4a_6^2 + 1579a_1^2a_5^6a_6 + 256968a_1a_2^2a_3a_6^5 - 237844a_1a_2^2a_4a_5a_6^4 \\
& - 10604a_1a_2^2a_5^3a_6^3 - 122359a_1a_2a_3^2a_5a_6^4 - 3666a_1a_2a_3a_4^2a_6^4 - 258023a_1a_2a_3a_4a_5^2a_6^3 \\
& + 21572a_1a_2a_3a_5^4a_6^2 - 12870a_1a_2a_4^3a_5a_6^3 + 42766a_1a_2a_4^2a_5^3a_6^2 + 5556a_1a_2a_4a_5^5a_6 \\
& + 4a_1a_2a_5^7 - 6126a_1a_3^3a_4a_6^4 + 60191a_1a_3^3a_5^2a_6^3 + 3747a_1a_3^2a_4^2a_5a_6^3 \\
& - 926a_1a_3^2a_4a_5^3a_6^2 - 1374a_1a_3^2a_5^5a_6 + 1142a_1a_3a_4^4a_6^3 + 2184a_1a_3a_4^3a_5^2a_6^2 \\
& + 364a_1a_3a_4^2a_5^4a_6 - 20a_1a_3a_4a_5^6 + 1685a_1a_4^5a_5a_6^2 + 270a_1a_4^4a_5^3a_6 \\
& - 108a_1a_4^3a_5^5 - 72144a_2^4a_5^5 + 26581a_2^3a_3a_5a_6^4 + 27653a_2^3a_4^2a_6^4 \\
& + 46392a_2^3a_4a_5^2a_6^3 - 5366a_2^3a_5^4a_6^2 - 64865a_2^2a_3^2a_4a_6^4 - 6633a_2^2a_3^2a_5^2a_6^3 \\
& - 10841a_2^2a_3a_4^2a_5a_6^3 + 4024a_2^2a_3a_4a_5^3a_6^2 + 230a_2^2a_3a_5^5a_6 - 5075a_2^2a_4^4a_6^3 \\
& - 14218a_2^2a_4^3a_5^2a_6^2 - 1415a_2^2a_4^2a_5^4a_6 + 22a_2^2a_4a_5^6 + 9237a_2a_3^4a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 4459a_2a_3^3a_4a_5a_6^3 - 1938a_2a_3^3a_5^3a_6^2 - 6080a_2a_3^2a_4^3a_6^3 + 21670a_2a_3^2a_4^2a_5^2a_6^2 \\
& + 1373a_2a_3^2a_4a_5^4a_6 + 6a_2a_3^2a_5^6 + 1955a_2a_3a_4^4a_5a_6^2 + 2398a_2a_3a_4^3a_5^3a_6 \\
& - 68a_2a_3a_4^2a_5^5 - 330a_2a_4^6a_6^2 + 816a_2a_4^5a_5^2a_6 + 168a_2a_4^4a_5^4 \\
& + 1884a_3^5a_5a_6^3 + 2685a_3^4a_4^2a_6^3 - 4122a_3^4a_4a_5^2a_6^2 - 147a_3^4a_5^4a_6 \\
& + 2056a_3^3a_4^3a_5a_6^2 - 1008a_3^3a_4^2a_5^3a_6 + 60a_3^3a_4a_5^5 + 729a_3^2a_4^5a_6^2 \\
& + 90a_3^2a_4^4a_5^3a_6 - 78a_3^2a_4^3a_5^4 + 312a_3a_4^6a_5a_6 - 24a_3a_4^5a_5^3 \\
& + 21a_4^8a_6 + 24a_4^7a_5^2 + 1018770a_0^2a_1a_6^5 + 78784a_0^2a_2a_5a_6^4 \\
& - 196542a_0^2a_3a_4a_6^4 - 655621a_0^2a_3a_5^2a_6^3 - 335032a_0^2a_4^2a_5a_6^3 - 65082a_0^2a_4a_5^3a_6^2 \\
& + 7996a_0^2a_5^5a_6 - 230291a_0a_1^2a_5a_6^4 - 685922a_0a_1a_2a_4a_6^4 + 103241a_0a_1a_2a_5^2a_6^3 \\
& + 415554a_0a_1a_3^2a_6^4 + 228296a_0a_1a_3a_4a_5a_6^3 - 35652a_0a_1a_3a_5^3a_6^2 + 66476a_0a_1a_4^3a_6^3 \\
& + 218070a_0a_1a_4^2a_5^2a_6^2 - 26442a_0a_1a_4a_5^4a_6 - 112a_0a_1a_5^6 - 265336a_0a_2^2a_3a_6^4 \\
& + 167022a_0a_2^2a_4a_5a_6^3 - 37735a_0a_2^2a_5^3a_6^2 - 141214a_0a_2a_3^2a_5a_6^3 + 117304a_0a_2a_3a_4^2a_6^3 \\
& + 131280a_0a_2a_3a_4a_5^2a_6^2 + 24432a_0a_2a_3a_5^4a_6 + 35922a_0a_2a_4^3a_5a_6^2 + 6868a_0a_2a_4^2a_5^3a_6 \\
& + 630a_0a_2a_4a_5^5 - 104427a_0a_3^3a_4a_6^3 - 24539a_0a_3^3a_5^2a_6^2 + 48129a_0a_3^2a_4^2a_5a_6^2 \\
& + 8354a_0a_3^2a_4a_5^3a_6 + 642a_0a_3^2a_5^5 - 13434a_0a_3a_4^4a_6^2 + 6246a_0a_3a_4^3a_5^2a_6 \\
& - 1804a_0a_3a_4^2a_5^4 - 2525a_0a_4^5a_5a_6 - 864a_0a_4^4a_5^3 - 95763a_1^3a_4a_6^4 \\
& + 196852a_1^3a_5^2a_6^3 + 13822a_1^2a_2a_3a_6^4 - 126659a_1^2a_2a_4a_5a_6^3 + 17478a_1^2a_2a_5^3a_6^2 \\
& - 91884a_1^2a_3^2a_5a_6^3 - 139629a_1^2a_3a_4^2a_6^3 + 87762a_1^2a_3a_4a_5^2a_6^2 - 7330a_1^2a_3a_5^4a_6 \\
& - 37796a_1^2a_4^3a_5a_6^2 + 26568a_1^2a_4^2a_5^3a_6 - 162a_1^2a_4a_5^5 + 98712a_1a_2^2a_6^4 \\
& + 228128a_1a_2^2a_3a_5a_6^3 + 45934a_1a_2^2a_4^2a_6^3 - 92221a_1a_2^2a_4a_5^2a_6^2 - 5052a_1a_2^2a_5^4a_6 \\
& + 101629a_1a_2a_3^2a_4a_6^3 - 24257a_1a_2a_3^2a_5^2a_6^2 + 13132a_1a_2a_3a_4^2a_5a_6^2 - 24420a_1a_2a_3a_4a_5^3a_6 \\
& + 12a_1a_2a_3a_5^5 - 5104a_1a_2a_4^4a_6^2 - 10966a_1a_2a_4^3a_5^2a_6 + 378a_1a_2a_4^2a_5^4 \\
& - 3648a_1a_3^4a_6^3 - 6908a_1a_3^3a_4a_5a_6^2 + 4116a_1a_3^3a_5^3a_6 + 10383a_1a_3^2a_4^3a_6^2 \\
& + 2774a_1a_3^2a_4^2a_5^2a_6 + 270a_1a_3^2a_4a_5^4 + 1690a_1a_3a_4^4a_5a_6 - 36a_1a_3a_4^3a_5^3 \\
& + 289a_1a_4^6a_6 + 300a_1a_4^5a_5^2 - 52625a_2^4a_5a_6^3 - 24841a_2^3a_3a_4a_6^3 \\
& + 7468a_2^3a_3a_5^2a_6^2 + 9097a_2^3a_4^2a_5a_6^2 + 6536a_2^3a_4a_5^3a_6 - 26a_2^3a_5^5 \\
& - 4744a_2^2a_3^3a_6^3 - 25561a_2^2a_3^2a_4a_5a_6^2 - 2854a_2^2a_3^2a_5^3a_6 + 2159a_2^2a_3a_4^3a_6^2 \\
& - 2700a_2^2a_3a_4^2a_5^2a_6 + 50a_2^2a_3a_4a_5^4 + 155a_2^2a_4^4a_5a_6 - 510a_2^2a_4^3a_5^3 \\
& + 5336a_2a_3^4a_5a_6^2 - 13981a_2a_3^3a_4^2a_6^2 + 1818a_2a_3^3a_4a_5^2a_6 - 30a_2a_3^3a_5^4 \\
& - 3260a_2a_3^2a_4^3a_5a_6 + 532a_2a_3^2a_4^2a_5^3 - 1427a_2a_3a_4^5a_6 + 12a_2a_3a_4^4a_5^2 \\
& - 168a_2a_4^6a_5 + 2040a_3^5a_4a_6^2 - 696a_3^5a_5^2a_6 + 924a_3^4a_4^2a_5a_6 \\
& - 192a_3^4a_4a_5^3 + 14a_3^3a_4^4a_6 - 12a_3^3a_4^3a_5^2 + 90a_3^2a_4^5a_5 \\
& - 12a_3a_4^7 - 1093812a_0^3a_6^4 + 693274a_0^2a_1a_5a_6^3 + 17298a_0^2a_2a_4a_6^3 \\
& + 209180a_0^2a_2a_5^2a_6^2 + 186567a_0^2a_3^2a_6^3 - 59598a_0^2a_3a_4a_5a_6^2 - 20889a_0^2a_3a_5^3a_6 \\
& + 55266a_0^2a_4^3a_6^2 - 74736a_0^2a_4^2a_5^2a_6 - 5162a_0^2a_4a_5^4 - 46401a_0a_1^2a_4a_6^3
\end{aligned}$$

$$\begin{aligned}
& -24081a_0a_1^2a_5^2a_6^2 + 101378a_0a_1a_2a_3a_6^3 - 446192a_0a_1a_2a_4a_5a_6^2 - 6735a_0a_1a_2a_5^3a_6 \\
& + 207138a_0a_1a_3^2a_5a_6^2 + 30438a_0a_1a_3a_4^2a_6^2 + 72723a_0a_1a_3a_4a_5^2a_6 - 428a_0a_1a_3a_5^4 \\
& + 11232a_0a_1a_4^3a_5a_6 + 9252a_0a_1a_4^2a_5^3 - 127806a_0a_2^3a_6^3 - 49711a_0a_2^2a_3a_5a_6^2 \\
& - 65059a_0a_2^2a_4^2a_6^2 + 18160a_0a_2^2a_4a_5^2a_6 - 900a_0a_2^2a_5^4 - 46062a_0a_2a_3^2a_4a_6^2 \\
& - 56317a_0a_2a_3^2a_5^2a_6 + 36326a_0a_2a_3a_4^2a_5a_6 + 4904a_0a_2a_3a_4a_5^3 + 11498a_0a_2a_4^4a_6 \\
& + 4928a_0a_2a_4^3a_5^2 + 2166a_0a_3^4a_6^2 - 19570a_0a_3^3a_4a_5a_6 - 2104a_0a_3^3a_5^3 \\
& - 18132a_0a_3^2a_4^3a_6 + 532a_0a_3^2a_4^2a_5^2 - 1564a_0a_3a_4^4a_5 - 640a_0a_4^6 \\
& - 6468a_1^3a_3a_6^3 - 63951a_1^3a_4a_5a_6^2 + 17288a_1^3a_5^3a_6 - 15048a_1^2a_2^2a_6^3 \\
& - 32119a_1^2a_2a_3a_5a_6^2 + 27204a_1^2a_2a_4^2a_6^2 - 41079a_1^2a_2a_4a_5^2a_6 - 110a_1^2a_2a_5^4 \\
& - 20649a_1^2a_3^2a_4a_6^2 + 3463a_1^2a_3^2a_5^2a_6 - 58442a_1^2a_3a_4^2a_5a_6 + 144a_1^2a_3a_4a_5^3 \\
& + 90a_1^2a_4^4a_6 - 2256a_1^2a_4^3a_5^2 + 48077a_1a_2^3a_5a_6^2 - 20582a_1a_2^2a_3a_4a_6^2 \\
& + 31080a_1a_2^2a_3a_5^2a_6 + 35305a_1a_2^2a_4^2a_5a_6 - 1160a_1a_2^2a_4a_5^3 + 21510a_1a_2a_3^3a_6^2 \\
& + 12722a_1a_2a_3^2a_4a_5a_6 - 128a_1a_2a_3^2a_5^3 - 930a_1a_2a_3a_4^3a_6 + 1412a_1a_2a_3a_4^2a_5^2 \\
& - 1272a_1a_2a_4^4a_5 - 592a_1a_3^4a_5a_6 + 6420a_1a_3^3a_4^2a_6 - 592a_1a_3^3a_4a_5^2 \\
& + 1488a_1a_3^2a_4^3a_5 + 504a_1a_3a_4^5 + 10597a_2^4a_4a_6^2 - 6096a_2^4a_5^2a_6 \\
& + 3224a_2^3a_3^2a_6^2 - 9541a_2^3a_3a_4a_5a_6 - 56a_2^3a_3a_5^3 - 2408a_2^3a_4^3a_6 \\
& + 22a_2^3a_4^2a_5^2 + 2772a_2^2a_3^3a_5a_6 + 7063a_2^2a_3^2a_4^2a_6 - 512a_2^2a_3^2a_4a_5^2 \\
& + 412a_2^2a_3a_4^3a_5 + 300a_2^2a_4^5 - 4792a_2a_3^4a_4a_6 - 16a_2a_3^4a_5^2 \\
& - 696a_2a_3^3a_4^2a_5 - 674a_2a_3^2a_4^4 + 912a_3^6a_6 + 192a_3^5a_4a_5 \\
& + 112a_3^4a_4^3 - 251356a_0^3a_5a_6^2 - 80274a_0^2a_1a_4a_6^2 - 8132a_0^2a_1a_5^2a_6 \\
& - 437508a_0^2a_2a_3a_6^2 + 51516a_0^2a_2a_4a_5a_6 + 11426a_0^2a_2a_5^3 - 50358a_0^2a_3^2a_5a_6 \\
& + 106680a_0^2a_3a_4^2a_6 + 11334a_0^2a_3a_4a_5^2 + 7364a_0^2a_4^3a_5 + 69918a_0a_1^2a_3a_6^2 \\
& - 8167a_0a_1^2a_4a_5a_6 - 1244a_0a_1^2a_5^3 + 274902a_0a_1a_2^2a_6^2 + 29014a_0a_1a_2a_3a_5a_6 \\
& - 9164a_0a_1a_2a_4^2a_6 - 23338a_0a_1a_2a_4a_5^2 - 55500a_0a_1a_3^2a_4a_6 + 1718a_0a_1a_3^2a_5^2 \\
& - 1930a_0a_1a_3a_4^2a_5 + 1830a_0a_1a_4^4 - 4244a_0a_2^3a_5a_6 - 77973a_0a_2^2a_3a_4a_6 \\
& - 2432a_0a_2^2a_3a_5^2 - 5547a_0a_2^2a_4^2a_5 + 50152a_0a_2a_3^3a_6 - 4626a_0a_2a_3^2a_4a_5 \\
& + 364a_0a_2a_3a_4^3 + 1088a_0a_3^4a_5 + 410a_0a_3^3a_4^2 - 40070a_1^3a_2a_6^2 \\
& - 18986a_1^3a_3a_5a_6 + 13311a_1^3a_4^2a_6 - 1356a_1^3a_4a_5^2 + 5407a_1^2a_2^2a_5a_6 \\
& + 71987a_1^2a_2a_3a_4a_6 + 940a_1^2a_2a_3a_5^2 + 2556a_1^2a_2a_4^2a_5 - 7228a_1^2a_3^3a_6 \\
& + 864a_1^2a_3^2a_4a_5 + 276a_1^2a_3a_4^3 - 26289a_1a_2^3a_4a_6 + 1268a_1a_2^3a_5^2 \\
& - 18896a_1a_2^2a_3^2a_6 - 2434a_1a_2^2a_3a_4a_5 + 597a_1a_2^2a_4^3 + 256a_1a_2a_3^3a_5 \\
& - 2342a_1a_2a_3^2a_4^2 + 352a_1a_3^4a_4 + 7594a_2^4a_3a_6 + 589a_2^4a_4a_5 \\
& + 440a_2^3a_3^2a_5 + 599a_2^3a_3a_4^2 + 320a_2^2a_3^3a_4 + 160a_2a_3^5 \\
& - 201068a_0^3a_4a_6 - 11504a_0^3a_5^2 + 237600a_0^2a_1a_3a_6 + 3094a_0^2a_1a_4a_5 \\
& - 34095a_0^2a_2^2a_6 - 28586a_0^2a_2a_3a_5 - 3492a_0^2a_2a_4^2 - 16548a_0^2a_3^2a_4
\end{aligned}$$

$$\begin{aligned}
& -52326a_0a_1^2a_2a_6 + 5900a_0a_1^2a_3a_5 - 11310a_0a_1^2a_4^2 + 14024a_0a_1a_2^2a_5 \\
& + 18632a_0a_1a_2a_3a_4 + 2920a_0a_1a_3^3 - 3034a_0a_2^3a_4 + 2314a_0a_2^2a_3^2 \\
& + 24071a_1^4a_6 - 1608a_1^3a_2a_5 - 3216a_1^3a_3a_4 + 3402a_1^2a_2^2a_4 \\
& - 80a_1^2a_2a_3^2 - 998a_1a_2^3a_3 - 113a_2^5 + 1046a_0^3a_3 \\
& + 6918a_0^2a_1a_2 + 1354a_0a_1^3
\end{aligned} \tag{A.25}$$

$$\begin{aligned}
e_{26} = & 570a_0a_2a_5a_6^{12} - 285a_0a_3a_5^2a_6^{11} + 420a_0a_4^2a_5a_6^{11} + 3540a_0a_4a_5^3a_6^{10} \\
& + 2601a_0a_5^5a_6^9 - 615a_1^2a_5a_6^{12} + 315a_1a_2a_5^2a_6^{11} + 480a_1a_3a_4a_5a_6^{11} \\
& + 735a_1a_3a_5^3a_6^{10} - 390a_1a_4^2a_5^2a_6^{10} - 1005a_1a_4a_5^4a_6^9 - 291a_1a_5^6a_6^8 \\
& - 60a_2^2a_4a_5a_6^{11} + 60a_2a_3^2a_5a_6^{11} - 180a_2a_3a_4a_5^2a_6^{10} - 270a_2a_3a_5^4a_6^9 \\
& - 60a_2a_4^3a_5a_6^{10} - 570a_2a_4^2a_5^3a_6^9 - 522a_2a_4a_5^5a_6^8 - 69a_2a_5^7a_6^7 \\
& - 30a_3^3a_5^2a_6^{10} - 90a_3^2a_4^2a_5a_6^{10} - 180a_3^2a_4a_5^3a_6^9 - 3a_3^2a_5^5a_6^8 \\
& + 150a_3a_4^3a_5^2a_6^9 + 600a_3a_4^2a_5^4a_6^8 + 315a_3a_4a_5^6a_6^7 + 24a_3a_5^8a_6^6 \\
& + 30a_4^5a_5a_6^9 + 300a_4^4a_5^3a_6^8 + 420a_4^3a_5^5a_6^7 + 120a_4^2a_5^7a_6^6 \\
& + 6a_4a_5^9a_6^5 + 1935a_0^2a_6^{12} + 15a_0a_1a_5a_6^{11} + 750a_0a_2a_4a_6^{11} \\
& + 14775a_0a_2a_5^2a_6^{10} + 2070a_0a_3^2a_6^{11} - 4170a_0a_3a_4a_5a_6^{10} - 11160a_0a_3a_5^3a_6^9 \\
& - 120a_0a_4^3a_6^{10} + 2190a_0a_4^2a_5^2a_6^9 + 25245a_0a_4a_5^4a_6^8 + 11517a_0a_5^6a_6^7 \\
& - 795a_1^2a_4a_6^{11} - 16125a_1^2a_5^2a_6^{10} - 2070a_1a_2a_3a_6^{11} + 1110a_1a_2a_4a_5a_6^{10} \\
& + 7305a_1a_2a_5^3a_6^9 + 150a_1a_3^2a_5a_6^{10} + 840a_1a_3a_4^2a_6^{10} + 10275a_1a_3a_4a_5^2a_6^9 \\
& + 6120a_1a_3a_5^4a_6^8 - 90a_1a_4^3a_5a_6^9 - 5490a_1a_4^2a_5^3a_6^8 - 6237a_1a_4a_5^5a_6^7 \\
& - 888a_1a_5^7a_6^6 + 360a_2^3a_6^{11} + 90a_2^2a_3a_5a_6^{10} - 330a_2^2a_4^2a_6^{10} \\
& - 1560a_2^2a_4a_5^2a_6^9 - 120a_2^2a_5^4a_6^8 + 240a_2a_3^2a_4a_6^{10} + 1620a_2a_3^2a_5^2a_6^9 \\
& + 270a_2a_3a_4^2a_5a_6^9 - 1980a_2a_3a_4a_5^3a_6^8 - 1617a_2a_3a_5^5a_6^7 + 90a_2a_4^4a_6^9 \\
& + 90a_2a_4^3a_5^2a_6^8 - 2310a_2a_4^2a_5^4a_6^7 - 1251a_2a_4a_5^6a_6^6 - 81a_2a_5^8a_6^5 \\
& + 45a_3^4a_6^{10} - 30a_3^3a_4a_5a_6^9 - 555a_3^3a_5^3a_6^8 - 90a_3^2a_4^3a_6^9 \\
& - 1890a_3^2a_4^2a_5^2a_6^8 - 1365a_3^2a_4a_5^4a_6^7 + 54a_3^2a_5^6a_6^6 - 225a_3a_4^4a_5a_6^8 \\
& + 300a_3a_4^3a_5^3a_6^7 + 1800a_3a_4^2a_5^5a_6^6 + 513a_3a_4a_5^7a_6^5 + 15a_3a_5^9a_6^4 \\
& - 15a_4^6a_6^8 - 120a_4^5a_5^2a_6^7 + 375a_4^4a_5^4a_6^6 + 420a_4^3a_5^6a_6^5 \\
& + 51a_4^2a_5^8a_6^4 + 40590a_0^2a_5a_6^{10} + 1485a_0a_1a_4a_6^{10} + 3845a_0a_1a_5^2a_6^9 \\
& - 2820a_0a_2a_3a_6^{10} + 3280a_0a_2a_4a_5a_6^9 + 77990a_0a_2a_5^3a_6^8 + 47970a_0a_3^2a_5a_6^9 \\
& + 2655a_0a_3a_4^2a_6^9 - 72630a_0a_3a_4a_5^2a_6^8 - 61065a_0a_3a_5^4a_6^7 - 6070a_0a_4^3a_5a_6^8 \\
& - 20790a_0a_4^2a_5^3a_6^7 + 36435a_0a_4a_5^5a_6^6 + 10931a_0a_5^7a_6^5 + 2865a_1^2a_3a_6^{10} \\
& - 1825a_1^2a_4a_5a_6^9 - 84720a_1^2a_5^3a_6^8 + 990a_1a_2^2a_6^{10} - 46920a_1a_2a_3a_5a_6^9 \\
& - 1965a_1a_2a_4^2a_6^9 + 16775a_1a_2a_4a_5^2a_6^8 + 33050a_1a_2a_5^4a_6^7 - 2610a_1a_3^2a_4a_6^9
\end{aligned}$$

$$\begin{aligned}
& + 3775a_1a_3^2a_5^2a_6^8 + 10705a_1a_3a_4^2a_5a_6^8 + 39135a_1a_3a_4a_5^3a_6^7 + 9916a_1a_3a_5^5a_6^6 \\
& + 450a_1a_4^4a_6^8 + 1390a_1a_4^3a_5^2a_6^7 - 11950a_1a_4^2a_5^4a_6^6 - 6991a_1a_4a_5^6a_6^5 \\
& - 455a_1a_5^8a_6^4 + 7650a_2^3a_5a_6^9 + 30a_2^2a_3a_4a_6^9 + 1930a_2^2a_3a_5^2a_6^8 \\
& - 5720a_2^2a_4^2a_5a_6^8 - 8540a_2^2a_4a_5^3a_6^7 - 800a_2^2a_5^5a_6^6 - 420a_2a_3^3a_6^9 \\
& + 4370a_2a_3^2a_4a_5a_6^8 + 8055a_2a_3^2a_5^3a_6^7 + 90a_2a_3a_4^3a_6^8 + 4470a_2a_3a_4^2a_5^2a_6^7 \\
& - 3805a_2a_3a_4a_5^4a_6^6 - 1949a_2a_3a_5^6a_6^5 + 1985a_2a_4^4a_5a_6^7 + 4880a_2a_4^3a_5^3a_6^6 \\
& - 293a_2a_4^2a_5^5a_6^5 - 385a_2a_4a_5^7a_6^4 - 10a_2a_5^9a_6^3 + 1140a_3^4a_5a_6^8 \\
& + 510a_3^3a_4^2a_6^8 - 735a_3^3a_4a_5^2a_6^7 - 2135a_3^3a_5^4a_6^6 - 510a_3^2a_4^3a_5a_6^7 \\
& - 6330a_3^2a_4^2a_5^3a_6^6 - 1491a_3^2a_4a_5^5a_6^5 + 147a_3^2a_5^7a_6^4 + 15a_3a_4^5a_6^7 \\
& - 1800a_3a_4^4a_5^2a_6^6 - 1425a_3a_4^3a_5^4a_6^5 + 807a_3a_4^2a_5^6a_6^4 + 108a_3a_4a_5^8a_6^3 \\
& - 200a_4^6a_5a_6^6 - 1140a_4^5a_5^3a_6^5 - 510a_4^4a_5^5a_6^4 - 4a_4^3a_5^7a_6^3 \\
& + 26088a_0^2a_4a_6^9 + 123275a_0^2a_5^2a_6^8 - 157095a_0a_1a_3a_6^9 + 109519a_0a_1a_4a_5a_6^8 \\
& + 22280a_0a_1a_5^3a_6^7 + 38230a_0a_2^2a_6^9 - 36230a_0a_2a_3a_5a_6^8 - 28839a_0a_2a_4^2a_6^8 \\
& - 37564a_0a_2a_4a_5^2a_6^7 + 116025a_0a_2a_5^4a_6^6 + 42936a_0a_3^2a_4a_6^8 + 210430a_0a_3^2a_5^2a_6^7 \\
& + 31894a_0a_3a_4^2a_5a_6^7 - 208956a_0a_3a_4a_5^3a_6^6 - 71117a_0a_3a_5^5a_6^5 + 5960a_0a_4^4a_6^7 \\
& - 11638a_0a_4^3a_5^2a_6^6 - 51034a_0a_4^2a_5^4a_6^5 + 10903a_0a_4a_5^6a_6^4 + 2240a_0a_5^8a_6^3 \\
& + 38915a_1^2a_2a_6^9 + 53790a_1^2a_3a_5a_6^8 - 6028a_1^2a_4^2a_6^8 + 68828a_1^2a_4a_5^2a_6^7 \\
& - 118700a_1^2a_5^4a_6^6 - 10010a_1a_2^2a_5a_6^8 + 8614a_1a_2a_3a_4a_6^8 - 201420a_1a_2a_3a_5^2a_6^7 \\
& - 25565a_1a_2a_4^2a_5a_6^7 + 32229a_1a_2a_4a_5^3a_6^6 + 35916a_1a_2a_5^5a_6^5 - 14430a_1a_3^3a_6^8 \\
& - 18558a_1a_3^2a_4a_5a_6^7 + 17330a_1a_3^2a_5^3a_6^6 - 3689a_1a_3a_4^3a_6^7 + 15185a_1a_3a_4^2a_5^2a_6^6 \\
& + 30001a_1a_3a_4a_5^4a_6^5 + 2832a_1a_3a_5^6a_6^4 + 4466a_1a_4^4a_5a_6^6 + 11286a_1a_4^3a_5^3a_6^5 \\
& - 2543a_1a_4^2a_5^5a_6^4 - 1308a_1a_4a_5^7a_6^3 - 30a_1a_5^9a_6^2 + 458a_2^3a_4a_6^8 \\
& + 32240a_2^3a_5^2a_6^7 - 100a_2^2a_3^2a_6^8 - 1150a_2^2a_3a_4a_5a_6^7 + 6935a_2^2a_3a_5^3a_6^6 \\
& + 468a_2^2a_4^3a_6^7 - 16740a_2^2a_4^2a_5^2a_6^6 - 12642a_2^2a_4a_5^4a_6^5 - 927a_2^2a_5^6a_6^4 \\
& - 2800a_2a_3^3a_5a_6^7 - 4570a_2a_3^2a_4^2a_6^7 + 11953a_2a_3^2a_4a_5^2a_6^6 + 8750a_2a_3^2a_5^4a_6^5 \\
& - 190a_2a_3a_4^3a_5a_6^6 + 13062a_2a_3a_4^2a_5^3a_6^5 - 619a_2a_3a_4a_5^5a_6^4 - 446a_2a_3a_5^7a_6^3 \\
& - 431a_2a_4^5a_6^6 + 3984a_2a_4^4a_5^2a_6^5 + 6305a_2a_4^3a_5^4a_6^4 + 747a_2a_4^2a_5^6a_6^3 \\
& + 1116a_3^4a_4a_6^7 + 5115a_3^4a_5^2a_6^6 + 4329a_3^3a_4^2a_5a_6^6 - 3228a_3^3a_4a_5^3a_6^5 \\
& - 1920a_3^3a_5^5a_6^4 + 891a_3^2a_4^4a_6^6 + 2406a_3^2a_4^3a_5^2a_6^5 - 3765a_3^2a_4^2a_5^4a_6^4 \\
& + 42a_3^2a_4a_5^6a_6^3 + 39a_3^2a_5^8a_6^2 + 1020a_3a_4^5a_5a_6^5 - 1500a_3a_4^4a_5^3a_6^4 \\
& - 1179a_3a_4^3a_5^5a_6^3 + 21a_3a_4^2a_5^7a_6^2 + 72a_4^7a_6^5 + 270a_4^6a_5^2a_6^4 \\
& - 300a_4^5a_5^4a_6^3 - 84a_4^4a_5^6a_6^2 + 130548a_0^2a_3a_6^8 - 2726a_0^2a_4a_5a_6^7 \\
& - 40910a_0^2a_5^3a_6^6 - 15a_0a_1a_2a_6^8 - 756736a_0a_1a_3a_5a_6^7 - 54084a_0a_1a_4^2a_6^7 \\
& + 506716a_0a_1a_4a_5^2a_6^6 + 7395a_0a_1a_5^4a_6^5 + 191330a_0a_2^2a_5a_6^7 - 46326a_0a_2a_3a_4a_6^7 \\
& - 93704a_0a_2a_3a_5^2a_6^6 - 95174a_0a_2a_4^2a_5a_6^6 - 80340a_0a_2a_4a_5^3a_6^5 + 55803a_0a_2a_5^5a_6^4
\end{aligned}$$

$$\begin{aligned}
& -25959a_0a_3^3a_6^7 + 155560a_0a_3^2a_4a_5a_6^6 + 210069a_0a_3^2a_5^3a_6^5 + 20050a_0a_3a_4^3a_6^6 \\
& + 143853a_0a_3a_4^2a_5^2a_6^5 - 108970a_0a_3a_4a_5^4a_6^4 - 14904a_0a_3a_5^6a_6^3 + 28215a_0a_4^4a_5a_6^5 \\
& + 13758a_0a_4^3a_5^3a_6^4 - 20260a_0a_4^2a_5^5a_6^3 + 36a_0a_4a_5^7a_6^2 + 55a_0a_5^9a_6 \\
& - 40265a_1^3a_6^8 + 159555a_1^2a_2a_5a_6^7 - 439a_1^2a_3a_4a_6^7 + 162573a_1^2a_3a_5^2a_6^6 \\
& - 70887a_1^2a_4^2a_5a_6^6 + 167265a_1^2a_4a_5^3a_6^5 - 44706a_1^2a_5^5a_6^4 + 25912a_1a_2^2a_4a_6^7 \\
& - 52975a_1a_2^2a_5^2a_6^6 + 39289a_1a_2a_3^2a_6^7 + 133024a_1a_2a_3a_4a_5a_6^6 - 207706a_1a_2a_3a_5^3a_6^5 \\
& - 3097a_1a_2a_4^3a_6^6 - 93332a_1a_2a_4^2a_5^2a_6^5 - 7791a_1a_2a_4a_5^4a_6^4 + 8026a_1a_2a_5^6a_6^3 \\
& - 73763a_1a_3^3a_5a_6^6 - 8173a_1a_3^2a_4^2a_6^6 - 15477a_1a_3^2a_4a_5^2a_6^5 + 17846a_1a_3^2a_5^4a_6^4 \\
& - 17499a_1a_3a_4^3a_5a_6^5 - 6182a_1a_3a_4^2a_5^3a_6^4 + 2232a_1a_3a_4a_5^5a_6^3 - 60a_1a_3a_5^7a_6^2 \\
& - 435a_1a_4^5a_6^5 + 6757a_1a_4^4a_5^2a_6^4 + 9030a_1a_4^3a_5^4a_6^3 + 801a_1a_4^2a_5^6a_6^2 \\
& - 11a_1a_4a_5^8a_6 + 1128a_2^3a_3a_6^7 - 22134a_2^3a_4a_5a_6^6 + 36345a_2^3a_5^3a_6^5 \\
& - 4440a_2^2a_3^2a_5a_6^6 + 5822a_2^2a_3a_4^2a_6^6 - 7721a_2^2a_3a_4a_5^2a_6^5 + 4113a_2^2a_3a_5^4a_6^4 \\
& + 10904a_2^2a_3^3a_5a_6^5 - 7764a_2^2a_4^2a_5^3a_6^4 - 4372a_2^2a_4a_5^5a_6^3 - 173a_2^2a_5^7a_6^2 \\
& - 4064a_2a_3^3a_4a_6^6 + 168a_2a_3^3a_5^2a_6^5 - 29673a_2a_3^2a_4^2a_5a_6^5 + 3356a_2a_3^2a_4a_5^3a_6^4 \\
& + 1410a_2a_3^2a_5^5a_6^3 - 3144a_2a_3a_4^4a_6^5 - 3106a_2a_3a_4^3a_5^2a_6^4 + 8080a_2a_3a_4^2a_5^4a_6^3 \\
& + 450a_2a_3a_4a_5^6a_6^2 - 7a_2a_3a_5^8a_6 - 3742a_2a_4^5a_5a_6^4 - 1335a_2a_4^4a_5^3a_6^3 \\
& + 681a_2a_4^3a_5^5a_6^2 + 38a_2a_4^2a_5^7a_6 - 1464a_3^5a_6^6 + 6228a_3^4a_4a_5a_6^5 \\
& + 4422a_3^4a_5^3a_6^4 - 651a_3^3a_4^3a_6^5 + 5934a_3^3a_4^2a_5^2a_6^4 - 2820a_3^3a_4a_5^4a_6^3 \\
& - 255a_3^3a_5^6a_6^2 + 2937a_3^2a_4^4a_5a_6^4 + 3270a_3^2a_4^3a_5^3a_6^3 - 396a_3^2a_4^2a_5^5a_6^2 \\
& + 39a_3^2a_4a_5^7a_6 + 1665a_3a_4^5a_5^2a_6^3 - 45a_3a_4^4a_5^4a_6^2 - 96a_3a_4^3a_5^6a_6 \\
& + 180a_4^7a_5a_6^3 + 360a_4^6a_5^3a_6^2 + 36a_4^5a_5^5a_6 + 342304a_0^2a_2a_6^7 \\
& + 598605a_0^2a_3a_5a_6^6 + 21689a_0^2a_4^2a_6^6 - 134208a_0^2a_4a_5^2a_6^5 - 149040a_0^2a_5^4a_6^4 \\
& + 201958a_0a_1^2a_6^7 - 207215a_0a_1a_2a_5a_6^6 - 59245a_0a_1a_3a_4a_6^6 - 449098a_0a_1a_3a_5^2a_6^5 \\
& - 277067a_0a_1a_4^2a_5a_6^5 + 425667a_0a_1a_4a_5^3a_6^4 - 9508a_0a_1a_5^5a_6^3 - 85232a_0a_2^2a_4a_6^6 \\
& + 202720a_0a_2^2a_5^2a_6^5 + 45193a_0a_2a_3^2a_6^6 - 161288a_0a_2a_3a_4a_5a_6^5 - 100047a_0a_2a_3a_5^3a_6^4 \\
& + 13310a_0a_2a_4^3a_6^5 - 5853a_0a_2a_4^2a_5^2a_6^4 - 22424a_0a_2a_4a_5^4a_6^3 + 8992a_0a_2a_5^6a_6^2 \\
& - 58083a_0a_3^3a_5a_6^5 - 52686a_0a_3^2a_4^2a_6^5 + 52920a_0a_3^2a_4a_5^2a_6^4 + 30384a_0a_3^2a_5^4a_6^3 \\
& + 30928a_0a_3a_4^3a_5a_6^4 + 119848a_0a_3a_4^2a_5^3a_6^3 - 5268a_0a_3a_4a_5^5a_6^2 - 101a_0a_3a_5^7a_6 \\
& - 3793a_0a_4^5a_6^4 + 11239a_0a_4^4a_5^2a_6^3 + 6926a_0a_4^3a_5^4a_6^2 - 1063a_0a_4^2a_5^6a_6 \\
& - 5a_0a_4a_5^8 - 226053a_1^3a_5a_6^6 - 13136a_1^2a_2a_4a_6^6 + 69529a_1^2a_2a_5^2a_6^5 \\
& + 114082a_1^2a_3^2a_6^6 - 179587a_1^2a_3a_4a_5a_6^5 + 106440a_1^2a_3a_5^3a_6^4 + 11603a_1^2a_4^3a_6^5 \\
& - 149913a_1^2a_4^2a_5^2a_6^4 + 68132a_1^2a_4a_5^4a_6^3 - 4129a_1^2a_5^6a_6^2 - 139638a_1a_2^2a_3a_6^6 \\
& + 184370a_1a_2^2a_4a_5a_6^5 - 20067a_1a_2^2a_5^3a_6^4 + 119936a_1a_2a_3^2a_5a_6^5 + 20984a_1a_2a_3a_4^2a_6^5 \\
& + 222475a_1a_2a_3a_4a_5^2a_6^4 - 45234a_1a_2a_3a_5^4a_6^3 - 1509a_1a_2a_4^3a_5a_6^4 - 66660a_1a_2a_4^2a_5^3a_6^3 \\
& - 8640a_1a_2a_4a_5^5a_6^2 + 154a_1a_2a_5^7a_6 + 4857a_1a_3^3a_4a_6^5 - 66522a_1a_3^3a_5^2a_6^4
\end{aligned}$$

$$\begin{aligned}
& -2574a_1a_3^2a_4^2a_5a_6^4 + 1358a_1a_3^2a_4a_5^3a_6^3 + 3356a_1a_3^2a_5^5a_6^2 - 188a_1a_3a_4^4a_6^4 \\
& -5772a_1a_3a_4^3a_5^3a_6^3 - 1534a_1a_3a_4^2a_5^4a_6^2 - 17a_1a_3a_4a_5^6a_6 - 2a_1a_3a_5^8 \\
& -1655a_1a_4^5a_5a_6^3 + 526a_1a_4^4a_5^3a_6^2 + 683a_1a_4^3a_5^5a_6 + 21a_1a_4^2a_5^7 \\
& + 46867a_2^4a_6^6 - 28670a_2^3a_3a_5a_6^5 - 22142a_2^3a_4^2a_6^5 - 43611a_2^3a_4a_5^2a_6^4 \\
& + 10036a_2^3a_5^4a_6^3 + 45016a_2^2a_3^2a_4a_6^5 - 151a_2^2a_3^2a_5^2a_6^4 + 4693a_2^2a_3a_4^2a_5a_6^4 \\
& - 7506a_2^2a_3a_4a_5^3a_6^3 - 231a_2^2a_3a_5^5a_6^2 + 4814a_2^2a_4^4a_6^4 + 15208a_2^2a_4^3a_5^2a_6^3 \\
& + 1224a_2^2a_4^2a_5^4a_6^2 - 164a_2^2a_4a_5^6a_6 - 2a_2^2a_5^8 - 8124a_2a_3^4a_6^5 \\
& - 11178a_2a_3^3a_4a_5a_6^4 + 3670a_2a_3^3a_5^3a_6^3 + 137a_2a_3^2a_4^3a_6^4 - 27402a_2a_3^2a_4^2a_5^2a_6^3 \\
& - 902a_2a_3^2a_4a_5^4a_6^2 - 16a_2a_3^2a_5^6a_6 - 5569a_2a_3a_4^4a_5a_6^3 - 2650a_2a_3a_4^3a_5^3a_6^2 \\
& + 627a_2a_3a_4^2a_5^5a_6 + 13a_2a_3a_4a_5^7 - 12a_2a_4^6a_6^3 - 2209a_2a_4^5a_5^2a_6^2 \\
& - 578a_2a_4^4a_5^4a_6 - 8a_2a_4^3a_5^6 - 1260a_3^5a_5a_6^4 - 2493a_3^4a_4^2a_6^4 \\
& + 5622a_3^4a_4a_5^2a_6^3 + 126a_3^4a_5^4a_6^2 - 1698a_3^3a_4^3a_5a_6^3 + 1644a_3^3a_4^2a_5^3a_6^2 \\
& - 282a_3^3a_4a_5^5a_6 + 4a_3^3a_5^7 - 981a_3^2a_4^5a_6^3 + 726a_3^2a_4^4a_5^2a_6^2 \\
& + 213a_3^2a_4^3a_5^4a_6 - 18a_3^2a_4^2a_5^6 - 411a_3a_4^6a_5a_6^2 + 228a_3a_4^5a_5^3a_6 \\
& + 12a_3a_4^4a_5^5 - 39a_4^8a_6^2 - 48a_4^7a_5^2a_6 + 2a_4^6a_5^4 \\
& - 677225a_0^2a_1a_6^6 + 72239a_0^2a_2a_5a_6^5 + 201723a_0^2a_3a_4a_6^5 + 642729a_0^2a_3a_5^2a_6^4 \\
& + 271710a_0^2a_4^2a_5a_6^4 - 15732a_0^2a_4a_5^3a_6^3 - 22972a_0^2a_5^5a_6^2 + 224888a_0a_1^2a_5a_6^5 \\
& + 491718a_0a_1a_2a_4a_6^5 - 195640a_0a_1a_2a_5^2a_6^4 - 282147a_0a_1a_3^2a_6^5 - 146834a_0a_1a_3a_4a_5a_6^4 \\
& + 72064a_0a_1a_3a_5^3a_6^3 - 57557a_0a_1a_4^3a_6^4 - 264712a_0a_1a_4^2a_5^2a_6^3 + 72928a_0a_1a_4a_5^4a_6^2 \\
& - 458a_0a_1a_5^6a_6 + 183747a_0a_2^2a_3a_6^5 - 219540a_0a_2^2a_4a_5a_6^4 + 64615a_0a_2^2a_5^3a_6^3 \\
& + 111796a_0a_2a_3^2a_5a_6^4 - 129150a_0a_2a_3a_4^2a_6^4 - 132640a_0a_2a_3a_4a_5^2a_6^3 - 43918a_0a_2a_3a_5^4a_6^2 \\
& - 5316a_0a_2a_4^3a_5a_6^3 + 9386a_0a_2a_4^2a_5^3a_6^2 - 824a_0a_2a_4a_5^5a_6 + 179a_0a_2a_5^7 \\
& + 90663a_0a_3^3a_4a_6^4 + 5231a_0a_3^3a_5^2a_6^3 - 97581a_0a_3^2a_4^2a_5a_6^3 - 11334a_0a_3^2a_4a_5^3a_6^2 \\
& - 1900a_0a_3^2a_5^5a_6 + 7567a_0a_3a_4^4a_6^3 - 8334a_0a_3a_4^3a_5^2a_6^2 + 10664a_0a_3a_4^2a_5^4a_6 \\
& + 34a_0a_3a_4a_5^6 - 1239a_0a_4^5a_5a_6^2 + 1209a_0a_4^4a_5^3a_6 + 455a_0a_4^3a_5^5 \\
& + 65024a_1^3a_4a_6^5 - 204286a_1^3a_5^2a_6^4 - 23757a_1^2a_2a_3a_6^5 + 55813a_1^2a_2a_4a_5a_6^4 \\
& - 27938a_1^2a_2a_5^3a_6^3 + 99594a_1^2a_3^2a_5a_6^4 + 117403a_1^2a_3a_4^2a_6^4 - 174086a_1^2a_3a_4a_5^2a_6^3 \\
& + 18030a_1^2a_3a_5^4a_6^2 + 28320a_1^2a_4^3a_5a_6^3 - 54670a_1^2a_4^2a_5^3a_6^2 + 2750a_1^2a_4a_5^5a_6 \\
& - 68a_1^2a_5^7 - 77853a_1a_2^3a_6^5 - 205381a_1a_2^2a_3a_5a_6^4 - 36943a_1a_2^2a_4^2a_6^4 \\
& + 155801a_1a_2^2a_4a_5^2a_6^3 + 6172a_1a_2^2a_5^4a_6^2 - 93460a_1a_2a_3^2a_4a_6^4 + 55202a_1a_2a_3^2a_5^2a_6^3 \\
& + 736a_1a_2a_3a_4^2a_5a_6^3 + 53144a_1a_2a_3a_4a_5^3a_6^2 - 940a_1a_2a_3a_5^5a_6 + 7087a_1a_2a_4^4a_6^3 \\
& + 14642a_1a_2a_4^3a_5^2a_6^2 - 4418a_1a_2a_4^2a_5^4a_6 - 221a_1a_2a_4a_5^6 + 6993a_1a_3^4a_6^4 \\
& + 15269a_1a_3^3a_4a_5a_6^3 - 10572a_1a_3^3a_5^3a_6^2 - 7687a_1a_3^2a_4^3a_6^3 - 1074a_1a_3^2a_4^2a_5^2a_6^2 \\
& - 468a_1a_3^2a_4a_5^4a_6 + 36a_1a_3^2a_5^6 + 612a_1a_3a_4^4a_5a_6^2 + 914a_1a_3a_4^3a_5^3a_6 \\
& + 106a_1a_3a_4^2a_5^5 - 288a_1a_4^6a_6^2 - 697a_1a_4^5a_5^2a_6 - 121a_1a_4^4a_5^4
\end{aligned}$$

$$\begin{aligned}
& + 58602a_2^4a_5a_6^4 + 27270a_2^3a_3a_4a_6^4 - 18566a_2^3a_3a_5^2a_6^3 - 14969a_2^3a_4^2a_5a_6^3 \\
& - 14350a_2^3a_4a_5^3a_6^2 + 321a_2^3a_5^5a_6 + 8056a_2^2a_3^2a_6^4 + 35868a_2^2a_3^2a_4a_5a_6^3 \\
& + 4476a_2^2a_3^2a_5^3a_6^2 - 691a_2^2a_3a_4^3a_6^3 + 4208a_2^2a_3a_4^2a_5^2a_6^2 - 755a_2^2a_3a_4a_5^4a_6 \\
& - 25a_2^2a_3a_5^6 + 1008a_2^2a_4^4a_5a_6^2 + 2272a_2^2a_4^3a_5^3a_6 + 103a_2^2a_4^2a_5^5 \\
& - 9182a_2a_3^4a_5a_6^3 + 15634a_2a_3^3a_4^2a_6^3 - 6330a_2a_3^3a_4a_5^2a_6^2 + 292a_2a_3^3a_4^4a_6 \\
& + 1746a_2a_3^2a_4^3a_5a_6^2 - 2666a_2a_3^2a_4^2a_5^3a_6 + 2a_2a_3^2a_4a_5^5 + 2055a_2a_3a_4^5a_6^2 \\
& - 1193a_2a_3a_4^4a_5^2a_6 - 213a_2a_3a_4^3a_5^4 + 418a_2a_4^6a_5a_6 + 4a_2a_4^5a_5^3 \\
& - 2382a_3^5a_4a_6^3 + 1518a_3^5a_5^2a_6^2 - 1668a_3^4a_4^2a_5a_6^2 + 606a_3^4a_4a_5^3a_6 \\
& - 42a_3^4a_5^5 - 369a_3^3a_4^4a_6^2 + 234a_3^3a_4^3a_5^2a_6 + 84a_3^3a_4^2a_5^4 \\
& - 501a_3^2a_4^5a_5a_6 + 24a_3^2a_4^4a_5^3 - 12a_3a_4^7a_6 - 36a_3a_4^6a_5^2 \\
& - 6a_4^8a_5 + 897186a_0^3a_6^5 - 801868a_0^2a_1a_5a_6^4 - 92533a_0^2a_2a_4a_6^4 \\
& - 235457a_0^2a_2a_5^2a_6^3 - 276723a_0^2a_3^2a_6^4 + 221223a_0^2a_3a_4a_5a_6^3 + 70693a_0^2a_3a_5^3a_6^2 \\
& - 28516a_0^2a_4^3a_6^3 + 165494a_0^2a_4^2a_5^2a_6^2 + 14380a_0^2a_4a_5^4a_6 + 273a_0^2a_5^6 \\
& + 49322a_0a_1^2a_4a_6^4 + 32898a_0a_1^2a_5^2a_6^3 - 51081a_0a_1a_2a_3a_6^4 + 593272a_0a_1a_2a_4a_5a_6^3 \\
& - 13686a_0a_1a_2a_5^3a_6^2 - 314131a_0a_1a_2^3a_5a_6^3 - 67535a_0a_1a_3a_4^2a_6^3 - 147616a_0a_1a_3a_4a_5^2a_6^2 \\
& + 8082a_0a_1a_3a_5^4a_6 - 9252a_0a_1a_4^3a_5a_6^2 - 32494a_0a_1a_4^2a_5^3a_6 + 1204a_0a_1a_4a_5^5 \\
& + 163555a_0a_2^3a_6^4 + 26517a_0a_2^2a_3a_5a_6^3 + 56194a_0a_2^2a_4^2a_6^3 - 59841a_0a_2^2a_4a_5^2a_6^2 \\
& + 4238a_0a_2^2a_5^4a_6 + 57980a_0a_2a_3^2a_4a_6^3 + 107116a_0a_2a_3^2a_5^2a_6^2 - 94809a_0a_2a_3a_4^2a_5a_6^2 \\
& - 22640a_0a_2a_3a_4a_5^3a_6 - 1550a_0a_2a_3a_5^5 - 22035a_0a_2a_4^4a_6^2 - 10632a_0a_2a_4^3a_5^2a_6 \\
& - 934a_0a_2a_4^2a_5^4 + 12093a_0a_3^4a_6^3 + 37803a_0a_3^3a_4a_5a_6^2 + 6442a_0a_3^3a_5^3a_6 \\
& + 36678a_0a_3^2a_4^3a_6^2 - 9504a_0a_3^2a_4^2a_5^2a_6 - 262a_0a_3^2a_4a_5^4 + 2799a_0a_3a_4^4a_5a_6 \\
& - 1042a_0a_3a_4^3a_5^3 + 2384a_0a_4^6a_6 + 865a_0a_4^5a_5^2 - 6081a_1^3a_3a_6^4 \\
& + 95342a_1^3a_4a_5a_6^3 - 38620a_1^3a_5^3a_6^2 + 16235a_1^2a_2^2a_6^4 + 67832a_1^2a_2a_3a_5a_6^3 \\
& - 4507a_1^2a_2a_4^2a_6^3 + 70452a_1^2a_2a_4a_5^2a_6^2 - 1854a_1^2a_2a_5^4a_6 + 33407a_1^2a_3^2a_4a_6^3 \\
& - 7376a_1^2a_3^2a_5^2a_6^2 + 119231a_1^2a_3a_4^2a_5a_6^2 - 9672a_1^2a_3a_4a_5^3a_6 + 444a_1^2a_3a_5^5 \\
& + 326a_1^2a_4^4a_6^2 + 5414a_1^2a_4^3a_5^2a_6 - 1737a_1^2a_4^2a_5^4 - 77500a_1a_2^3a_5a_6^3 \\
& + 8611a_1a_2^2a_3a_4a_6^3 - 61381a_1a_2^2a_3a_5^2a_6^2 - 69371a_1a_2^2a_4^2a_5a_6^2 + 11782a_1a_2^2a_4a_5^3a_6 \\
& + 314a_1a_2^2a_5^5 - 38651a_1a_2a_3^3a_6^3 - 35242a_1a_2a_3^2a_4a_5a_6^2 + 1964a_1a_2a_3^2a_5^3a_6 \\
& - 1727a_1a_2a_3a_4^3a_6^2 - 5622a_1a_2a_3a_4^2a_5^2a_6 + 822a_1a_2a_3a_4a_5^4 + 5322a_1a_2a_4^4a_5a_6 \\
& + 1612a_1a_2a_4^3a_5^3 + 2916a_1a_3^4a_5a_6^2 - 12855a_1a_3^3a_4^2a_6^2 + 3034a_1a_3^3a_4a_5^2a_6 \\
& - 124a_1a_3^3a_5^4 - 4550a_1a_3^2a_4^3a_5a_6 - 554a_1a_3^2a_4^2a_5^3 - 1553a_1a_3a_4^5a_6 \\
& - 234a_1a_3a_4^4a_5^2 - 135a_1a_4^6a_5 - 16514a_2^4a_4a_6^3 + 14622a_2^4a_5^2a_6^2 \\
& + 2162a_2^3a_3^2a_6^3 + 24220a_2^3a_3a_4a_5a_6^2 - 442a_2^3a_3a_5^3a_6 + 6107a_2^3a_4^3a_6^2 \\
& - 692a_2^3a_4^2a_5^2a_6 - 419a_2^3a_4a_5^4 - 4234a_2^2a_3^3a_5a_6^2 - 12799a_2^2a_3^2a_4^2a_6^2 \\
& + 3742a_2^2a_3^2a_4a_5^2a_6 + 179a_2^2a_3^2a_5^4 + 104a_2^2a_3a_4^3a_5a_6 + 448a_2^2a_3a_4^2a_5^3
\end{aligned}$$

$$\begin{aligned}
& -1058a_2^2a_4^5a_6 - 281a_2^2a_4^4a_5^2 + 11394a_2a_3^4a_4a_6^2 - 388a_2a_3^4a_5^2a_6 \\
& + 1886a_2a_3^3a_4^2a_5a_6 - 394a_2a_3^3a_4a_5^3 + 2726a_2a_3^2a_4^4a_6 + 390a_2a_3^2a_4^3a_5^2 \\
& + 281a_2a_3a_4^5a_5 + 36a_2a_4^7 - 1800a_3^6a_6^2 - 360a_3^5a_4a_5a_6 \\
& + 144a_3^5a_5^3 - 174a_3^4a_4^3a_6 - 18a_3^4a_4^2a_5^2 - 108a_3^3a_4^4a_5 \\
& + 42a_3^2a_4^6 + 235475a_0^3a_5a_6^3 + 155123a_0^2a_1a_4a_6^3 + 14337a_0^2a_1a_5^2a_6^2 \\
& + 620214a_0^2a_2a_3a_6^3 - 119853a_0^2a_2a_4a_5a_6^2 - 42234a_0^2a_2a_5^3a_6 + 48432a_0^2a_3^2a_5a_6^2 \\
& - 255576a_0^2a_3a_4^2a_6^2 - 27490a_0^2a_3a_4a_5^2a_6 - 4120a_0^2a_3a_5^4 - 6140a_0^2a_4^3a_5a_6 \\
& + 7068a_0^2a_4^2a_5^3 - 62847a_0a_1^2a_3a_6^3 - 7974a_0a_1^2a_4a_5a_6^2 + 2376a_0a_1^2a_5^3a_6 \\
& - 405698a_0a_1a_2^2a_6^3 - 27654a_0a_1a_2a_3a_5a_6^2 + 22041a_0a_1a_2a_4^2a_6^2 + 90738a_0a_1a_2a_4a_5^2a_6 \\
& + 622a_0a_1a_2a_5^4 + 121629a_0a_1a_3^2a_4a_6^2 - 19953a_0a_1a_3^2a_5^2a_6 - 3818a_0a_1a_3a_4^2a_5a_6 \\
& - 7472a_0a_1a_3a_4a_5^3 - 7620a_0a_1a_4^4a_6 - 2290a_0a_1a_4^3a_5^2 + 11006a_0a_2^3a_5a_6^2 \\
& + 166083a_0a_2^2a_3a_4a_6^2 + 5026a_0a_2^2a_3a_5^2a_6 + 10547a_0a_2^2a_4^2a_5a_6 - 782a_0a_2^2a_4a_5^3 \\
& - 101591a_0a_2a_3^3a_6^2 + 23936a_0a_2a_3^2a_4a_5a_6 + 4442a_0a_2a_3^2a_5^3 - 276a_0a_2a_3a_4^3a_6 \\
& - 454a_0a_2a_3a_4^2a_5^2 - 3351a_0a_2a_4^4a_5 - 2360a_0a_3^4a_5a_6 + 1253a_0a_3^3a_4^2a_6 \\
& + 940a_0a_3^3a_4a_5^2 + 2710a_0a_3^2a_4^3a_5 + 628a_0a_3a_4^5 + 69169a_1^3a_2a_6^3 \\
& + 36608a_1^3a_3a_5a_6^2 - 32483a_1^3a_4^2a_6^2 + 8560a_1^3a_4a_5^2a_6 - 964a_1^3a_5^4 \\
& - 2076a_1^2a_2^2a_5a_6^2 - 138552a_1^2a_2a_3a_4a_6^2 + 4360a_1^2a_2a_3a_5^2a_6 + 932a_1^2a_2a_4^2a_5a_6 \\
& + 4288a_1^2a_2a_4a_5^3 + 16433a_1^2a_3^3a_6^2 + 2459a_1^2a_3^2a_4a_5a_6 - 336a_1^2a_3^2a_5^3 \\
& - 496a_1^2a_3a_4^3a_6 + 6068a_1^2a_3a_4^2a_5^2 + 390a_1^2a_4^4a_5 + 56756a_1a_2^3a_4a_6^2 \\
& - 8441a_1a_2^3a_5^2a_6 + 36465a_1a_2^2a_3^2a_6^2 + 3705a_1a_2^2a_3a_4a_5a_6 - 2116a_1a_2^2a_3a_5^3 \\
& - 3357a_1a_2^2a_4^3a_6 - 5558a_1a_2^2a_4^2a_5^2 - 2760a_1a_2a_3^3a_5a_6 + 8959a_1a_2a_3^2a_4^2a_6 \\
& - 62a_1a_2a_3^2a_4a_5^2 + 468a_1a_2a_3a_4^3a_5 + 33a_1a_2a_4^5 - 2408a_1a_3^4a_4a_6 \\
& - 32a_1a_3^4a_5^2 - 372a_1a_3^3a_4^2a_5 - 622a_1a_3^2a_4^4 - 17308a_2^4a_3a_6^2 \\
& - 2566a_2^4a_4a_5a_6 + 438a_2^4a_5^3 - 1226a_2^3a_3^2a_5a_6 - 2866a_2^3a_3a_4^2a_6 \\
& + 750a_2^3a_3a_4a_5^2 + 820a_2^3a_4^3a_5 - 1250a_2^2a_3^3a_4a_6 - 90a_2^2a_3^3a_5^2 \\
& - 1638a_2^2a_3^2a_4^2a_5 - 415a_2^2a_3a_4^4 - 400a_2a_3^5a_6 + 760a_2a_3^4a_4a_5 \\
& + 406a_2a_3^3a_4^3 - 160a_3^6a_5 - 168a_3^5a_4^2 + 427425a_0^3a_4a_6^2 \\
& + 38215a_0^3a_5^2a_6 - 492108a_0^2a_1a_3a_6^2 - 27201a_0^2a_1a_4a_5a_6 + 3589a_0^2a_1a_5^3 \\
& + 77034a_0^2a_2^2a_6^2 + 103793a_0^2a_2a_3a_5a_6 - 9794a_0^2a_2a_4^2a_6 - 10864a_0^2a_2a_4a_5^2 \\
& + 68457a_0^2a_3^2a_4a_6 + 14919a_0^2a_3^2a_5^2 - 23435a_0^2a_3a_4^2a_5 - 4210a_0^2a_4^4 \\
& + 111099a_0a_1^2a_2a_6^2 - 21139a_0a_1^2a_3a_5a_6 + 46318a_0a_1^2a_4^2a_6 + 2702a_0a_1^2a_4a_5^2 \\
& - 54311a_0a_1a_2^2a_5a_6 - 60192a_0a_1a_2a_3a_4a_6 + 1711a_0a_1a_2a_3a_5^2 + 6905a_0a_1a_2a_4^2a_5 \\
& - 4828a_0a_1a_3^3a_6 + 10632a_0a_1a_3^2a_4a_5 + 1715a_0a_1a_3a_4^3 + 16902a_0a_2^3a_4a_6 \\
& - 221a_0a_2^3a_5^2 - 7665a_0a_2^2a_3^2a_6 + 7269a_0a_2^2a_3a_4a_5 + 3313a_0a_2^2a_4^3 \\
& - 4440a_0a_2a_3^3a_5 - 4037a_0a_2a_3^2a_4^2 - 560a_0a_3^4a_4 - 49413a_1^4a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 5626a_1^3a_2a_5a_6 + 6909a_1^3a_3a_4a_6 + 2232a_1^3a_3a_5^2 - 3087a_1^3a_4^2a_5 \\
& - 19175a_1^2a_2^2a_4a_6 - 1393a_1^2a_2^2a_5^2 - 942a_1^2a_2a_3^2a_6 - 13093a_1^2a_2a_3a_4a_5 \\
& - 1281a_1^2a_2a_4^3 - 928a_1^2a_3^3a_5 - 335a_1^2a_3^2a_4^2 + 5419a_1a_2^3a_3a_6 \\
& + 5368a_1a_2^3a_4a_5 + 3172a_1a_2^2a_3^2a_5 + 1850a_1a_2^2a_3a_4^2 + 152a_1a_2a_3^3a_4 \\
& + 320a_1a_3^5 + 718a_2^5a_6 - 1186a_2^4a_3a_5 - 570a_2^4a_4^2 \\
& + 654a_2^3a_3^2a_4 - 440a_2^2a_3^4 - 11262a_0^3a_3a_6 + 15523a_0^3a_4a_5 \\
& - 24318a_0^2a_1a_2a_6 - 21958a_0^2a_1a_3a_5 + 13885a_0^2a_1a_4^2 - 1165a_0^2a_2^2a_5 \\
& + 36387a_0^2a_2a_3a_4 - 7660a_0^2a_3^3 + 252a_0a_1^3a_6 + 10253a_0a_1^2a_2a_5 \\
& - 3174a_0a_1^2a_3a_4 - 15845a_0a_1a_2^2a_4 - 12828a_0a_1a_2a_3^2 + 1099a_0a_2^3a_3 \\
& - 5100a_1^4a_5 + 7881a_1^3a_2a_4 + 3456a_1^3a_3^2 + 181a_1^2a_2^2a_3 \\
& - 818a_1a_2^4 + 15818a_0^3a_2 - 21192a_0^2a_1^2
\end{aligned} \tag{A.26}$$

$$\begin{aligned}
e_{27} = & - 129a_0^2a_6^{13} + 43a_0a_1a_5a_6^{12} - 88a_0a_2a_4a_6^{12} - 1253a_0a_2a_5^2a_6^{11} \\
& - 138a_0a_3^2a_6^{12} + 228a_0a_3a_4a_5a_6^{11} + 573a_0a_3a_5^3a_6^{10} - 20a_0a_4^3a_6^{11} \\
& - 996a_0a_4^2a_5^2a_6^{10} - 3663a_0a_4a_5^4a_6^9 - 1442a_0a_5^6a_6^8 + 94a_1^2a_4a_6^{12} \\
& + 1341a_1^2a_5^2a_6^{11} + 138a_1a_2a_3a_6^{12} - 140a_1a_2a_4a_5a_6^{11} - 662a_1a_2a_5^3a_6^{10} \\
& - 46a_1a_3^2a_5a_6^{11} - 88a_1a_3a_4^2a_6^{11} - 960a_1a_3a_4a_5^2a_6^{10} - 568a_1a_3a_5^4a_6^9 \\
& + 62a_1a_4^3a_5a_6^{10} + 802a_1a_4^2a_5^3a_6^9 + 835a_1a_4a_5^5a_6^8 + 122a_1a_5^7a_6^7 \\
& - 24a_2^3a_6^{12} - 10a_2^2a_3a_5a_6^{11} + 26a_2^2a_4^2a_6^{11} + 140a_2^2a_4a_5^2a_6^{10} \\
& + 26a_2^2a_5^4a_6^9 - 20a_2a_3^2a_4a_6^{11} - 114a_2a_3^2a_5^2a_6^{10} + 30a_2a_3a_4^2a_5a_6^{10} \\
& + 364a_2a_3a_4a_5^3a_6^9 + 215a_2a_3a_5^5a_6^8 - 2a_2a_4^4a_6^{10} + 118a_2a_4^3a_5^2a_6^9 \\
& + 440a_2a_4^2a_5^4a_6^8 + 199a_2a_4a_5^6a_6^7 + 13a_2a_5^8a_6^6 - 3a_3^4a_6^{11} \\
& + 18a_3^3a_4a_5a_6^{10} + 59a_3^3a_5^3a_6^9 + 12a_3^2a_4^3a_6^{10} + 162a_3^2a_4^2a_5^2a_6^9 \\
& + 60a_3^2a_4a_5^4a_6^8 - 24a_3^2a_5^6a_6^7 - 15a_3a_4^4a_5a_6^9 - 300a_3a_4^3a_5^3a_6^8 \\
& - 450a_3a_4^2a_5^5a_6^7 - 105a_3a_4a_5^7a_6^6 - 3a_3a_5^9a_6^5 - a_4^6a_6^9 \\
& - 60a_4^5a_5^2a_6^8 - 225a_4^4a_5^4a_6^7 - 140a_4^3a_5^6a_6^6 - 15a_4^2a_5^8a_6^5 \\
& - 6751a_0^2a_5a_6^{11} - 143a_0a_1a_4a_6^{11} + 850a_0a_1a_5^2a_6^{10} + 226a_0a_2a_3a_6^{11} \\
& - 2246a_0a_2a_4a_5a_6^{10} - 16273a_0a_2a_5^3a_6^9 - 7496a_0a_3^2a_5a_6^{10} - 80a_0a_3a_4^2a_6^{10} \\
& + 13545a_0a_3a_4a_5^2a_6^9 + 10096a_0a_3a_5^4a_6^8 + 850a_0a_4^3a_5a_6^9 - 3130a_0a_4^2a_5^3a_6^8 \\
& - 15331a_0a_4a_5^5a_6^7 - 3656a_0a_5^7a_6^6 - 232a_1^2a_3a_6^{11} + 2440a_1^2a_4a_5a_6^{10} \\
& + 17726a_1^2a_5^3a_6^9 - 66a_1a_2^2a_6^{11} + 7566a_1a_2a_3a_5a_6^{10} + 176a_1a_2a_4^2a_6^{10} \\
& - 4894a_1a_2a_4a_5^2a_6^9 - 7544a_1a_2a_5^4a_6^8 + 242a_1a_3^2a_4a_6^{10} - 1410a_1a_3^2a_5^2a_6^9 \\
& - 3048a_1a_3a_4^2a_5a_6^9 - 9296a_1a_3a_4a_5^3a_6^8 - 2348a_1a_3a_5^5a_6^7 - 60a_1a_4^4a_6^9 \\
& + 886a_1a_4^3a_5^2a_6^8 + 5197a_1a_4^2a_5^4a_6^7 + 2622a_1a_4a_5^6a_6^6 + 178a_1a_5^8a_6^5
\end{aligned}$$

$$\begin{aligned}
& -1262a_2^3a_5a_6^{10} - 2a_2^2a_3a_4a_6^{10} - 452a_2^2a_3a_5^2a_6^9 + 1094a_2^2a_4^2a_5a_6^9 \\
& + 1832a_2^2a_4a_5^3a_6^8 + 251a_2^2a_5^5a_6^7 + 32a_2a_3^3a_6^{10} - 966a_2a_3^2a_4a_5a_6^9 \\
& - 1411a_2a_3^2a_5^3a_6^8 - 46a_2a_3a_4^3a_6^9 - 120a_2a_3a_4^2a_5^2a_6^8 + 2109a_2a_3a_4a_5^4a_6^7 \\
& + 674a_2a_3a_5^6a_6^6 - 331a_2a_4^4a_5a_6^8 - 252a_2a_4^3a_5^3a_6^7 + 756a_2a_4^2a_5^5a_6^6 \\
& + 203a_2a_4a_5^7a_6^5 + 5a_2a_5^9a_6^4 - 180a_3^4a_5a_6^9 - 54a_3^3a_4^2a_6^9 \\
& + 489a_3^3a_4a_5^2a_6^8 + 544a_3^3a_5^4a_6^7 + 396a_3^2a_4^3a_5a_6^8 + 1632a_3^2a_4^2a_5^3a_6^7 \\
& + 144a_3^2a_4a_5^5a_6^6 - 72a_3^2a_5^7a_6^5 + 21a_3a_4^5a_6^8 + 360a_3a_4^4a_5^2a_6^7 \\
& - 450a_3a_4^3a_5^4a_6^6 - 645a_3a_4^2a_5^6a_6^5 - 63a_3a_4a_5^8a_6^4 + 64a_4^6a_5a_6^7 \\
& + 180a_4^5a_5^3a_6^6 - 60a_4^4a_5^5a_6^5 - 40a_4^3a_5^7a_6^4 - 7619a_0^2a_4a_6^{10} \\
& - 44834a_0^2a_5^2a_6^9 + 36053a_0a_1a_3a_6^{10} - 26343a_0a_1a_4a_5a_6^9 + 3226a_0a_1a_5^3a_6^8 \\
& - 8560a_0a_2^2a_6^{10} + 6332a_0a_2a_3a_5a_6^9 + 6831a_0a_2a_4^2a_6^9 + 2309a_0a_2a_4a_5^2a_6^8 \\
& - 45772a_0a_2a_5^4a_6^7 - 12614a_0a_2^3a_4a_6^9 - 60533a_0a_3^2a_5^2a_6^8 + 2389a_0a_3a_4^2a_5a_6^8 \\
& + 86729a_0a_3a_4a_5^3a_6^7 + 26235a_0a_3a_5^5a_6^6 - 986a_0a_4^4a_6^8 + 10042a_0a_4^3a_5^2a_6^7 \\
& + 9961a_0a_4^2a_5^4a_6^6 - 12141a_0a_4a_5^6a_6^5 - 1685a_0a_5^8a_6^4 - 9572a_1^2a_2a_6^{10} \\
& - 11138a_1^2a_3a_5a_6^9 + 1153a_1^2a_4^2a_6^9 - 5180a_1^2a_4a_5^2a_6^8 + 49724a_1^2a_5^4a_6^7 \\
& + 4200a_1a_2^2a_5a_6^9 + 1128a_1a_2a_3a_4a_6^9 + 62050a_1a_2a_3a_5^2a_6^8 + 4130a_1a_2a_4^2a_5a_6^8 \\
& - 23094a_1a_2a_4a_5^3a_6^7 - 16918a_1a_2a_5^5a_6^6 + 3614a_1a_3^3a_6^9 + 3232a_1a_3^2a_4a_5a_6^8 \\
& - 8604a_1a_3^2a_5^3a_6^7 + 374a_1a_3a_4^3a_6^8 - 11938a_1a_3a_4^2a_5^2a_6^7 - 15221a_1a_3a_4a_5^4a_6^6 \\
& - 1446a_1a_3a_5^6a_6^5 - 1480a_1a_4^4a_5a_6^7 - 1102a_1a_4^3a_5^3a_6^6 + 4515a_1a_4^2a_5^5a_6^5 \\
& + 1224a_1a_4a_5^7a_6^4 + 30a_1a_5^9a_6^3 - 482a_2^3a_4a_6^9 - 10350a_2^3a_5^2a_6^8 \\
& + 148a_2^2a_3^2a_6^9 + 38a_2^2a_3a_4a_5a_6^8 - 3043a_2^2a_3a_5^3a_6^7 + 134a_2^2a_4^3a_6^8 \\
& + 6258a_2^2a_4^2a_5^2a_6^7 + 5157a_2^2a_4a_5^4a_6^6 + 482a_2^2a_5^6a_6^5 + 164a_2a_3^3a_5a_6^8 \\
& + 706a_2a_3^2a_4^2a_6^8 - 6071a_2a_3^2a_4a_5^2a_6^7 - 3027a_2a_3^2a_5^4a_6^6 - 1122a_2a_3a_4^3a_5a_6^7 \\
& - 3458a_2a_3a_4^2a_5^3a_6^6 + 1923a_2a_3a_4a_5^5a_6^5 + 350a_2a_3a_5^7a_6^4 - 11a_2a_4^5a_6^7 \\
& - 2278a_2a_4^4a_5^2a_6^6 - 2409a_2a_4^3a_5^4a_6^5 - 123a_2a_4^2a_5^6a_6^4 + 14a_2a_4a_5^8a_6^3 \\
& - 408a_3^4a_4a_6^8 - 1491a_3^4a_5^2a_6^7 - 993a_3^3a_4^2a_5a_6^7 + 2470a_3^3a_4a_5^3a_6^6 \\
& + 924a_3^3a_5^5a_6^5 - 225a_3^2a_4^4a_6^7 + 948a_3^2a_4^3a_5^2a_6^6 + 2394a_3^2a_4^2a_5^4a_6^5 \\
& - 183a_3^2a_4a_5^6a_6^4 - 33a_3^2a_5^8a_6^3 + 42a_3a_4^5a_5a_6^6 + 1620a_3a_4^4a_5^3a_6^5 \\
& + 435a_3a_4^3a_5^5a_6^4 - 93a_3a_4^2a_5^7a_6^3 - 8a_4^7a_6^6 + 198a_4^6a_5^2a_6^5 \\
& + 450a_4^5a_5^4a_6^4 + 80a_4^4a_5^6a_6^3 - 28411a_0^2a_3a_6^9 - 9567a_0^2a_4a_5a_6^8 \\
& - 25336a_0^2a_5^3a_6^7 + 443a_0a_1a_2a_6^9 + 295558a_0a_1a_3a_5a_6^8 + 12257a_0a_1a_4^2a_6^8 \\
& - 215256a_0a_1a_4a_5^2a_6^7 + 14364a_0a_1a_5^4a_6^6 - 72316a_0a_2^2a_5a_6^8 + 13106a_0a_2a_3a_4a_6^8 \\
& + 29712a_0a_2a_3a_5^2a_6^7 + 45227a_0a_2a_4^2a_5a_6^7 + 41351a_0a_2a_4a_5^3a_6^6 - 37511a_0a_2a_5^5a_6^5 \\
& + 8670a_0a_3^3a_6^8 - 84658a_0a_3^2a_4a_5a_6^7 - 108756a_0a_3^2a_5^3a_6^6 - 9473a_0a_3a_4^3a_6^7 \\
& - 25937a_0a_3a_4^2a_5^2a_6^6 + 99894a_0a_3a_4a_5^4a_6^5 + 11982a_0a_3a_5^6a_6^4 - 11302a_0a_4^4a_5a_6^6
\end{aligned}$$

$$\begin{aligned}
& + 8442a_0a_4^3a_5^3a_6^5 + 14445a_0a_4^2a_5^5a_6^4 - 1472a_0a_4a_5^7a_6^3 - 110a_0a_5^9a_6^2 \\
& + 9662a_1^3a_6^9 - 73180a_1^2a_2a_5a_6^8 - 2366a_1^2a_3a_4a_6^8 - 67902a_1^2a_3a_5^2a_6^7 \\
& + 14554a_1^2a_4a_5a_6^7 - 68164a_1^2a_4a_5^3a_6^6 + 36496a_1^2a_5^5a_6^5 - 7814a_1a_2^2a_4a_6^8 \\
& + 34148a_1a_2^2a_5^2a_6^7 - 12666a_1a_2a_3^2a_6^8 - 26512a_1a_2a_3a_4a_5a_6^7 + 118506a_1a_2a_3a_5^3a_6^6 \\
& + 2340a_1a_2a_4^3a_6^7 + 35540a_1a_2a_4^2a_5^2a_6^6 - 14029a_1a_2a_4a_5^4a_6^5 - 8264a_1a_2a_5^6a_6^4 \\
& + 31876a_1a_3^3a_5a_6^7 + 4574a_1a_3^2a_4^2a_6^7 + 4668a_1a_3^2a_4a_5^2a_6^6 - 13184a_1a_3^2a_5^4a_6^5 \\
& + 7420a_1a_3a_4^3a_5a_6^6 - 3020a_1a_3a_4^2a_5^3a_6^5 - 3096a_1a_3a_4a_5^5a_6^4 + 58a_1a_3a_5^7a_6^3 \\
& + 16a_1a_4^5a_6^6 - 5570a_1a_4^4a_5^2a_6^5 - 5975a_1a_4^3a_5^4a_6^4 - 50a_1a_4^2a_5^6a_6^3 \\
& + 58a_1a_4a_5^8a_6^2 + 76a_2^3a_3a_6^8 + 5554a_2^3a_4a_5a_6^7 - 21497a_2^3a_5^3a_6^6 \\
& + 3004a_2^2a_3^2a_5a_6^7 - 1366a_2^2a_3a_4^2a_6^7 + 1799a_2^2a_3a_4a_5^2a_6^6 - 3778a_2^2a_3a_5^4a_6^5 \\
& - 3226a_2^2a_4^3a_5a_6^6 + 6414a_2^2a_4^2a_5^3a_6^5 + 3459a_2^2a_4a_5^5a_6^4 + 176a_2^2a_5^7a_6^3 \\
& + 2016a_2a_3^3a_4a_6^7 - 2120a_2a_3^3a_5^2a_6^6 + 9785a_2a_3^2a_4^2a_5a_6^6 - 6544a_2a_3^2a_4a_5^3a_6^5 \\
& - 912a_2a_3^2a_5^5a_6^4 + 1188a_2a_3a_4^4a_6^6 - 3118a_2a_3a_4^3a_5^2a_6^5 - 5865a_2a_3a_4^2a_5^4a_6^4 \\
& + 22a_2a_3a_4a_5^6a_6^3 + 20a_2a_3a_5^8a_6^2 + 1232a_2a_4^5a_5a_6^5 - 1060a_2a_4^4a_5^3a_6^4 \\
& - 977a_2a_4^3a_5^5a_6^3 - 49a_2a_4^2a_5^7a_6^2 + 444a_3^5a_6^7 - 3702a_3^4a_4a_5a_6^6 \\
& - 2232a_3^4a_5^3a_6^5 - 243a_3^3a_4^3a_6^6 - 2892a_3^3a_4^2a_5^2a_6^5 + 2820a_3^3a_4a_5^4a_6^4 \\
& + 226a_3^3a_5^6a_6^3 - 2280a_3^2a_4^4a_5a_6^5 - 840a_3^2a_4^3a_5^3a_6^4 + 474a_3^2a_4^2a_5^5a_6^3 \\
& - 84a_3^2a_4a_5^7a_6^2 - 132a_3a_4^6a_6^5 - 900a_3a_4^5a_5^2a_6^4 + 675a_3a_4^4a_5^4a_6^3 \\
& + 147a_3a_4^3a_5^6a_6^2 - 180a_4^7a_5a_6^4 - 240a_4^6a_5^3a_6^3 - 161823a_0^2a_2a_6^8 \\
& - 208239a_0^2a_3a_5a_6^7 + 12555a_0^2a_4^2a_6^7 + 109730a_0^2a_4a_5^2a_6^6 + 74891a_0^2a_5^4a_6^5 \\
& - 91749a_0a_1^2a_6^8 + 110523a_0a_1a_2a_5a_6^7 + 28241a_0a_1a_3a_4a_6^7 + 318694a_0a_1a_3a_5^2a_6^6 \\
& + 105967a_0a_1a_4^2a_5a_6^6 - 315983a_0a_1a_4a_5^3a_6^5 + 20906a_0a_1a_5^5a_6^4 + 42928a_0a_2^2a_4a_6^7 \\
& - 129325a_0a_2^2a_5^2a_6^6 - 12049a_0a_2a_3^2a_6^7 + 49868a_0a_2a_3a_4a_5a_6^6 + 53519a_0a_2a_3a_5^3a_6^5 \\
& - 13913a_0a_2a_4^3a_6^6 + 12572a_0a_2a_4^2a_5^2a_6^5 + 30192a_0a_2a_4a_5^4a_6^4 - 10738a_0a_2a_5^6a_6^3 \\
& + 41920a_0a_3^3a_5a_6^6 + 26791a_0a_3^2a_4^2a_6^6 - 81036a_0a_3^2a_4a_5^2a_6^5 - 35812a_0a_3^2a_5^4a_6^4 \\
& - 41627a_0a_3a_4^3a_5a_6^5 - 86650a_0a_3a_4^2a_5^3a_6^4 + 16944a_0a_3a_4a_5^5a_6^3 + 343a_0a_3a_5^7a_6^2 \\
& + 1859a_0a_4^5a_6^5 - 13924a_0a_4^4a_5^2a_6^4 - 1744a_0a_4^3a_5^4a_6^3 + 2804a_0a_4^2a_5^6a_6^2 \\
& + 11a_0a_4a_5^8a_6 + 95033a_1^3a_5a_6^7 + 11806a_1^2a_2a_4a_6^7 - 79138a_1^2a_2a_5^2a_6^6 \\
& - 57212a_1^2a_3^2a_6^7 + 89731a_1^2a_3a_4a_5a_6^6 - 88456a_1^2a_3a_5^3a_6^5 - 1440a_1^2a_4^3a_6^6 \\
& + 63126a_1^2a_4^2a_5^2a_6^5 - 67280a_1^2a_4a_5^4a_6^4 + 6811a_1^2a_5^6a_6^3 + 53972a_1a_2^2a_3a_6^7 \\
& - 94680a_1a_2^2a_4a_5a_6^6 + 37575a_1a_2^2a_5^3a_6^5 - 72407a_1a_2a_3^2a_5a_6^6 - 19078a_1a_2a_3a_4^2a_6^6 \\
& - 113603a_1a_2a_3a_4a_5^2a_6^5 + 53070a_1a_2a_3a_5^4a_6^4 + 10824a_1a_2a_4^3a_5a_6^5 + 58570a_1a_2a_4^2a_5^3a_6^4 \\
& + 4744a_1a_2a_4a_5^5a_6^3 - 608a_1a_2a_5^7a_6^2 - 670a_1a_3^3a_4a_6^6 + 50694a_1a_3^3a_5^2a_6^5 \\
& + 5838a_1a_3^2a_4^2a_5a_6^5 - 1280a_1a_3^2a_4a_5^3a_6^4 - 4580a_1a_3^2a_5^5a_6^3 + 703a_1a_3a_4^4a_6^5 \\
& + 9102a_1a_3a_4^3a_5^2a_6^4 + 2386a_1a_3a_4^2a_5^4a_6^3 + 223a_1a_3a_4a_5^6a_6^2 + 16a_1a_3a_5^8a_6
\end{aligned}$$

$$\begin{aligned}
& + 1067a_1a_4^5a_5a_6^4 - 2036a_1a_4^4a_5^3a_6^3 - 1453a_1a_4^3a_5^5a_6^2 - 59a_1a_4^2a_5^7a_6 \\
& - 22045a_2^4a_6^7 + 20314a_2^3a_3a_5a_6^6 + 12718a_2^3a_4^2a_6^6 + 25533a_2^3a_4a_5^2a_6^5 \\
& - 11719a_2^3a_5^4a_6^4 - 21084a_2^2a_3^2a_4a_6^6 + 3341a_2^2a_3^2a_5^2a_6^5 + 351a_2^2a_3a_4^2a_5a_6^5 \\
& + 5560a_2^2a_3a_4a_5^3a_6^4 - 243a_2^2a_3a_5^5a_6^3 - 2994a_2^2a_4^4a_6^5 - 10174a_2^2a_4^3a_5^2a_6^4 \\
& - 68a_2^2a_4^2a_5^4a_6^3 + 371a_2^2a_4a_5^6a_6^2 + 7a_2^2a_5^8a_6 + 5072a_2a_3^4a_6^6 \\
& + 9606a_2a_3^3a_4a_5a_6^5 - 4720a_2a_3^3a_5^3a_6^4 + 2717a_2a_3^2a_4^3a_6^5 + 18366a_2a_3^2a_4^2a_5^2a_6^4 \\
& - 808a_2a_3^2a_4a_5^4a_6^3 + 59a_2a_3^2a_5^6a_6^2 + 4969a_2a_3a_4^4a_5a_6^4 - 582a_2a_3a_4^3a_5^3a_6^3 \\
& - 1437a_2a_3a_4^2a_5^5a_6^2 - 33a_2a_3a_4a_5^7a_6 + 254a_2a_4^6a_6^4 + 2357a_2a_4^5a_5^2a_6^3 \\
& + 550a_2a_4^4a_5^4a_6^2 - 6a_2a_4^3a_5^6a_6 + 426a_3^5a_5a_6^5 + 1299a_3^4a_4^2a_6^5 \\
& - 5250a_3^4a_4a_5^2a_6^4 - 36a_3^4a_5^4a_6^3 - 192a_3^3a_4^3a_5a_6^4 - 1512a_3^3a_4^2a_5^3a_6^3 \\
& + 510a_3^3a_4a_5^5a_6^2 - 14a_3^3a_5^7a_6 + 498a_3^2a_4^5a_6^4 - 2046a_3^2a_4^4a_5^2a_6^3 \\
& - 165a_3^2a_4^3a_5^4a_6^2 + 39a_3^2a_4^2a_5^6a_6 - 21a_3a_4^6a_5a_6^3 - 546a_3a_4^5a_5^3a_6^2 \\
& + 12a_3a_4^4a_5^5a_6 + 21a_4^8a_6^3 - 48a_4^7a_5^2a_6^2 - 34a_4^6a_5^4a_6 \\
& + 326754a_0^2a_1a_6^7 - 138938a_0^2a_2a_5a_6^6 - 138714a_0^2a_3a_4a_6^6 - 404020a_0^2a_3a_5^2a_6^5 \\
& - 107657a_0^2a_4^2a_5a_6^5 + 94867a_0^2a_4a_5^3a_6^4 + 30952a_0^2a_5^5a_6^3 - 161405a_0a_1^2a_5a_6^6 \\
& - 245814a_0a_1a_2a_4a_6^6 + 194312a_0a_1a_2a_5^2a_6^5 + 126350a_0a_1a_3^2a_6^6 + 36646a_0a_1a_3a_4a_5a_6^5 \\
& - 67646a_0a_1a_3a_5^3a_6^4 + 36679a_0a_1a_4^3a_6^5 + 212395a_0a_1a_4^2a_5^2a_6^4 - 103196a_0a_1a_4a_5^4a_6^3 \\
& + 3484a_0a_1a_5^6a_6^2 - 88930a_0a_2^2a_3a_6^6 + 174190a_0a_2^2a_4a_5a_6^5 - 72298a_0a_2^2a_5^3a_6^4 \\
& - 58155a_0a_2a_3^2a_5a_6^5 + 86812a_0a_2a_3a_4^2a_6^5 + 61584a_0a_2a_3a_4a_5^2a_6^4 + 45386a_0a_2a_3a_5^4a_6^3 \\
& - 32605a_0a_2a_4^3a_5a_6^4 - 25780a_0a_2a_4^2a_5^3a_6^3 + 1424a_0a_2a_4a_5^5a_6^2 - 747a_0a_2a_5^7a_6 \\
& - 48582a_0a_3^3a_4a_6^5 + 18748a_0a_3^3a_5^2a_6^4 + 103308a_0a_3^2a_4^2a_5a_6^4 - 1966a_0a_3^2a_4a_5^3a_6^3 \\
& + 1868a_0a_3^2a_5^5a_6^2 + 266a_0a_3a_4^4a_6^4 - 9340a_0a_3a_4^3a_5^2a_6^3 - 24022a_0a_3a_4^2a_5^4a_6^2 \\
& + 206a_0a_3a_4a_5^6a_6 - 18a_0a_3a_5^8 + 6047a_0a_4^5a_5a_6^3 - 1522a_0a_4^4a_5^3a_6^2 \\
& - 795a_0a_4^3a_5^5a_6 + 13a_0a_4^2a_5^7 - 29241a_1^3a_4a_6^6 + 146202a_1^3a_5^2a_6^5 \\
& + 24008a_1^2a_2a_3a_6^6 + 16901a_1^2a_2a_4a_5a_6^5 + 16138a_1^2a_2a_5^3a_6^4 - 79200a_1^2a_3^2a_5a_6^5 \\
& - 61981a_1^2a_3a_4^2a_6^5 + 187184a_1^2a_3a_4a_5^2a_6^4 - 28942a_1^2a_3a_5^4a_6^3 - 6926a_1^2a_4^3a_5a_6^4 \\
& + 54500a_1^2a_4^2a_5^3a_6^3 - 9392a_1^2a_4a_5^5a_6^2 + 284a_1^2a_5^7a_6 + 42944a_1a_2^3a_6^6 \\
& + 122000a_1a_2^2a_3a_5a_6^5 + 17584a_1a_2^2a_4^2a_6^5 - 158922a_1a_2^2a_4a_5^2a_6^4 + 1210a_1a_2^2a_5^4a_6^3 \\
& + 52557a_1a_2a_3^2a_4a_6^5 - 68084a_1a_2a_3^2a_5^2a_6^4 - 21690a_1a_2a_3a_4^2a_5a_6^4 - 62552a_1a_2a_3a_4a_5^3a_6^3 \\
& + 3708a_1a_2a_3a_5^5a_6^2 - 6129a_1a_2a_4^4a_6^4 - 5704a_1a_2a_4^3a_5^2a_6^3 + 12642a_1a_2a_4^2a_5^4a_6^2 \\
& + 793a_1a_2a_4a_5^6a_6 - 7182a_1a_3^4a_6^5 - 16196a_1a_3^3a_4a_5a_6^4 + 16016a_1a_3^3a_5^3a_6^3 \\
& + 1798a_1a_3^2a_4^3a_6^4 - 474a_1a_3^2a_4^2a_5^2a_6^3 + 56a_1a_3^2a_4a_5^4a_6^2 - 214a_1a_3^2a_5^6a_6 \\
& - 2632a_1a_3a_4^4a_5a_6^3 - 896a_1a_3a_4^3a_5^3a_6^2 - 320a_1a_3a_4^2a_5^5a_6 - 8a_1a_3a_4a_5^7 \\
& + 187a_1a_4^6a_6^3 + 1140a_1a_4^5a_5^2a_6^2 + 215a_1a_4^4a_5^4a_6 - 9a_1a_4^3a_5^6 \\
& - 45182a_2^4a_5a_6^5 - 19250a_2^3a_3a_4a_6^5 + 26278a_2^3a_3a_5^2a_6^4 + 17585a_2^3a_4^2a_5a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 17818a_2^3a_4a_5^3a_6^3 - 1145a_2^3a_5^5a_6^2 - 6964a_2^2a_3^3a_5^5 - 29700a_2^2a_3^2a_4a_5a_6^4 \\
& - 3594a_2^2a_3^3a_5^3a_6^3 - 159a_2^2a_3a_4^3a_6^4 - 1158a_2^2a_3a_4^2a_5^2a_6^3 + 2169a_2^2a_3a_4a_5^4a_6^2 \\
& + 94a_2^2a_3a_5^6a_6 - 2049a_2^2a_4^4a_5a_6^3 - 3818a_2^2a_4^3a_5^3a_6^2 - 277a_2^2a_4^2a_5^5a_6 \\
& + a_2^2a_4a_5^7 + 10412a_2a_3^4a_5a_6^4 - 10006a_2a_3^3a_4^2a_6^4 + 9138a_2a_3^3a_4a_5^2a_6^3 \\
& - 962a_2a_3^3a_5^4a_6^2 + 3106a_2a_3^2a_4^3a_5a_6^3 + 4292a_2a_3^2a_4^2a_5^3a_6^2 - 106a_2a_3^2a_4a_5^5a_6 \\
& - 1363a_2a_3a_4^5a_6^3 + 3168a_2a_3a_4^4a_5^2a_6^2 + 449a_2a_3a_4^3a_5^4a_6 - 11a_2a_3a_4^2a_5^6 \\
& - 285a_2a_4^6a_5a_6^2 + 208a_2a_4^5a_5^3a_6 + 16a_2a_4^4a_5^5 + 2010a_3^5a_4a_6^4 \\
& - 1848a_3^5a_5^2a_6^3 + 1944a_3^4a_4^2a_5a_6^3 - 816a_3^4a_4a_5^3a_6^2 + 144a_3^4a_5^5a_6 \\
& + 900a_3^3a_4^4a_6^3 - 600a_3^3a_4^3a_5^2a_6^2 - 144a_3^3a_4^2a_5^4a_6 - 2a_3^3a_4a_5^6 \\
& + 972a_3^2a_4^5a_5a_6^2 - 180a_3^2a_4^4a_5^3a_6 + 18a_3^2a_4^3a_5^5 + 99a_3a_4^7a_6^2 \\
& + 96a_3a_4^6a_5^2a_6 - 24a_3a_4^5a_5^4 + 36a_4^8a_5a_6 + 8a_4^7a_5^3 \\
& - 561633a_0^3a_6^6 + 659838a_0^2a_1a_5a_6^5 + 126207a_0^2a_2a_4a_6^5 + 161186a_0^2a_2a_5^2a_6^4 \\
& + 261564a_0^2a_3^3a_6^5 - 348611a_0^2a_3a_4a_5a_6^4 - 108527a_0^2a_3a_5^3a_6^3 - 13749a_0^2a_4^3a_6^4 \\
& - 163958a_0^2a_4^2a_5^2a_6^3 - 6576a_0^2a_4a_5^4a_6^2 + 87a_0^2a_5^6a_6 - 36234a_0a_1^2a_4a_6^5 \\
& - 35956a_0a_1^2a_5^2a_6^4 + 381a_0a_1a_2a_3a_6^5 - 510200a_0a_1a_2a_4a_5a_6^4 + 47434a_0a_1a_2a_5^3a_6^3 \\
& + 297833a_0a_1a_3^2a_5a_6^4 + 77689a_0a_1a_3a_4^2a_6^4 + 145848a_0a_1a_3a_4a_5^2a_6^3 - 29422a_0a_1a_3a_5^4a_6^2 \\
& - 2164a_0a_1a_4^3a_5a_6^3 + 54286a_0a_1a_4^2a_5^3a_6^2 - 7132a_0a_1a_4a_5^5a_6 - 22a_0a_1a_5^7 \\
& - 146291a_0a_2^3a_6^5 + 14958a_0a_2^2a_3a_5a_6^4 - 23545a_0a_2^2a_4^2a_6^4 + 85579a_0a_2^2a_4a_5^2a_6^3 \\
& - 10126a_0a_2^2a_5^4a_6^2 - 45305a_0a_2a_3^2a_4a_6^4 - 120163a_0a_2a_3^2a_5^2a_6^3 + 132827a_0a_2a_3a_4^2a_5a_6^3 \\
& + 38044a_0a_2a_3a_4a_5^3a_6^2 + 5402a_0a_2a_3a_5^5a_6 + 24599a_0a_2a_4^4a_6^3 + 3126a_0a_2a_4^3a_5^2a_6^2 \\
& - 820a_0a_2a_4^2a_5^4a_6 + 88a_0a_2a_4a_5^6 - 24768a_0a_3^4a_6^4 - 35888a_0a_3^3a_4a_5a_6^3 \\
& - 7014a_0a_3^3a_5^3a_6^2 - 39382a_0a_3^2a_4^3a_6^3 + 30414a_0a_3^2a_4^2a_5^2a_6^2 + 550a_0a_3^2a_4a_5^4a_6 \\
& + 172a_0a_3^2a_5^6 + 4917a_0a_3a_4^4a_5a_6^2 + 3896a_0a_3a_4^3a_5^3a_6 - 400a_0a_3a_4^2a_5^5 \\
& - 3441a_0a_4^6a_6^2 - 831a_0a_4^5a_5^2a_6 - 256a_0a_4^4a_5^4 + 17745a_1^3a_3a_6^5 \\
& - 93712a_1^3a_4a_5a_6^4 + 50176a_1^3a_5^3a_6^3 - 8632a_1^2a_2^2a_6^5 - 70326a_1^2a_2a_3a_5a_6^4 \\
& - 19199a_1^2a_2a_4^2a_6^4 - 46060a_1^2a_2a_4a_5^2a_6^3 + 7678a_1^2a_2a_5^4a_6^2 - 30097a_1^2a_3^2a_4a_6^4 \\
& + 12216a_1^2a_3^2a_5^2a_6^3 - 129977a_1^2a_3a_4^2a_5a_6^3 + 36308a_1^2a_3a_4a_5^3a_6^2 - 1806a_1^2a_3a_5^5a_6 \\
& - 752a_1^2a_4^4a_6^3 - 2800a_1^2a_4^3a_5^2a_6^2 + 7253a_1^2a_4^2a_5^4a_6 - 26a_1^2a_4a_5^6 \\
& + 79867a_1a_2^3a_5a_6^4 + 9826a_1a_2^2a_3a_4a_6^4 + 64475a_1a_2^2a_3a_5^2a_6^3 + 73843a_1a_2^2a_4^2a_5a_6^3 \\
& - 36360a_1a_2^2a_4a_5^3a_6^2 - 1028a_1a_2^2a_5^5a_6 + 42325a_1a_2a_3^3a_6^4 + 47127a_1a_2a_3^2a_4a_5a_6^3 \\
& - 7516a_1a_2a_3^2a_5^3a_6^2 + 5169a_1a_2a_3a_4^3a_6^3 + 6410a_1a_2a_3a_4^2a_5^2a_6^2 - 3766a_1a_2a_3a_4a_5^4a_6 \\
& + 8a_1a_2a_3a_5^6 - 10334a_1a_2a_4^4a_5a_6^2 - 4802a_1a_2a_4^3a_5^3a_6 + 40a_1a_2a_4^2a_5^5 \\
& - 6608a_1a_3^4a_5a_6^3 + 13182a_1a_3^3a_4^2a_6^3 - 7530a_1a_3^3a_4a_5^2a_6^2 + 680a_1a_3^3a_5^4a_6 \\
& + 3816a_1a_3^2a_4^3a_5a_6^2 + 1246a_1a_3^2a_4^2a_5^3a_6 + 66a_1a_3^2a_4a_5^5 + 1617a_1a_3a_4^5a_6^2 \\
& - 510a_1a_3a_4^4a_5^2a_6 - 58a_1a_3a_4^3a_5^4 + 137a_1a_4^6a_5a_6 + 90a_1a_4^5a_5^3
\end{aligned}$$

$$\begin{aligned}
& + 16654a_2^4a_4a_6^4 - 20324a_2^4a_5^2a_6^3 - 9242a_2^3a_3^2a_6^4 - 34684a_2^3a_3a_4a_5a_6^3 \\
& + 2992a_2^3a_3a_5^2a_6^2 - 8769a_2^3a_4^3a_6^3 + 2864a_2^3a_4^2a_5^2a_6^2 + 1891a_2^3a_4a_5^4a_6 \\
& - a_2^3a_5^6 + 4126a_2^2a_3^3a_5a_6^3 + 13209a_2^2a_3^2a_4^2a_6^3 - 8378a_2^2a_3^2a_4a_5^2a_6^2 \\
& - 641a_2^2a_3^2a_5^4a_6 - 2330a_2^2a_3a_4^3a_5a_6^2 - 1440a_2^2a_3a_4^2a_5^3a_6 + 2a_2^2a_3a_4a_5^5 \\
& + 1617a_2^2a_4^5a_6^2 + 448a_2^2a_4^4a_5^2a_6 - 62a_2^2a_4^3a_5^4 - 15050a_2a_3^4a_4a_6^3 \\
& + 1674a_2a_3^4a_5^2a_6^2 - 1128a_2a_3^3a_4^2a_5a_6^2 + 1602a_2a_3^3a_4a_5^3a_6 + 2a_2a_3^3a_5^5 \\
& - 4527a_2a_3^2a_4^4a_6^2 - 162a_2a_3^2a_4^3a_5^2a_6 + 52a_2a_3^2a_4^2a_5^4 - 597a_2a_3a_4^5a_5a_6 \\
& + 134a_2a_3a_4^4a_5^3 - 122a_2a_4^7a_6 - 76a_2a_4^6a_5^2 + 2144a_3^6a_6^3 \\
& + 288a_3^5a_4a_5a_6^2 - 480a_3^5a_5^3a_6 + 180a_3^4a_4^3a_6^2 - 72a_3^4a_4^2a_5^2a_6 \\
& + 24a_3^4a_4a_5^4 + 294a_3^3a_4^4a_5a_6 - 120a_3^3a_4^3a_5^3 - 87a_3^2a_4^6a_6 \\
& + 66a_3^2a_4^5a_5^2 - 2a_4^9 - 105906a_0^3a_5a_6^4 - 191592a_0^2a_1a_4a_6^4 \\
& - 14018a_0^2a_1a_5^2a_6^3 - 583179a_0^2a_2a_3a_6^4 + 132841a_0^2a_2a_4a_5a_6^3 + 67109a_0^2a_2a_5^3a_6^2 \\
& + 11053a_0^2a_3^2a_5a_6^3 + 337908a_0^2a_3a_4^3a_6^3 - 6495a_0^2a_3a_4a_5^2a_6^2 + 8396a_0^2a_3a_5^4a_6 \\
& - 32788a_0^2a_4^3a_5a_6^2 - 28190a_0^2a_4^2a_5^3a_6 - 1596a_0^2a_4a_5^5 + 25131a_0a_1^2a_3a_6^4 \\
& + 36526a_0a_1^2a_4a_5a_6^3 - 2004a_0a_1^2a_5^3a_6^2 + 397613a_0a_1a_2^2a_6^4 + 7668a_0a_1a_2a_3a_5a_6^3 \\
& - 28705a_0a_1a_2a_4^2a_6^3 - 157346a_0a_1a_2a_4a_5^2a_6^2 + 1414a_0a_1a_2a_5^4a_6 - 139469a_0a_1a_3^2a_4a_6^3 \\
& + 66204a_0a_1a_3^2a_5^2a_6^2 + 29034a_0a_1a_3a_4^2a_5a_6^2 + 32660a_0a_1a_3a_4a_5^3a_6 + 312a_0a_1a_3a_5^5 \\
& + 12370a_0a_1a_4^4a_6^2 - 226a_0a_1a_4^3a_5^2a_6 + 1936a_0a_1a_4^2a_5^4 - 15171a_0a_2^3a_5a_6^3 \\
& - 202584a_0a_2^2a_3a_4a_6^3 - 55a_0a_2^2a_3a_5^2a_6^2 + 1628a_0a_2^2a_4^2a_5a_6^2 + 6510a_0a_2^2a_4a_5^3a_6 \\
& - 268a_0a_2^2a_5^5 + 122955a_0a_2a_3^3a_6^3 - 46323a_0a_2a_3^2a_4a_5a_6^2 - 14954a_0a_2a_3^2a_5^3a_6 \\
& - 704a_0a_2a_3a_4^3a_6^2 + 6982a_0a_2a_3a_4^2a_5^2a_6 + 1004a_0a_2a_3a_4a_5^4 + 9407a_0a_2a_4^4a_5a_6 \\
& + 1538a_0a_2a_4^3a_5^3 - 312a_0a_3^4a_5a_6^2 - 8088a_0a_3^3a_4^2a_6^2 - 3516a_0a_3^3a_4a_5^2a_6 \\
& - 554a_0a_3^3a_5^4 - 10686a_0a_3^2a_4^3a_5a_6 + 686a_0a_3^2a_4^2a_5^3 - 3264a_0a_3a_4^5a_6 \\
& - 462a_0a_3a_4^4a_5^2 - 523a_0a_4^6a_5 - 76889a_1^3a_2a_6^4 - 41892a_1^3a_3a_5a_6^3 \\
& + 43044a_1^3a_4^3a_6^3 - 20808a_1^3a_4a_5^2a_6^2 + 3962a_1^3a_5^4a_6 - 12084a_1^2a_2^2a_5a_6^3 \\
& + 146940a_1^2a_2a_3a_4a_6^3 - 23404a_1^2a_2a_3a_5^2a_6^2 - 26343a_1^2a_2a_4^2a_5a_6^2 - 16428a_1^2a_2a_4a_5^3a_6 \\
& - 16a_1^2a_2a_5^5 - 22036a_1^2a_3^3a_6^3 - 16032a_1^2a_3^2a_4a_5a_6^2 + 1488a_1^2a_3^2a_5^3a_6 \\
& - 1219a_1^2a_3a_4^3a_6^2 - 24432a_1^2a_3a_4^2a_5^2a_6 + 78a_1^2a_3a_4a_5^4 - 84a_1^2a_4^4a_5a_6 \\
& - 408a_1^2a_4^3a_5^3 - 70661a_1a_2^3a_4a_6^3 + 22753a_1a_2^3a_5^2a_6^2 - 38873a_1a_2^2a_3^2a_6^3 \\
& + 9022a_1a_2^2a_3a_4a_5a_6^2 + 8172a_1a_2^2a_3a_5^3a_6 + 8238a_1a_2^2a_4^3a_6^2 + 22200a_1a_2^2a_4^2a_5^2a_6 \\
& - 190a_1a_2^2a_4a_5^4 + 9484a_1a_2a_3^3a_5a_6^2 - 12747a_1a_2a_3^2a_4^2a_6^2 + 2654a_1a_2a_3^2a_4a_5^2a_6 \\
& - 46a_1a_2a_3^2a_5^4 - 632a_1a_2a_3a_4^3a_5a_6 + 436a_1a_2a_3a_4^2a_5^3 + 55a_1a_2a_4^5a_6 \\
& - 578a_1a_2a_4^4a_5^2 + 6456a_1a_3^4a_4a_6^2 - 32a_1a_3^4a_5^2a_6 + 2216a_1a_3^3a_4^2a_5a_6 \\
& - 144a_1a_3^3a_4a_5^3 + 2532a_1a_3^2a_4^4a_6 + 398a_1a_3^2a_4^3a_5^2 + 356a_1a_3a_4^5a_5 \\
& + 51a_1a_4^7 + 23244a_2^4a_3a_6^3 + 4652a_2^4a_4a_5a_6^2 - 2042a_2^4a_5^3a_6
\end{aligned}$$

$$\begin{aligned}
& -130a_2^3a_3^2a_5a_6^2 + 5298a_2^3a_3a_4^2a_6^2 - 4282a_2^3a_3a_4a_5^2a_6 - 16a_2^3a_3a_5^4 \\
& - 3068a_2^3a_4^3a_5a_6 - 148a_2^3a_4^2a_5^3 + 1466a_2^2a_3^3a_4a_6^2 + 212a_2^2a_3^3a_5^2a_6 \\
& + 4734a_2^2a_3^2a_4^2a_5a_6 - 22a_2^2a_3^2a_4a_5^3 + 1096a_2^2a_3a_4^4a_6 - 92a_2^2a_3a_4^3a_5^2 \\
& + 239a_2^2a_4^5a_5 + 168a_2a_3^5a_6^2 - 2776a_2a_3^4a_4a_5a_6 - 16a_2a_3^4a_5^3 \\
& - 1578a_2a_3^3a_4^3a_6 + 10a_2a_3^3a_4^2a_5^2 - 488a_2a_3^2a_4^4a_5 - 55a_2a_3a_4^6 \\
& + 512a_3^6a_5a_6 + 288a_3^5a_4^2a_6 - 96a_3^5a_4a_5^2 + 192a_3^4a_4^3a_5 \\
& - 30a_3^3a_4^5 - 543141a_0^3a_4a_6^3 - 56396a_0^3a_5^2a_6^2 + 595152a_0^2a_1a_3a_6^3 \\
& + 70845a_0^2a_1a_4a_5a_6^2 - 15431a_0^2a_1a_5^3a_6 - 108870a_0^2a_2^2a_6^3 - 158975a_0^2a_2a_3a_5a_6^2 \\
& + 59174a_0^2a_2a_4^2a_6^2 + 29070a_0^2a_2a_4a_5^2a_6 + 3596a_0^2a_2a_5^4 - 115572a_0^2a_3^2a_4a_6^2 \\
& - 40416a_0^2a_3^2a_5^2a_6 + 91923a_0^2a_3a_4^2a_5a_6 + 8336a_0^2a_3a_4a_5^3 + 13930a_0^2a_4^4a_6 \\
& - 1790a_0^2a_4^3a_5^2 - 128771a_0a_1^2a_2a_6^3 + 31535a_0a_1^2a_3a_5a_6^2 - 82349a_0a_1^2a_4^2a_6^2 \\
& + 326a_0a_1^2a_4a_5^2a_6 - 286a_0a_1^2a_5^4 + 92251a_0a_1a_2^2a_5a_6^2 + 75346a_0a_1a_2a_3a_4a_6^2 \\
& - 12733a_0a_1a_2a_3a_5^2a_6 - 13773a_0a_1a_2a_4^2a_5a_6 - 6548a_0a_1a_2a_4a_5^3 - 5766a_0a_1a_3^3a_6^2 \\
& - 46806a_0a_1a_3^2a_4a_5a_6 - 1948a_0a_1a_3^2a_5^3 - 4099a_0a_1a_3a_4^3a_6 - 832a_0a_1a_3a_4^2a_5^2 \\
& + 3516a_0a_1a_4^4a_5 - 37189a_0a_2^3a_4a_6^2 + 1906a_0a_2^3a_5^2a_6 + 14219a_0a_2^2a_3^2a_6^2 \\
& - 26620a_0a_2^2a_3a_4a_5a_6 + 360a_0a_2^2a_3a_5^3 - 10545a_0a_2^2a_4^3a_6 - 1280a_0a_2^2a_4^2a_5^2 \\
& + 14520a_0a_2a_3^3a_5a_6 + 11379a_0a_2a_3^2a_4^2a_6 - 2704a_0a_2a_3^2a_4a_5^2 - 376a_0a_2a_3a_4^3a_5 \\
& + 764a_0a_2a_4^5 + 3024a_0a_3^4a_4a_6 + 784a_0a_3^4a_5^2 - 520a_0a_3^3a_4^2a_5 \\
& + 450a_0a_3^3a_4^4 + 58187a_1^4a_6^3 - 8566a_1^3a_2a_5a_6^2 - 1285a_1^3a_3a_4a_6^2 \\
& - 9008a_1^3a_3a_5^2a_6 + 10101a_1^3a_4^2a_5a_6 - 284a_1^3a_4a_5^3 + 44185a_1^2a_2^2a_4a_6^2 \\
& + 4003a_1^2a_2^2a_5^2a_6 + 2581a_1^2a_2a_3^2a_6^2 + 50295a_1^2a_2a_3a_4a_5a_6 + 68a_1^2a_2a_3a_5^3 \\
& + 5799a_1^2a_2a_4^3a_6 + 720a_1^2a_2a_4^2a_5^2 + 2848a_1^2a_3^3a_5a_6 + 1884a_1^2a_3^2a_4^2a_6 \\
& + 80a_1^2a_3^2a_4a_5^2 + 348a_1^2a_3a_4^3a_5 - 594a_1^2a_4^5 - 13029a_1a_2^3a_3a_6^2 \\
& - 22661a_1a_2^3a_4a_5a_6 + 264a_1a_2^3a_5^3 - 12584a_1a_2^2a_3^2a_5a_6 - 8889a_1a_2^2a_3a_4^2a_6 \\
& - 616a_1a_2^2a_3a_4a_5^2 - 298a_1a_2^2a_4^3a_5 - 1848a_1a_2a_3^3a_4a_6 + 48a_1a_2a_3^3a_5^2 \\
& - 1140a_1a_2a_3^2a_4^2a_5 + 84a_1a_2a_3a_4^4 - 1152a_1a_3^5a_6 + 32a_1a_4^4a_4a_5 \\
& - 136a_1a_3^3a_4^3 - 1826a_2^5a_6^2 + 5440a_2^4a_3a_5a_6 + 2306a_2^4a_4^2a_6 \\
& + 388a_2^4a_4a_5^2 - 1254a_2^3a_3^2a_4a_6 + 90a_2^3a_3^2a_5^2 + 558a_2^3a_3a_4^2a_5 \\
& - 115a_2^3a_4^4 + 1760a_2^2a_3^4a_6 + 8a_2^2a_3^3a_4a_5 + 550a_2^2a_3^2a_4^3 \\
& + 32a_2a_3^5a_5 - 184a_2a_3^4a_4^2 + 128a_3^6a_4 + 40038a_0^3a_3a_6^2 \\
& - 49111a_0^3a_4a_5a_6 + 183a_0^3a_5^3 + 41418a_0^2a_1a_2a_6^2 + 90726a_0^2a_1a_3a_5a_6 \\
& - 49833a_0^2a_1a_4^2a_6 + 2113a_0^2a_1a_4a_5^2 + 20586a_0^2a_2^2a_5a_6 - 138409a_0^2a_2a_3a_4a_6 \\
& - 15807a_0^2a_2a_3a_5^2 + 14884a_0^2a_2a_4^2a_5 + 22656a_0^2a_3^3a_6 - 14496a_0^2a_3^2a_4a_5 \\
& + 4040a_0^2a_3a_4^3 - 11712a_0a_1^3a_6^2 - 51341a_0a_1^2a_2a_5a_6 + 14002a_0a_1^2a_3a_4a_6 \\
& + 2652a_0a_1^2a_3a_5^2 - 9007a_0a_1^2a_4^2a_5 + 59667a_0a_1a_2^2a_4a_6 + 3395a_0a_1a_2^2a_5^2
\end{aligned}$$

$$\begin{aligned}
& + 47608a_0a_1a_2a_3^2a_6 + 8714a_0a_1a_2a_3a_4a_5 - 7908a_0a_1a_2a_4^3 + 3824a_0a_1a_3^3a_5 \\
& - 246a_0a_1a_3^2a_4^2 - 9511a_0a_2^3a_3a_6 - 2833a_0a_2^3a_4a_5 - 288a_0a_2^2a_3^2a_5 \\
& + 45a_0a_2^2a_3a_4^2 + 2464a_0a_2a_3^3a_4 - 672a_0a_5^5 + 20656a_1^4a_5a_6 \\
& - 29225a_1^3a_2a_4a_6 + 20a_1^3a_2a_5^2 - 11872a_1^3a_3^2a_6 + 16a_1^3a_3a_4a_5 \\
& + 1439a_1^3a_4^3 + 1882a_1^2a_2^2a_3a_6 + 1215a_1^2a_2^2a_4a_5 + 272a_1^2a_2a_3^2a_5 \\
& - 1683a_1^2a_2a_3a_4^2 + 96a_1^2a_3^3a_4 + 3335a_1a_2^4a_6 - 324a_1a_2^3a_3a_5 \\
& + 1296a_1a_2^3a_4^2 + 912a_1a_2^2a_3^2a_4 + 32a_1a_2a_3^4 - 82a_2^5a_5 \\
& - 836a_2^4a_3a_4 - 56a_2^3a_3^3 - 68534a_0^3a_2a_6 - 8476a_0^3a_3a_5 \\
& - 4144a_0^3a_4^2 + 70458a_0^2a_1^2a_6 + 84a_0^2a_1a_2a_5 - 3408a_0^2a_1a_3a_4 \\
& - 22563a_0^2a_2^2a_4 + 16076a_0^2a_2a_3^2 + 2708a_0a_1^3a_5 + 13228a_0a_1^2a_2a_4 \\
& - 5648a_0a_1^2a_3^2 + 590a_0a_1a_2^2a_3 + 2977a_0a_4^2 - 1920a_1^4a_4 \\
& - 1520a_1^3a_2a_3 - 1059a_1^2a_2^3 + 26326a_0^3a_1
\end{aligned} \tag{A.27}$$

$$\begin{aligned}
e_{28} = & 516a_0^2a_5a_6^{12} - 172a_0a_1a_5^2a_6^{11} + 352a_0a_2a_4a_5a_6^{11} + 1592a_0a_2a_5^3a_6^{10} \\
& + 552a_0a_3^2a_5a_6^{11} - 912a_0a_3a_4a_5^2a_6^{10} - 582a_0a_3a_5^4a_6^9 + 80a_0a_4^3a_5a_6^{10} \\
& + 1464a_0a_4^2a_5^3a_6^9 + 2520a_0a_4a_5^5a_6^8 + 524a_0a_5^7a_6^7 - 376a_1^2a_4a_5a_6^{11} \\
& - 1674a_1^2a_5^3a_6^{10} - 552a_1a_2a_3a_5a_6^{11} + 560a_1a_2a_4a_5^2a_6^{10} + 758a_1a_2a_5^4a_6^9 \\
& + 184a_1a_3^2a_5^2a_6^{10} + 352a_1a_3a_4^2a_5a_6^{10} + 960a_1a_3a_4a_5^3a_6^9 + 238a_1a_3a_5^5a_6^8 \\
& - 248a_1a_4^3a_5^2a_6^9 - 868a_1a_4^2a_5^4a_6^8 - 412a_1a_4a_5^6a_6^7 - 29a_1a_5^8a_6^6 \\
& + 96a_2^3a_5a_6^{11} + 40a_2^2a_3a_5^2a_6^{10} - 104a_2^2a_4^2a_5a_6^{10} - 200a_2^2a_4a_5^3a_6^9 \\
& - 38a_2^2a_5^5a_6^8 + 80a_2a_3^2a_4a_5a_6^{10} + 96a_2a_3^2a_5^3a_6^9 - 120a_2a_3a_4^2a_5^2a_6^9 \\
& - 376a_2a_3a_4a_5^4a_6^8 - 98a_2a_3a_5^6a_6^7 + 8a_2a_4^4a_5a_6^9 - 112a_2a_4^3a_5^3a_6^8 \\
& - 188a_2a_4^2a_5^5a_6^7 - 40a_2a_4a_5^7a_6^6 - a_2a_5^9a_6^5 + 12a_3^4a_5a_6^{10} \\
& - 72a_3^3a_4a_5^2a_6^9 - 56a_3^3a_5^4a_6^8 - 48a_3^2a_4^3a_5a_6^9 - 108a_3^2a_4^2a_5^3a_6^8 \\
& + 48a_3^2a_4a_5^5a_6^7 + 15a_3^2a_5^7a_6^6 + 60a_3a_4^4a_5^2a_6^8 + 300a_3a_4^3a_5^4a_6^7 \\
& + 180a_3a_4^2a_5^6a_6^6 + 15a_3a_4a_5^8a_6^5 + 4a_4^6a_5a_6^8 + 60a_4^5a_5^3a_6^7 \\
& + 90a_4^4a_5^5a_6^6 + 20a_4^3a_5^7a_6^5 + 1464a_0^2a_4a_6^{11} + 9280a_0^2a_5^2a_6^{10} \\
& - 5130a_0a_1a_3a_6^{11} + 3504a_0a_1a_4a_5a_6^{10} - 2402a_0a_1a_5^3a_6^9 + 1200a_0a_2^2a_6^{11} \\
& - 394a_0a_2a_3a_5a_6^{10} - 892a_0a_2a_4^2a_6^{10} + 2524a_0a_2a_4a_5^2a_6^9 + 11304a_0a_2a_5^4a_6^8 \\
& + 2238a_0a_3^2a_4a_6^{10} + 10616a_0a_3^2a_5^2a_6^9 - 2218a_0a_3a_4^2a_5a_6^9 - 18420a_0a_3a_4a_5^3a_6^8 \\
& - 4915a_0a_3a_5^5a_6^7 + 70a_0a_4^4a_6^9 - 1544a_0a_4^3a_5^2a_6^8 + 3322a_0a_4^2a_5^4a_6^7 \\
& + 5924a_0a_4a_5^6a_6^6 + 676a_0a_5^8a_6^5 + 1410a_1^2a_2a_6^{11} + 1078a_1^2a_3a_5a_6^{10} \\
& - 284a_1^2a_4^2a_6^{10} - 2882a_1^2a_4a_5^2a_6^9 - 12143a_1^2a_5^4a_6^8 - 876a_1a_2^2a_5a_6^{10} \\
& - 678a_1a_2a_3a_4a_6^{10} - 11196a_1a_2a_3a_5^2a_6^9 + 86a_1a_2a_4^2a_5a_6^9 + 7288a_1a_2a_4a_5^3a_6^8
\end{aligned}$$

$$\begin{aligned}
& + 4471a_1a_2a_5^5a_6^7 - 540a_1a_3^3a_6^{10} + 78a_1a_3^2a_4a_5a_6^9 + 2419a_1a_3^2a_5^3a_6^8 \\
& + 128a_1a_3a_4^3a_6^9 + 3994a_1a_3a_4^2a_5^2a_6^8 + 4162a_1a_3a_4a_5^4a_6^7 + 381a_1a_3a_5^6a_6^6 \\
& + 143a_1a_4^4a_5a_6^8 - 1274a_1a_4^3a_5^3a_6^7 - 2566a_1a_4^2a_5^5a_6^6 - 584a_1a_4a_5^7a_6^5 \\
& - 15a_1a_5^9a_6^4 + 144a_2^3a_4a_6^{10} + 1994a_2^3a_5^2a_6^9 - 30a_2^2a_3^2a_6^{10} \\
& + 58a_2^2a_3a_4a_5a_6^9 + 678a_2^2a_3a_5^3a_6^8 - 76a_2^2a_4^3a_6^9 - 1520a_2^2a_4^2a_5^2a_6^8 \\
& - 1394a_2^2a_4a_5^4a_6^7 - 163a_2^2a_5^6a_6^6 + 82a_2a_3^3a_5a_6^9 - 20a_2a_3^2a_4^2a_6^9 \\
& + 1358a_2a_3^2a_4a_5^2a_6^8 + 505a_2a_3^2a_5^4a_6^7 + 244a_2a_3a_4^3a_5a_6^8 - 164a_2a_3a_4^2a_5^3a_6^7 \\
& - 1064a_2a_3a_4a_5^5a_6^6 - 139a_2a_3a_5^7a_6^5 + 22a_2a_4^5a_6^8 + 469a_2a_4^4a_5^2a_6^7 \\
& + 256a_2a_4^3a_5^4a_6^6 - 94a_2a_4^2a_5^6a_6^5 - 11a_2a_4a_5^8a_6^4 + 78a_3^4a_4a_6^9 \\
& + 237a_3^4a_5^2a_6^8 + 18a_3^3a_4^2a_5a_6^8 - 847a_3^3a_4a_5^3a_6^7 - 240a_3^3a_5^5a_6^6 \\
& + 3a_3^2a_4^4a_6^8 - 624a_3^2a_4^3a_5^2a_6^7 - 546a_3^2a_4^2a_5^4a_6^6 + 159a_3^2a_4a_5^6a_6^5 \\
& + 15a_3^2a_5^8a_6^4 - 99a_3a_4^5a_5a_6^7 - 330a_3a_4^4a_5^3a_6^6 + 210a_3a_4^3a_5^5a_6^5 \\
& + 90a_3a_4^2a_5^7a_6^4 - 4a_4^7a_6^7 - 102a_4^6a_5^2a_6^6 - 120a_4^5a_5^4a_6^5 \\
& - 5a_4^4a_5^6a_6^4 + 3666a_0^2a_3a_6^{10} + 9244a_0^2a_4a_5a_6^9 + 21122a_0^2a_5^3a_6^8 \\
& - 90a_0a_1a_2a_6^{10} - 79554a_0a_1a_3a_5a_6^9 - 1472a_0a_1a_4^2a_6^9 + 58450a_0a_1a_4a_5^2a_6^8 \\
& - 10858a_0a_1a_5^4a_6^7 + 18870a_0a_2^2a_5a_6^9 - 2524a_0a_2a_3a_4a_6^9 - 5490a_0a_2a_3a_5^2a_6^8 \\
& - 14744a_0a_2a_4^2a_5a_6^8 - 8996a_0a_2a_4a_5^3a_6^7 + 17740a_0a_2a_5^5a_6^6 - 1698a_0a_3^3a_6^9 \\
& + 29004a_0a_3^2a_4a_5a_6^8 + 36413a_0a_3^2a_5^3a_6^7 + 2130a_0a_3a_4^3a_6^8 - 11446a_0a_3a_4^2a_5^2a_6^7 \\
& - 49993a_0a_3a_4a_5^4a_6^6 - 5335a_0a_3a_5^6a_6^5 + 1789a_0a_4^4a_5a_6^7 - 8030a_0a_4^3a_5^3a_6^6 \\
& - 2714a_0a_4^2a_5^5a_6^5 + 2168a_0a_4a_5^7a_6^4 + 110a_0a_5^9a_6^3 - 1410a_1^3a_6^{10} \\
& + 21618a_1^2a_2a_5a_6^9 + 1096a_1^2a_3a_4a_6^9 + 16756a_1^2a_3a_5^2a_6^8 - 1784a_1^2a_4^2a_5a_6^8 \\
& + 11014a_1^2a_4a_5^3a_6^7 - 18586a_1^2a_5^5a_6^6 + 1386a_1a_2^2a_4a_6^9 - 12646a_1a_2^2a_5^2a_6^8 \\
& + 2358a_1a_2a_3^2a_6^9 - 1442a_1a_2a_3a_4a_5a_6^8 - 42350a_1a_2a_3a_5^3a_6^7 - 766a_1a_2a_4^3a_6^8 \\
& - 5450a_1a_2a_4^2a_5^2a_6^7 + 13957a_1a_2a_4a_5^4a_6^6 + 4827a_1a_2a_5^6a_6^5 - 9052a_1a_3^3a_5a_6^8 \\
& - 1410a_1a_3^2a_4^2a_6^8 + 1297a_1a_3^2a_4a_5^2a_6^7 + 6466a_1a_3^2a_5^4a_6^6 - 932a_1a_3a_4^3a_5a_6^7 \\
& + 5236a_1a_3a_4^2a_5^3a_6^6 + 2180a_1a_3a_4a_5^5a_6^5 - 32a_1a_3a_5^7a_6^4 + 75a_1a_4^5a_6^7 \\
& + 2298a_1a_4^4a_5^2a_6^6 + 1250a_1a_4^3a_5^4a_6^5 - 704a_1a_4^2a_5^6a_6^4 - 82a_1a_4a_5^8a_6^3 \\
& - 84a_2^3a_3a_6^9 - 158a_2^3a_4a_5a_6^8 + 8270a_2^3a_5^3a_6^7 - 902a_2^2a_3^2a_5a_6^8 \\
& + 218a_2^2a_3a_4^2a_6^8 + 398a_2^2a_3a_4a_5^2a_6^7 + 1806a_2^2a_3a_5^4a_6^6 + 372a_2^2a_4^3a_5a_6^7 \\
& - 3408a_2^2a_4^2a_5^3a_6^6 - 1884a_2^2a_4a_5^5a_6^5 - 118a_2^2a_5^7a_6^4 - 482a_2a_3^3a_4a_6^8 \\
& + 1402a_2a_3^3a_5^2a_6^7 - 1398a_2a_3^2a_4^2a_5a_6^7 + 3435a_2a_3^2a_4a_5^3a_6^6 + 232a_2a_3^2a_5^5a_6^5 \\
& - 176a_2a_3a_4^4a_6^7 + 2300a_2a_3a_4^3a_5^2a_6^6 + 1626a_2a_3a_4^2a_5^4a_6^5 - 351a_2a_3a_4a_5^6a_6^4 \\
& - 21a_2a_3a_5^8a_6^3 - 69a_2a_4^5a_5a_6^6 + 1090a_2a_4^4a_5^3a_6^5 + 542a_2a_4^3a_5^5a_6^4 \\
& + 22a_2a_4^2a_5^7a_6^3 - 78a_3^5a_6^8 + 1314a_3^4a_4a_5a_6^7 + 652a_3^4a_5^3a_6^6 \\
& + 186a_3^3a_4^3a_6^7 + 267a_3^3a_4^2a_5^2a_6^6 - 1716a_3^3a_4a_5^4a_6^5 - 117a_3^3a_5^6a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 588a_3^2a_4^4a_5a_6^6 - 780a_3^2a_4^3a_5^3a_6^5 - 237a_3^2a_4^2a_5^5a_6^4 + 87a_3^2a_4a_5^7a_6^3 \\
& + 43a_3a_4^6a_6^6 - 162a_3a_4^5a_5^2a_6^5 - 750a_3a_4^4a_5^4a_6^4 - 70a_3a_4^3a_5^6a_6^3 \\
& + 36a_4^7a_5a_6^5 - 60a_4^6a_5^3a_6^4 - 60a_4^5a_5^5a_6^3 + 54744a_0^2a_2a_6^9 \\
& + 43502a_0^2a_3a_5a_6^8 - 10130a_0^2a_4a_6^8 - 35456a_0^2a_4a_5^2a_6^7 - 16562a_0^2a_5^4a_6^6 \\
& + 30330a_0a_1^2a_6^9 - 39190a_0a_1a_2a_5a_6^8 - 14840a_0a_1a_3a_4a_6^8 - 164727a_0a_1a_3a_5^2a_6^7 \\
& - 20302a_0a_1a_4^2a_5a_6^7 + 156988a_0a_1a_4a_5^3a_6^6 - 18817a_0a_1a_5^5a_6^5 - 13862a_0a_2^2a_4a_6^8 \\
& + 57872a_0a_2^2a_5^2a_6^7 + 1244a_0a_2a_3^2a_6^8 - 8772a_0a_2a_3a_4a_5a_6^7 - 21087a_0a_2a_3a_5^3a_6^6 \\
& + 5756a_0a_2a_4^3a_6^7 - 22536a_0a_2a_4^2a_5^2a_6^6 - 23162a_0a_2a_4a_5^4a_6^5 + 8573a_0a_2a_5^6a_6^4 \\
& - 17473a_0a_3^3a_5a_6^7 - 7020a_0a_3^2a_4^2a_6^7 + 59480a_0a_3^2a_4a_5^2a_6^6 + 24657a_0a_3^2a_5^4a_6^5 \\
& + 24494a_0a_3a_4^3a_5a_6^6 + 22650a_0a_3a_4^2a_5^3a_6^5 - 21734a_0a_3a_4a_5^5a_6^4 - 426a_0a_3a_5^7a_6^3 \\
& - 249a_0a_5^4a_6^6 + 7653a_0a_4^4a_5^2a_6^5 - 4542a_0a_4^3a_5^4a_6^4 - 2684a_0a_4^2a_5^6a_6^3 \\
& + 66a_0a_4a_5^8a_6^2 - 27253a_1^3a_5a_6^8 - 3034a_1^2a_2a_4a_6^8 + 51022a_1^2a_2a_5^2a_6^7 \\
& + 20743a_1^2a_3^2a_6^8 - 25711a_1^2a_3a_4a_5a_6^7 + 46196a_1^2a_3a_5^3a_6^6 - 198a_1^2a_4^3a_6^7 \\
& - 10273a_1^2a_4^2a_5^2a_6^6 + 35717a_1^2a_4a_5^4a_6^5 - 7085a_1^2a_5^6a_6^4 - 14784a_1a_2^2a_3a_6^8 \\
& + 30892a_1a_2^2a_4a_5a_6^7 - 29322a_1a_2^2a_5^3a_6^6 + 26653a_1a_2a_3^2a_5a_6^7 + 8538a_1a_2a_3a_4^2a_6^7 \\
& + 28725a_1a_2a_3a_4a_5^2a_6^6 - 37516a_1a_2a_3a_5^4a_6^5 - 8730a_1a_2a_4^3a_5a_6^6 - 28124a_1a_2a_4^2a_5^3a_6^5 \\
& + 2070a_1a_2a_4a_5^5a_6^4 + 924a_1a_2a_5^7a_6^3 - 1374a_1a_3^3a_4a_6^7 - 26144a_1a_3^3a_5^2a_6^6 \\
& - 5487a_1a_3^2a_4^2a_5a_6^6 + 2631a_1a_3^2a_4a_5^3a_6^5 + 3986a_1a_3^2a_5^5a_6^4 - 831a_1a_3a_4^4a_6^6 \\
& - 5850a_1a_3a_4^3a_5^2a_6^5 - 792a_1a_3a_4^2a_5^4a_6^4 - 290a_1a_3a_4a_5^6a_6^3 - 29a_1a_3a_5^8a_6^2 \\
& - 81a_1a_4^5a_5a_6^5 + 3052a_1a_4^4a_5^3a_6^4 + 1544a_1a_4^3a_5^5a_6^3 + 38a_1a_4^2a_5^7a_6^2 \\
& + 7394a_2^4a_6^8 - 9282a_2^3a_3a_5a_6^7 - 5020a_2^3a_4^2a_6^7 - 8242a_2^3a_4a_5^2a_6^6 \\
& + 8773a_2^3a_5^4a_6^5 + 6676a_2^2a_3^2a_4a_6^7 - 2061a_2^2a_3^2a_5^2a_6^6 - 788a_2^2a_3a_4^2a_5a_6^6 \\
& - 1199a_2^2a_3a_4a_5^3a_6^5 + 563a_2^2a_3a_5^5a_6^4 + 1276a_2^2a_4^4a_6^6 + 4524a_2^2a_4^3a_5^2a_6^5 \\
& - 734a_2^2a_4^2a_5^4a_6^4 - 448a_2^2a_4a_5^6a_6^3 - 11a_2^2a_5^8a_6^2 - 2150a_2a_3^4a_6^7 \\
& - 4050a_2a_3^3a_4a_5a_6^6 + 3727a_2a_3^3a_5^3a_6^5 - 1856a_2a_3^2a_4^2a_6^6 - 6183a_2a_3^2a_4^2a_5^2a_6^5 \\
& + 1326a_2a_3^2a_4a_5^4a_6^4 - 117a_2a_3^2a_5^6a_6^3 - 1839a_2a_3a_4^4a_5a_6^5 + 2724a_2a_3a_4^3a_5^3a_6^4 \\
& + 1279a_2a_3a_4^2a_5^5a_6^3 + 17a_2a_3a_4a_5^7a_6^2 - 188a_2a_4^6a_6^5 - 1126a_2a_4^5a_5^2a_6^4 \\
& - 65a_2a_4^4a_5^4a_6^3 + 38a_2a_4^3a_5^6a_6^2 + 27a_3^5a_5a_6^6 - 270a_3^4a_4^2a_6^6 \\
& + 3231a_3^4a_4a_5^2a_6^5 - 48a_3^4a_5^4a_6^4 + 1101a_3^3a_4^3a_5a_6^5 + 474a_3^3a_4^2a_5^3a_6^4 \\
& - 519a_3^3a_4a_5^5a_6^3 + 19a_3^3a_5^7a_6^2 + 9a_3^2a_4^5a_6^5 + 1842a_3^2a_4^4a_5^2a_6^4 \\
& - 135a_3^2a_4^3a_5^4a_6^3 - 9a_3^2a_4^2a_5^6a_6^2 + 309a_3a_4^6a_5a_6^4 + 369a_3a_4^5a_5^3a_6^3 \\
& - 105a_3a_4^4a_5^5a_6^2 + 6a_4^8a_6^4 + 132a_4^7a_5^2a_6^3 + 56a_4^6a_5^4a_6^2 \\
& - 112494a_0^2a_1a_6^8 + 109978a_0^2a_2a_5a_6^7 + 63286a_0^2a_3a_4a_6^7 + 159605a_0^2a_3a_5^2a_6^6 \\
& + 2136a_0^2a_4^2a_5a_6^6 - 86442a_0^2a_4a_5^3a_6^5 - 22319a_0^2a_5^5a_6^4 + 86029a_0a_1^2a_5a_6^7 \\
& + 85180a_0a_1a_2a_4a_6^7 - 120657a_0a_1a_2a_5^2a_6^6 - 34514a_0a_1a_3^2a_6^7 + 7876a_0a_1a_3a_4a_5a_6^6
\end{aligned}$$

$$\begin{aligned}
& + 23686a_0a_1a_3a_5^3a_6^5 - 17678a_0a_1a_4^3a_6^6 - 109918a_0a_1a_4^2a_5^2a_6^5 + 90309a_0a_1a_4a_5^4a_6^4 \\
& - 6946a_0a_1a_5^6a_6^3 + 29654a_0a_2^2a_3a_6^7 - 90496a_0a_2^2a_4a_5a_6^6 + 53437a_0a_2^2a_5^3a_6^5 \\
& + 18436a_0a_2a_3^2a_5a_6^6 - 36274a_0a_2a_3a_4^2a_6^6 - 2044a_0a_2a_3a_4a_5^2a_6^5 - 31524a_0a_2a_3a_5^4a_6^4 \\
& + 38720a_0a_2a_4^3a_5a_6^5 + 16222a_0a_2a_4^2a_5^3a_6^4 - 4356a_0a_2a_4a_5^5a_6^3 + 1270a_0a_2a_5^7a_6^2 \\
& + 14873a_0a_3^3a_4a_6^6 - 23298a_0a_3^3a_5^2a_6^5 - 61443a_0a_3^2a_4^2a_5a_6^5 + 19269a_0a_3^2a_4a_5^3a_6^4 \\
& - 176a_0a_3^2a_5^5a_6^3 - 1407a_0a_3a_4^4a_6^5 + 31152a_0a_3a_4^3a_5^2a_6^4 + 25698a_0a_3a_4^2a_5^4a_6^3 \\
& - 1128a_0a_3a_4a_5^6a_6^2 + 53a_0a_3a_5^8a_6 - 4755a_0a_4^5a_5a_6^4 + 3366a_0a_4^4a_5^3a_6^3 \\
& - 36a_0a_4^3a_5^5a_6^2 - 140a_0a_4^2a_5^7a_6 + 7919a_1^3a_4a_6^7 - 73354a_1^3a_5^2a_6^6 \\
& - 14654a_1^2a_2a_3a_6^7 - 31253a_1^2a_2a_4a_5a_6^6 + 6460a_1^2a_2a_5^3a_6^5 + 48326a_1^2a_3^2a_5a_6^6 \\
& + 20729a_1^2a_3a_4^2a_6^6 - 116810a_1^2a_3a_4a_5^2a_6^5 + 30154a_1^2a_3a_5^4a_6^4 - 3496a_1^2a_4^3a_5a_6^5 \\
& - 25220a_1^2a_4^2a_5^3a_6^4 + 13530a_1^2a_4a_5^5a_6^3 - 614a_1^2a_5^7a_6^2 - 16298a_1a_2^3a_6^7 \\
& - 48630a_1a_2^2a_3a_5a_6^6 - 4138a_1a_2^2a_4^2a_6^6 + 100835a_1a_2^2a_4a_5^2a_6^5 - 10572a_1a_2^2a_5^4a_6^4 \\
& - 16769a_1a_2a_3^2a_4a_6^6 + 49638a_1a_2a_3^2a_5^2a_6^5 + 25308a_1a_2a_3a_4^2a_5a_6^5 + 40784a_1a_2a_3a_4a_5^3a_6^4 \\
& - 6340a_1a_2a_3a_5^5a_6^3 + 3139a_1a_2a_4^4a_6^5 - 6822a_1a_2a_4^3a_5^2a_6^4 - 16562a_1a_2a_4^2a_5^4a_6^3 \\
& - 850a_1a_2a_4a_5^6a_6^2 + 13a_1a_2a_5^8a_6 + 4444a_1a_3^4a_6^6 + 7956a_1a_3^3a_4a_5a_6^5 \\
& - 15778a_1a_3^3a_5^3a_6^4 + 591a_1a_3^2a_4^3a_6^5 - 2373a_1a_3^2a_4^2a_5^2a_6^4 + 420a_1a_3^2a_4a_5^4a_6^3 \\
& + 428a_1a_3^2a_5^6a_6^2 + 882a_1a_3a_4^4a_5a_6^4 - 1280a_1a_3a_4^3a_5^3a_6^3 + 158a_1a_3a_4^2a_5^5a_6^2 \\
& - 14a_1a_3a_4a_5^7a_6 - 175a_1a_4^6a_6^4 - 1358a_1a_4^5a_5^2a_6^3 - 37a_1a_4^4a_5^4a_6^2 \\
& + 72a_1a_4^3a_5^6a_6 + 24602a_2^4a_5a_6^6 + 8622a_2^3a_3a_4a_6^6 - 22816a_2^3a_3a_5^2a_6^5 \\
& - 14244a_2^3a_4^2a_5a_6^5 - 13033a_2^3a_4a_5^3a_6^4 + 2017a_2^3a_5^5a_6^3 + 3532a_2^2a_3^3a_6^6 \\
& + 15408a_2^2a_3^2a_4a_5a_6^5 + 1967a_2^2a_3^2a_5^3a_6^4 - 8a_2^2a_3a_4^3a_6^5 - 1741a_2^2a_3a_4^2a_5^2a_6^4 \\
& - 2253a_2^2a_3a_4a_5^4a_6^3 - 115a_2^2a_3a_5^6a_6^2 + 2003a_2^2a_4^4a_5a_6^4 + 3688a_2^2a_4^3a_5^3a_6^3 \\
& + 300a_2^2a_4^2a_5^5a_6^2 - 8a_2^2a_4a_5^7a_6 - 7743a_2a_3^4a_5a_6^5 + 3504a_2a_3^3a_4^2a_6^5 \\
& - 6663a_2a_3^3a_4a_5^2a_6^4 + 1496a_2a_3^3a_5^4a_6^3 - 5073a_2a_3^2a_4^3a_5a_6^4 - 2952a_2a_3^2a_4^2a_5^3a_6^3 \\
& + 217a_2a_3^2a_4a_5^5a_6^2 - 5a_2a_3^2a_5^7a_6 + 327a_2a_3a_4^5a_6^4 - 3022a_2a_3a_4^4a_5^2a_6^3 \\
& + 91a_2a_3a_4^3a_5^4a_6^2 + 67a_2a_3a_4^2a_5^6a_6 - 79a_2a_4^6a_5a_6^3 - 437a_2a_4^5a_5^3a_6^2 \\
& - 40a_2a_4^4a_5^5a_6 - 1197a_3^5a_4a_6^5 + 1488a_3^5a_5^2a_6^4 - 1068a_3^4a_4^2a_5a_6^4 \\
& + 768a_3^4a_4a_5^3a_6^3 - 204a_3^4a_5^5a_6^2 - 801a_3^3a_4^4a_6^4 + 1200a_3^3a_4^3a_5^2a_6^3 \\
& + 66a_3^3a_4^2a_5^4a_6^2 + 17a_3^3a_4a_5^6a_6 - 606a_3^2a_4^5a_5a_6^3 + 408a_3^2a_4^4a_5^3a_6^2 \\
& - 81a_3^2a_4^3a_5^5a_6 - 105a_3a_4^7a_6^3 + 51a_3a_4^6a_5^2a_6^2 + 78a_3a_4^5a_5^4a_6 \\
& - 42a_4^8a_5a_6^2 - 8a_4^7a_5^3a_6 + 275766a_0^3a_6^7 - 394955a_0^2a_1a_5a_6^6 \\
& - 102792a_0^2a_2a_4a_6^6 - 62545a_0^2a_2a_5^2a_6^5 - 168029a_0^2a_3^2a_6^6 + 313455a_0^2a_3a_4a_5a_6^5 \\
& + 89635a_0^2a_3a_5^3a_6^4 + 28256a_0^2a_4^3a_6^5 + 68033a_0^2a_4^2a_5^2a_6^4 - 12092a_0^2a_4a_5^4a_6^3 \\
& - 1596a_0^2a_5^6a_6^2 + 18861a_0a_1^2a_4a_6^6 + 29492a_0a_1^2a_5^2a_6^5 + 19706a_0a_1a_2a_3a_6^6 \\
& + 296172a_0a_1a_2a_4a_5a_6^5 - 59424a_0a_1a_2a_5^3a_6^4 - 177379a_0a_1a_3^2a_5a_6^5 - 51229a_0a_1a_3a_4^2a_6^5
\end{aligned}$$

$$\begin{aligned}
& -66200a_0a_1a_3a_4a_5^2a_6^4 + 46854a_0a_1a_3a_5^4a_6^3 + 1764a_0a_1a_4^3a_5a_6^4 - 63782a_0a_1a_4^2a_5^3a_6^3 \\
& + 14088a_0a_1a_4a_5^5a_6^2 - 119a_0a_1a_5^7a_6 + 94400a_0a_2^3a_6^6 - 34623a_0a_2^2a_3a_5a_6^5 \\
& - 276a_0a_2^2a_4^2a_6^5 - 69235a_0a_2^2a_4a_5^2a_6^4 + 13745a_0a_2^2a_5^4a_6^3 + 23812a_0a_2a_3^2a_4a_5^5 \\
& + 88737a_0a_2a_3^2a_5^2a_6^4 - 107727a_0a_2a_3a_4^2a_5a_6^4 - 27294a_0a_2a_3a_4a_5^3a_6^3 - 8210a_0a_2a_3a_5^5a_6^2 \\
& - 17069a_0a_2a_4^4a_6^4 + 16340a_0a_2a_4^3a_5^2a_6^3 + 6506a_0a_2a_4^2a_5^4a_6^2 - 86a_0a_2a_4a_5^6a_6 \\
& + 17a_0a_2a_5^8 + 24195a_0a_3^4a_6^5 + 13791a_0a_3^3a_4a_5a_6^4 + 1684a_0a_3^3a_5^3a_6^3 \\
& + 23490a_0a_3^2a_4^3a_6^4 - 43629a_0a_3^2a_4^2a_5^2a_6^3 + 234a_0a_3^2a_4a_5^4a_6^2 - 491a_0a_3^2a_5^6a_6 \\
& - 15642a_0a_3a_4^4a_5a_6^3 - 1958a_0a_3a_4^3a_5^3a_6^2 + 1495a_0a_3a_4^2a_5^5a_6 + 2a_0a_3a_4a_5^7 \\
& + 2338a_0a_4^6a_6^3 - 1630a_0a_4^5a_5^2a_6^2 + 601a_0a_4^4a_5^4a_6 + 44a_0a_4^3a_5^6 \\
& - 18935a_1^3a_3a_6^6 + 61772a_1^3a_4a_5a_6^5 - 42574a_1^3a_5^3a_6^4 - 159a_1^2a_2^2a_6^6 \\
& + 39452a_1^2a_2a_3a_5a_6^5 + 19951a_1^2a_2a_4^2a_5^5 - 9760a_1^2a_2a_4a_5^2a_6^4 - 10406a_1^2a_2a_5^4a_6^3 \\
& + 15455a_1^2a_3^2a_4a_5^5 - 12262a_1^2a_3^2a_5^2a_6^4 + 80777a_1^2a_3a_4^2a_5a_6^4 - 56570a_1^2a_3a_4a_5^3a_6^3 \\
& + 3917a_1^2a_3a_5^5a_6^2 + 618a_1^2a_4^4a_6^4 - 4054a_1^2a_4^3a_5^2a_6^3 - 10457a_1^2a_4^2a_5^4a_6^2 \\
& + 447a_1^2a_4a_5^6a_6 - 7a_1^2a_5^8 - 54303a_1a_2^3a_5a_6^5 - 15394a_1a_2^2a_3a_4a_5^5 \\
& - 36919a_1a_2^2a_3a_5^2a_6^4 - 44362a_1a_2^2a_4^2a_5a_6^4 + 54693a_1a_2^2a_4a_5^3a_6^3 + 713a_1a_2^2a_5^5a_6^2 \\
& - 29805a_1a_2a_3^3a_6^5 - 32113a_1a_2a_3^2a_4a_5a_6^4 + 13282a_1a_2a_3^2a_5^3a_6^3 - 4587a_1a_2a_3a_4^3a_6^4 \\
& + 940a_1a_2a_3a_4^2a_5^2a_6^3 + 7044a_1a_2a_3a_4a_5^4a_6^2 - 78a_1a_2a_3a_5^6a_6 + 11696a_1a_2a_4^4a_5a_6^3 \\
& + 5006a_1a_2a_4^3a_5^3a_6^2 - 785a_1a_2a_4^2a_5^5a_6 - 16a_1a_2a_4a_5^7 + 8490a_1a_3^4a_5a_6^4 \\
& - 6696a_1a_3^3a_4^2a_6^4 + 10326a_1a_3^3a_4a_5^2a_6^3 - 1490a_1a_3^3a_5^4a_6^2 + 1131a_1a_3^2a_4^3a_5a_6^3 \\
& - 1236a_1a_3^2a_4^2a_5^3a_6^2 - 43a_1a_3^2a_4a_5^5a_6 + 2a_1a_3^2a_5^7 - 474a_1a_3a_4^5a_6^3 \\
& + 1819a_1a_3a_4^4a_5^2a_6^2 + 368a_1a_3a_4^3a_5^4a_6 + 32a_1a_3a_4^2a_5^6 - 5a_1a_4^6a_5a_6^2 \\
& - 241a_1a_4^5a_5^3a_6 - 23a_1a_4^4a_5^5 - 11454a_2^4a_4a_5^5 + 18080a_2^4a_5^2a_6^4 \\
& + 11134a_2^3a_3^2a_6^5 + 29840a_2^3a_3a_4a_5a_6^4 - 6913a_2^3a_3a_5^3a_6^3 + 7772a_2^3a_4^3a_6^4 \\
& - 5893a_2^3a_4^2a_5^2a_6^3 - 3563a_2^3a_4a_5^4a_6^2 + 42a_2^3a_5^6a_6 - 3634a_2^2a_3^3a_5a_6^4 \\
& - 9133a_2^2a_3^2a_4^2a_6^4 + 8677a_2^2a_3^2a_4a_5^2a_6^3 + 977a_2^2a_3^2a_5^4a_6^2 + 2731a_2^2a_3a_4^3a_5a_6^3 \\
& + 962a_2^2a_3a_4^2a_5^3a_6^2 - 140a_2^2a_3a_4a_5^5a_6 - 4a_2^2a_3a_5^7 - 1492a_2^2a_4^5a_6^3 \\
& - 327a_2^2a_4^4a_5^2a_6^2 + 286a_2^2a_4^3a_5^4a_6 + 8a_2^2a_4^2a_5^6 + 12201a_2a_3^4a_4a_6^4 \\
& - 2922a_2a_3^4a_5^3a_6^3 - 1587a_2a_3^3a_4^2a_5a_6^3 - 2380a_2a_3^3a_4a_5^3a_6^2 + 23a_2a_3^3a_5^5a_6 \\
& + 4002a_2a_3^2a_4^3a_6^3 - 1012a_2a_3^2a_4^3a_5^2a_6^2 - 24a_2a_3^2a_4^2a_5^4a_6 + 11a_2a_3^2a_4a_5^6 \\
& + 240a_2a_3a_4^5a_5a_6^2 - 675a_2a_3a_4^4a_5^3a_6 - 27a_2a_3a_4^3a_5^5 + 167a_2a_4^7a_6^2 \\
& + 142a_2a_4^6a_5^2a_6 + 2a_2a_4^5a_5^4 - 1725a_3^6a_6^4 - 258a_3^5a_4a_5a_6^3 \\
& + 708a_3^5a_5^3a_6^2 - 441a_3^4a_4^3a_6^3 - 18a_3^4a_4^2a_5^2a_6^2 - 141a_3^4a_4a_5^4a_6 \\
& + a_3^4a_5^6 - 687a_3^3a_4^4a_5a_6^2 + 348a_3^3a_4^3a_5^3a_6 - 12a_3^3a_4^2a_5^5 \\
& - 39a_3^2a_4^6a_6^2 - 159a_3^2a_4^5a_5^2a_6 + 18a_3^2a_4^4a_5^4 - 60a_3a_4^7a_5a_6 \\
& - 4a_3a_4^6a_5^3 - 3a_4^8a_5^2 - 5557a_0^3a_5a_6^5 + 158217a_0^2a_1a_4a_5^5
\end{aligned}$$

$$\begin{aligned}
& -1868a_0^2a_1a_5^2a_6^4 + 382452a_0^2a_2a_3a_6^5 - 92991a_0^2a_2a_4a_5a_6^4 - 67261a_0^2a_2a_5^3a_6^3 \\
& - 61279a_0^2a_3^2a_5a_6^4 - 277546a_0^2a_3a_4^2a_6^4 + 78367a_0^2a_3a_4a_5^2a_6^3 - 3966a_0^2a_3a_5^4a_6^2 \\
& + 75200a_0^2a_4^3a_5a_6^3 + 32700a_0^2a_4^2a_5^3a_6^2 + 3354a_0^2a_4a_5^5a_6 + 83a_0^2a_5^7 \\
& + 5523a_0a_1^2a_3a_6^5 - 43512a_0a_1^2a_4a_5a_6^4 + 2890a_0a_1^2a_5^3a_6^3 - 272700a_0a_1a_2^2a_6^5 \\
& + 2068a_0a_1a_2a_3a_5a_6^4 + 25397a_0a_1a_2a_4^2a_6^4 + 164714a_0a_1a_2a_4a_5^2a_6^3 - 7298a_0a_1a_2a_5^4a_6^2 \\
& + 88559a_0a_1a_3^2a_4a_6^4 - 105514a_0a_1a_3^2a_5^2a_6^3 - 50182a_0a_1a_3a_4^2a_5a_6^3 - 52872a_0a_1a_3a_4a_5^3a_6^2 \\
& + 546a_0a_1a_3a_5^5a_6 - 8844a_0a_1a_4^4a_6^3 + 19146a_0a_1a_4^3a_5^2a_6^2 - 4356a_0a_1a_4^2a_5^4a_6 \\
& + 239a_0a_1a_4a_5^6 + 15264a_0a_2^3a_5a_6^4 + 160157a_0a_2^2a_3a_4a_6^4 - 7457a_0a_2^2a_3a_5^2a_6^3 \\
& - 19447a_0a_2^2a_4^2a_5a_6^3 - 12247a_0a_2^2a_4a_5^3a_6^2 + 1007a_0a_2^2a_5^5a_6 - 97583a_0a_2a_3^4a_6^2 \\
& + 47806a_0a_2a_3^2a_4a_5a_6^3 + 23112a_0a_2a_3^2a_5^3a_6^2 - 1378a_0a_2a_3a_4^3a_6^3 - 26352a_0a_2a_3a_4^2a_5^2a_6^2 \\
& - 3418a_0a_2a_3a_4a_5^4a_6 - 132a_0a_2a_3a_6^6 - 14874a_0a_2a_4^4a_5a_6^2 - 2976a_0a_2a_4^3a_5^3a_6 \\
& - 146a_0a_2a_4^2a_5^5 + 6726a_0a_3^4a_5a_6^3 + 16149a_0a_3^3a_4^2a_6^3 + 5220a_0a_3^3a_4a_5^2a_6^2 \\
& + 1564a_0a_3^3a_5^4a_6 + 16299a_0a_3^2a_4^3a_5a_6^2 - 2322a_0a_3^2a_4^2a_5^3a_6 - 59a_0a_3^2a_4a_5^5 \\
& + 5916a_0a_3a_4^5a_6^2 - 1933a_0a_3a_4^4a_5^2a_6 + 3a_0a_3a_4^3a_5^4 + 1637a_0a_4^6a_5a_6 \\
& + 77a_0a_4^5a_5^3 + 58693a_1^3a_2a_6^5 + 31298a_1^3a_3a_5a_6^4 - 34662a_1^3a_4^2a_6^4 \\
& + 29334a_1^3a_4a_5^2a_6^3 - 6738a_1^3a_5^4a_6^2 + 22398a_1^2a_2^2a_5a_6^4 - 93784a_1^2a_2a_3a_4a_6^4 \\
& + 37640a_1^2a_2a_3a_5^2a_6^3 + 53367a_1^2a_2a_4^2a_5a_6^3 + 19654a_1^2a_2a_4a_5^3a_6^2 - 543a_1^2a_2a_5^5a_6 \\
& + 18506a_1^2a_3^3a_6^4 + 25354a_1^2a_3^2a_4a_5a_6^3 - 4668a_1^2a_3^2a_5^3a_6^2 + 4035a_1^2a_3a_4^3a_6^3 \\
& + 40368a_1^2a_3a_4^2a_5^2a_6^2 - 2327a_1^2a_3a_4a_5^4a_6 + 34a_1^2a_3a_5^6 - 1092a_1^2a_4^4a_5a_6^2 \\
& - 358a_1^2a_4^3a_5^3a_6 - 288a_1^2a_4^2a_5^5 + 55509a_1a_2^3a_4a_6^4 - 33217a_1a_2^3a_5^2a_6^3 \\
& + 24281a_1a_2^2a_3^2a_6^4 - 29314a_1a_2^2a_3a_4a_5a_6^3 - 11736a_1a_2^2a_3a_5^3a_6^2 - 11295a_1a_2^2a_4^3a_6^3 \\
& - 36453a_1a_2^2a_4^2a_5^2a_6^2 + 2883a_1a_2^2a_4a_5^4a_6 + 40a_1a_2^2a_5^6 - 15792a_1a_2a_3^3a_5a_6^3 \\
& + 7349a_1a_2a_3^2a_4^2a_6^3 - 7434a_1a_2a_3^2a_4a_5^2a_6^2 + 192a_1a_2a_3^2a_5^4a_6 - 234a_1a_2a_3a_4^3a_5a_6^2 \\
& - 1500a_1a_2a_3a_4^2a_5^3a_6 + 22a_1a_2a_3a_4a_5^5 - 210a_1a_2a_4^5a_6^2 + 2319a_1a_2a_4^4a_5^2a_6 \\
& + 383a_1a_2a_4^3a_5^4 - 9030a_1a_3^4a_4a_6^3 + 556a_1a_3^4a_5^2a_6^2 - 3766a_1a_3^3a_4^2a_5a_6^2 \\
& + 540a_1a_3^3a_4a_5^3a_6 - 10a_1a_3^3a_5^5 - 3909a_1a_3^2a_4^4a_6^2 - 630a_1a_3^2a_4^3a_5^2a_6 \\
& - 117a_1a_3^2a_4^2a_5^4 - 478a_1a_3a_4^5a_5a_6 - 70a_1a_3a_4^4a_5^3 - 81a_1a_4^7a_6 \\
& - 9a_1a_4^6a_5^2 - 20274a_2^4a_3a_6^4 - 4788a_2^4a_4a_5a_6^3 + 3995a_2^4a_5^3a_6^2 \\
& + 4474a_2^3a_3^2a_5a_6^3 - 4604a_2^3a_3a_4^2a_6^3 + 10269a_2^3a_3a_4a_5^2a_6^2 - 102a_2^3a_3a_5^4a_6 \\
& + 5641a_2^3a_4^3a_5a_6^2 + 124a_2^3a_4^2a_5^3a_6 - 70a_2^3a_4a_5^5 + 38a_2^2a_3^3a_4a_6^3 \\
& - 512a_2^2a_3^3a_5^2a_6^2 - 5835a_2^2a_3^2a_4^2a_5a_6^2 + 322a_2^2a_3^2a_4a_5^3a_6 + 21a_2^2a_3^2a_5^5 \\
& - 939a_2^2a_3a_4^4a_6^2 + 1144a_2^2a_3a_4^3a_5^2a_6 + 135a_2^2a_3a_4^2a_5^4 - 550a_2^2a_4^5a_5a_6 \\
& - 91a_2^2a_4^4a_5^3 + 516a_2a_3^5a_6^3 + 4514a_2a_3^4a_4a_5a_6^2 + 32a_2a_3^4a_5^3a_6 \\
& + 2619a_2a_3^3a_4^3a_6^2 - 714a_2a_3^3a_4^2a_5^2a_6 - 63a_2a_3^3a_4a_5^4 + 1105a_2a_3^2a_4^4a_5a_6 \\
& - 60a_2a_3^2a_4^3a_5^3 + 170a_2a_3a_4^6a_6 + 107a_2a_3a_4^5a_5^2 + 12a_2a_4^7a_5
\end{aligned}$$

$$\begin{aligned}
& -784a_3^6a_5a_6^2 - 54a_3^5a_4^2a_6^2 + 360a_3^5a_4a_5^2a_6 - 12a_3^5a_5^4 \\
& -306a_3^4a_4^3a_5a_6 + 84a_3^4a_4^2a_5^3 + 183a_3^3a_4^5a_6 - 36a_3^3a_4^4a_5^2 \\
& -12a_3^2a_4^6a_5 + 12a_3a_4^8 + 455323a_0^3a_4a_6^4 + 38302a_0^3a_5^2a_6^3 \\
& -460767a_0^2a_1a_3a_6^4 - 77048a_0^2a_1a_4a_5a_6^3 + 36615a_0^2a_1a_5^3a_6^2 + 111795a_0^2a_2^2a_6^4 \\
& + 143540a_0^2a_2a_3a_5a_6^3 - 95284a_0^2a_2a_4^2a_6^3 - 4305a_0^2a_2a_4a_5^2a_6^2 - 10410a_0^2a_2a_5^4a_6 \\
& + 104972a_0^2a_3^2a_4a_6^3 + 41255a_0^2a_3^2a_5^2a_6^2 - 148738a_0^2a_3a_4^2a_5a_6^2 - 17754a_0^2a_3a_4a_5^3a_6 \\
& - 866a_0^2a_3a_5^5 - 19702a_0^2a_4^4a_6^2 + 16392a_0^2a_4^3a_5^2a_6 + 1829a_0^2a_4^2a_5^4 \\
& + 86409a_0a_1^2a_2a_6^4 - 22357a_0a_1^2a_3a_5a_6^3 + 82658a_0a_1^2a_4^2a_6^3 - 20112a_0a_1^2a_4a_5^2a_6^2 \\
& + 614a_0a_1^2a_5^4a_6 - 94573a_0a_1a_2^2a_5a_6^3 - 39002a_0a_1a_2a_3a_4a_6^3 + 26905a_0a_1a_2a_3a_5^2a_6^2 \\
& - 1042a_0a_1a_2a_4^2a_5a_6^2 + 17176a_0a_1a_2a_4a_5^3a_6 + 24a_0a_1a_2a_5^5 + 22879a_0a_1a_3^3a_6^3 \\
& + 83353a_0a_1a_3^2a_4a_5a_6^2 + 2606a_0a_1a_3^2a_5^3a_6 + 2120a_0a_1a_3a_4^3a_6^2 - 3418a_0a_1a_3a_4^2a_5^2a_6 \\
& - 1930a_0a_1a_3a_4a_5^4 - 13372a_0a_1a_4^4a_5a_6 - 794a_0a_1a_4^3a_5^3 + 42466a_0a_2^3a_4a_6^3 \\
& - 7206a_0a_2^3a_5^2a_6^2 - 21749a_0a_2^2a_3^2a_6^3 + 41237a_0a_2^2a_3a_4a_5a_6^2 - 2068a_0a_2^2a_3a_5^3a_6 \\
& + 14377a_0a_2^2a_4^3a_6^2 - 767a_0a_2^2a_4^2a_5^2a_6 - 133a_0a_2^2a_4a_5^4 - 22310a_0a_2a_3^3a_5a_6^2 \\
& - 12935a_0a_2a_3^2a_4^2a_6^2 + 7564a_0a_2a_3^2a_4a_5^2a_6 + 196a_0a_2a_3^2a_5^4 + 9802a_0a_2a_3a_4^3a_5a_6 \\
& + 292a_0a_2a_3a_4^2a_5^3 - 824a_0a_2a_4^5a_6 - 783a_0a_2a_4^4a_5^2 - 7452a_0a_3^4a_4a_6^2 \\
& - 2464a_0a_3^4a_5^2a_6 + 2170a_0a_3^3a_4^2a_5a_6 + 352a_0a_3^3a_4a_5^3 - 1146a_0a_3^3a_4^4a_6 \\
& - 354a_0a_3^2a_4^3a_5^2 + 1192a_0a_3a_4^5a_5 - 105a_0a_4^7 - 43195a_1^4a_6^4 \\
& + 8372a_1^3a_2a_5a_6^3 - 10739a_1^3a_3a_4a_6^3 + 15000a_1^3a_3a_5^2a_6^2 - 17037a_1^3a_4^2a_5a_6^2 \\
& + 770a_1^3a_4a_5^3a_6 - 174a_1^3a_5^5 - 54739a_1^2a_2^2a_4a_6^3 - 1591a_1^2a_2^2a_5^2a_6^2 \\
& - 741a_1^2a_2a_3^2a_6^3 - 77135a_1^2a_2a_3a_4a_5a_6^2 + 2754a_1^2a_2a_3a_5^3a_6 - 10017a_1^2a_2a_4^3a_6^2 \\
& + 6132a_1^2a_2a_4^2a_5^2a_6 + 1041a_1^2a_2a_4a_5^4 - 1660a_1^2a_3^3a_5a_6^2 - 1507a_1^2a_3^2a_4^2a_6^2 \\
& + 2948a_1^2a_3^2a_4a_5^2a_6 + 168a_1^2a_3^2a_5^4 - 2582a_1^2a_3a_4^3a_5a_6 + 1188a_1^2a_3a_4^2a_5^3 \\
& + 1440a_1^2a_4^5a_6 + 417a_1^2a_4^4a_5^2 + 17805a_1a_2^3a_3a_6^3 + 41147a_1a_2^3a_4a_5a_6^2 \\
& - 2260a_1a_2^3a_5^3a_6 + 19592a_1a_2^2a_3^2a_5a_6^2 + 16715a_1a_2^2a_3a_4^2a_6^2 - 2320a_1a_2^2a_3a_4a_5^2a_6 \\
& - 368a_1a_2^2a_3a_5^4 - 281a_1a_2^2a_4^3a_5a_6 - 1818a_1a_2^2a_4^2a_5^3 + 4062a_1a_2a_3^3a_4a_6^2 \\
& - 408a_1a_2a_3^3a_5^2a_6 + 3588a_1a_2a_3^2a_4^2a_5a_6 + 220a_1a_2a_3^2a_4a_5^3 - 568a_1a_2a_3a_4^4a_6 \\
& + 276a_1a_2a_3a_4^3a_5^2 - 174a_1a_2a_4^5a_5 + 1680a_1a_3^5a_6^2 - 608a_1a_3^4a_4a_5a_6 \\
& - 16a_1a_3^4a_5^3 - 158a_1a_3^3a_4^3a_6 - 108a_1a_3^3a_4^2a_5^2 - 216a_1a_3^2a_4^4a_5 \\
& - 138a_1a_3a_4^6 + 2556a_2^5a_6^3 - 10689a_2^4a_3a_5a_6^2 - 3871a_2^4a_4^2a_6^2 \\
& - 994a_2^4a_4a_5^2a_6 + 107a_2^4a_5^4 - 230a_2^3a_3^2a_4a_6^2 - 50a_2^3a_3^2a_5^2a_6 \\
& - 2601a_2^3a_3a_4^2a_5a_6 + 110a_2^3a_3a_4a_5^3 + 46a_2^3a_4^4a_6 + 392a_2^3a_4^3a_5^2 \\
& - 2862a_2^2a_3^4a_6^2 + 626a_2^2a_3^3a_4a_5a_6 + 12a_2^2a_3^3a_5^3 - 1283a_2^2a_3^2a_4^3a_6 \\
& - 330a_2^2a_3^2a_4^2a_5^2 - 362a_2^2a_3a_4^4a_5 - 3a_2^2a_4^6 - 176a_2a_3^5a_5a_6 \\
& + 412a_2a_3^4a_4^2a_6 + 24a_2a_3^4a_4a_5^2 + 438a_2a_3^3a_4^3a_5 + 53a_2a_3^2a_4^5
\end{aligned}$$

$$\begin{aligned}
& -272a_3^6a_4a_6 + 48a_3^6a_5^2 - 144a_3^5a_4^2a_5 - 45a_3^4a_4^4 \\
& -64765a_0^3a_3a_6^3 + 90900a_0^3a_4a_5a_6^2 + 5259a_0^3a_5^3a_6 - 51747a_0^2a_1a_2a_6^3 \\
& -172106a_0^2a_1a_3a_5a_6^2 + 70688a_0^2a_1a_4^2a_6^2 - 31119a_0^2a_1a_4a_5^2a_6 - 102a_0^2a_1a_5^4 \\
& -68981a_0^2a_2^2a_5a_6^2 + 213554a_0^2a_2a_3a_4a_6^2 + 40949a_0^2a_2a_3a_5^2a_6 - 69134a_0^2a_2a_4^2a_5a_6 \\
& -2646a_0^2a_2a_4a_5^3 - 28737a_0^2a_3^3a_6^2 + 45583a_0^2a_3^2a_4a_5a_6 + 2314a_0^2a_3^2a_5^3 \\
& -11830a_0^2a_3a_4^3a_6 - 8636a_0^2a_3a_4^2a_5^2 - 1862a_0^2a_4^4a_5 + 26008a_0a_1^3a_6^3 \\
& + 103083a_0a_1^2a_2a_5a_6^2 - 29380a_0a_1^2a_3a_4a_6^2 - 8790a_0a_1^2a_3a_5^2a_6 + 36117a_0a_1^2a_4^2a_5a_6 \\
& + 554a_0a_1^2a_4a_5^3 - 93529a_0a_1a_2^2a_4a_6^2 - 2893a_0a_1a_2^2a_5^2a_6 - 74403a_0a_1a_2a_3^2a_6^2 \\
& -23742a_0a_1a_2a_3a_4a_5a_6 + 1014a_0a_1a_2a_3a_5^3 + 26052a_0a_1a_2a_4^3a_6 + 3594a_0a_1a_2a_4^2a_5^2 \\
& -10232a_0a_1a_3^3a_5a_6 + 923a_0a_1a_3^2a_4^2a_6 + 4168a_0a_1a_3^2a_4a_5^2 + 1136a_0a_1a_3a_4^3a_5 \\
& + 336a_0a_1a_4^5 + 28307a_0a_2^3a_3a_6^2 + 12430a_0a_2^3a_4a_5a_6 - 68a_0a_2^3a_5^3 \\
& + 1586a_0a_2^2a_3^2a_5a_6 - 2612a_0a_2^2a_3a_4^2a_6 + 884a_0a_2^2a_3a_4a_5^2 + 1691a_0a_2^2a_4^3a_5 \\
& -6152a_0a_2a_3^3a_4a_6 + 464a_0a_2a_3^3a_5^2 - 726a_0a_2a_3^2a_4^2a_5 - 3310a_0a_2a_3a_4^4 \\
& + 2688a_0a_3^5a_6 - 592a_0a_3^4a_4a_5 + 596a_0a_3^3a_4^3 - 35506a_1^4a_5a_6^2 \\
& + 46333a_1^3a_2a_4a_6^2 - 1744a_1^3a_2a_5^2a_6 + 17114a_1^3a_3^2a_6^2 + 1638a_1^3a_3a_4a_5a_6 \\
& + 668a_1^3a_3a_5^3 - 2961a_1^3a_4^3a_6 - 180a_1^3a_4^2a_5^2 - 9020a_1^2a_2^2a_3a_6^2 \\
& -9885a_1^2a_2^2a_4a_5a_6 - 612a_1^2a_2^2a_5^3 - 3424a_1^2a_2a_3^2a_5a_6 + 2685a_1^2a_2a_3a_4^2a_6 \\
& -4452a_1^2a_2a_3a_4a_5^2 - 2994a_1^2a_2a_4^3a_5 - 2656a_1^2a_3^3a_4a_6 - 944a_1^2a_3^3a_5^2 \\
& -204a_1^2a_3^2a_4^2a_5 + 960a_1^2a_3a_4^4 - 5925a_1a_2^2a_6^2 + 2944a_1a_2^2a_3a_5a_6 \\
& -4566a_1a_2^2a_4^2a_6 + 2400a_1a_2^2a_4a_5^2 + 182a_1a_2^2a_3^2a_4a_6 + 1036a_1a_2^2a_3^2a_5^2 \\
& + 1806a_1a_2^2a_3a_4^2a_5 + 609a_1a_2^2a_4^4 + 288a_1a_2a_3^4a_6 - 240a_1a_2a_3^3a_4a_5 \\
& -278a_1a_2a_3^2a_4^3 + 96a_1a_3^5a_5 + 288a_1a_3^4a_4^2 + 333a_2^5a_5a_6 \\
& + 2783a_2^4a_3a_4a_6 - 476a_2^4a_3a_5^2 - 525a_2^4a_4^2a_5 - 220a_2^3a_3^3a_6 \\
& + 366a_2^3a_3^2a_4a_5 + 305a_2^3a_3a_4^3 - 128a_2^2a_3^4a_5 - 534a_2^2a_3^3a_4^2 \\
& + 208a_2a_3^5a_4 - 64a_3^7 + 124373a_0^3a_2a_6^2 + 4339a_0^3a_3a_5a_6 \\
& -1240a_0^3a_4^2a_6 - 8241a_0^3a_4a_5^2 - 99084a_0^2a_1^2a_6^2 + 19312a_0^2a_1a_2a_5a_6 \\
& + 25439a_0^2a_1a_3a_4a_6 + 2826a_0^2a_1a_3a_5^2 + 18054a_0^2a_1a_4^2a_5 + 89482a_0^2a_2^2a_4a_6 \\
& -1197a_0^2a_2^2a_5^2 - 46102a_0^2a_2a_3^2a_6 + 18851a_0^2a_2a_3a_4a_5 + 1014a_0^2a_2a_4^3 \\
& -520a_0^2a_3^3a_5 - 2149a_0^2a_3^2a_4^2 - 11152a_0a_1^3a_5a_6 - 43186a_0a_1^2a_2a_4a_6 \\
& + 2770a_0a_1^2a_2a_5^2 + 21656a_0a_1^2a_3^2a_6 - 3216a_0a_1^2a_3a_4a_5 - 3993a_0a_1^2a_4^3 \\
& -11231a_0a_1a_2^2a_3a_6 - 13983a_0a_1a_2^2a_4a_5 - 4664a_0a_1a_2a_3^2a_5 + 5343a_0a_1a_2a_3a_4^2 \\
& -1024a_0a_1a_3^3a_4 - 12402a_0a_2^4a_6 + 542a_0a_2^3a_3a_5 + 678a_0a_2^3a_4^2 \\
& + 52a_0a_2^2a_3^2a_4 - 896a_0a_2a_3^4 + 5652a_1^4a_4a_6 - 1314a_1^4a_5^2 \\
& + 6836a_1^3a_2a_3a_6 + 3894a_1^3a_2a_4a_5 + 528a_1^3a_3^2a_5 - 2454a_1^3a_3a_4^2 \\
& + 4719a_1^2a_2^3a_6 + 1476a_1^2a_2^2a_3a_5 + 2475a_1^2a_2^2a_4^2 + 2016a_1^2a_2a_3^2a_4
\end{aligned}$$

$$\begin{aligned}
& + 704a_1^2a_3^4 - 594a_1a_2^4a_5 - 2430a_1a_2^3a_3a_4 - 784a_1a_2^2a_3^3 \\
& + 165a_2^5a_4 + 371a_2^4a_3^2 - 86084a_0^3a_1a_6 + 12627a_0^3a_2a_5 \\
& + 27034a_0^3a_3a_4 - 7594a_0^2a_1^2a_5 - 28941a_0^2a_1a_2a_4 - 11320a_0^2a_1a_2^3 \\
& - 6745a_0^2a_2^2a_3 + 5652a_0a_1^3a_4 + 2200a_0a_1^2a_2a_3 + 7683a_0a_1a_2^3 \\
& + 216a_1^4a_3 - 2550a_1^3a_2^2 - 7309a_0^4
\end{aligned} \tag{A.28}$$

$$\begin{aligned}
e_{29} = & -132a_0^2a_4a_6^{12} - 848a_0^2a_5^2a_6^{11} + 342a_0a_1a_3a_6^{12} - 184a_0a_1a_4a_5a_6^{11} \\
& + 346a_0a_1a_5^3a_6^{10} - 80a_0a_2^2a_6^{12} - 34a_0a_2a_3a_5a_6^{11} + 36a_0a_2a_4a_6^{11} \\
& - 604a_0a_2a_4a_5^2a_6^{10} - 1296a_0a_2a_5^4a_6^9 - 186a_0a_3^2a_4a_6^{11} - 880a_0a_3^2a_5^2a_6^{10} \\
& + 230a_0a_3a_4^2a_5a_6^{10} + 1428a_0a_3a_4a_5^3a_6^9 + 299a_0a_3a_5^5a_6^8 - 10a_0a_4^4a_6^{10} \\
& - 200a_0a_4^3a_5^2a_6^9 - 1394a_0a_4^2a_5^4a_6^8 - 1116a_0a_4a_5^6a_6^7 - 113a_0a_5^8a_6^6 \\
& - 94a_1^2a_2a_6^{12} - 10a_1^2a_3a_5a_6^{11} + 44a_1^2a_4a_6^{11} + 640a_1^2a_4a_5^2a_6^{10} \\
& + 1317a_1^2a_5^4a_6^9 + 76a_1a_2^2a_5a_6^{11} + 82a_1a_2a_3a_4a_6^{11} + 900a_1a_2a_3a_5^2a_6^{10} \\
& - 90a_1a_2a_4^2a_5a_6^{10} - 912a_1a_2a_4a_5^3a_6^9 - 511a_1a_2a_5^5a_6^8 + 36a_1a_3^3a_6^{11} \\
& - 82a_1a_3^2a_4a_5a_6^{10} - 293a_1a_3^2a_5^3a_6^9 - 32a_1a_3a_4^3a_6^{10} - 518a_1a_3a_4^2a_5^2a_6^9 \\
& - 472a_1a_3a_4a_5^4a_6^8 - 39a_1a_3a_5^6a_6^7 + 23a_1a_4^4a_5a_6^9 + 390a_1a_4^3a_5^3a_6^8 \\
& + 522a_1a_4^2a_5^5a_6^7 + 112a_1a_4a_5^7a_6^6 + 3a_1a_5^9a_6^5 - 16a_2^3a_4a_6^{11} \\
& - 174a_2^3a_5^2a_6^{10} + 2a_2^2a_3^2a_6^{11} - 6a_2^2a_3a_4a_5a_6^{10} - 58a_2^2a_3a_5^3a_6^9 \\
& + 12a_2^2a_3^4a_6^{10} + 184a_2^2a_4^2a_5^2a_6^9 + 184a_2^2a_4a_5^4a_6^8 + 25a_2^2a_5^6a_6^7 \\
& - 14a_2a_3^3a_5a_6^{10} - 4a_2a_3^2a_4^2a_6^{10} - 106a_2a_3^2a_4a_5^2a_6^9 - 23a_2a_3^2a_5^4a_6^8 \\
& + 4a_2a_3a_4^3a_5a_6^9 + 180a_2a_3a_4^2a_5^3a_6^8 + 204a_2a_3a_4a_5^5a_6^7 + 23a_2a_3a_5^7a_6^6 \\
& - 2a_2a_4^5a_6^9 - 15a_2a_4^4a_5^2a_6^8 + 48a_2a_4^3a_5^4a_6^7 + 38a_2a_4^2a_5^6a_6^6 \\
& + 3a_2a_4a_5^8a_6^5 - 6a_3^4a_4a_6^{10} - 15a_3^4a_5^2a_6^9 + 18a_3^3a_4^2a_5a_6^9 \\
& + 105a_3^3a_4a_5^3a_6^8 + 24a_3^3a_5^5a_6^7 + 3a_3^2a_4^4a_6^9 + 60a_3^2a_4^3a_5^2a_6^8 \\
& - 18a_3^2a_4^2a_5^4a_6^7 - 48a_3^2a_4a_5^6a_6^6 - 3a_3^2a_5^8a_6^5 - 3a_3a_4^5a_5a_6^8 \\
& - 90a_3a_4^4a_5^3a_6^7 - 150a_3a_4^3a_5^5a_6^6 - 30a_3a_4^2a_5^7a_6^5 - 6a_4^6a_5^2a_6^7 \\
& - 30a_4^5a_5^4a_6^6 - 15a_4^4a_5^6a_6^5 - 210a_0^2a_3a_6^{11} - 3136a_0^2a_4a_5a_6^{10} \\
& - 6328a_0^2a_5^3a_6^9 + 6a_0a_1a_2a_6^{11} + 13234a_0a_1a_3a_5a_6^{10} + 60a_0a_1a_4^2a_6^{10} \\
& - 8890a_0a_1a_4a_5^2a_6^9 + 3472a_0a_1a_5^4a_6^8 - 3082a_0a_2^2a_5a_6^{10} + 252a_0a_2a_3a_4a_6^{10} \\
& + 138a_0a_2a_3a_5^2a_6^9 + 2472a_0a_2a_4^2a_5a_6^9 - 1012a_0a_2a_4a_5^3a_6^8 - 5254a_0a_2a_5^5a_6^7 \\
& + 150a_0a_3^3a_6^{10} - 5872a_0a_3^2a_4a_5a_6^9 - 7355a_0a_3^2a_5^3a_6^8 - 158a_0a_3a_4^3a_6^9 \\
& + 6034a_0a_3a_4^2a_5^2a_6^8 + 12433a_0a_3a_4a_5^4a_6^7 + 1113a_0a_3a_5^6a_6^6 - 23a_0a_4^4a_5a_6^8 \\
& + 1186a_0a_4^3a_5^3a_6^7 - 1974a_0a_4^2a_5^5a_6^6 - 1296a_0a_4a_5^7a_6^5 - 55a_0a_5^9a_6^4 \\
& + 94a_1^3a_6^{11} - 3714a_1^2a_2a_5a_6^{10} - 160a_1^2a_3a_4a_6^{10} - 1930a_1^2a_3a_5^2a_6^9
\end{aligned}$$

$$\begin{aligned}
& + 526a_1^2a_4^2a_5a_6^9 + 1504a_1^2a_4a_5^3a_6^8 + 5368a_1^2a_5^5a_6^7 - 110a_1a_2^2a_4a_6^{10} \\
& + 2586a_1a_2^2a_5^2a_6^9 - 194a_1a_2a_3^2a_6^{10} + 1754a_1a_2a_3a_4a_5a_6^9 + 8634a_1a_2a_3a_5^3a_6^8 \\
& + 98a_1a_2a_4^3a_6^9 - 714a_1a_2a_4^2a_5^2a_6^8 - 5333a_1a_2a_4a_5^4a_6^7 - 1523a_1a_2a_5^6a_6^6 \\
& + 1500a_1a_3^3a_5a_6^9 + 182a_1a_3^2a_4^2a_6^9 - 1259a_1a_3^2a_4a_5^2a_6^8 - 1858a_1a_3^2a_5^4a_6^7 \\
& - 340a_1a_3a_4^3a_5a_6^8 - 2220a_1a_3a_4^2a_5^3a_6^7 - 678a_1a_3a_4a_5^5a_6^6 + 18a_1a_3a_5^7a_6^5 \\
& - 21a_1a_4^5a_6^8 - 212a_1a_4^4a_5^2a_6^7 + 702a_1a_4^3a_5^4a_6^6 + 624a_1a_4^2a_5^6a_6^5 \\
& + 53a_1a_4a_5^8a_6^4 + 12a_2^3a_3a_6^{10} - 270a_2^3a_4a_5a_6^9 - 1862a_2^3a_5^3a_6^8 \\
& + 126a_2^2a_3^2a_5a_6^9 - 22a_2^2a_3a_4^2a_6^9 - 222a_2^2a_3a_4a_5^2a_6^8 - 416a_2^2a_3a_5^4a_6^7 \\
& + 108a_2^2a_3^4a_5a_6^8 + 1168a_2^2a_4^2a_5^3a_6^7 + 654a_2^2a_4a_5^5a_6^6 + 47a_2^2a_5^7a_6^5 \\
& + 46a_2a_3^3a_4a_6^9 - 394a_2a_3^3a_5^2a_6^8 - 14a_2a_3^2a_4^2a_5a_6^8 - 729a_2a_3^2a_4a_5^3a_6^7 \\
& + 18a_2a_3^2a_5^5a_6^6 - 436a_2a_3a_4^3a_5^2a_6^7 + 132a_2a_3a_4^2a_5^4a_6^6 + 231a_2a_3a_4a_5^6a_6^5 \\
& + 10a_2a_3a_5^8a_6^4 - 49a_2a_4^5a_5a_6^7 - 312a_2a_4^4a_5^3a_6^6 - 114a_2a_4^3a_5^5a_6^5 \\
& - 2a_2a_4^2a_5^7a_6^4 + 6a_3^5a_6^9 - 246a_3^4a_4a_5a_6^8 - 90a_3^4a_5^3a_6^7 \\
& - 30a_3^3a_4^3a_6^8 + 243a_3^3a_4^2a_5^2a_6^7 + 546a_3^3a_4a_5^4a_6^6 + 30a_3^3a_5^6a_6^5 \\
& + 18a_3^2a_4^4a_5a_6^7 + 432a_3^2a_4^3a_5^3a_6^6 - 36a_3^2a_4^2a_5^5a_6^5 - 48a_3^2a_4a_5^7a_6^4 \\
& + 3a_3a_4^6a_6^7 + 180a_3a_4^5a_5^2a_6^6 + 180a_3a_4^4a_5^4a_6^5 - 30a_3a_4^3a_5^6a_6^4 \\
& + 12a_4^7a_5a_6^6 + 72a_4^6a_5^3a_6^5 + 30a_4^5a_5^5a_6^4 - 12644a_0^2a_2a_6^{10} \\
& - 3814a_0^2a_3a_5a_6^9 + 2906a_0^2a_4^2a_6^9 + 966a_0^2a_4a_5^2a_6^8 - 2072a_0^2a_5^4a_6^7 \\
& - 6950a_0a_1^2a_6^{10} + 9086a_0a_1a_2a_5a_6^9 + 6260a_0a_1a_3a_4a_6^9 + 57777a_0a_1a_3a_5^2a_6^8 \\
& - 690a_0a_1a_4^2a_5a_6^8 - 49640a_0a_1a_4a_5^3a_6^7 + 9531a_0a_1a_5^5a_6^6 + 2682a_0a_2^2a_4a_6^9 \\
& - 17676a_0a_2^2a_5^2a_6^8 + 268a_0a_2a_3^2a_6^9 + 768a_0a_2a_3a_4a_5a_6^8 + 5181a_0a_2a_3a_5^3a_6^7 \\
& - 932a_0a_2a_4^3a_6^8 + 13848a_0a_2a_4^2a_5^2a_6^7 + 7736a_0a_2a_4a_5^4a_6^6 - 4801a_0a_2a_5^6a_6^5 \\
& + 4059a_0a_3^3a_5a_6^8 + 408a_0a_3^2a_4^2a_6^8 - 24520a_0a_3^2a_4a_5^2a_6^7 - 10239a_0a_3^2a_5^4a_6^6 \\
& - 5874a_0a_3a_4^3a_5a_6^7 + 8066a_0a_3a_4^2a_5^3a_6^6 + 13956a_0a_3a_4a_5^5a_6^5 + 236a_0a_3a_5^7a_6^4 \\
& - 29a_0a_4^5a_6^7 - 1005a_0a_4^4a_5^2a_6^6 + 4098a_0a_4^3a_5^4a_6^5 + 628a_0a_4^2a_5^6a_6^4 \\
& - 154a_0a_4a_5^8a_6^3 + 4887a_1^3a_5a_6^9 - 238a_1^2a_2a_4a_6^9 - 19268a_1^2a_2a_5^2a_6^8 \\
& - 5129a_1^2a_3^2a_6^9 + 3945a_1^2a_3a_4a_5a_6^8 - 13888a_1^2a_3a_5^3a_6^7 + 40a_1^2a_4^3a_6^8 \\
& - 947a_1^2a_4^2a_5^2a_6^7 - 8369a_1^2a_4a_5^4a_6^6 + 4433a_1^2a_5^6a_6^5 + 2832a_1a_2^2a_3a_6^9 \\
& - 5744a_1a_2^2a_4a_5a_6^8 + 12562a_1a_2^2a_5^3a_6^7 - 5447a_1a_2a_3^2a_5a_6^8 - 1922a_1a_2a_3a_4^2a_6^8 \\
& + 41a_1a_2a_3a_4a_5^2a_6^7 + 15860a_1a_2a_3a_5^4a_6^6 + 3062a_1a_2a_4^3a_5a_6^7 + 5460a_1a_2a_4^2a_5^3a_6^6 \\
& - 4136a_1a_2a_4a_5^5a_6^5 - 706a_1a_2a_5^7a_6^4 + 954a_1a_3^3a_4a_6^8 + 8592a_1a_3^3a_5^2a_6^7 \\
& + 2009a_1a_3^2a_4^2a_5a_6^7 - 3085a_1a_3^2a_4a_5^3a_6^6 - 2222a_1a_3^2a_5^5a_6^5 + 329a_1a_3a_4^4a_6^7 \\
& + 1054a_1a_3a_4^3a_5^2a_6^6 - 698a_1a_3a_4^2a_5^4a_6^5 + 164a_1a_3a_4a_5^6a_6^4 + 23a_1a_3a_5^8a_6^3 \\
& - 281a_1a_4^5a_5a_6^6 - 1892a_1a_4^4a_5^3a_6^5 - 640a_1a_4^3a_5^5a_6^4 + 30a_1a_4^2a_5^7a_6^3 \\
& - 1694a_2^4a_6^9 + 2598a_2^3a_3a_5a_6^8 + 1252a_2^3a_4^2a_6^8 + 742a_2^3a_4a_5^2a_6^7
\end{aligned}$$

$$\begin{aligned}
& -4093a_2^3a_5^4a_6^6 - 1408a_2^2a_3^2a_4a_6^8 + 463a_2^2a_3^2a_5^2a_6^7 + 108a_2^2a_3a_4^2a_5a_6^7 \\
& - 415a_2^2a_3a_4a_5^3a_6^6 - 345a_2^2a_3a_5^5a_6^5 - 356a_2^2a_4^4a_6^7 - 1108a_2^2a_4^3a_5^2a_6^6 \\
& + 904a_2^2a_4^2a_5^4a_6^5 + 353a_2^2a_4a_5^6a_6^4 + 10a_2^2a_5^8a_6^3 + 582a_2a_3^4a_6^8 \\
& + 742a_2a_3^3a_4a_5a_6^7 - 1725a_2a_3^3a_5^3a_6^6 + 544a_2a_3^2a_4^3a_6^7 + 701a_2a_3^2a_4^2a_5^2a_6^6 \\
& - 586a_2a_3^2a_4a_5^4a_6^5 + 106a_2a_3^2a_5^6a_6^4 + 137a_2a_3a_4^4a_5a_6^6 - 1708a_2a_3a_4^3a_5^3a_6^5 \\
& - 447a_2a_3a_4^2a_5^5a_6^4 + 11a_2a_3a_4a_5^7a_6^3 + 56a_2a_4^6a_6^6 + 182a_2a_4^5a_5^2a_6^5 \\
& - 155a_2a_4^4a_5^4a_6^4 - 34a_2a_4^3a_5^6a_6^3 - 81a_3^5a_5a_6^7 - 42a_3^4a_4^2a_6^7 \\
& - 1167a_3^4a_4a_5^2a_6^6 + 72a_3^4a_5^4a_6^5 - 531a_3^3a_4^3a_5a_6^6 + 390a_3^3a_4^2a_5^3a_6^5 \\
& + 327a_3^3a_4a_5^5a_6^4 - 13a_3^3a_5^7a_6^3 - 69a_3^2a_4^5a_6^6 - 462a_3^2a_4^4a_5^2a_6^5 \\
& + 330a_3^2a_4^3a_5^4a_6^4 - 36a_3^2a_4^2a_5^6a_6^3 - 111a_3a_4^6a_5a_6^5 + 129a_3a_4^5a_5^3a_6^4 \\
& + 135a_3a_4^4a_5^5a_6^3 - 6a_4^8a_6^5 - 48a_4^7a_5^2a_6^4 - 4a_4^6a_5^4a_6^3 \\
& + 26350a_0^2a_1a_6^9 - 52874a_0^2a_2a_5a_6^8 - 18474a_0^2a_3a_4a_6^8 - 35189a_0^2a_3a_5^2a_6^7 \\
& + 17258a_0^2a_4^2a_5a_6^7 + 36592a_0^2a_4a_5^3a_6^6 + 8879a_0^2a_5^5a_6^5 - 33855a_0a_1^2a_5a_6^8 \\
& - 19768a_0a_1a_2a_4a_6^8 + 49215a_0a_1a_2a_5^2a_6^7 + 4410a_0a_1a_3^2a_6^8 - 3628a_0a_1a_3a_4a_5a_6^7 \\
& + 9146a_0a_1a_3a_5^3a_6^6 + 5878a_0a_1a_4^3a_6^7 + 29734a_0a_1a_4^2a_5^2a_6^6 - 52979a_0a_1a_4a_5^4a_6^5 \\
& + 6722a_0a_1a_5^6a_6^4 - 6418a_0a_2^2a_3a_6^8 + 31224a_0a_2^2a_4a_5a_6^7 - 26667a_0a_2^2a_5^3a_6^6 \\
& - 2220a_0a_2a_3^2a_5a_6^7 + 9374a_0a_2a_3a_4^2a_6^7 - 9572a_0a_2a_3a_4a_5^2a_6^6 + 15772a_0a_2a_3a_5^4a_6^5 \\
& - 19656a_0a_2a_4^3a_5a_6^6 + 3798a_0a_2a_4^2a_5^3a_6^5 + 5546a_0a_2a_4a_5^5a_6^4 - 1266a_0a_2a_5^7a_6^3 \\
& - 1731a_0a_3^3a_4a_6^7 + 12912a_0a_3^3a_5^2a_6^6 + 19537a_0a_3^2a_4^2a_5a_6^6 - 20511a_0a_3^2a_4a_5^3a_6^5 \\
& - 1084a_0a_3^2a_5^5a_6^4 - 229a_0a_3a_4^4a_6^6 - 26608a_0a_3a_4^3a_5^2a_6^5 - 10942a_0a_3a_4^2a_5^4a_6^4 \\
& + 1928a_0a_3a_4a_5^6a_6^3 - 73a_0a_3a_5^8a_6^2 + 1261a_0a_4^5a_5a_6^5 - 2422a_0a_4^4a_5^3a_6^4 \\
& + 1472a_0a_4^3a_5^5a_6^3 + 214a_0a_4^2a_5^7a_6^2 - 885a_1^3a_4a_6^8 + 25226a_1^3a_5^2a_6^7 \\
& + 5338a_1^2a_2a_3a_6^8 + 13739a_1^2a_2a_4a_5a_6^7 - 14540a_1^2a_2a_5^3a_6^6 - 22150a_1^2a_3^2a_5a_6^7 \\
& - 4593a_1^2a_3a_4^2a_6^7 + 42046a_1^2a_3a_4a_5^2a_6^6 - 19390a_1^2a_3a_5^4a_6^5 + 2402a_1^2a_4^3a_5a_6^6 \\
& + 1566a_1^2a_4^2a_5^3a_6^5 - 9456a_1^2a_4a_5^5a_6^4 + 814a_1^2a_5^7a_6^3 + 4062a_1a_2^3a_6^8 \\
& + 13426a_1a_2^2a_3a_5a_6^7 - 38a_1a_2^2a_4^2a_6^7 - 38881a_1a_2^2a_4a_5^2a_6^6 + 11644a_1a_2^2a_5^4a_6^5 \\
& + 2179a_1a_2a_3^2a_4a_6^7 - 21276a_1a_2a_3^2a_5^2a_6^6 - 13312a_1a_2a_3a_4^2a_5a_6^6 - 12624a_1a_2a_3a_4a_5^3a_6^5 \\
& + 5892a_1a_2a_3a_5^5a_6^4 - 783a_1a_2a_4^4a_6^6 + 9354a_1a_2a_4^3a_5^2a_6^5 + 10892a_1a_2a_4^2a_5^4a_6^4 \\
& + 30a_1a_2a_4a_5^6a_6^3 - 37a_1a_2a_5^8a_6^2 - 1656a_1a_3^4a_6^7 - 480a_1a_3^3a_4a_5a_6^6 \\
& + 10074a_1a_3^3a_5^3a_6^5 - 125a_1a_3^2a_4^3a_6^6 + 3735a_1a_3^2a_4^2a_5^2a_6^5 - 1064a_1a_3^2a_4a_5^4a_6^4 \\
& - 478a_1a_3^2a_5^6a_6^3 + 818a_1a_3a_4^4a_5a_6^5 + 1716a_1a_3a_4^3a_5^3a_6^4 + 70a_1a_3a_4^2a_5^5a_6^3 \\
& + 62a_1a_3a_4a_5^7a_6^2 + 111a_1a_4^6a_5^5 + 470a_1a_4^5a_5^2a_6^4 - 586a_1a_4^4a_5^4a_6^3 \\
& - 126a_1a_4^3a_5^6a_6^2 - 9446a_2^4a_5a_6^7 - 2314a_2^3a_3a_4a_6^7 + 12292a_2^3a_3a_5^2a_6^6 \\
& + 7440a_2^3a_4^2a_5a_6^6 + 5051a_2^3a_4a_5^3a_6^5 - 2029a_2^3a_5^5a_6^4 - 1060a_2^2a_3^3a_6^7 \\
& - 5360a_2^2a_3^2a_4a_5a_6^6 - 1149a_2^2a_3^2a_5^3a_6^5 + 92a_2^2a_3a_4^3a_6^6 + 1035a_2^2a_3a_4^2a_5^2a_6^5
\end{aligned}$$

$$\begin{aligned}
& + 885a_2^2a_3a_4a_5^4a_6^4 + 64a_2^2a_3a_5^6a_6^3 - 1359a_2^2a_4^4a_5a_6^5 - 2452a_2^2a_4^3a_5^3a_6^4 \\
& - 159a_2^2a_4^2a_5^5a_6^3 + 19a_2^2a_4a_5^7a_6^2 + 3705a_2^2a_3^4a_5a_6^6 - 508a_2^2a_3^3a_4^2a_6^6 \\
& + 2301a_2^2a_3^3a_4^2a_5^5a_6^5 - 1274a_2^2a_3^3a_5^4a_6^4 + 3019a_2^2a_3^3a_4^5a_6^5 + 768a_2^2a_3^2a_4^2a_5^3a_6^4 \\
& - 97a_2^2a_3^2a_4^2a_5^3a_6^3 + 13a_2^2a_3^2a_5^7a_6^2 + 63a_2^2a_3^2a_4^5a_6^5 + 1138a_2^2a_3^2a_4^4a_5^2a_6^4 \\
& - 627a_2^2a_3^2a_4^3a_5^4a_6^3 - 93a_2^2a_3^2a_4^2a_5^6a_6^2 + 187a_2^2a_4^6a_5a_6^4 + 343a_2^2a_4^5a_5^3a_6^3 \\
& + 27a_2^2a_4^4a_5^5a_6^2 + 459a_2^2a_3^5a_4a_6^6 - 834a_2^2a_3^5a_5^2a_6^5 + 12a_2^2a_3^4a_4^2a_5^5a_6^4 \\
& - 510a_2^2a_3^4a_4^3a_5^4a_6^3 + 162a_2^2a_3^4a_5^5a_6^3 + 267a_2^2a_3^3a_4^4a_5^5 - 1284a_2^2a_3^3a_4^3a_5^2a_6^4 \\
& + 63a_2^2a_3^3a_4^2a_5^4a_6^3 - 33a_2^2a_3^3a_4^2a_5^6a_6^2 - 114a_2^2a_3^3a_4^5a_5^4a_6^4 - 426a_2^2a_3^3a_4^4a_5^3a_6^3 \\
& + 108a_2^2a_3^3a_4^3a_5^5a_6^2 + 21a_2^2a_3^3a_4^7a_6^4 - 249a_2^2a_3^3a_4^6a_5^2a_6^3 - 63a_2^2a_3^3a_4^5a_5^4a_6^2 \\
& - 24a_2^2a_3^3a_4^5a_6^2 - 106854a_0^3a_6^8 + 170481a_0^2a_1a_5a_6^7 + 55136a_0^2a_2a_4a_6^7 \\
& + 2897a_0^2a_2a_5^2a_6^6 + 75603a_0^2a_3^2a_6^7 - 177449a_0^2a_3a_4a_5a_6^6 - 41605a_0^2a_3a_5^3a_6^5 \\
& - 17870a_0^2a_4^3a_6^6 + 7279a_0^2a_4^2a_5^2a_6^5 + 15502a_0^2a_4a_5^4a_6^4 + 2234a_0^2a_5^6a_6^3 \\
& - 7023a_0a_1^2a_4a_6^7 - 18048a_0a_1^2a_5^2a_6^6 - 16574a_0a_1a_2a_3a_6^7 - 116276a_0a_1a_2a_4a_5a_6^6 \\
& + 42780a_0a_1a_2a_5^3a_6^5 + 62725a_0a_1a_3^2a_5a_6^6 + 19179a_0a_1a_3a_4^2a_6^6 + 1876a_0a_1a_3a_4a_5^2a_6^5 \\
& - 39002a_0a_1a_3a_5^4a_6^4 + 7040a_0a_1a_4^3a_5a_6^5 + 51766a_0a_1a_4^2a_5^3a_6^4 - 14496a_0a_1a_4a_5^5a_6^3 \\
& + 611a_0a_1a_5^7a_6^2 - 44256a_0a_2^3a_6^7 + 28021a_0a_2^2a_3a_5a_6^6 + 6168a_0a_2^2a_4^2a_6^6 \\
& + 35769a_0a_2^2a_4a_5^2a_6^5 - 11107a_0a_2^2a_5^4a_6^4 - 8572a_0a_2a_3^2a_4a_6^6 - 44091a_0a_2a_3^2a_5^2a_6^5 \\
& + 49273a_0a_2a_3a_4^2a_5^5a_6^5 + 4810a_0a_2a_3a_4a_5^3a_6^4 + 7686a_0a_2a_3a_5^5a_6^3 + 6635a_0a_2a_4^4a_6^5 \\
& - 27620a_0a_2a_4^3a_5^2a_6^4 - 7586a_0a_2a_4^2a_5^4a_6^3 + 286a_0a_2a_4a_5^6a_6^2 - 57a_0a_2a_5^8a_6 \\
& - 14337a_0a_3^4a_6^6 + 4599a_0a_3^3a_4a_5^5a_6^5 + 3124a_0a_3^3a_5^3a_6^4 - 7266a_0a_3^2a_4^3a_6^5 \\
& + 31311a_0a_3^2a_4^2a_5^2a_6^4 - 2158a_0a_3^2a_4a_5^4a_6^3 + 605a_0a_3^2a_5^6a_6^2 + 12898a_0a_3a_4^4a_5^4a_6^4 \\
& - 8250a_0a_3a_4^3a_5^3a_6^3 - 2317a_0a_3a_4^2a_5^5a_6^2 + 16a_0a_3a_4a_5^7a_6 - 740a_0a_4^6a_6^4 \\
& + 2194a_0a_4^5a_5^2a_6^3 - 1030a_0a_4^4a_5^4a_6^2 - 8a_0a_4^3a_5^6a_6 + 12177a_1^3a_3a_6^7 \\
& - 26404a_1^3a_4a_5a_6^6 + 25246a_1^3a_5^3a_6^5 + 3369a_1^2a_2^2a_6^7 - 11316a_1^2a_2a_3a_5a_6^6 \\
& - 8703a_1^2a_2a_4^2a_6^6 + 32508a_1^2a_2a_4a_5^2a_6^5 + 4926a_1^2a_2a_5^4a_6^4 - 4557a_1^2a_3^2a_4a_6^6 \\
& + 5376a_1^2a_3^2a_5^2a_6^5 - 27811a_1^2a_3a_4^2a_5^5a_6^5 + 45710a_1^2a_3a_4a_5^3a_6^4 - 5297a_1^2a_3a_5^5a_6^3 \\
& - 116a_1^2a_4^4a_6^6 + 6588a_1^2a_4^3a_5^2a_6^4 + 5891a_1^2a_4^2a_5^4a_6^3 - 1183a_1^2a_4a_5^6a_6^2 \\
& + 23a_1^2a_5^8a_6 + 24261a_1a_2^3a_5a_6^6 + 9206a_1a_2^2a_3a_4a_6^6 + 9685a_1a_2^2a_3a_5^2a_6^5 \\
& + 13430a_1a_2^2a_4^2a_5a_6^5 - 45743a_1a_2^2a_4a_5^3a_6^4 + 1153a_1a_2^2a_5^5a_6^3 + 13691a_1a_2a_3^3a_6^6 \\
& + 8775a_1a_2a_3^2a_4a_5a_6^5 - 12678a_1a_2a_3^2a_5^3a_6^4 + 1461a_1a_2a_3a_4^3a_6^5 - 6924a_1a_2a_3a_4^2a_5^2a_6^4 \\
& - 6548a_1a_2a_3a_4a_5^4a_6^3 + 208a_1a_2a_3a_5^6a_6^2 - 7508a_1a_2a_4^4a_5a_6^4 - 278a_1a_2a_4^3a_5^3a_6^3 \\
& + 1907a_1a_2a_4^2a_5^5a_6^2 + 42a_1a_2a_4a_5^7a_6 - 6486a_1a_3^4a_5a_6^5 + 1020a_1a_3^3a_4^2a_6^5 \\
& - 7022a_1a_3^3a_4a_5^2a_6^4 + 2032a_1a_3^3a_5^4a_6^3 - 2753a_1a_3^2a_4^3a_5^4a_6^4 + 1896a_1a_3^2a_4^2a_5^3a_6^3 \\
& - 19a_1a_3^2a_4a_5^5a_6^2 - 2a_1a_3^2a_5^7a_6 - 178a_1a_3a_4^5a_6^4 - 921a_1a_3a_4^4a_5^2a_6^3 \\
& - 368a_1a_3a_4^3a_5^4a_6^2 - 70a_1a_3a_4^2a_5^6a_6 + 207a_1a_4^6a_5a_6^3 + 591a_1a_4^5a_5^3a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 27a_1a_4^4a_5^5a_6 + 5458a_2^4a_4a_6^6 - 10800a_2^4a_5^2a_6^5 - 7506a_2^3a_3^2a_6^6 \\
& - 15256a_2^3a_3a_4a_5a_6^5 + 8299a_2^3a_3a_5^3a_6^4 - 4276a_2^3a_4^3a_6^5 + 7079a_2^3a_4^2a_5^2a_6^4 \\
& + 3517a_2^3a_4a_5^4a_6^3 - 151a_2^3a_5^6a_6^2 + 2990a_2^2a_3^3a_5a_6^5 + 4695a_2^2a_3^2a_4^2a_6^5 \\
& - 5127a_2^2a_3^2a_4a_5^2a_6^4 - 1033a_2^2a_3^2a_5^4a_6^3 - 949a_2^2a_3a_4^3a_5a_6^4 + 318a_2^2a_3a_4^2a_5^3a_6^3 \\
& + 302a_2^2a_3a_4a_5^5a_6^2 + 12a_2^2a_3a_5^7a_6 + 926a_2^2a_4^5a_6^4 + 97a_2^2a_4^4a_5^2a_6^3 \\
& - 477a_2^2a_4^3a_5^4a_6^2 - 23a_2^2a_4^2a_5^6a_6 - 6267a_2a_3^4a_4a_6^5 + 2780a_2a_3^4a_5^2a_6^4 \\
& + 2969a_2a_3^3a_4^2a_5a_6^4 + 1668a_2a_3^3a_4a_5^3a_6^3 - 65a_2a_3^3a_5^5a_6^2 - 2006a_2a_3^2a_4^4a_6^4 \\
& + 1380a_2a_3^2a_4^3a_5^2a_6^3 - 247a_2a_3^2a_4^2a_5^4a_6^2 - 29a_2a_3^2a_4a_5^6a_6 + 296a_2a_3a_4^5a_5a_6^3 \\
& + 801a_2a_3a_4^4a_5^3a_6^2 + 27a_2a_3a_4^3a_5^5a_6 - 117a_2a_4^7a_6^3 - 90a_2a_4^6a_5^2a_6^2 \\
& + 12a_2a_4^5a_5^4a_6 + 975a_3^6a_6^5 + 210a_3^5a_4a_5a_6^4 - 636a_3^5a_5^3a_6^3 \\
& + 579a_3^4a_4^3a_6^4 + 18a_3^4a_4^2a_5^2a_6^3 + 237a_3^4a_4a_5^4a_6^2 - 3a_3^4a_5^6a_6 \\
& + 705a_3^3a_4^4a_5a_6^3 - 432a_3^3a_4^3a_5^3a_6^2 + 24a_3^3a_4^2a_5^5a_6 + 165a_3^2a_4^6a_6^3 \\
& + 135a_3^2a_4^5a_5^2a_6^2 - 9a_3^2a_4^4a_5^4a_6 + 108a_3a_4^7a_5a_6^2 - 24a_3a_4^6a_5^3a_6 \\
& + 6a_4^9a_6^2 + 15a_4^8a_5^2a_6 + 32123a_0^3a_5a_6^6 - 87039a_0^2a_1a_4a_6^6 \\
& + 23276a_0^2a_1a_5^2a_6^5 - 177128a_0^2a_2a_3a_6^6 + 56593a_0^2a_2a_4a_5a_6^5 + 47903a_0^2a_2a_5^3a_6^4 \\
& + 59241a_0^2a_3^2a_5a_6^5 + 149256a_0^2a_3a_4^2a_6^5 - 105497a_0^2a_3a_4a_5^2a_6^4 - 1766a_0^2a_3a_5^4a_6^3 \\
& - 73310a_0^2a_4^3a_5a_6^4 - 8990a_0^2a_4^2a_5^3a_6^3 - 2772a_0^2a_4a_5^5a_6^2 + 11a_0^2a_5^7a_6 \\
& - 11829a_0a_1^2a_3a_6^6 + 28240a_0a_1^2a_4a_5a_6^5 - 2726a_0a_1^2a_5^3a_6^4 + 133488a_0a_1a_2^2a_6^6 \\
& - 176a_0a_1a_2a_3a_5a_6^5 - 17059a_0a_1a_2a_4^2a_6^5 - 115906a_0a_1a_2a_4a_5^2a_6^4 + 10218a_0a_1a_2a_5^4a_6^3 \\
& - 28161a_0a_1a_3^2a_4a_6^5 + 92780a_0a_1a_3^2a_5^2a_6^4 + 41374a_0a_1a_3a_4^2a_5a_6^4 + 40404a_0a_1a_3a_4a_5^3a_6^3 \\
& - 4034a_0a_1a_3a_5^5a_6^2 + 2002a_0a_1a_4^4a_6^4 - 28530a_0a_1a_4^3a_5^2a_6^3 + 7262a_0a_1a_4^2a_5^4a_6^2 \\
& - 1177a_0a_1a_4a_5^6a_6 - 4a_0a_1a_5^8 - 13472a_0a_2^3a_5a_6^5 - 87751a_0a_2^2a_3a_4a_6^5 \\
& + 9213a_0a_2^2a_3a_5^2a_6^4 + 21977a_0a_2^2a_4^2a_5a_6^4 + 9845a_0a_2^2a_4a_5^3a_6^3 - 1577a_0a_2^2a_5^5a_6^2 \\
& + 52189a_0a_2a_3^3a_6^5 - 30578a_0a_2a_3^2a_4a_5a_6^4 - 22680a_0a_2a_3^2a_5^3a_6^3 + 5922a_0a_2a_3a_4^3a_6^4 \\
& + 39908a_0a_2a_3a_4^2a_5^2a_6^3 + 3374a_0a_2a_3a_4a_5^4a_6^2 + 324a_0a_2a_3a_5^6a_6 + 16422a_0a_2a_4^4a_5a_6^3 \\
& - 44a_0a_2a_4^3a_5^3a_6^2 - 156a_0a_2a_4^2a_5^5a_6 + 10a_0a_2a_4a_5^7 - 10418a_0a_3^4a_5a_6^4 \\
& - 16801a_0a_3^3a_4^2a_6^4 - 2012a_0a_3^3a_4a_5^2a_6^3 - 1864a_0a_3^3a_5^4a_6^2 - 9189a_0a_3^2a_4^3a_5a_6^3 \\
& + 4542a_0a_3^2a_4^2a_5^3a_6^2 + 241a_0a_3^2a_4a_5^5a_6 - 8a_0a_3^2a_5^7 - 4612a_0a_3a_4^5a_6^3 \\
& + 8907a_0a_3a_4^4a_5^2a_6^2 + a_0a_3a_4^3a_5^4a_6 - 12a_0a_3a_4^2a_5^6 - 1655a_0a_4^6a_5a_6^2 \\
& + 121a_0a_4^5a_5^3a_6 - 41a_0a_4^4a_5^5 - 31567a_1^3a_2a_6^6 - 15418a_1^3a_3a_5a_6^5 \\
& + 17358a_1^3a_4^2a_6^5 - 27366a_1^3a_4a_5^2a_6^4 + 6250a_1^3a_5^4a_6^3 - 17638a_1^2a_2^2a_5a_6^5 \\
& + 37564a_1^2a_2a_3a_4a_6^5 - 28544a_1^2a_2a_3a_5^2a_6^4 - 48465a_1^2a_2a_4^2a_5a_6^4 - 2994a_1^2a_2a_4a_5^3a_6^3 \\
& + 1595a_1^2a_2a_5^5a_6^2 - 9550a_1^2a_3^3a_6^5 - 17182a_1^2a_3^2a_4a_5a_6^4 + 8228a_1^2a_3^2a_5^3a_6^3 \\
& - 4521a_1^2a_3a_4^3a_6^4 - 34470a_1^2a_3a_4^2a_5^2a_6^3 + 6481a_1^2a_3a_4a_5^4a_6^2 - 116a_1^2a_3a_5^6a_6 \\
& + 1194a_1^2a_4^4a_5a_6^3 + 2400a_1^2a_4^3a_5^3a_6^2 + 890a_1^2a_4^2a_5^5a_6 - 2a_1^2a_4a_5^7
\end{aligned}$$

$$\begin{aligned}
& -28287a_1a_2^3a_4a_6^5 + 28905a_1a_2^3a_5^2a_6^4 - 8523a_1a_2^2a_3^2a_6^5 + 32814a_1a_2^2a_3a_4a_5a_6^4 \\
& + 6504a_1a_2^2a_3a_5^3a_6^3 + 9217a_1a_2^2a_4^3a_6^4 + 30327a_1a_2^2a_4^2a_5^2a_6^3 - 8121a_1a_2^2a_4a_5^4a_6^2 \\
& - 94a_1a_2^2a_5^6a_6 + 14672a_1a_2a_3^3a_5a_6^4 + 243a_1a_2a_3^2a_4^2a_6^4 + 7598a_1a_2a_3^2a_4a_5^2a_6^3 \\
& - 466a_1a_2a_3^2a_5^4a_6^2 + 18a_1a_2a_3a_4^3a_5a_6^3 + 1616a_1a_2a_3a_4^2a_5^3a_6^2 - 114a_1a_2a_3a_4a_5^5a_6 \\
& - 10a_1a_2a_4^5a_6^3 - 4701a_1a_2a_4^4a_5^2a_6^2 - 887a_1a_2a_4^3a_5^4a_6 + 4a_1a_2a_4^2a_5^6 \\
& + 7066a_1a_3^4a_4a_6^4 - 1684a_1a_3^4a_5^2a_6^3 + 1322a_1a_3^3a_4^2a_5a_6^3 - 1516a_1a_3^3a_4a_5^3a_6^2 \\
& - 18a_1a_3^3a_5^5a_6 + 2689a_1a_3^2a_4^4a_6^3 - 598a_1a_3^2a_4^3a_5^2a_6^2 + 163a_1a_3^2a_4^2a_5^4a_6 \\
& - 4a_1a_3^2a_4a_5^6 - 322a_1a_3a_4^5a_5a_6^2 - 46a_1a_3a_4^4a_5^3a_6 - 10a_1a_3a_4^3a_5^5 \\
& + 3a_1a_4^7a_6^2 - 117a_1a_4^6a_5^2a_6 + 15a_1a_4^5a_5^4 + 11946a_2^4a_3a_6^5 \\
& + 3348a_2^4a_4a_5a_6^4 - 4235a_2^4a_5^3a_6^3 - 7478a_2^3a_3^2a_5a_6^4 + 1512a_2^3a_3a_4^2a_6^4 \\
& - 12815a_2^3a_3a_4a_5^2a_6^3 + 612a_2^3a_3a_5^4a_6^2 - 6339a_2^3a_4^3a_5a_6^3 + 624a_2^3a_4^2a_5^3a_6^2 \\
& + 284a_2^3a_4a_5^5a_6 - 1426a_2^2a_3^3a_4a_6^4 + 1396a_2^2a_3^3a_5^2a_6^3 + 5081a_2^2a_3^2a_4^2a_5a_6^3 \\
& - 338a_2^2a_3^2a_4a_5^3a_6^2 - 57a_2^2a_3^2a_5^5a_6 + 297a_2^2a_3a_4^4a_6^3 - 1696a_2^2a_3a_4^3a_5^2a_6^2 \\
& - 316a_2^2a_3a_4^2a_5^4a_6 + 706a_2^2a_4^5a_5a_6^2 + 187a_2^2a_4^4a_5^3a_6 - a_2^2a_4^3a_5^5 \\
& - 868a_2a_3^5a_6^4 - 4182a_2a_3^4a_4a_5a_6^3 - 32a_2a_3^4a_5^3a_6^2 - 2401a_2a_3^3a_4^3a_6^3 \\
& + 1666a_2a_3^3a_4^2a_5^2a_6^2 + 161a_2a_3^3a_4a_5^4a_6 - 1037a_2a_3^2a_4^4a_5a_6^2 + 244a_2a_3^2a_4^3a_5^3a_6 \\
& - 2a_2a_3^2a_4^2a_5^5 - 210a_2a_3a_4^6a_6^2 - 21a_2a_3a_4^5a_5^2a_6 + 21a_2a_3a_4^4a_5^4 \\
& - 36a_2a_4^7a_5a_6 - 8a_2a_4^6a_5^3 + 784a_3^6a_5a_6^3 - 126a_3^5a_4^2a_6^3 \\
& - 456a_3^5a_4a_5^2a_6^2 + 36a_3^5a_5^4a_6 + 462a_3^4a_4^3a_5a_6^2 - 156a_3^4a_4^2a_5^3a_6 \\
& - 195a_3^3a_4^5a_6^2 + 18a_3^3a_4^4a_5^2a_6 + 6a_3^3a_4^3a_5^4 + 54a_3^2a_4^6a_5a_6 \\
& - 18a_3^2a_4^5a_5^3 - 18a_3a_4^8a_6 + 12a_3a_4^7a_5^2 - 2a_4^9a_5 \\
& - 258297a_0^3a_4a_6^5 + 4918a_0^3a_5^2a_6^4 + 235209a_0^2a_1a_3a_6^5 + 27856a_0^2a_1a_4a_5a_6^4 \\
& - 51637a_0^2a_1a_5^3a_6^3 - 88177a_0^2a_2^2a_6^5 - 88912a_0^2a_2a_3a_5a_6^4 + 77682a_0^2a_2a_4^2a_6^4 \\
& - 42069a_0^2a_2a_4a_5^2a_6^3 + 16034a_0^2a_2a_5^4a_6^2 - 56772a_0^2a_3^2a_4a_6^4 - 19665a_0^2a_3^2a_5^2a_6^3 \\
& + 137606a_0^2a_3a_4^2a_5a_6^3 + 11982a_0^2a_3a_4a_5^3a_6^2 + 412a_0^2a_3a_5^5a_6 + 16050a_0^2a_4^4a_6^3 \\
& - 33162a_0^2a_4^3a_5^2a_6^2 - 2017a_0^2a_4^2a_5^4a_6 - 364a_0^2a_4a_5^6 - 30643a_0a_1^2a_2a_5^5 \\
& + 391a_0a_1^2a_3a_5a_6^4 - 52802a_0a_1^2a_4^2a_6^4 + 33008a_0a_1^2a_4a_5^2a_6^3 - 1922a_0a_1^2a_5^4a_6^2 \\
& + 69855a_0a_1a_2^2a_5a_6^4 - 2366a_0a_1a_2a_3a_4a_6^4 - 22359a_0a_1a_2a_3a_5^2a_6^3 + 21506a_0a_1a_2a_4^2a_5a_6^3 \\
& - 21508a_0a_1a_2a_4a_5^3a_6^2 + 728a_0a_1a_2a_5^5a_6 - 26617a_0a_1a_3^3a_6^4 - 72343a_0a_1a_3^2a_4a_5a_6^3 \\
& + 4782a_0a_1a_3^2a_5^3a_6^2 + 2684a_0a_1a_3a_4^3a_6^3 + 7842a_0a_1a_3a_4^2a_5^2a_6^2 + 7738a_0a_1a_3a_4a_5^4a_6 \\
& + 92a_0a_1a_3a_5^6 + 14472a_0a_1a_4^4a_5a_6^2 - 3190a_0a_1a_4^3a_5^3a_6 + 358a_0a_1a_4^2a_5^5 \\
& - 27070a_0a_2^3a_4a_6^4 + 12218a_0a_2^3a_5^2a_6^3 + 25659a_0a_2^2a_3^2a_6^4 - 41487a_0a_2^2a_3a_4a_5a_6^3 \\
& + 2268a_0a_2^2a_3a_5^3a_6^2 - 11587a_0a_2^2a_4^3a_6^3 + 7481a_0a_2^2a_4^2a_5^2a_6^2 + 399a_0a_2^2a_4a_5^4a_6 \\
& - 70a_0a_2^2a_5^6 + 20318a_0a_2a_3^3a_5a_6^3 + 6469a_0a_2a_3^2a_4^2a_6^3 - 5220a_0a_2a_3^2a_4a_5^2a_6^2 \\
& - 256a_0a_2a_3^2a_5^4a_6 - 20746a_0a_2a_3a_4^3a_5a_6^2 + 428a_0a_2a_3a_4^2a_5^3a_6 + 148a_0a_2a_3a_4a_5^5
\end{aligned}$$

$$\begin{aligned}
& -656a_0a_2a_4^5a_6^2 + 3161a_0a_2a_4^4a_5^2a_6 + 190a_0a_2a_4^3a_5^4 + 9808a_0a_4^3a_5a_6^3 \\
& + 3372a_0a_3^4a_5^2a_6^2 - 7506a_0a_3^3a_4^2a_5a_6^2 - 1700a_0a_3^3a_4a_5^3a_6 + 84a_0a_3^3a_5^5 \\
& - 204a_0a_3^2a_4^4a_6^2 + 590a_0a_3^2a_4^3a_5^2a_6 - 190a_0a_3^2a_4^2a_5^4 - 3964a_0a_3a_4^5a_5a_6 \\
& + 44a_0a_3a_4^4a_5^3 + 163a_0a_4^7a_6 - 88a_0a_4^6a_5^2 + 20825a_1^4a_6^5 \\
& - 7116a_1^3a_2a_5a_6^4 + 15245a_1^3a_3a_4a_6^4 - 14408a_1^3a_3a_5^2a_6^3 + 18699a_1^3a_4^2a_5a_6^3 \\
& - 674a_1^3a_4a_5^3a_6^2 + 518a_1^3a_5^5a_6 + 39565a_1^2a_2^2a_4a_6^4 - 6501a_1^2a_2^2a_5^2a_6^3 \\
& - 3981a_1^2a_2^2a_5^4a_6 + 60013a_1^2a_2a_3a_4a_5a_6^3 - 7970a_1^2a_2a_3a_5^3a_6^2 + 8535a_1^2a_2a_4^3a_6^3 \\
& - 22122a_1^2a_2a_4^2a_5^2a_6^2 - 2967a_1^2a_2a_4a_5^4a_6 + 2a_1^2a_2a_5^6 - 2636a_1^2a_3^3a_5a_6^3 \\
& - 1969a_1^2a_3^2a_4^2a_6^3 - 8976a_1^2a_3^2a_4a_5^2a_6^2 - 448a_1^2a_3^2a_5^4a_6 + 4886a_1^2a_3a_4^3a_5a_6^2 \\
& - 3712a_1^2a_3a_4^2a_5^3a_6 - 1274a_1^2a_4^5a_6^2 - 359a_1^2a_4^4a_5^2a_6 - 36a_1^2a_4^3a_5^4 \\
& - 14931a_1a_2^3a_3a_6^4 - 41185a_1a_2^3a_4a_5a_6^3 + 5996a_1a_2^3a_5^3a_6^2 - 14492a_1a_2^2a_3^2a_5a_6^3 \\
& - 15993a_1a_2^2a_3a_4^2a_6^3 + 12064a_1a_2^2a_3a_4a_5^2a_6^2 + 1050a_1a_2^2a_3a_5^4a_6 + 3423a_1a_2^2a_4^3a_5a_6^2 \\
& + 6114a_1a_2^2a_4^2a_5^3a_6 - 24a_1a_2^2a_4a_5^5 - 3214a_1a_2a_3^3a_4a_6^3 + 1172a_1a_2a_3^3a_5^2a_6^2 \\
& - 4200a_1a_2a_3^2a_4^2a_5a_6^2 - 376a_1a_2a_3^2a_4a_5^3a_6 + 4a_1a_2a_3^2a_5^5 + 1384a_1a_2a_3a_4^4a_6^2 \\
& - 740a_1a_2a_3a_4^3a_5^2a_6 + 30a_1a_2a_3a_4^2a_5^4 + 806a_1a_2a_4^5a_5a_6 - 100a_1a_2a_4^4a_5^3 \\
& - 1168a_1a_3^5a_6^3 + 2496a_1a_3^4a_4a_5a_6^2 + 240a_1a_3^4a_5^3a_6 + 1482a_1a_3^3a_4^3a_6^2 \\
& + 772a_1a_3^3a_4^2a_5^2a_6 + 48a_1a_3^3a_4a_5^4 + 648a_1a_3^2a_4^4a_5a_6 + 56a_1a_3^2a_4^3a_5^3 \\
& + 338a_1a_3a_4^6a_6 + 64a_1a_3a_4^5a_5^2 + 24a_1a_4^7a_5 - 2208a_2^5a_6^4 \\
& + 11771a_2^4a_3a_5a_6^3 + 3441a_2^4a_4^2a_6^3 + 640a_2^4a_4a_5^2a_6^2 - 423a_2^4a_5^4a_6 \\
& + 2586a_2^3a_3^2a_4a_6^3 - 814a_2^3a_3^2a_5^2a_6^2 + 4059a_2^3a_3a_4^2a_5a_6^2 - 690a_2^3a_3a_4a_5^3a_6 \\
& + 366a_2^3a_4^4a_6^2 - 1312a_2^3a_4^3a_5^2a_6 - 47a_2^3a_4^2a_5^4 + 2298a_2^2a_3^4a_6^3 \\
& - 2414a_2^2a_3^3a_4a_5a_6^2 - 84a_2^2a_3^3a_5^3a_6 + 617a_2^2a_3^2a_4^3a_6^2 + 618a_2^2a_3^2a_4^2a_5^2a_6 \\
& - 10a_2^2a_3^2a_4a_5^4 + 414a_2^2a_3a_4^4a_5a_6 - 12a_2^2a_3a_4^3a_5^3 - 67a_2^2a_4^6a_6 \\
& + 34a_2^2a_4^5a_5^2 + 336a_2a_3^5a_5a_6^2 - 372a_2a_3^4a_4^2a_6^2 - 40a_2a_3^4a_4a_5^2a_6 \\
& - 882a_2a_3^3a_4^3a_5a_6 + 16a_2a_3^3a_4^2a_5^3 - 59a_2a_3^2a_4^5a_6 - 34a_2a_3^2a_4^4a_5^2 \\
& - 72a_2a_3a_4^6a_5 + 6a_2a_4^8 + 144a_3^6a_4a_6^2 - 144a_3^6a_5^2a_6 \\
& + 240a_3^5a_4^2a_5a_6 - 81a_3^4a_4^4a_6 - 48a_3^4a_4^3a_5^2 + 60a_3^3a_4^5a_5 \\
& - 18a_3^2a_4^7 + 54327a_0^3a_3a_6^4 - 123116a_0^3a_4a_5a_6^3 - 18137a_0^3a_5^3a_6^2 \\
& + 52909a_0^2a_1a_2a_6^4 + 187014a_0^2a_1a_3a_5a_6^3 - 48988a_0^2a_1a_4^2a_6^3 + 79101a_0^2a_1a_4a_5^2a_6^2 \\
& - 1068a_0^2a_1a_5^4a_6 + 105447a_0^2a_2^2a_5a_6^3 - 179662a_0^2a_2a_3a_4a_6^3 - 52767a_0^2a_2a_3a_5^2a_6^2 \\
& + 107082a_0^2a_2a_4^2a_5a_6^2 - 2450a_0^2a_2a_4a_5^3a_6 + 284a_0^2a_2a_5^5 + 21287a_0^2a_3^3a_6^3 \\
& - 55209a_0^2a_3^2a_4a_5a_6^2 - 1058a_0^2a_3^2a_5^3a_6 + 11490a_0^2a_3a_4^3a_6^2 + 15154a_0^2a_3a_4^2a_5^2a_6 \\
& + 2890a_0^2a_3a_4a_5^4 + 5122a_0^2a_4^4a_5a_6 - 2390a_0^2a_4^3a_5^3 - 28288a_0a_1^3a_6^4 \\
& - 113029a_0a_1^2a_2a_5a_6^3 + 39664a_0a_1^2a_3a_4a_6^3 + 18774a_0a_1^2a_3a_5^2a_6^2 - 53007a_0a_1^2a_4^2a_5a_6^2 \\
& + 1442a_0a_1^2a_4a_5^3a_6 + 298a_0a_1^2a_5^5 + 78107a_0a_1a_2^2a_4a_6^3 - 9935a_0a_1a_2^2a_5^2a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 61113a_0a_1a_2a_3^2a_6^3 + 21402a_0a_1a_2a_3a_4a_5a_6^2 - 6246a_0a_1a_2a_3a_5^3a_6 - 34488a_0a_1a_2a_4^3a_6^2 \\
& - 14a_0a_1a_2a_4^2a_5^2a_6 - 1472a_0a_1a_2a_4a_5^4 + 6484a_0a_1a_3^3a_5a_6^2 - 6519a_0a_1a_3^2a_4^2a_6^2 \\
& - 16468a_0a_1a_3^2a_4a_5^2a_6 - 770a_0a_1a_3^2a_5^4 + 5292a_0a_1a_3a_4^3a_5a_6 - 884a_0a_1a_3a_4^2a_5^3 \\
& + 736a_0a_1a_4^5a_6 + 2366a_0a_1a_4^4a_5^2 - 43137a_0a_2^3a_3a_6^3 - 19582a_0a_2^3a_4a_5a_6^2 \\
& + 696a_0a_2^3a_5^3a_6 + 714a_0a_2^2a_3^2a_5a_6^2 + 12030a_0a_2^2a_3a_4^2a_6^2 + 1056a_0a_2^2a_3a_4a_5^2a_6 \\
& + 526a_0a_2^2a_3a_5^4 - 6289a_0a_2^2a_4^3a_5a_6 + 84a_0a_2^2a_4^2a_5^3 + 7892a_0a_2a_3^3a_4a_6^2 \\
& - 1240a_0a_2a_3^3a_5^2a_6 - 3010a_0a_2a_3^2a_4^2a_5a_6 - 788a_0a_2a_3^2a_4a_5^3 + 6854a_0a_2a_3a_4^4a_6 \\
& - 104a_0a_2a_3a_4^3a_5^2 - 400a_0a_2a_4^5a_5 - 4464a_0a_3^5a_6^2 + 2688a_0a_3^4a_4a_5a_6 \\
& - 288a_0a_3^4a_5^3 + 784a_0a_3^3a_4^3a_6 + 1148a_0a_3^3a_4^2a_5^2 - 508a_0a_3^3a_4^4a_5 \\
& + 266a_0a_3a_4^6 + 34418a_1^4a_5a_6^3 - 40371a_1^3a_2a_4a_6^3 + 5248a_1^3a_2a_5^2a_6^2 \\
& - 12750a_1^3a_3a_6^3 - 2982a_1^3a_3a_4a_5a_6^2 - 1780a_1^3a_3a_5^3a_6 + 1983a_1^3a_4^3a_6^2 \\
& + 24a_1^3a_4^2a_5^2a_6 - 54a_1^3a_4a_5^4 + 15948a_1^2a_2^2a_3a_6^3 + 23671a_1^2a_2^2a_4a_5a_6^2 \\
& + 1668a_1^2a_2^2a_5^3a_6 + 7788a_1^2a_2a_3^2a_5a_6^2 + 417a_1^2a_2a_3a_4^2a_6^2 + 12296a_1^2a_2a_3a_4a_5^2a_6 \\
& - 34a_1^2a_2a_3a_5^4 + 8674a_1^2a_2a_4^3a_5a_6 + 84a_1^2a_2a_4^2a_5^3 + 5968a_1^2a_3^3a_4a_6^2 \\
& + 2544a_1^2a_3^2a_5^2a_6 + 1108a_1^2a_3^2a_4^2a_5a_6 + 96a_1^2a_3^2a_4a_5^3 - 2754a_1^2a_3a_4^4a_6 \\
& + 60a_1^2a_3a_4^3a_5^2 - 354a_1^2a_4^5a_5 + 5931a_1a_2^4a_6^3 - 7944a_1a_2^3a_3a_5a_6^2 \\
& + 6456a_1a_2^3a_4^2a_6^2 - 8924a_1a_2^3a_4a_5^2a_6 + 32a_1a_2^3a_5^4 - 6054a_1a_2^2a_3^2a_4a_6^2 \\
& - 3100a_1a_2^2a_3^2a_5^2a_6 - 6422a_1a_2^2a_3a_4^2a_5a_6 - 60a_1a_2^2a_3a_4a_5^3 - 2045a_1a_2^2a_4^4a_6 \\
& - 320a_1a_2^2a_4^3a_5^2 - 1008a_1a_2a_3^4a_6^2 + 480a_1a_2a_3^3a_4a_5a_6 - 32a_1a_2a_3^3a_5^3 \\
& + 1054a_1a_2a_3^2a_4^3a_6 - 84a_1a_2a_3^2a_4^2a_5^2 + 92a_1a_2a_3a_4^4a_5 - 18a_1a_2a_4^6 \\
& - 544a_1a_3^5a_5a_6 - 1312a_1a_3^4a_4^2a_6 - 192a_1a_3^4a_4a_5^2 - 64a_1a_3^3a_4^3a_5 \\
& + 40a_1a_3^2a_4^5 - 619a_2^5a_5a_6^2 - 3969a_2^4a_3a_4a_6^2 + 1858a_2^4a_3a_5^2a_6 \\
& + 1999a_2^4a_4^2a_5a_6 + 130a_2^4a_4a_5^3 + 1220a_2^3a_3^3a_6^2 - 730a_2^3a_3^2a_4a_5a_6 \\
& - 587a_2^3a_3a_4^3a_6 + 108a_2^3a_3a_4^2a_5^2 + 58a_2^3a_4^4a_5 + 480a_2^2a_3^4a_5a_6 \\
& + 1762a_2^2a_3^3a_4^2a_6 + 80a_2^2a_3^3a_4a_5^2 + 56a_2^2a_3^2a_4^3a_5 + 142a_2^2a_3a_4^5 \\
& - 560a_2a_3^5a_4a_6 - 32a_2a_3^4a_4^2a_5 - 136a_2a_3^3a_4^4 + 192a_3^7a_6 \\
& + 96a_3^5a_4^3 - 126023a_0^3a_2a_6^3 + 36163a_0^3a_3a_5a_6^2 + 24548a_0^3a_4^2a_6^2 \\
& + 23867a_0^3a_4a_5^2a_6 + 1006a_0^3a_5^4 + 76088a_0^2a_1^3a_6^3 - 53508a_0^2a_1a_2a_5a_6^2 \\
& - 50397a_0^2a_1a_3a_4a_6^2 - 754a_0^2a_1a_3a_5^2a_6 - 54626a_0^2a_1a_4^2a_5a_6 + 1320a_0^2a_1a_4a_5^3 \\
& - 149694a_0^2a_2^2a_4a_6^2 + 7475a_0^2a_2^2a_5^2a_6 + 57894a_0^2a_2a_3^2a_6^2 - 29321a_0^2a_2a_3a_4a_5a_6 \\
& - 996a_0^2a_2a_3a_5^3 + 6022a_0^2a_2a_4^3a_6 + 7406a_0^2a_2a_4^2a_5^2 - 2872a_0^2a_3^3a_5a_6 \\
& + 2979a_0^2a_3^2a_4^2a_6 - 7640a_0^2a_3^2a_4a_5^2 + 7612a_0^2a_3a_4^3a_5 - 1088a_0^2a_4^5 \\
& + 15620a_0a_1^3a_5a_6^2 + 61986a_0a_1^2a_2a_4a_6^2 - 7314a_0a_1^2a_2a_5^2a_6 - 35880a_0a_1^2a_3^2a_6^2 \\
& - 2348a_0a_1^2a_3a_4a_5a_6 - 2392a_0a_1^2a_3a_5^3 + 8003a_0a_1^2a_4^3a_6 - 1480a_0a_1^2a_4^2a_5^2 \\
& + 35025a_0a_1a_2^2a_3a_6^2 + 39201a_0a_1a_2^2a_4a_5a_6 + 1008a_0a_1a_2^2a_5^3 + 16112a_0a_1a_2a_3^2a_5a_6
\end{aligned}$$

$$\begin{aligned}
& -16417a_0a_1a_2a_3a_4^2a_6 + 5588a_0a_1a_2a_3a_4a_5^2 - 7108a_0a_1a_2a_4^3a_5 + 9888a_0a_1a_3^3a_4a_6 \\
& + 2672a_0a_1a_3^3a_5^2 - 580a_0a_1a_3^2a_4^2a_5 - 2508a_0a_1a_3a_4^4 + 21770a_0a_2^4a_6^2 \\
& - 3310a_0a_2^3a_3a_5a_6 - 3894a_0a_2^3a_4^2a_6 - 1092a_0a_2^3a_4a_5^2 - 4760a_0a_2^2a_3^2a_4a_6 \\
& - 1828a_0a_2^2a_3^2a_5^2 - 1268a_0a_2^2a_3a_4^2a_5 + 1390a_0a_2^2a_4^4 + 1376a_0a_2a_3^4a_6 \\
& + 912a_0a_2a_3^3a_4a_5 + 2276a_0a_2a_3^2a_4^3 + 320a_0a_3^5a_5 - 1552a_0a_3^4a_4^2 \\
& - 7708a_1^4a_4a_6^2 + 3814a_1^4a_3^2a_6 - 13756a_1^3a_2a_3a_6^2 - 11750a_1^3a_2a_4a_5a_6 \\
& - 72a_1^3a_2a_5^3 - 2416a_1^3a_3^2a_5a_6 + 6178a_1^3a_3a_4^2a_6 + 208a_1^3a_3a_4a_5^2 \\
& + 284a_1^3a_4^3a_5 - 8633a_1^2a_2^3a_6^2 - 3380a_1^2a_2^2a_3a_5a_6 - 8043a_1^2a_2^2a_4^2a_6 \\
& + 120a_1^2a_2^2a_4a_5^2 - 4192a_1^2a_2a_3^2a_4a_6 + 208a_1^2a_2a_3^2a_5^2 - 180a_1^2a_2a_3a_4^2a_5 \\
& + 414a_1^2a_2a_4^4 - 1472a_1^2a_3^4a_6 - 256a_1^2a_3^3a_4a_5 + 48a_1^2a_3^2a_4^3 \\
& + 2106a_1a_2^4a_5a_6 + 8430a_1a_2^3a_3a_4a_6 - 100a_1a_2^3a_3a_5^2 + 1308a_1a_2^3a_4^2a_5 \\
& + 2032a_1a_2^2a_3^3a_6 + 368a_1a_2^2a_3^2a_4a_5 - 380a_1a_2^2a_3a_4^3 + 64a_1a_2a_3^4a_5 \\
& - 144a_1a_2a_3^3a_4^2 + 256a_1a_3^5a_4 - 407a_2^5a_4a_6 - 42a_2^5a_5^2 \\
& - 1377a_2^4a_3^2a_6 - 436a_2^4a_3a_4a_5 - 256a_2^4a_4^3 + 192a_2^3a_3^2a_4^2 \\
& - 160a_2^2a_3^4a_4 + 119112a_0^3a_1a_6^2 - 33929a_0^3a_2a_5a_6 - 84070a_0^3a_3a_4a_6 \\
& - 6272a_0^3a_3a_5^2 + 3108a_0^3a_4^2a_5 + 20930a_0^2a_1^2a_5a_6 + 83471a_0^2a_1a_2a_4a_6 \\
& - 4644a_0^2a_1a_2a_5^2 + 24840a_0^2a_1a_3^2a_6 - 4264a_0^2a_1a_3a_4a_5 + 732a_0^2a_1a_4^3 \\
& + 17783a_0^2a_2^2a_3a_6 - 3580a_0^2a_2^2a_4a_5 + 208a_0^2a_2a_3^2a_5 - 16228a_0^2a_2a_3a_4^2 \\
& + 6080a_0^2a_3^3a_4 - 12028a_0a_1^3a_4a_6 + 2816a_0a_1^3a_5^2 - 9544a_0a_1^2a_2a_3a_6 \\
& - 2916a_0a_1^2a_2a_4a_5 + 4416a_0a_1^2a_3^2a_5 + 6592a_0a_1^2a_3a_4^2 - 25841a_0a_1a_2^3a_6 \\
& - 3220a_0a_1a_2^2a_3a_5 + 5094a_0a_1a_2^2a_4^2 - 2720a_0a_1a_2a_3^2a_4 - 3232a_0a_1a_3^4 \\
& + 1224a_0a_2^4a_5 + 708a_0a_2^3a_3a_4 + 2608a_0a_2^2a_3^3 + 888a_1^4a_3a_6 \\
& - 384a_1^4a_4a_5 + 8162a_1^3a_2^2a_6 + 416a_1^3a_2a_3a_5 + 480a_1^3a_2a_4^2 \\
& + 32a_1^3a_3^2a_4 - 228a_1^2a_2^3a_5 - 912a_1^2a_2^2a_3a_4 - 416a_1^2a_2a_3^3 \\
& - 516a_1a_2^4a_4 + 144a_1a_2^3a_3^2 + 104a_2^5a_3 + 21927a_0^4a_6 \\
& + 8748a_0^3a_1a_5 - 9032a_0^3a_2a_4 + 5984a_0^3a_3^2 - 2656a_0^2a_1^2a_4 \\
& + 15296a_0^2a_1a_2a_3 - 1492a_0^2a_2^3 - 7296a_0a_1^3a_3 + 5632a_0a_1^2a_2^2 \\
& - 1152a_1^4a_2
\end{aligned} \tag{A.29}$$

$$\begin{aligned}
e_{30} = & 396a_0^2a_4a_5a_6^{11} + 738a_0^2a_3^3a_6^{10} - 1026a_0a_1a_3a_5a_6^{11} + 552a_0a_1a_4a_5^2a_6^{10} \\
& - 436a_0a_1a_3^4a_6^9 + 240a_0a_2^2a_5a_6^{11} + 102a_0a_2a_3a_5^2a_6^{10} - 108a_0a_2a_4^2a_5a_6^{10} \\
& + 580a_0a_2a_4a_5^3a_6^9 + 710a_0a_2a_5^5a_6^8 + 558a_0a_3^2a_4a_5a_6^{10} + 708a_0a_3^2a_5^3a_6^9 \\
& - 690a_0a_3a_4^2a_5^2a_6^9 - 1092a_0a_3a_4a_5^4a_6^8 - 57a_0a_3a_5^6a_6^7 + 30a_0a_4^4a_5a_6^9 \\
& + 320a_0a_4^3a_5^3a_6^8 + 822a_0a_4^2a_5^5a_6^7 + 288a_0a_4a_5^7a_6^6 + 11a_0a_5^9a_6^5
\end{aligned}$$

$$\begin{aligned}
& + 282a_1^2a_2a_5a_6^{11} + 30a_1^2a_3a_5^2a_6^{10} - 132a_1^2a_4^2a_5a_6^{10} - 604a_1^2a_4a_5^3a_6^9 \\
& - 675a_1^2a_5^5a_6^8 - 228a_1a_2^2a_5^2a_6^{10} - 246a_1a_2a_3a_4a_5a_6^{10} - 768a_1a_2a_3a_5^3a_6^9 \\
& + 270a_1a_2a_4^2a_5^2a_6^9 + 776a_1a_2a_4a_5^4a_6^8 + 203a_1a_2a_5^6a_6^7 - 108a_1a_3^3a_5a_6^{10} \\
& + 246a_1a_3^2a_4a_5^2a_6^9 + 235a_1a_3^2a_5^4a_6^8 + 96a_1a_3a_4^3a_5a_6^9 + 322a_1a_3a_4^2a_5^3a_6^8 \\
& + 72a_1a_3a_4a_5^5a_6^7 - 5a_1a_3a_5^7a_6^6 - 69a_1a_4^4a_5^2a_6^8 - 302a_1a_4^3a_5^4a_6^7 \\
& - 166a_1a_4^2a_5^6a_6^6 - 13a_1a_4a_5^8a_6^5 + 48a_2^3a_4a_5a_6^{10} + 186a_2^3a_5^3a_6^9 \\
& - 6a_2^2a_3^2a_5a_6^{10} + 18a_2^2a_3a_4a_5^2a_6^9 + 34a_2^2a_3a_5^4a_6^8 - 36a_2^2a_4^3a_5a_6^9 \\
& - 188a_2^2a_4^2a_5^3a_6^8 - 104a_2^2a_4a_5^5a_6^7 - 8a_2^2a_5^7a_6^6 + 42a_2a_3^3a_5^2a_6^9 \\
& + 12a_2a_3^2a_4^2a_5a_6^9 + 38a_2a_3^2a_4a_5^3a_6^8 - 15a_2a_3^2a_5^5a_6^7 - 12a_2a_3a_4^3a_5^2a_6^8 \\
& - 120a_2a_3a_4^2a_5^4a_6^7 - 52a_2a_3a_4a_5^6a_6^6 - 2a_2a_3a_5^8a_6^5 + 6a_2a_4^5a_5a_6^8 \\
& + 17a_2a_4^4a_5^3a_6^7 - 4a_2a_4^3a_5^5a_6^6 - 2a_2a_4^2a_5^7a_6^5 + 18a_3^4a_4a_5a_6^9 \\
& + 3a_3^4a_5^3a_6^8 - 54a_3^3a_4^2a_5^2a_6^8 - 63a_3^3a_4a_5^4a_6^7 - 2a_3^3a_5^6a_6^6 \\
& - 9a_3^2a_4^4a_5a_6^8 - 12a_3^2a_4^3a_5^3a_6^7 + 54a_3^2a_4^2a_5^5a_6^6 + 12a_3^2a_4a_5^7a_6^5 \\
& + 9a_3a_4^5a_5^2a_6^7 + 60a_3a_4^4a_5^4a_6^6 + 30a_3a_4^3a_5^6a_6^5 + 4a_4^6a_5^3a_6^6 \\
& + 6a_4^5a_5^5a_6^5 + 1800a_0^2a_2a_6^{11} - 270a_0^2a_3a_5a_6^{10} - 306a_0^2a_4^2a_6^{10} \\
& + 2508a_0^2a_4a_5^2a_6^9 + 2071a_0^2a_5^4a_6^8 + 990a_0a_1^2a_6^{11} - 1278a_0a_1a_2a_5a_6^{10} \\
& - 1584a_0a_1a_3a_4a_6^{10} - 12243a_0a_1a_3a_5^2a_6^9 + 978a_0a_1a_4^2a_5a_6^9 + 8642a_0a_1a_4a_5^3a_6^8 \\
& - 2746a_0a_1a_5^5a_6^7 - 240a_0a_2^2a_4a_6^{10} + 3396a_0a_2^2a_5^2a_6^9 - 90a_0a_2a_3^2a_6^{10} \\
& + 132a_0a_2a_3a_4a_5a_6^9 - 343a_0a_2a_3a_5^3a_6^8 - 12a_0a_2a_4^3a_6^9 - 3270a_0a_2a_4^2a_5^2a_6^8 \\
& - 64a_0a_2a_4a_5^4a_6^7 + 1684a_0a_2a_5^6a_6^6 - 405a_0a_3^3a_5a_6^9 + 252a_0a_3^2a_4^2a_6^9 \\
& + 5715a_0a_3^2a_4a_5^2a_6^8 + 2510a_0a_3^2a_5^4a_6^7 + 144a_0a_3a_4^3a_5a_6^8 - 5734a_0a_3a_4^2a_5^3a_6^7 \\
& - 4173a_0a_3a_4a_5^5a_6^6 - 34a_0a_3a_5^7a_6^5 - 171a_0a_4^4a_5^2a_6^7 - 538a_0a_4^3a_5^4a_6^6 \\
& + 556a_0a_4^2a_5^6a_6^5 + 121a_0a_4a_5^8a_6^4 - 447a_1^3a_5a_6^{10} + 258a_1^2a_2a_4a_6^{10} \\
& + 3984a_1^2a_2a_5^2a_6^9 + 765a_1^2a_3^2a_6^{10} - 405a_1^2a_3a_4a_5a_6^9 + 1942a_1^2a_3a_5^3a_6^8 \\
& - 18a_1^2a_4^3a_6^9 + 57a_1^2a_4^2a_5^2a_6^8 - 250a_1^2a_4a_5^4a_6^7 - 1523a_1^2a_5^6a_6^6 \\
& - 360a_1a_2^2a_3a_6^{10} + 438a_1a_2^2a_4a_5a_6^9 - 2895a_1a_2^2a_5^3a_6^8 + 447a_1a_2a_3^2a_5a_6^9 \\
& + 156a_1a_2a_3a_4^2a_6^9 - 1719a_1a_2a_3a_4a_5^2a_6^8 - 3687a_1a_2a_3a_5^4a_6^7 - 384a_1a_2a_4^3a_5a_6^8 \\
& + 640a_1a_2a_4^2a_5^3a_6^7 + 2065a_1a_2a_4a_5^5a_6^6 + 275a_1a_2a_5^7a_6^5 - 252a_1a_3^3a_4a_6^9 \\
& - 1593a_1a_3^3a_5^2a_6^8 - 162a_1a_3^2a_4^2a_5a_6^8 + 1519a_1a_3^2a_4a_5^3a_6^7 + 708a_1a_3^2a_5^5a_6^6 \\
& - 36a_1a_3a_4^4a_6^8 + 258a_1a_3a_4^3a_5^2a_6^7 + 405a_1a_3a_4^2a_5^4a_6^6 - 85a_1a_3a_4a_5^6a_6^5 \\
& - 10a_1a_3a_5^8a_6^4 + 87a_1a_4^5a_5a_6^7 + 258a_1a_4^4a_5^3a_6^6 - 136a_1a_4^3a_5^5a_6^5 \\
& - 58a_1a_4^2a_5^7a_6^4 + 240a_2^4a_6^{10} - 396a_2^3a_3a_5a_6^9 - 168a_2^3a_4^2a_6^9 \\
& + 324a_2^3a_4a_5^2a_6^8 + 1082a_2^3a_5^4a_6^7 + 186a_2^2a_3^2a_4a_6^9 - 9a_2^2a_3^2a_5^2a_6^8 \\
& + 18a_2^2a_3a_4^2a_5a_6^8 + 182a_2^2a_3a_4a_5^3a_6^7 + 77a_2^2a_3a_5^5a_6^6 + 51a_2^2a_4^4a_6^8 \\
& - 521a_2^2a_4^2a_5^4a_6^6 - 164a_2^2a_4a_5^6a_6^5 - 5a_2^2a_5^8a_6^4 - 90a_2a_3^4a_6^9
\end{aligned}$$

$$\begin{aligned}
& + 427a_2a_3^3a_5^3a_6^7 - 72a_2a_3^2a_4^3a_6^8 + 66a_2a_3^2a_4^2a_5^2a_6^7 + 45a_2a_3^2a_4a_5^4a_6^6 \\
& - 47a_2a_3^2a_5^6a_6^5 + 57a_2a_3a_4^4a_5a_6^7 + 352a_2a_3a_4^3a_5^3a_6^6 + 6a_2a_3a_4^2a_5^5a_6^5 \\
& - 12a_2a_3a_4a_5^7a_6^4 - 6a_2a_4^6a_6^7 + 27a_2a_4^5a_5^2a_6^6 + 91a_2a_4^4a_5^4a_6^5 \\
& + 18a_2a_4^3a_5^6a_6^4 + 27a_3^5a_5a_6^8 + 27a_3^4a_4^2a_6^8 + 207a_3^4a_4a_5^2a_6^7 \\
& - 36a_3^4a_4^5a_6^6 + 54a_3^3a_4^3a_5a_6^7 - 315a_3^3a_4^2a_5^3a_6^6 - 105a_3^3a_4a_5^5a_6^5 \\
& + 5a_3^3a_5^7a_6^4 + 9a_3^2a_4^5a_6^7 - 72a_3^2a_4^4a_5^2a_6^6 - 126a_3^2a_4^3a_5^4a_6^5 \\
& + 42a_3^2a_4^2a_5^6a_6^4 - 18a_3a_4^6a_5a_6^6 - 147a_3a_4^5a_5^3a_6^5 - 45a_3a_4^4a_5^5a_6^4 \\
& - 12a_4^7a_5^2a_6^5 - 19a_4^6a_5^4a_6^4 - 3780a_0^2a_1a_6^{10} + 16092a_0^2a_2a_5a_6^9 \\
& + 3096a_0^2a_3a_4a_6^9 + 1983a_0^2a_3a_5^2a_6^8 - 7212a_0^2a_4^2a_5a_6^8 - 6178a_0^2a_4a_3^3a_6^7 \\
& - 1593a_0^2a_5^5a_6^6 + 9405a_0a_1^2a_5a_6^9 + 2808a_0a_1a_2a_4a_6^9 - 13161a_0a_1a_2a_5^2a_6^8 \\
& + 144a_0a_1a_3^2a_6^9 - 2928a_0a_1a_3a_4a_5a_6^8 - 12140a_0a_1a_3a_5^3a_6^7 - 1110a_0a_1a_4^3a_6^8 \\
& - 534a_0a_1a_4^2a_5^2a_6^7 + 19826a_0a_1a_4a_5^4a_6^6 - 3693a_0a_1a_5^6a_6^5 + 780a_0a_2^2a_3a_6^9 \\
& - 6702a_0a_2^2a_4a_5a_6^8 + 9114a_0a_2^2a_5^3a_6^7 - 582a_0a_2a_3^2a_5a_6^8 - 1512a_0a_2a_3a_4^2a_6^8 \\
& + 3642a_0a_2a_3a_4a_5^2a_6^7 - 5111a_0a_2a_3a_5^4a_6^6 + 4300a_0a_2a_4^3a_5a_6^7 - 8008a_0a_2a_4^2a_5^3a_6^6 \\
& - 2508a_0a_2a_4a_5^5a_6^5 + 848a_0a_2a_5^7a_6^4 - 297a_0a_3^3a_4a_6^8 - 3693a_0a_3^3a_5^2a_6^7 \\
& - 2520a_0a_3^2a_4^2a_5a_6^7 + 9995a_0a_3^2a_4a_5^3a_6^6 + 889a_0a_3^2a_5^5a_6^5 + 366a_0a_3a_4^4a_6^7 \\
& + 8134a_0a_3a_4^3a_5^2a_6^6 - 1757a_0a_3a_4^2a_5^4a_6^5 - 1610a_0a_3a_4a_5^6a_6^4 + 51a_0a_3a_5^8a_6^3 \\
& - 157a_0a_4^5a_5a_6^6 - 119a_0a_4^4a_5^3a_6^5 - 1392a_0a_4^3a_5^5a_6^4 - 82a_0a_4^2a_5^7a_6^3 \\
& - 93a_1^3a_4a_6^9 - 5468a_1^3a_5^2a_6^8 - 1068a_1^2a_2a_3a_6^9 - 1959a_1^2a_2a_4a_5a_6^8 \\
& + 8422a_1^2a_2a_3^3a_6^7 + 6972a_1^2a_3^2a_5a_6^8 + 783a_1^2a_3a_4^2a_6^8 - 8358a_1^2a_3a_4a_5^2a_6^7 \\
& + 7079a_1^2a_3a_5^4a_6^6 - 268a_1^2a_4^3a_5a_6^7 + 2880a_1^2a_4^2a_5^3a_6^6 + 2865a_1^2a_4a_5^5a_6^5 \\
& - 630a_1^2a_5^7a_6^4 - 600a_1a_2^3a_6^9 - 2814a_1a_2^2a_3a_5a_6^8 + 240a_1a_2^2a_4^2a_6^8 \\
& + 8266a_1a_2^2a_4a_5^2a_6^7 - 6172a_1a_2^2a_5^4a_6^6 + 207a_1a_2a_3^2a_4a_6^8 + 4869a_1a_2a_3^2a_5^2a_6^7 \\
& + 3342a_1a_2a_3a_4^2a_5a_6^7 + 186a_1a_2a_3a_4a_5^3a_6^6 - 3064a_1a_2a_3a_5^5a_6^5 + 24a_1a_2a_4^4a_6^7 \\
& - 4224a_1a_2a_4^3a_5^2a_6^6 - 2939a_1a_2a_4^2a_5^4a_6^5 + 567a_1a_2a_4a_5^6a_6^4 + 40a_1a_2a_5^8a_6^3 \\
& + 342a_1a_3^4a_6^8 - 1182a_1a_3^3a_4a_5a_6^7 - 3914a_1a_3^3a_5^3a_6^6 - 168a_1a_3^2a_4^3a_6^7 \\
& - 1491a_1a_3^2a_4^2a_5^2a_6^6 + 1351a_1a_3^2a_4a_5^4a_6^5 + 328a_1a_3^2a_5^6a_6^4 - 500a_1a_3a_4^4a_5a_6^6 \\
& - 286a_1a_3a_4^3a_5^3a_6^5 - 13a_1a_3a_4^2a_5^5a_6^4 - 61a_1a_3a_4a_5^7a_6^3 - 18a_1a_4^6a_6^6 \\
& + 291a_1a_5^4a_5^2a_6^5 + 717a_1a_4^4a_5^4a_6^4 + 94a_1a_4^3a_5^6a_6^3 + 2482a_2^4a_5a_6^8 \\
& + 324a_2^3a_3a_4a_6^8 - 3966a_2^3a_3a_5^2a_6^7 - 2288a_2^3a_4^2a_5a_6^7 - 516a_2^3a_4a_5^3a_6^6 \\
& + 1186a_2^3a_5^5a_6^5 + 174a_2^2a_3^3a_6^8 + 1368a_2^2a_3^2a_4a_5a_6^7 + 627a_2^2a_3^2a_5^3a_6^6 \\
& - 12a_2^2a_3a_4^3a_6^7 + 16a_2^2a_3a_4^2a_5^2a_6^6 - 98a_2^2a_3a_4a_5^4a_6^5 - 30a_2^2a_3a_5^6a_6^4 \\
& + 592a_2^2a_4^4a_5a_6^6 + 916a_2^2a_4^3a_5^3a_6^5 - 62a_2^2a_4^2a_5^5a_6^4 - 24a_2^2a_4a_5^7a_6^3 \\
& - 1089a_2a_3^4a_5a_6^7 - 30a_2a_3^3a_4^2a_6^7 - 123a_2a_3^3a_4a_5^2a_6^6 + 617a_2a_3^3a_5^4a_6^5 \\
& - 832a_2a_3^2a_4^3a_5a_6^6 + 55a_2a_3^2a_4^2a_5^3a_6^5 - 47a_2a_3^2a_4a_5^5a_6^4 - 12a_2a_3^2a_5^7a_6^3
\end{aligned}$$

$$\begin{aligned}
& -39a_2a_3a_4^5a_6^6 - 15a_2a_3a_4^4a_5^2a_6^5 + 454a_2a_3a_4^3a_5^4a_6^4 + 50a_2a_3a_4^2a_5^6a_6^3 \\
& - 87a_2a_4^6a_5a_6^5 - 129a_2a_4^5a_3^3a_6^4 - 14a_2a_4^4a_5^5a_6^3 - 99a_3^5a_4a_6^7 \\
& + 309a_3^5a_5^2a_6^6 + 219a_3^4a_4^2a_5a_6^6 + 159a_3^4a_4a_5^3a_6^5 - 81a_3^4a_5^5a_6^4 \\
& - 9a_3^3a_4^4a_6^6 + 507a_3^3a_4^3a_5^2a_6^5 - 171a_3^3a_4^2a_5^4a_6^4 + 26a_3^3a_4a_5^6a_6^3 \\
& + 171a_3^2a_4^5a_5a_6^5 + 57a_3^2a_4^4a_5^3a_6^4 - 66a_3^2a_4^3a_5^5a_6^3 + 9a_3a_4^7a_6^5 \\
& + 111a_3a_4^6a_5^2a_6^4 - 24a_3a_4^5a_5^4a_6^3 + 12a_4^8a_5a_6^4 + 20a_4^7a_5^3a_6^3 \\
& + 31910a_0^3a_6^9 - 50905a_0^2a_1a_5a_6^8 - 19418a_0^2a_2a_4a_6^8 + 11403a_0^2a_2a_5^2a_6^7 \\
& - 23955a_0^2a_3^2a_6^8 + 64563a_0^2a_3a_4a_5a_6^7 + 9185a_0^2a_3a_5^3a_6^6 + 5828a_0^2a_4^3a_6^7 \\
& - 21037a_0^2a_4^2a_5^2a_6^6 - 7596a_0^2a_4a_5^4a_6^5 - 1526a_0^2a_5^6a_6^4 + 2137a_0a_1^2a_4a_6^8 \\
& + 9000a_0a_1^2a_5^2a_6^7 + 7788a_0a_1a_2a_3a_6^8 + 29676a_0a_1a_2a_4a_5a_6^7 - 19780a_0a_1a_2a_5^3a_6^6 \\
& - 10651a_0a_1a_3^2a_5a_6^7 - 3445a_0a_1a_3a_4^2a_6^7 + 10898a_0a_1a_3a_4a_5^2a_6^6 + 17363a_0a_1a_3a_5^4a_6^5 \\
& - 6920a_0a_1a_4^3a_5a_6^6 - 22598a_0a_1a_4^2a_5^3a_6^5 + 9755a_0a_1a_4a_5^5a_6^4 - 892a_0a_1a_5^7a_6^3 \\
& + 14830a_0a_2^3a_6^8 - 13777a_0a_2^2a_3a_5a_6^7 - 3866a_0a_2^2a_4^2a_6^7 - 12692a_0a_2^2a_4a_5^2a_6^6 \\
& + 5685a_0a_2^2a_5^4a_6^5 + 1828a_0a_2a_3^2a_4a_6^7 + 13948a_0a_2a_3^2a_5^2a_6^6 - 11433a_0a_2a_3a_4^2a_5a_6^6 \\
& + 2830a_0a_2a_3a_4a_5^3a_6^5 - 5074a_0a_2a_3a_5^5a_6^4 - 906a_0a_2a_4^4a_6^6 + 18400a_0a_2a_4^3a_5^2a_6^5 \\
& + 1077a_0a_2a_4^2a_5^4a_6^4 - 612a_0a_2a_4a_5^6a_6^3 + 81a_0a_2a_5^8a_6^2 + 5415a_0a_3^4a_6^7 \\
& - 7389a_0a_3^3a_4a_5a_6^6 - 3131a_0a_3^3a_5^3a_6^5 + 879a_0a_3^2a_4^3a_6^6 - 9867a_0a_3^2a_4^2a_5^2a_6^5 \\
& + 3215a_0a_3^2a_4a_5^4a_6^4 - 346a_0a_3^2a_5^6a_6^3 - 3021a_0a_3a_4^4a_5a_6^5 + 12304a_0a_3a_4^3a_5^3a_6^4 \\
& + 1498a_0a_3a_4^2a_5^5a_6^3 + 112a_0a_4^6a_6^5 - 615a_0a_4^5a_5^2a_6^4 + 580a_0a_4^4a_5^4a_6^3 \\
& - 142a_0a_4^3a_5^6a_6^2 - 5107a_1^3a_3a_6^8 + 6680a_1^3a_4a_5a_6^7 - 10570a_1^3a_5^3a_6^6 \\
& - 2257a_1^2a_2^2a_6^8 + 1266a_1^2a_2a_3a_5a_6^7 + 1719a_1^2a_2a_4^2a_6^7 - 19134a_1^2a_2a_4a_5^2a_6^6 \\
& + 1163a_1^2a_2a_5^4a_6^5 + 1085a_1^2a_3^2a_4a_6^7 + 808a_1^2a_3^2a_5^2a_6^6 + 4739a_1^2a_3a_4^2a_5a_6^6 \\
& - 19702a_1^2a_3a_4a_5^3a_6^5 + 4312a_1^2a_3a_5^5a_6^4 - 85a_1^2a_4^4a_6^6 - 3370a_1^2a_4^3a_5^2a_6^5 \\
& + 117a_1^2a_4^2a_5^4a_6^4 + 1160a_1^2a_4a_5^6a_6^3 - 40a_1^2a_5^8a_6^2 - 6853a_1a_2^3a_5a_6^7 \\
& - 2926a_1a_2^2a_3a_4a_6^7 + 78a_1a_2^2a_3a_5^2a_6^6 - 677a_1a_2^2a_4^2a_5a_6^6 + 21695a_1a_2^2a_4a_5^3a_6^5 \\
& - 2306a_1a_2^2a_5^5a_6^4 - 4065a_1a_2a_3^3a_6^7 + 1559a_1a_2a_3^2a_4a_5a_6^6 + 6608a_1a_2a_3^2a_5^3a_6^5 \\
& + 163a_1a_2a_3a_4^3a_6^6 + 4790a_1a_2a_3a_4^2a_5^2a_6^5 + 2686a_1a_2a_3a_4a_5^4a_6^4 - 294a_1a_2a_3a_5^6a_6^3 \\
& + 2252a_1a_2a_4^4a_5a_6^5 - 3338a_1a_2a_4^3a_5^3a_6^4 - 1847a_1a_2a_4^2a_5^5a_6^3 - 19a_1a_2a_4a_5^7a_6^2 \\
& + 2906a_1a_3^4a_5a_6^6 + 397a_1a_3^3a_4^2a_6^6 + 1567a_1a_3^3a_4a_5^2a_6^5 - 1728a_1a_3^3a_5^4a_6^4 \\
& + 725a_1a_3^2a_4^3a_5a_6^5 - 2155a_1a_3^2a_4^2a_5^3a_6^4 + 96a_1a_3^2a_4a_5^5a_6^3 + 4a_1a_3^2a_5^7a_6^2 \\
& + 77a_1a_3a_4^5a_6^5 - 432a_1a_3a_4^4a_5^2a_6^4 + 113a_1a_3a_4^3a_5^4a_6^3 + 19a_1a_3a_4^2a_5^6a_6^2 \\
& - 277a_1a_4^6a_5a_6^4 - 444a_1a_4^5a_5^3a_6^3 + 18a_1a_4^4a_5^5a_6^2 - 1758a_4^4a_4a_6^7 \\
& + 4461a_2^4a_5^2a_6^6 + 3130a_2^3a_3^2a_6^7 + 4236a_2^3a_3a_4a_5a_6^6 - 5655a_2^3a_3a_5^3a_6^5 \\
& + 1398a_2^3a_4^3a_6^6 - 4864a_2^3a_4^2a_5^2a_6^5 - 1724a_2^3a_4a_5^4a_6^4 + 229a_2^3a_5^6a_6^3 \\
& - 1784a_2^2a_3^3a_5a_6^6 - 1783a_2^2a_3^2a_4^2a_6^6 + 2287a_2^2a_3^2a_4a_5^2a_6^5 + 858a_2^2a_3^2a_5^4a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 48a_2^2a_3a_4^3a_5a_6^5 - 229a_2^2a_3a_4^2a_5^3a_6^4 - 231a_2^2a_3a_4a_5^5a_6^3 - 15a_2^2a_3a_5^7a_6^2 \\
& - 358a_2^2a_4^5a_6^5 + 321a_2^2a_4^4a_5^2a_6^4 + 552a_2^2a_4^3a_5^4a_6^3 + 26a_2^2a_4^2a_5^6a_6^2 \\
& + 2023a_2a_3^4a_4a_6^6 - 1546a_2a_3^4a_4^2a_5^5 - 1975a_2a_3^3a_4^2a_5a_6^5 - 479a_2a_3^3a_4a_5^3a_6^4 \\
& + 68a_2a_3^3a_5^3a_6^3 + 562a_2a_3^2a_4^4a_6^5 - 723a_2a_3^2a_4^3a_5^2a_6^4 + 272a_2a_3^2a_4^2a_5^4a_6^3 \\
& + 20a_2a_3^2a_4a_5^6a_6^2 - 333a_2a_3a_4^5a_5a_6^4 - 384a_2a_3a_4^4a_5^3a_6^3 + 18a_2a_3a_4^3a_5^5a_6^2 \\
& + 43a_2a_4^7a_6^4 + 5a_2a_4^6a_5^2a_6^3 - 12a_2a_4^5a_5^4a_6^2 - 383a_3^6a_6^6 \\
& - 48a_3^5a_4a_5a_6^5 + 384a_3^5a_5^3a_6^4 - 327a_3^4a_4^3a_6^5 + 198a_3^4a_4^2a_5^2a_6^4 \\
& - 192a_3^4a_4a_5^4a_6^3 + 3a_3^4a_5^6a_6^2 - 147a_3^3a_4^4a_5a_6^4 + 406a_3^3a_4^3a_5^3a_6^3 \\
& - 6a_3^3a_4^2a_5^5a_6^2 - 87a_3^2a_4^6a_6^4 + 120a_3^2a_4^5a_5^2a_6^3 - 27a_3^2a_4^4a_5^4a_6^2 \\
& - 21a_3a_4^7a_5a_6^3 + 63a_3a_4^6a_5^3a_6^2 - 4a_4^9a_6^3 - 6a_4^8a_5^2a_6^2 \\
& - 14493a_0^3a_5a_6^7 + 30697a_0^2a_1a_4a_6^7 - 27332a_0^2a_1a_5^2a_6^6 + 57092a_0^2a_2a_3a_6^7 \\
& - 32467a_0^2a_2a_4a_5a_6^6 - 21883a_0^2a_2a_5^3a_6^5 - 31775a_0^2a_3^2a_5a_6^6 - 52622a_0^2a_3a_4^2a_6^6 \\
& + 75693a_0^2a_3a_4a_5^2a_6^5 + 2096a_0^2a_3a_5^4a_6^4 + 41096a_0^2a_4^3a_5a_6^5 - 9260a_0^2a_4^2a_5^3a_6^4 \\
& + 2368a_0^2a_4a_5^5a_6^3 - 222a_0^2a_5^7a_6^2 + 5891a_0a_1^2a_3a_6^7 - 13036a_0a_1^2a_4a_5a_6^6 \\
& + 206a_0a_1^2a_5^3a_6^5 - 46252a_0a_1a_2^2a_6^7 + 328a_0a_1a_2a_3a_5a_6^6 + 8609a_0a_1a_2a_4^2a_6^6 \\
& + 55710a_0a_1a_2a_4a_5^2a_6^5 - 7287a_0a_1a_2a_5^4a_6^4 + 1651a_0a_1a_3^2a_4a_6^6 - 46070a_0a_1a_3^2a_5^2a_6^5 \\
& - 18368a_0a_1a_3a_4^2a_5a_6^5 - 14320a_0a_1a_3a_4a_5^3a_6^4 + 6120a_0a_1a_3a_5^5a_6^3 + 434a_0a_1a_4^4a_6^5 \\
& + 12210a_0a_1a_4^3a_5^2a_6^4 - 10342a_0a_1a_4^2a_5^4a_6^3 + 1722a_0a_1a_4a_5^6a_6^2 - 9a_0a_1a_5^8a_6 \\
& + 9772a_0a_2^3a_5a_6^6 + 34183a_0a_2^2a_3a_4a_6^6 - 7402a_0a_2^2a_3a_5^2a_6^5 - 12590a_0a_2^2a_4^2a_5a_6^5 \\
& - 3705a_0a_2^2a_4a_5^3a_6^4 + 1135a_0a_2^2a_5^5a_6^3 - 18631a_0a_2a_3^3a_6^6 + 13316a_0a_2a_3^2a_4a_5a_6^5 \\
& + 15433a_0a_2a_3^2a_5^3a_6^4 - 5932a_0a_2a_3a_4^3a_6^5 - 26000a_0a_2a_3a_4^2a_5^2a_6^4 + 74a_0a_2a_3a_4a_5^4a_6^3 \\
& - 436a_0a_2a_3a_5^6a_6^2 - 9753a_0a_2a_4^4a_5a_6^4 + 7100a_0a_2a_4^3a_5^3a_6^3 + 676a_0a_2a_4^2a_5^5a_6^2 \\
& - 18a_0a_2a_4a_5^7a_6 + 8104a_0a_3^4a_5a_6^5 + 9896a_0a_3^3a_4^2a_6^5 - 3418a_0a_3^3a_4a_5^2a_6^4 \\
& + 1006a_0a_3^3a_5^4a_6^3 - 1181a_0a_3^2a_4^3a_5a_6^4 - 4529a_0a_3^2a_4^2a_5^3a_6^3 - 606a_0a_3^2a_4a_5^5a_6^2 \\
& + 17a_0a_3^2a_5^7a_6 + 1390a_0a_3a_4^5a_6^4 - 10206a_0a_3a_4^4a_5^2a_6^3 + 960a_0a_3a_4^3a_5^4a_6^2 \\
& - 37a_0a_3a_4^2a_5^6a_6 + 640a_0a_4^6a_5a_6^3 - 36a_0a_4^5a_5^3a_6^2 + 114a_0a_4^4a_5^5a_6 \\
& + 11889a_1^3a_2a_6^7 + 4208a_1^3a_3a_5a_6^6 - 5286a_1^3a_4^2a_6^6 + 16418a_1^3a_4a_5^2a_6^5 \\
& - 3866a_1^3a_5^4a_6^4 + 6778a_1^2a_2^2a_5a_6^6 - 10166a_1^2a_2a_3a_4a_6^6 + 10520a_1^2a_2a_3a_5^2a_6^5 \\
& + 22699a_1^2a_2^2a_4^2a_5a_6^5 - 11378a_1^2a_2a_4a_5^3a_6^4 - 1400a_1^2a_2a_5^5a_6^3 + 2716a_1^2a_3^3a_6^6 \\
& + 4186a_1^2a_3^2a_4a_5a_6^5 - 7618a_1^2a_3^2a_5^3a_6^4 + 2449a_1^2a_3a_4^3a_6^5 + 15134a_1^2a_3a_4^2a_5^2a_6^4 \\
& - 7298a_1^2a_3a_4a_5^4a_6^3 + 255a_1^2a_3a_5^6a_6^2 - 762a_1^2a_4^4a_5a_6^4 - 3230a_1^2a_4^3a_5^3a_6^3 \\
& - 711a_1^2a_4^2a_5^5a_6^2 + 31a_1^2a_4a_5^7a_6 + 9259a_1a_2^3a_4a_6^6 - 15340a_1a_2^3a_5^2a_6^5 \\
& + 1333a_1a_2^2a_3^2a_6^6 - 18308a_1a_2^2a_3a_4a_5a_6^5 + 962a_1a_2^2a_3a_5^3a_6^4 - 4476a_1a_2^2a_4^3a_6^5 \\
& - 12595a_1a_2^2a_4^2a_5^2a_6^4 + 9807a_1a_2^2a_4a_5^4a_6^3 - 21a_1a_2^2a_5^6a_6^2 - 7914a_1a_2a_3^3a_5a_6^5 \\
& - 2298a_1a_2a_3^2a_4^2a_6^5 - 2141a_1a_2a_3^2a_4a_5^2a_6^4 + 704a_1a_2a_3^2a_5^4a_6^3 + 1372a_1a_2a_3a_4^3a_5a_6^4
\end{aligned}$$

$$\begin{aligned}
& -302a_1a_2a_3a_4^2a_5^3a_6^3 + 344a_1a_2a_3a_4a_5^5a_6^2 + 6a_1a_2a_3a_5^7a_6 + 301a_1a_2a_4^5a_6^4 \\
& + 4676a_1a_2a_4^4a_5^2a_6^3 + 499a_1a_2a_4^3a_5^4a_6^2 - 62a_1a_2a_4^2a_5^6a_6 - 3050a_1a_3^4a_4a_5^5 \\
& + 2132a_1a_3^4a_5^2a_6^4 + 1794a_1a_3^3a_4^2a_5^4a_6 + 1630a_1a_3^3a_4a_5^3a_6^3 + 18a_1a_3^3a_5^5a_6^2 \\
& - 682a_1a_3^3a_4^4a_6 + 1113a_1a_3^2a_4^3a_5^2a_6^3 - 192a_1a_3^2a_4^2a_5^4a_6^2 - 11a_1a_3^2a_4a_5^6a_6 \\
& + 612a_1a_3a_4^5a_5a_6^3 + 36a_1a_3a_4^4a_5^3a_6^2 + 87a_1a_3a_4^3a_5^5a_6 + 30a_1a_4^7a_6^3 \\
& - 9a_1a_4^6a_5^2a_6^2 - 48a_1a_4^5a_5^4a_6 - 4790a_2^4a_3a_6^6 - 1910a_2^4a_4a_5a_6^5 \\
& + 2626a_2^4a_5^3a_6^4 + 6092a_2^3a_3^2a_5a_6^5 + 402a_2^3a_3a_4^2a_6^5 + 8623a_2^3a_3a_4a_5^2a_6^4 \\
& - 1167a_2^3a_3a_5^4a_6^3 + 4266a_2^3a_4^3a_5a_6^4 - 1693a_2^3a_4^2a_5^3a_6^3 - 435a_2^3a_4a_5^5a_6^2 \\
& + 3a_2^3a_5^7a_6 + 1260a_2^2a_3^3a_4a_6^5 - 2016a_2^2a_3^3a_5^2a_6^4 - 3961a_2^2a_3^2a_4^2a_5a_6^4 \\
& + 151a_2^2a_3^2a_4a_5^3a_6^3 + 97a_2^2a_3^2a_5^5a_6^2 - 105a_2^2a_3a_4^4a_6^4 + 671a_2^2a_3a_4^3a_5^2a_6^3 \\
& + 219a_2^2a_3a_4^2a_5^4a_6^2 - 9a_2^2a_3a_4a_5^6a_6 - 720a_2^2a_4^5a_5a_6^3 - 265a_2^2a_4^4a_5^3a_6^2 \\
& + 14a_2^2a_4^3a_5^5a_6 + 620a_2a_3^5a_6^5 + 2330a_2a_3^4a_4a_5a_6^4 + 14a_2a_3^4a_5^3a_6^3 \\
& + 1263a_2a_3^3a_4^3a_6^4 - 1697a_2a_3^3a_4^2a_5^2a_6^3 - 168a_2a_3^3a_4a_5^4a_6^2 - a_2a_3^3a_5^6a_6 \\
& + 546a_2a_3^2a_4^4a_5a_6^3 - 78a_2a_3^2a_4^3a_5^2a_6^2 + 33a_2a_3^2a_4^2a_5^5a_6 + 134a_2a_3a_4^6a_6^3 \\
& - 108a_2a_3a_4^5a_5^2a_6^2 - 63a_2a_3a_4^4a_5^4a_6 + 51a_2a_4^7a_5a_6^2 + 10a_2a_4^6a_5^3a_6 \\
& - 554a_3^6a_5a_6^4 + 39a_3^5a_4^2a_6^4 + 318a_3^5a_4a_5^2a_6^3 - 39a_3^5a_5^4a_6^2 \\
& - 591a_3^4a_4^3a_5a_6^3 + 96a_3^4a_4^2a_5^3a_6^2 + 3a_3^4a_4a_5^5a_6 - 63a_3^3a_4^4a_5^2a_6^2 \\
& - 33a_3^3a_4^3a_5^4a_6 - 156a_3^2a_4^6a_5a_6^2 + 45a_3^2a_4^5a_5^3a_6 - 3a_3a_4^8a_6^2 \\
& - 24a_3a_4^7a_5^2a_6 - 2a_4^9a_5a_6 + 96915a_0^3a_4a_6^6 - 29386a_0^3a_5^2a_6^5 \\
& - 78613a_0^2a_1a_3a_6^6 + 14376a_0^2a_1a_4a_5a_6^5 + 41403a_0^2a_1a_5^3a_6^4 + 53151a_0^2a_2^2a_6^6 \\
& + 38186a_0^2a_2a_3a_5a_6^5 - 38060a_0^2a_2a_4^2a_6^5 + 45337a_0^2a_2a_4a_5^2a_6^4 - 16910a_0^2a_2a_5^4a_6^3 \\
& + 19410a_0^2a_3^2a_4a_6^5 + 4717a_0^2a_3^2a_5^2a_6^4 - 86264a_0^2a_3a_4^2a_5a_6^4 - 3412a_0^2a_3a_4a_5^3a_6^3 \\
& + 1363a_0^2a_3a_5^5a_6^2 - 8263a_0^2a_4^4a_6^4 + 32628a_0^2a_4^3a_5^2a_6^3 - 2278a_0^2a_4^2a_5^4a_6^2 \\
& + 486a_0^2a_4a_5^6a_6 + 10a_0^2a_5^8 + 1057a_0a_1^2a_2a_6^6 + 13053a_0a_1^2a_3a_5a_6^5 \\
& + 23610a_0a_1^2a_4a_6^5 - 23156a_0a_1^2a_4a_5^2a_6^4 + 3118a_0a_1^2a_5^4a_6^3 - 41161a_0a_1a_2^2a_5a_6^5 \\
& + 12922a_0a_1a_2a_3a_4a_6^5 + 3597a_0a_1a_2a_3a_5^2a_6^4 - 20754a_0a_1a_2a_4^2a_5a_6^4 + 18420a_0a_1a_2a_4a_5^3a_6^3 \\
& - 1688a_0a_1a_2a_5^5a_6^2 + 15801a_0a_1a_3^3a_6^5 + 28393a_0a_1a_3^2a_4a_5a_6^4 - 12210a_0a_1a_3^2a_5^3a_6^3 \\
& - 4202a_0a_1a_3a_4^3a_6^4 - 2598a_0a_1a_3a_4^2a_5^2a_6^3 - 10462a_0a_1a_3a_4a_5^4a_6^2 - 35a_0a_1a_3a_5^6a_6 \\
& - 2982a_0a_1a_4^4a_5a_6^3 + 9802a_0a_1a_4^3a_5^3a_6^2 - 472a_0a_1a_4^2a_5^5a_6 + 23a_0a_1a_4a_5^7 \\
& + 8926a_0a_2^3a_4a_6^5 - 10853a_0a_2^3a_5^2a_6^4 - 20629a_0a_2^2a_3^2a_6^5 + 33781a_0a_2^2a_3a_4a_5a_6^4 \\
& + 823a_0a_2^2a_3a_5^3a_6^3 + 6172a_0a_2^2a_4^3a_6^4 - 11761a_0a_2^2a_4^2a_5^2a_6^3 + 167a_0a_2^2a_4a_5^4a_6^2 \\
& + 197a_0a_2^2a_5^6a_6 - 10788a_0a_2a_3^3a_5a_6^4 - 216a_0a_2a_3^2a_4^2a_6^4 - 3626a_0a_2a_3^2a_4a_5^2a_6^3 \\
& + 572a_0a_2a_3^2a_5^4a_6^2 + 12354a_0a_2a_3a_4^3a_5a_6^3 - 3938a_0a_2a_3a_4^2a_5^3a_6^2 - 252a_0a_2a_3a_4a_5^5a_6 \\
& + 10a_0a_2a_3a_5^7 + 852a_0a_2a_4^5a_6^3 - 7272a_0a_2a_4^4a_5^2a_6^2 - 280a_0a_2a_4^3a_5^4a_6 \\
& - 14a_0a_2a_4^2a_5^6 - 7288a_0a_3^4a_4a_6^4 - 2276a_0a_3^4a_5^2a_6^3 + 12219a_0a_3^3a_4^2a_5a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 3168a_0a_3^3a_4a_5^3a_6^2 - 171a_0a_3^3a_5^5a_6 + 1924a_0a_3^2a_4^4a_6^3 - 1975a_0a_3^2a_4^3a_5^2a_6^2 \\
& + 699a_0a_3^2a_4^2a_5^4a_6 - a_0a_3^2a_4a_5^6 + 4358a_0a_3a_4^5a_5a_6^2 - 963a_0a_3a_4^4a_5^3a_6 \\
& + 35a_0a_3a_4^3a_5^5 - 58a_0a_4^7a_6^2 + 79a_0a_4^6a_5^2a_6 - 8a_0a_4^5a_5^4 \\
& - 6269a_1^4a_6^6 + 5790a_1^3a_2a_5a_6^5 - 9735a_1^3a_3a_4a_6^5 + 9920a_1^3a_3a_5^2a_6^4 \\
& - 12517a_1^3a_4^2a_5^4a_6^4 + 1478a_1^3a_4a_5^3a_6^3 - 492a_1^3a_5^5a_6^2 - 16579a_1^2a_2^2a_4a_6^5 \\
& + 10177a_1^2a_2^2a_5^2a_6^4 + 5809a_1^2a_2a_3^2a_6^5 - 25115a_1^2a_2a_3a_4a_5a_6^4 + 8166a_1^2a_2a_3a_5^3a_6^3 \\
& - 3637a_1^2a_2a_4^3a_6^4 + 27398a_1^2a_2a_4^2a_5^2a_6^3 + 1606a_1^2a_2a_4a_5^4a_6^2 - 75a_1^2a_2a_5^6a_6 \\
& + 4100a_1^2a_3^3a_5a_6^4 + 3831a_1^2a_3^2a_4^2a_6^4 + 10184a_1^2a_3^2a_4a_5^2a_6^3 + 83a_1^2a_3^2a_5^4a_6^2 \\
& - 3312a_1^2a_3a_4^3a_5a_6^3 + 4740a_1^2a_3a_4^2a_5^3a_6^2 - 194a_1^2a_3a_4a_5^5a_6 - 6a_1^2a_3a_5^7 \\
& + 582a_1^2a_4^5a_6^3 + 127a_1^2a_4^4a_5^2a_6^2 - 278a_1^2a_4^3a_5^4a_6 - 18a_1^2a_4^2a_5^6 \\
& + 7785a_1a_2^3a_3a_6^5 + 24423a_1a_2^3a_4a_5a_6^4 - 7520a_1a_2^3a_5^3a_6^3 + 4126a_1a_2^2a_3^2a_5a_6^4 \\
& + 8118a_1a_2^2a_3a_4^2a_6^4 - 18099a_1a_2^2a_3a_4a_5^2a_6^3 - 805a_1a_2^2a_3a_5^4a_6^2 - 5557a_1a_2^2a_4^3a_5a_6^3 \\
& - 7517a_1a_2^2a_4^2a_5^2a_6^2 + 401a_1a_2^2a_4a_5^5a_6 + 2a_1a_2^2a_5^7 + 182a_1a_2a_3^3a_4a_6^4 \\
& - 1532a_1a_2a_3^3a_5^2a_6^3 + 1676a_1a_2a_3^2a_4^2a_5a_6^3 - 206a_1a_2a_3^2a_4a_5^3a_6^2 - 48a_1a_2a_3^2a_5^5a_6 \\
& - 1588a_1a_2a_3a_4^4a_6^3 + 1154a_1a_2a_3a_4^3a_5^2a_6^2 - 347a_1a_2a_3a_4^2a_5^4a_6 - 8a_1a_2a_3a_4a_5^6 \\
& - 846a_1a_2a_4^5a_5a_6^2 + 540a_1a_2a_4^4a_5^3a_6 + 32a_1a_2a_4^3a_5^5 + 316a_1a_3^5a_6^4 \\
& - 3336a_1a_3^4a_4a_5a_6^3 - 324a_1a_3^4a_5^2a_6^2 - 2061a_1a_3^3a_4^3a_6^3 - 582a_1a_3^3a_4^2a_5^2a_6^2 \\
& + 63a_1a_3^3a_4a_5^4a_6 + 2a_1a_3^3a_5^6 - 289a_1a_3^2a_4^4a_5a_6^2 - 74a_1a_3^2a_4^3a_5^3a_6 \\
& + 15a_1a_3^2a_4^2a_5^5 - 236a_1a_3a_4^6a_6^2 + 43a_1a_3a_4^5a_5^2a_6 - 34a_1a_3a_4^4a_5^4 \\
& + 53a_1a_4^7a_5a_6 + 7a_1a_4^6a_5^3 + 1224a_2^5a_6^5 - 7963a_2^4a_3a_5a_6^4 \\
& - 1624a_2^4a_4^2a_6^4 + 486a_2^4a_4a_5^2a_6^3 + 638a_2^4a_5^4a_6^2 - 2952a_2^3a_3^2a_4a_6^4 \\
& + 1954a_2^3a_3^2a_5^2a_6^3 - 2403a_2^3a_3a_4^2a_5a_6^3 + 1555a_2^3a_3a_4a_5^3a_6^2 - 11a_2^3a_3a_5^5a_6 \\
& - 518a_2^3a_4^4a_6^3 + 2145a_2^3a_4^3a_5^2a_6^2 + 40a_2^3a_4^2a_5^4a_6 - 5a_2^3a_4a_5^6 \\
& - 766a_2^2a_3^4a_6^4 + 3338a_2^2a_3^3a_4a_5a_6^3 + 6a_2^2a_3^3a_5^3a_6^2 + 679a_2^2a_3^2a_4^3a_6^3 \\
& - 1043a_2^2a_3^2a_4^2a_5^2a_6^2 - 13a_2^2a_3^2a_4a_5^4a_6 - 2a_2^2a_3^2a_5^6 + 176a_2^2a_3a_4^3a_5^3a_6 \\
& + 14a_2^2a_3a_4^2a_5^5 + 156a_2^2a_4^6a_6^2 - 48a_2^2a_4^5a_5^2a_6 - 10a_2^2a_4^4a_5^4 \\
& - 352a_2a_3^5a_5a_6^3 + 192a_2a_3^4a_4^2a_6^3 + 150a_2a_3^4a_4a_5^2a_6^2 + 6a_2a_3^4a_5^4a_6 \\
& + 805a_2a_3^3a_4^3a_5a_6^2 - 78a_2a_3^3a_4^2a_5^3a_6 + 3a_2a_3^3a_4a_5^5 - 10a_2a_3^2a_4^5a_6^2 \\
& - 136a_2a_3^2a_4^4a_5^2a_6 - 23a_2a_3^2a_4^3a_5^4 + 97a_2a_3a_4^6a_5a_6 + 15a_2a_3a_4^5a_5^3 \\
& - 14a_2a_4^8a_6 + 88a_3^6a_4a_6^3 + 168a_3^6a_5^2a_6^2 - 252a_3^5a_4^2a_5a_6^2 \\
& - 24a_3^5a_4a_5^3a_6 + 246a_3^4a_4^4a_6^2 + 150a_3^4a_4^3a_5^2a_6 - 3a_3^4a_4^2a_5^4 \\
& - 66a_3^3a_4^5a_5a_6 + 12a_3^3a_4^4a_5^3 + 51a_3^2a_4^7a_6 - 6a_3^2a_4^6a_5^2 \\
& + a_4^{10} - 23667a_0^3a_3a_6^5 + 112264a_0^3a_4a_5a_6^4 + 21037a_0^3a_5^3a_6^3 \\
& - 40725a_0^2a_1a_2a_6^5 - 123140a_0^2a_1a_3a_5a_6^4 + 15686a_0^2a_1a_4^2a_6^4 - 83805a_0^2a_1a_4a_5^2a_6^3 \\
& + 5254a_0^2a_1a_5^4a_6^2 - 93951a_0^2a_2^2a_5a_6^4 + 93792a_0^2a_2a_3a_4a_6^4 + 50411a_0^2a_2a_3a_5^2a_6^3
\end{aligned}$$

$$\begin{aligned}
& -78786a_0^2a_2a_4^2a_5a_6^3 + 13424a_0^2a_2a_4a_5^3a_6^2 - 371a_0^2a_2a_5^5a_6 - 11199a_0^2a_3^3a_6^4 \\
& + 32593a_0^2a_3^2a_4a_5a_6^3 - 5128a_0^2a_3^2a_5^3a_6^2 - 2354a_0^2a_3a_4^3a_6^3 - 4216a_0^2a_3a_4^2a_5^2a_6^2 \\
& - 4213a_0^2a_3a_4a_5^4a_6 - 120a_0^2a_3a_5^6 - 8020a_0^2a_4^4a_5a_6^2 + 4944a_0^2a_4^3a_5^3a_6 \\
& + 363a_0^2a_4^2a_5^5 + 18288a_0a_1^3a_6^5 + 79817a_0a_1^2a_2a_5a_6^4 - 35272a_0a_1^2a_3a_4a_6^4 \\
& - 23392a_0a_1^2a_3a_5^2a_6^3 + 38967a_0a_1^2a_4^2a_5a_6^3 - 4126a_0a_1^2a_4a_5^3a_6^2 - 692a_0a_1^2a_5^5a_6 \\
& - 36345a_0a_1a_2^2a_4a_6^4 + 19467a_0a_1a_2^2a_5^2a_6^3 - 25853a_0a_1a_2a_3^2a_6^4 - 5214a_0a_1a_2a_3a_4a_5a_6^3 \\
& + 11320a_0a_1a_2a_3a_5^3a_6^2 + 23796a_0a_1a_2a_4^3a_6^3 - 11698a_0a_1a_2a_4^2a_5^2a_6^2 + 2570a_0a_1a_2a_4a_5^4a_6 \\
& + 33a_0a_1a_2a_5^6 + 4338a_0a_1a_3^3a_5a_6^3 + 12393a_0a_1a_3^2a_4^2a_6^3 + 23470a_0a_1a_3^2a_4a_5^2a_6^2 \\
& + 1220a_0a_1a_3^2a_5^4a_6 - 16080a_0a_1a_3a_4^3a_5a_6^2 + 576a_0a_1a_3a_4^2a_5^3a_6 - 254a_0a_1a_3a_4a_5^5 \\
& - 2602a_0a_1a_4^5a_6^2 - 4372a_0a_1a_4^4a_5^2a_6 - 190a_0a_1a_4^3a_5^4 + 39019a_0a_3^3a_6^4 \\
& + 15244a_0a_2^3a_4a_5a_6^3 - 1733a_0a_2^3a_5^3a_6^2 - 6552a_0a_2^2a_3^2a_5a_6^3 - 20263a_0a_2^2a_3a_4^2a_6^3 \\
& - 5902a_0a_2^2a_3a_4a_5^2a_6^2 - 1485a_0a_2^2a_3a_5^4a_6 + 11535a_0a_2^2a_4^3a_5a_6^2 - 761a_0a_2^2a_4^2a_5^3a_6 \\
& - 37a_0a_2^2a_4a_5^5 - 7632a_0a_2a_3^3a_4a_6^3 + 428a_0a_2a_3^3a_5^2a_6^2 + 12997a_0a_2a_3^2a_4^2a_5a_6^2 \\
& + 1304a_0a_2a_3^2a_4a_5^3a_6 - 115a_0a_2a_3^2a_5^5 - 2896a_0a_2a_3a_4^4a_6^2 + 2574a_0a_2a_3a_4^3a_5^2a_6 \\
& + 75a_0a_2a_3a_4^2a_5^4 + 1784a_0a_2a_4^5a_5a_6 - 69a_0a_2a_4^4a_5^3 + 3968a_0a_3^5a_6^3 \\
& - 4824a_0a_3^4a_4a_5a_6^2 + 552a_0a_3^4a_5^3a_6 - 3422a_0a_3^3a_4^3a_6^2 - 2664a_0a_3^3a_4^2a_5^2a_6 \\
& + 6a_0a_3^3a_4a_5^4 + 2170a_0a_3^2a_4^4a_5a_6 - 118a_0a_3^2a_4^3a_5^3 - 526a_0a_3a_4^6a_6 \\
& + 284a_0a_3a_4^5a_5^2 - 3a_0a_4^7a_5 - 21640a_1^4a_5a_6^4 + 20337a_1^3a_2a_4a_6^4 \\
& - 6912a_1^3a_2a_5^2a_6^3 + 4466a_1^3a_3^2a_6^4 + 808a_1^3a_3a_4a_5a_6^3 + 1204a_1^3a_3a_5^3a_6^2 \\
& - 519a_1^3a_4^3a_6^3 - 1710a_1^3a_4^2a_5^2a_6^2 - 104a_1^3a_4a_5^4a_6 - 34a_1^3a_5^6 \\
& - 15022a_1^2a_2^2a_3a_6^4 - 26249a_1^2a_2^2a_4a_5a_6^3 - 816a_1^2a_2^2a_5^3a_6^2 - 7078a_1^2a_2a_3^2a_5a_6^3 \\
& - 2945a_1^2a_2a_3a_4^2a_6^3 - 11090a_1^2a_2a_3a_4a_5^2a_6^2 + 574a_1^2a_2a_3a_5^4a_6 - 10116a_1^2a_2a_4^3a_5a_6^2 \\
& + 2272a_1^2a_2a_4^2a_5^3a_6 + 132a_1^2a_2a_4a_5^5 - 5128a_1^2a_3^3a_4a_6^3 - 2068a_1^2a_3^2a_5^2a_6^2 \\
& - 2308a_1^2a_3^2a_4^2a_5a_6^2 + 300a_1^2a_3^2a_4a_5^3a_6 + 82a_1^2a_3^2a_5^5 + 2752a_1^2a_3a_4^4a_6^2 \\
& - 712a_1^2a_3a_4^3a_5^2a_6 + 56a_1^2a_3a_4^2a_5^4 + 384a_1^2a_4^5a_5a_6 + 128a_1^2a_4^4a_5^3 \\
& - 3603a_1a_2^4a_6^4 + 10098a_1a_2^3a_3a_5a_6^3 - 4811a_1a_2^3a_4^2a_6^3 + 12913a_1a_2^3a_4a_5^2a_6^2 \\
& - 399a_1a_2^3a_5^4a_6 + 10518a_1a_2^2a_3^2a_4a_6^3 + 3060a_1a_2^2a_3^2a_5^2a_6^2 + 8628a_1a_2^2a_3a_4^2a_5a_6^2 \\
& - 766a_1a_2^2a_3a_4a_5^3a_6 - 34a_1a_2^2a_3a_5^5 + 2616a_1a_2^2a_4^4a_6^2 - 235a_1a_2^2a_4^3a_5^2a_6 \\
& - 273a_1a_2^2a_4^2a_5^4 + 1232a_1a_2a_3^4a_6^3 - 320a_1a_2a_3^3a_4a_5a_6^2 + 120a_1a_2a_3^3a_5^3a_6 \\
& - 1479a_1a_2a_3^2a_4^3a_6^2 + 908a_1a_2a_3^2a_4^2a_5^2a_6 + 51a_1a_2a_3^2a_4a_5^4 - 590a_1a_2a_3a_4^4a_5a_6 \\
& + 188a_1a_2a_3a_4^3a_5^3 - 70a_1a_2a_4^6a_6 - 127a_1a_2a_4^5a_5^2 + 752a_1a_3^5a_5a_6^2 \\
& + 1616a_1a_3^4a_4^2a_6^2 - 24a_1a_3^4a_4a_5^2a_6 - 24a_1a_3^4a_5^4 - 232a_1a_3^3a_4^3a_5a_6 \\
& - 108a_1a_3^3a_4^2a_5^3 - 218a_1a_3^2a_4^5a_6 + 18a_1a_3^2a_4^4a_5^2 - 50a_1a_3a_4^6a_5 \\
& - 15a_1a_4^8 + 685a_2^5a_5a_6^3 + 3143a_2^4a_3a_4a_6^3 - 2925a_2^4a_3a_5^2a_6^2 \\
& - 3236a_2^4a_4^2a_5a_6^2 - 194a_2^4a_4a_5^3a_6 + 15a_2^4a_5^5 - 2100a_2^3a_3^3a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 426a_2^3a_3^2a_4a_5a_6^2 + 26a_2^3a_3^2a_5^3a_6 + 147a_2^3a_3a_4^3a_6^2 - 553a_2^3a_3a_4^2a_5^2a_6 \\
& - 7a_2^3a_3a_4a_5^4 - 255a_2^3a_4^4a_5a_6 + 76a_2^3a_4^3a_5^3 - 580a_2^2a_3^4a_5a_6^2 \\
& - 2164a_2^2a_3^3a_4^2a_6^2 + 90a_2^2a_3^3a_4a_5^2a_6 + 21a_2^2a_3^3a_5^4 + 373a_2^2a_3^2a_4^3a_5a_6 \\
& - 2a_2^2a_3^2a_4^2a_5^3 - 229a_2^2a_3a_4^5a_6 - 103a_2^2a_3a_4^4a_5^2 + 13a_2^2a_4^6a_5 \\
& + 664a_2a_3^5a_4a_6^2 - 144a_2a_3^4a_4^2a_5a_6 - 24a_2a_3^4a_4a_5^3 + 222a_2a_3^3a_4^4a_6 \\
& + 58a_2a_3^3a_4^3a_5^2 + 58a_2a_3^2a_4^5a_5 + 5a_2a_3a_4^7 - 240a_3^7a_6^2 \\
& + 48a_3^6a_4a_5a_6 - 120a_3^5a_4^3a_6 + 24a_3^5a_4^2a_5^2 - 42a_3^4a_4^4a_5 \\
& - 4a_3^3a_4^6 + 80075a_0^3a_2a_6^4 - 64961a_0^3a_3a_5a_6^3 - 39152a_0^3a_4^2a_6^3 \\
& - 22995a_0^3a_4a_5^2a_6^2 - 14a_0^3a_5^4a_6 - 34392a_0^2a_1^2a_6^4 + 61726a_0^2a_1a_2a_5a_6^3 \\
& + 46321a_0^2a_1a_3a_4a_6^3 - 11686a_0^2a_1a_3a_5^2a_6^2 + 62458a_0^2a_1a_4^2a_5a_6^2 - 9598a_0^2a_1a_4a_5^3a_6 \\
& - 210a_0^2a_1a_5^5 + 143726a_0^2a_2^2a_4a_6^3 - 9125a_0^2a_2^2a_5^2a_6^2 - 42490a_0^2a_2a_3^2a_6^3 \\
& - 1223a_0^2a_2a_3a_4a_5a_6^2 - 1366a_0^2a_2a_3a_5^3a_6 - 17470a_0^2a_2a_4^3a_6^2 - 10912a_0^2a_2a_4^2a_5^2a_6 \\
& - 419a_0^2a_2a_4a_5^4 + 9646a_0^2a_3^3a_5a_6^2 + 141a_0^2a_3^2a_4^2a_6^2 + 16318a_0^2a_3^2a_4a_5^2a_6 \\
& + 556a_0^2a_3^2a_5^4 - 17684a_0^2a_3a_4^3a_5a_6 - 2572a_0^2a_3a_4^2a_5^3 + 3184a_0^2a_4^5a_6 \\
& + 935a_0^2a_4^4a_5^2 - 10352a_0a_1^3a_5a_6^3 - 56006a_0a_1^2a_2a_4a_6^3 + 1528a_0a_1^2a_2a_5^2a_6^2 \\
& + 32656a_0a_1^2a_3^2a_6^3 + 14632a_0a_1^2a_3a_4a_5a_6^2 + 5496a_0a_1^2a_3a_5^3a_6 - 5535a_0a_1^2a_4^3a_6^2 \\
& + 2950a_0a_1^2a_4^2a_5^2a_6 - 20a_0a_1^2a_4a_5^4 - 48641a_0a_1a_2^2a_3a_6^3 - 42855a_0a_1a_2^2a_4a_5a_6^2 \\
& - 198a_0a_1a_2^2a_5^3a_6 - 21658a_0a_1a_2a_3^2a_5a_6^2 + 24185a_0a_1a_2a_3a_4^2a_6^2 - 11940a_0a_1a_2a_3a_4a_5^2a_6 \\
& - 184a_0a_1a_2a_3a_5^4 + 13568a_0a_1a_2a_4^3a_5a_6 + 1016a_0a_1a_2a_4^2a_5^3 - 19024a_0a_1a_3^3a_4a_6^2 \\
& - 5584a_0a_1a_3^3a_5^2a_6 + 2090a_0a_1a_3^2a_4^2a_5a_6 + 952a_0a_1a_3^2a_4a_5^3 + 5136a_0a_1a_3a_4^4a_6 \\
& + 424a_0a_1a_3a_4^3a_5^2 - 622a_0a_1a_4^5a_5 - 21270a_0a_2^4a_6^3 + 6076a_0a_2^3a_3a_5a_6^2 \\
& + 6551a_0a_2^3a_4^2a_6^2 + 2354a_0a_2^3a_4a_5^2a_6 + 15a_0a_2^3a_5^4 + 10972a_0a_2^2a_3^2a_4a_6^2 \\
& + 4876a_0a_2^2a_3^2a_5^2a_6 + 2313a_0a_2^2a_3a_4^2a_5a_6 + 232a_0a_2^2a_3a_4a_5^3 - 4268a_0a_2^2a_4^4a_6 \\
& + 57a_0a_2^2a_4^3a_5^2 + 128a_0a_2a_3^4a_6^2 - 1936a_0a_2a_3^3a_4a_5a_6 + 440a_0a_2a_3^3a_5^3 \\
& - 7892a_0a_2a_3^2a_4^3a_6 - 352a_0a_2a_3^2a_4^2a_5^2 - 726a_0a_2a_3a_4^4a_5 - 38a_0a_2a_4^6 \\
& - 560a_0a_3^5a_5a_6 + 3616a_0a_3^4a_4^2a_6 - 64a_0a_3^3a_4^3a_5 - 124a_0a_3^2a_4^5 \\
& + 7332a_1^4a_4a_6^3 - 3326a_1^4a_5^2a_6^2 + 15572a_1^3a_2a_3a_6^3 + 15384a_1^3a_2a_4a_5a_6^2 \\
& - 64a_1^3a_2a_5^3a_6 + 4180a_1^3a_3^2a_5a_6^2 - 6510a_1^3a_3a_4^2a_6^2 + 632a_1^3a_3a_4a_5^2a_6 \\
& + 280a_1^3a_3a_5^4 + 1130a_1^3a_4^3a_5a_6 + 60a_1^3a_4^2a_5^3 + 8529a_1^2a_3^3a_6^3 \\
& + 2000a_1^2a_2^2a_3a_5a_6^2 + 10363a_1^2a_2^2a_4^2a_6^2 - 2994a_1^2a_2^2a_4a_5^2a_6 - 127a_1^2a_2^2a_5^4 \\
& + 1928a_1^2a_2a_3^2a_4a_6^2 - 1384a_1^2a_2a_3^2a_5^2a_6 - 1398a_1^2a_2a_3a_4^2a_5a_6 - 828a_1^2a_2a_3a_4a_5^3 \\
& - 844a_1^2a_2a_4^4a_6 - 1032a_1^2a_2a_4^3a_5^2 + 880a_1^2a_4^4a_6^2 - 96a_1^2a_3^3a_4a_5a_6 \\
& - 368a_1^2a_3^3a_5^3 + 872a_1^2a_3^2a_4^3a_6 + 212a_1^2a_3^2a_4^2a_5^2 + 244a_1^2a_3a_4^4a_5 \\
& + 111a_1^2a_4^6 - 3120a_1a_2^4a_5a_6^2 - 12030a_1a_2^3a_3a_4a_6^2 + 1204a_1a_2^3a_3a_5^2a_6 \\
& - 2510a_1a_2^3a_4^2a_5a_6 + 542a_1a_2^3a_4a_5^3 - 1800a_1a_2^2a_3^3a_6^2 - 120a_1a_2^2a_3^2a_4a_5a_6
\end{aligned}$$

$$\begin{aligned}
& + 188a_1a_2^2a_3^2a_5^3 + 1191a_1a_2^2a_3a_4^3a_6 + 314a_1a_2^2a_3a_4^2a_5^2 + 511a_1a_2^2a_4^4a_5 \\
& - 96a_1a_2a_3^4a_5a_6 + 128a_1a_2a_3^3a_4^2a_6 - 72a_1a_2a_3^3a_4a_5^2 - 424a_1a_2a_3^2a_4^3a_5 \\
& + 102a_1a_2a_3a_4^5 - 208a_1a_3^5a_4a_6 + 96a_1a_3^5a_5^2 + 192a_1a_3^4a_4^2a_5 \\
& + 52a_1a_3^3a_4^4 + 369a_2^5a_4a_6^2 + 96a_2^5a_5^2a_6 + 2193a_2^4a_3^2a_6^2 \\
& + 1070a_2^4a_3a_4a_5a_6 - 94a_2^4a_3a_5^3 + 806a_2^4a_4^3a_6 - 169a_2^4a_4^2a_5^2 \\
& - 96a_2^3a_3^3a_5a_6 - 564a_2^3a_3^2a_4^2a_6 + 114a_2^3a_3^2a_4a_5^2 + 161a_2^3a_3a_4^3a_5 \\
& - 37a_2^3a_4^5 + 280a_2^2a_3^4a_4a_6 - 72a_2^2a_3^4a_5^2 - 180a_2^2a_3^3a_4^2a_5 \\
& - 103a_2^2a_3^2a_4^4 - 32a_2a_3^6a_6 + 48a_2a_3^5a_4a_5 + 88a_2a_3^4a_4^3 \\
& - 48a_3^6a_4^2 - 90100a_0^3a_1a_6^3 + 34419a_0^3a_2a_5a_6^2 + 105606a_0^3a_3a_4a_6^2 \\
& + 6830a_0^3a_3a_5^2a_6 - 7016a_0^3a_4^2a_5a_6 - 3602a_0^3a_4a_5^3 - 22718a_0^2a_1^2a_5^2a_6^2 \\
& - 103475a_0^2a_1a_2a_4a_6^2 + 14324a_0^2a_1a_2a_5^2a_6 - 21012a_0^2a_1a_3^2a_6^2 + 16018a_0^2a_1a_3a_4a_5a_6 \\
& + 1868a_0^2a_1a_3a_5^3 + 648a_0^2a_1a_4^3a_6 + 4676a_0^2a_1a_4^2a_5^2 - 23135a_0^2a_2^2a_3a_6^2 \\
& - 4044a_0^2a_2^2a_4a_5a_6 - 846a_0^2a_2^2a_5^3 + 5172a_0^2a_2a_3^2a_5a_6 + 42656a_0^2a_2a_3a_4^2a_6 \\
& + 5766a_0^2a_2a_3a_4a_5^2 - 5956a_0^2a_2a_4^3a_5 - 16432a_0^2a_3^3a_4a_6 - 1392a_0^2a_3^2a_5^2 \\
& + 2270a_0^2a_3^2a_4^2a_5 - 676a_0^2a_3a_4^4 + 11060a_0a_1^3a_4a_6^2 - 6136a_0a_1^3a_5^2a_6 \\
& + 20632a_0a_1^2a_2a_3a_6^2 + 19740a_0a_1^2a_2a_4a_5a_6 + 1096a_0a_1^2a_2a_5^3 - 9968a_0a_1^2a_3^2a_5a_6 \\
& - 16824a_0a_1^2a_3a_4^2a_6 - 488a_0a_1^2a_3a_4a_5^2 - 94a_0a_1^2a_4^3a_5 + 36597a_0a_1^2a_5^2a_6^2 \\
& + 4230a_0a_1a_2^2a_3a_5a_6 - 14676a_0a_1a_2^2a_4^2a_6 - 3890a_0a_1a_2^2a_4a_5^2 + 3584a_0a_1a_2a_3^2a_4a_6 \\
& - 48a_0a_1a_2a_3^2a_5^2 - 286a_0a_1a_2a_3a_4^2a_5 + 2320a_0a_1a_2a_4^4 + 7360a_0a_1a_3^4a_6 \\
& - 1216a_0a_1a_3^3a_4a_5 + 448a_0a_1a_3^2a_4^3 - 3064a_0a_2^4a_5a_6 + 344a_0a_2^3a_3a_4a_6 \\
& + 92a_0a_2^3a_3a_5^2 + 1059a_0a_2^3a_4^2a_5 - 6320a_0a_2^2a_3^3a_6 - 392a_0a_2^2a_3^2a_4a_5 \\
& + 54a_0a_2^2a_3a_4^3 - 560a_0a_2a_3^4a_5 + 1088a_0a_2a_3^3a_4^2 - 32a_0a_3^5a_4 \\
& - 3752a_1^4a_3a_6^2 - 1880a_1^4a_4a_5a_6 - 364a_1^4a_5^3 - 11070a_1^3a_2^2a_6^2 \\
& - 600a_1^3a_2a_3a_5a_6 - 1472a_1^3a_2a_4^2a_6 + 712a_1^3a_2a_4a_5^2 + 576a_1^3a_3^2a_4a_6 \\
& - 512a_1^3a_3^2a_5^2 - 572a_1^3a_3a_4^2a_5 - 532a_1^3a_4^4 + 1018a_1^2a_2^3a_5a_6 \\
& + 3352a_1^2a_2^2a_3a_4a_6 + 524a_1^2a_2^2a_3a_5^2 + 1632a_1^2a_2^2a_4^2a_5 + 1184a_1^2a_2a_3^3a_6 \\
& + 1184a_1^2a_2a_3^2a_4a_5 - 168a_1^2a_2a_3a_4^3 + 544a_1^2a_3^4a_5 - 576a_1^2a_3^3a_4^2 \\
& + 1452a_1a_2^4a_4a_6 - 208a_1a_2^4a_5^2 - 864a_1a_2^3a_3^2a_6 - 1264a_1a_2^2a_3a_4a_5 \\
& - 567a_1a_2^3a_4^3 - 336a_1a_2^2a_3^3a_5 + 528a_1a_2^2a_3^2a_4^2 - 16a_1a_2a_3^4a_4 \\
& - 128a_1a_3^6 - 336a_2^5a_3a_6 + 100a_2^5a_4a_5 + 142a_2^4a_3^2a_5 \\
& + 25a_2^4a_3a_4^2 - 72a_2^3a_3^3a_4 + 80a_2^2a_3^5 - 27303a_0^4a_6^2 \\
& - 23784a_0^3a_1a_5a_6 + 25344a_0^3a_2a_4a_6 + 4142a_0^3a_2a_5^2 - 13056a_0^3a_3^2a_6 \\
& + 10532a_0^3a_3a_4a_5 - 3264a_0^3a_4^3 + 4952a_0^2a_1^2a_4a_6 - 1008a_0^2a_1^2a_5^2 \\
& - 29896a_0^2a_1a_2a_3a_6 - 5014a_0^2a_1a_2a_4a_5 - 4016a_0^2a_1a_3^2a_5 - 1980a_0^2a_1a_3a_4^2 \\
& + 6336a_0^2a_2^2a_6 + 1094a_0^2a_2^2a_3a_5 + 9840a_0^2a_2^2a_4^2 - 7696a_0^2a_2a_3^2a_4
\end{aligned}$$

$$\begin{aligned}
& + 1792a_0^2a_3^4 + 14656a_0a_1^3a_3a_6 - 1144a_0a_1^3a_4a_5 - 19800a_0a_1^2a_2^2a_6 \\
& - 4400a_0a_1^2a_2a_3a_5 - 5120a_0a_1^2a_2a_4^2 + 2304a_0a_1^2a_3^2a_4 + 3218a_0a_1a_2^3a_5 \\
& + 2960a_0a_1a_2^2a_3a_4 + 1024a_0a_1a_2a_3^3 - 1404a_0a_2^4a_4 - 528a_0a_2^3a_3^2 \\
& + 4128a_1^4a_2a_6 + 1584a_1^4a_3a_5 + 1488a_1^4a_4^2 - 860a_1^3a_2^2a_5 \\
& - 1280a_1^3a_2a_3a_4 - 256a_1^3a_3^3 - 216a_1^2a_2^3a_4 - 48a_1^2a_2^2a_3^2 \\
& + 348a_1a_2^4a_3 - 12a_2^6 - 2074a_0^4a_5 + 3464a_0^3a_1a_4 \\
& - 5440a_0^3a_2a_3 + 688a_0^2a_1^2a_3 - 8748a_0^2a_1a_2^2 + 9120a_0a_1^3a_2 \\
& - 1728a_1^5
\end{aligned} \tag{A.30}$$

$$\begin{aligned}
e_{31} = & -120a_0^2a_2a_6^{12} + 60a_0^2a_3a_5a_6^{11} - 6a_0^2a_4^2a_6^{11} - 510a_0^2a_4a_5^2a_6^{10} \\
& - 355a_0^2a_5^4a_6^9 - 66a_0a_1^2a_6^{12} + 84a_0a_1a_2a_5a_6^{11} + 174a_0a_1a_3a_4a_6^{11} \\
& + 1179a_0a_1a_3a_5^2a_6^{10} - 114a_0a_1a_4^2a_5a_6^{10} - 602a_0a_1a_4a_5^3a_6^9 + 342a_0a_1a_5^5a_6^8 \\
& - 314a_0a_2^2a_5^2a_6^{10} + 6a_0a_2a_3^2a_6^{11} - 66a_0a_2a_3a_4a_5a_6^{10} - 93a_0a_2a_3a_5^3a_6^9 \\
& + 8a_0a_2a_4^3a_6^{10} + 166a_0a_2a_4^2a_5^2a_6^9 - 366a_0a_2a_4a_5^4a_6^8 - 264a_0a_2a_5^6a_6^7 \\
& - 3a_0a_3^3a_5a_6^{10} - 54a_0a_3^2a_4^2a_6^{10} - 633a_0a_3^2a_4a_5^2a_6^9 - 299a_0a_3^2a_5^4a_6^8 \\
& + 68a_0a_3a_4^3a_5a_6^9 + 772a_0a_3a_4^2a_5^3a_6^8 + 405a_0a_3a_4a_5^5a_6^7 - 9a_0a_3a_5^7a_6^6 \\
& - 2a_0a_4^5a_6^9 - 56a_0a_4^4a_5^2a_6^8 - 282a_0a_4^3a_5^4a_6^7 - 270a_0a_4^2a_5^6a_6^6 \\
& - 33a_0a_4a_5^8a_6^5 + 11a_1^3a_5a_6^{11} - 36a_1^2a_2a_4a_6^{11} - 350a_1^2a_2a_5^2a_6^{10} \\
& - 51a_1^2a_3^2a_6^{11} + 57a_1^2a_3a_4a_5a_6^{10} - 48a_1^2a_3a_5^3a_6^9 + 10a_1^2a_4^3a_6^{10} \\
& + 143a_1^2a_4^2a_5^2a_6^9 + 351a_1^2a_4a_5^4a_6^8 + 223a_1^2a_5^6a_6^7 + 24a_1a_2^2a_3a_6^{11} \\
& + 8a_1a_2^2a_4a_5a_6^{10} + 283a_1a_2^2a_5^3a_6^9 + 9a_1a_2a_3^2a_5a_6^{10} + 6a_1a_2a_3a_4^2a_6^{10} \\
& + 279a_1a_2a_3a_4a_5^2a_6^9 + 363a_1a_2a_3a_5^4a_6^8 - 12a_1a_2a_3^3a_5a_6^9 - 318a_1a_2a_4^2a_5^3a_6^8 \\
& - 365a_1a_2a_4a_5^5a_6^7 - 44a_1a_2a_5^7a_6^6 + 24a_1a_3^3a_4a_6^{10} + 123a_1a_3^3a_5^2a_6^9 \\
& - 42a_1a_3^2a_4^2a_5a_6^9 - 275a_1a_3^2a_4a_5^3a_6^8 - 98a_1a_3^2a_5^5a_6^7 - 4a_1a_3a_4^4a_6^9 \\
& - 88a_1a_3a_4^3a_5^2a_6^8 - 47a_1a_3a_4^2a_5^4a_6^7 + 24a_1a_3a_4a_5^6a_6^6 + 2a_1a_3a_5^8a_6^5 \\
& + 3a_1a_4^5a_5a_6^8 + 76a_1a_4^4a_5^3a_6^7 + 116a_1a_4^3a_5^5a_6^6 + 22a_1a_4^2a_5^7a_6^5 \\
& - 16a_2^4a_6^{11} + 24a_2^3a_3a_5a_6^{10} + 8a_2^3a_4^2a_6^{10} - 76a_2^3a_4a_5^2a_6^9 \\
& - 124a_2^3a_5^4a_6^8 - 12a_2^2a_3^2a_4a_6^{10} - 7a_2^2a_3^2a_5^2a_6^9 + 2a_2^2a_3a_4^2a_5a_6^9 \\
& - 6a_2^2a_3a_4a_5^3a_6^8 - 3a_2^2a_3a_5^5a_6^7 - a_2^2a_4^4a_6^9 + 48a_2^2a_4^3a_5^2a_6^8 \\
& + 115a_2^2a_4^2a_5^4a_6^7 + 32a_2^2a_4a_5^6a_6^6 + a_2^2a_5^8a_6^5 + 6a_2a_3^4a_6^{10} \\
& - 12a_2a_3^3a_4a_5a_6^9 - 43a_2a_3^3a_5^3a_6^8 + 4a_2a_3^2a_4^3a_6^9 - 2a_2a_3^2a_4^2a_5^2a_6^8 \\
& + 23a_2a_3^2a_4a_5^4a_6^7 + 9a_2a_3^2a_5^6a_6^6 - 3a_2a_3a_4^4a_5a_6^8 + 8a_2a_3a_4^3a_5^3a_6^7 \\
& + 30a_2a_3a_4^2a_5^5a_6^6 + 4a_2a_3a_4a_5^7a_6^5 - 7a_2a_4^5a_5^2a_6^7 - 11a_2a_4^4a_5^4a_6^6 \\
& - 2a_2a_4^3a_5^6a_6^5 - 3a_3^5a_5a_6^9 - 3a_3^4a_4^2a_6^9 - 12a_3^4a_4a_5^2a_6^8
\end{aligned}$$

$$\begin{aligned}
& + 6a_3^4a_5^4a_6^7 + 6a_3^3a_4^3a_5a_6^8 + 51a_3^3a_4^2a_5^3a_6^7 + 9a_3^3a_4a_5^5a_6^6 \\
& - a_3^3a_5^7a_6^5 + 6a_3^2a_4^4a_5^2a_6^7 - 24a_3^2a_4^3a_5^4a_6^6 - 18a_3^2a_4^2a_5^6a_6^5 \\
& - 9a_3a_4^5a_5^3a_6^6 - 15a_3a_4^4a_5^5a_6^5 - a_4^6a_5^4a_6^5 + 252a_0^2a_1a_6^{11} \\
& - 2904a_0^2a_2a_5a_6^{10} - 222a_0^2a_3a_4a_6^{10} + 771a_0^2a_3a_5^2a_6^9 + 990a_0^2a_4^2a_5a_6^9 \\
& - 828a_0^2a_4a_5^3a_6^8 - 152a_0^2a_5^5a_6^7 - 1635a_0a_1^2a_5a_6^{10} - 186a_0a_1a_2a_4a_6^{10} \\
& + 2165a_0a_1a_2a_5^2a_6^9 - 78a_0a_1a_3^2a_6^{10} + 1998a_0a_1a_3a_4a_5a_6^9 + 4616a_0a_1a_3a_5^3a_6^8 \\
& + 86a_0a_1a_4^3a_6^9 - 1496a_0a_1a_4^2a_5^2a_6^8 - 3898a_0a_1a_4a_5^4a_6^7 + 1162a_0a_1a_5^6a_6^6 \\
& - 36a_0a_2^2a_3a_6^{10} + 686a_0a_2^2a_4a_5a_6^9 - 2040a_0a_2^2a_5^3a_6^8 + 210a_0a_2a_3^2a_5a_6^9 \\
& + 144a_0a_2a_3a_4^2a_6^9 - 716a_0a_2a_3a_4a_5^2a_6^8 + 733a_0a_2a_3a_5^4a_6^7 - 188a_0a_2a_4^3a_5a_6^8 \\
& + 2626a_0a_2a_4^2a_5^3a_6^7 + 2a_0a_2a_4a_5^5a_6^6 - 349a_0a_2a_5^7a_6^5 + 87a_0a_3^3a_4a_6^9 \\
& + 435a_0a_3^3a_5^2a_6^8 - 246a_0a_3^2a_4^2a_5a_6^8 - 2581a_0a_3^2a_4a_5^3a_6^7 - 340a_0a_3^2a_5^5a_6^6 \\
& - 54a_0a_3a_4^4a_6^8 - 236a_0a_3a_4^3a_5^2a_6^7 + 2462a_0a_3a_4^2a_5^4a_6^6 + 579a_0a_3a_4a_5^6a_6^5 \\
& - 20a_0a_3a_5^8a_6^4 + 57a_0a_4^5a_5a_6^7 + 290a_0a_4^4a_5^3a_6^6 + 208a_0a_4^3a_5^5a_6^5 \\
& - 58a_0a_4^2a_5^7a_6^4 + 25a_1^3a_4a_6^{10} + 614a_1^3a_5^2a_6^9 + 90a_1^2a_2a_3a_6^{10} \\
& - 237a_1^2a_2a_4a_5a_6^9 - 2218a_1^2a_2a_5^3a_6^8 - 1290a_1^2a_3^2a_5a_6^9 - 93a_1^2a_3a_4^2a_6^9 \\
& + 1004a_1^2a_3a_4a_5^2a_6^8 - 1212a_1^2a_3a_4^4a_6^7 - 24a_1^2a_4^3a_5a_6^8 - 668a_1^2a_4^2a_5^3a_6^7 \\
& - 16a_1^2a_4a_5^5a_6^6 + 258a_1^2a_5^7a_6^5 + 40a_1a_2^2a_6^{10} + 478a_1a_2^2a_3a_5a_6^9 \\
& - 38a_1a_2^2a_4^2a_6^9 - 700a_1a_2^2a_4a_5^2a_6^8 + 1667a_1a_2^2a_5^4a_6^7 - 69a_1a_2a_3^2a_4a_6^9 \\
& - 409a_1a_2a_3^2a_5^2a_6^8 - 292a_1a_2a_3a_4^2a_5a_6^8 + 850a_1a_2a_3a_4a_5^3a_6^7 + 828a_1a_2a_3a_5^5a_6^6 \\
& + 18a_1a_2a_4^4a_6^8 + 634a_1a_2a_4^3a_5^2a_6^7 - 194a_1a_2a_4^2a_5^4a_6^6 - 400a_1a_2a_4a_5^6a_6^5 \\
& - 20a_1a_2a_5^8a_6^4 - 30a_1a_3^4a_6^9 + 450a_1a_3^3a_4a_5a_6^8 + 816a_1a_3^3a_5^3a_6^7 \\
& + 54a_1a_3^2a_4^3a_6^8 - a_1a_3^2a_4^2a_5^2a_6^7 - 714a_1a_3^2a_4a_5^4a_6^6 - 122a_1a_3^2a_5^6a_6^5 \\
& + 60a_1a_3a_4^4a_5a_6^7 - 100a_1a_3a_4^3a_5^3a_6^6 + 52a_1a_3a_4^2a_5^5a_6^5 + 32a_1a_3a_4a_5^7a_6^4 \\
& - 3a_1a_4^6a_6^7 - 140a_1a_4^5a_5^2a_6^6 - 166a_1a_4^4a_5^4a_6^5 + 2a_1a_4^3a_5^6a_6^4 \\
& - 408a_2^4a_5a_6^9 - 16a_2^3a_3a_4a_6^9 + 690a_2^3a_3a_5^2a_6^8 + 350a_2^3a_4^2a_5a_6^8 \\
& - 274a_2^3a_4a_5^3a_6^7 - 375a_2^3a_5^5a_6^6 - 12a_2^2a_3^3a_6^9 - 254a_2^2a_3^2a_4a_5a_6^8 \\
& - 197a_2^2a_3^2a_5^3a_6^7 - 6a_2^2a_3a_4^3a_6^8 - 70a_2^2a_3a_4^2a_5^2a_6^7 + 27a_2^2a_3a_4a_5^4a_6^6 \\
& + 16a_2^2a_3a_5^6a_6^5 - 119a_2^2a_4^4a_5a_6^7 - 72a_2^2a_4^3a_5^3a_6^6 + 118a_2^2a_4^2a_5^5a_6^5 \\
& + 16a_2^2a_4a_5^7a_6^4 + 177a_2a_3^4a_5a_6^8 + 12a_2a_3^3a_4^2a_6^8 - 131a_2a_3^3a_4a_5^2a_6^7 \\
& - 158a_2a_3^3a_5^4a_6^6 + 102a_2a_3^2a_4^3a_5a_6^7 - 23a_2a_3^2a_4^2a_5^3a_6^6 + 53a_2a_3^2a_4a_5^5a_6^5 \\
& + 5a_2a_3^2a_5^7a_6^4 + 3a_2a_3a_4^5a_6^7 - 82a_2a_3a_4^4a_5^2a_6^6 - 134a_2a_3a_4^3a_5^4a_6^5 \\
& - 16a_2a_3a_4^2a_5^6a_6^4 + 14a_2a_4^6a_5a_6^6 + 7a_2a_4^5a_5^3a_6^5 - 7a_2a_4^4a_5^5a_6^4 \\
& + 9a_3^5a_4a_6^8 - 66a_3^5a_5^2a_6^7 - 72a_3^4a_4^2a_5a_6^7 + 24a_3^4a_5^5a_6^5 \\
& - 6a_3^3a_4^4a_6^7 - 15a_3^3a_4^3a_5^2a_6^6 + 105a_3^3a_4^2a_5^4a_6^5 - 11a_3^3a_4a_5^6a_6^4 \\
& - 12a_3^2a_5^5a_6^6 + 84a_3^2a_4^4a_5^3a_6^5 + 12a_3^2a_4^3a_5^5a_6^4 + 27a_3a_4^6a_5^2a_6^5
\end{aligned}$$

$$\begin{aligned}
& + 45a_3a_4^5a_5^4a_6^4 + 4a_4^7a_5^3a_6^4 - 6906a_0^3a_6^{10} + 9669a_0^2a_1a_5a_6^9 \\
& + 4230a_0^2a_2a_4a_6^9 - 6549a_0^2a_2a_5^2a_6^8 + 5223a_0^2a_3^2a_6^9 - 14229a_0^2a_3a_4a_5a_6^8 \\
& + 693a_0^2a_3a_5^3a_6^7 - 906a_0^2a_4^3a_6^8 + 9693a_0^2a_4^2a_5^2a_6^7 + 1639a_0^2a_4a_5^4a_6^6 \\
& + 643a_0^2a_5^6a_6^5 - 663a_0a_1^2a_4a_6^9 - 3554a_0a_1^2a_5^2a_6^8 - 2244a_0a_1a_2a_3a_6^9 \\
& - 4322a_0a_1a_2a_4a_5a_6^8 + 6108a_0a_1a_2a_5^3a_6^7 - 69a_0a_1a_3^2a_5a_6^8 + 123a_0a_1a_3a_4^2a_6^8 \\
& - 3434a_0a_1a_3a_4a_5^2a_6^7 - 3289a_0a_1a_3a_5^4a_6^6 + 2092a_0a_1a_4^3a_5a_6^7 + 3042a_0a_1a_4^2a_5^3a_6^6 \\
& - 4231a_0a_1a_4a_5^5a_6^5 + 630a_0a_1a_5^7a_6^4 - 3394a_0a_2^3a_6^9 + 4309a_0a_2^2a_3a_5a_6^8 \\
& + 1308a_0a_2^2a_4^2a_6^8 + 2970a_0a_2^2a_4a_5^2a_6^7 - 2064a_0a_2^2a_5^4a_6^6 - 72a_0a_2a_3^2a_4a_6^8 \\
& - 2488a_0a_2a_3^2a_5^2a_6^7 + 1015a_0a_2a_3a_4^2a_5a_6^7 - 874a_0a_2a_3a_4a_5^3a_6^6 + 2136a_0a_2a_3a_5^5a_6^5 \\
& - 172a_0a_2a_4^4a_6^7 - 4898a_0a_2a_4^3a_5^2a_6^6 + 2597a_0a_2a_4^2a_5^4a_6^5 + 319a_0a_2a_4a_5^6a_6^4 \\
& - 70a_0a_2a_5^8a_6^3 - 1287a_0a_3^4a_6^8 + 3297a_0a_3^3a_4a_5a_6^7 + 1223a_0a_3^3a_5^3a_6^6 \\
& + 69a_0a_3^2a_4^3a_6^7 + 661a_0a_3^2a_4^2a_5^2a_6^6 - 1755a_0a_3^2a_4a_5^4a_6^5 + 92a_0a_3^2a_5^6a_6^4 \\
& - 417a_0a_3a_4^4a_5a_6^6 - 5150a_0a_3a_4^3a_5^3a_6^5 + 215a_0a_3a_4^2a_5^5a_6^4 + a_0a_3a_4a_5^7a_6^3 \\
& - 9a_0a_4^6a_6^6 + 125a_0a_4^5a_5^2a_6^5 + 295a_0a_4^4a_5^4a_6^4 + 190a_0a_4^3a_5^6a_6^3 \\
& + 1367a_1^3a_3a_6^9 - 860a_1^3a_4a_5a_6^8 + 2844a_1^3a_5^3a_6^7 + 729a_1^2a_2^2a_6^9 \\
& - 102a_1^2a_2a_3a_5a_6^8 - 121a_1^2a_2a_4^2a_6^8 + 4618a_1^2a_2a_4a_5^2a_6^7 - 2016a_1^2a_2a_5^4a_6^6 \\
& - 435a_1^2a_3^2a_4a_6^8 - 1730a_1^2a_3^2a_5^2a_6^7 - 361a_1^2a_3a_4^2a_5a_6^7 + 4232a_1^2a_3a_4a_5^3a_6^6 \\
& - 1954a_1^2a_3a_5^5a_6^5 + 21a_1^2a_4^4a_6^7 + 358a_1^2a_4^3a_5^2a_6^6 - 1492a_1^2a_4^2a_5^4a_6^5 \\
& - 426a_1^2a_4a_5^6a_6^4 + 40a_1^2a_5^8a_6^3 + 1113a_1a_2^3a_5a_6^8 + 538a_1a_2^2a_3a_4a_6^8 \\
& - 376a_1a_2^2a_3a_5^2a_6^7 - 697a_1a_2^2a_4^2a_5a_6^7 - 5431a_1a_2^2a_4a_5^3a_6^6 + 1606a_1a_2^2a_5^5a_6^5 \\
& + 765a_1a_2a_3^3a_6^8 - 1559a_1a_2a_3^2a_4a_5a_6^7 - 1704a_1a_2a_3^2a_5^3a_6^6 - 209a_1a_2a_3a_4^3a_6^7 \\
& - 1130a_1a_2a_3a_4^2a_5^2a_6^6 - 124a_1a_2a_3a_4a_5^4a_6^5 + 218a_1a_2a_3a_5^6a_6^4 - 74a_1a_2a_4^4a_5a_6^6 \\
& + 2320a_1a_2a_4^3a_5^3a_6^5 + 710a_1a_2a_4^2a_5^5a_6^4 - 26a_1a_2a_4a_5^7a_6^3 - 708a_1a_3^4a_5a_6^7 \\
& - 123a_1a_3^3a_4^2a_6^7 + 539a_1a_3^3a_4a_5^2a_6^6 + 830a_1a_3^3a_5^4a_6^5 + 255a_1a_3^2a_4^3a_5a_6^6 \\
& + 825a_1a_3^2a_4^3a_5^5a_6^5 - 212a_1a_3^2a_4a_5^5a_6^4 - 8a_1a_3^2a_5^7a_6^3 + 15a_1a_3a_4^5a_6^6 \\
& + 232a_1a_3a_4^4a_5^2a_6^5 - 190a_1a_3a_4^3a_5^4a_6^4 + 16a_1a_3a_4^2a_5^6a_6^3 + 61a_1a_4^6a_5a_6^5 \\
& - 76a_1a_4^5a_5^3a_6^4 - 86a_1a_4^4a_5^5a_6^3 + 348a_2^4a_4a_6^8 - 1291a_2^4a_5^2a_6^7 \\
& - 808a_2^3a_3^2a_6^8 - 440a_2^3a_3a_4a_5a_6^7 + 2183a_2^3a_3a_5^3a_6^6 - 242a_2^3a_4^3a_6^7 \\
& + 1792a_2^3a_4^2a_5^2a_6^6 + 224a_2^3a_4a_5^4a_6^5 - 177a_2^3a_5^6a_6^4 + 648a_2^2a_3^3a_5a_6^7 \\
& + 437a_2^2a_3^2a_4^2a_6^7 - 947a_2^2a_3^2a_4a_5^2a_6^6 - 469a_2^2a_3^2a_5^4a_6^5 - 64a_2^2a_3a_4^3a_5a_6^6 \\
& - 119a_2^2a_3a_4^2a_5^3a_6^5 + 117a_2^2a_3a_4a_5^5a_6^4 + 11a_2^2a_3a_5^7a_6^3 + 67a_2^2a_4^5a_6^6 \\
& - 360a_2^2a_4^4a_5^5a_6^5 - 320a_2^2a_4^3a_5^4a_6^4 - 8a_2^2a_4^2a_5^6a_6^3 - 393a_2a_3^4a_4a_6^7 \\
& + 493a_2a_3^4a_5^2a_6^6 + 615a_2a_3^3a_4^2a_5a_6^6 - 65a_2a_3^3a_4a_5^3a_6^5 - 38a_2a_3^3a_5^5a_6^4 \\
& - 87a_2a_3^2a_4^4a_6^6 + 143a_2a_3^2a_4^3a_5^2a_6^5 - 90a_2a_3^2a_4^2a_5^4a_6^4 - 2a_2a_3^2a_4a_5^6a_6^3 \\
& + 117a_2a_3a_4^5a_5a_6^5 + 72a_2a_3a_4^4a_5^3a_6^4 - 18a_2a_3a_4^3a_5^5a_6^3 - 7a_2a_4^7a_6^5
\end{aligned}$$

$$\begin{aligned}
& + 31a_2a_4^6a_5^2a_6^4 + 26a_2a_4^5a_5^4a_6^3 + 99a_3^6a_6^7 - 36a_3^5a_4a_5a_6^6 \\
& - 150a_3^5a_5^3a_6^5 + 78a_3^4a_4^3a_6^6 - 171a_3^4a_4^2a_5^2a_6^5 + 105a_3^4a_4a_5^4a_6^4 \\
& - a_3^4a_5^6a_6^3 - 93a_3^3a_4^4a_5a_6^5 - 156a_3^3a_4^3a_5^3a_6^4 - 6a_3^3a_4^2a_5^5a_6^3 \\
& + 6a_3^2a_4^6a_6^5 - 123a_3^2a_4^5a_5^2a_6^4 + 39a_3^2a_4^4a_5^4a_6^3 - 27a_3a_4^7a_5a_6^4 \\
& - 43a_3a_4^6a_5^3a_6^3 - 6a_4^8a_5^2a_6^3 + 795a_6^3a_5a_6^8 - 6369a_6^2a_1a_4a_6^8 \\
& + 15324a_6^2a_1a_5^2a_6^7 - 12234a_6^2a_2a_3a_6^8 + 14409a_6^2a_2a_4a_5a_6^7 + 5167a_6^2a_2a_5^3a_6^6 \\
& + 11163a_6^2a_3^2a_5a_6^7 + 11370a_6^2a_3a_4^2a_6^7 - 34821a_6^2a_3a_4a_5^2a_6^6 - 547a_6^2a_3a_5^4a_6^5 \\
& - 13694a_6^2a_4^3a_5a_6^6 + 10294a_6^2a_4^2a_5^3a_6^5 - 1855a_6^2a_4a_5^5a_6^4 + 230a_6^2a_5^7a_6^3 \\
& - 1311a_6a_1^2a_3a_6^8 + 4616a_6a_1^2a_4a_5a_6^7 + 572a_6a_1^2a_5^3a_6^6 + 10866a_6a_1a_2^2a_6^8 \\
& - 1822a_6a_1a_2a_3a_5a_6^7 - 3039a_6a_1a_2a_4^2a_6^7 - 17726a_6a_1a_2a_4a_5^2a_6^6 + 3059a_6a_1a_2a_5^4a_6^5 \\
& + 1341a_6a_1a_3^2a_4a_6^7 + 11672a_6a_1a_3^2a_5^2a_6^6 + 3406a_6a_1a_3a_4^2a_5a_6^6 + 10a_6a_1a_3a_4a_5^3a_6^5 \\
& - 4348a_6a_1a_3a_5^5a_6^4 - 86a_6a_1a_4^4a_6^6 + 1398a_6a_1a_4^3a_5^2a_6^5 + 7067a_6a_1a_4^2a_5^4a_6^4 \\
& - 1328a_6a_1a_4a_5^6a_6^3 + 40a_6a_1a_5^8a_6^2 - 5060a_6a_2^3a_5a_6^7 - 9165a_6a_2^2a_3a_4a_6^7 \\
& + 5030a_6a_2^2a_3a_5^2a_6^6 + 4580a_6a_2^2a_4^2a_5a_6^6 + 799a_6a_2^2a_4a_5^3a_6^5 - 290a_6a_2^2a_5^5a_6^4 \\
& + 4273a_6a_2a_3^3a_6^7 - 3726a_6a_2a_3^2a_4a_5a_6^6 - 6789a_6a_2a_3^2a_5^3a_6^5 + 2540a_6a_2a_3a_4^3a_6^6 \\
& + 6598a_6a_2a_3a_4^2a_5^2a_6^5 - 926a_6a_2a_3a_4a_5^4a_6^4 + 456a_6a_2a_3a_5^6a_6^3 + 2005a_6a_2a_4^4a_5a_6^5 \\
& - 7754a_6a_2a_4^3a_5^3a_6^4 - 39a_6a_2a_4^2a_5^5a_6^3 + 35a_6a_2a_4a_5^7a_6^2 - 3662a_6a_3^4a_5a_6^6 \\
& - 3286a_6a_3^3a_4^2a_6^6 + 4326a_6a_3^3a_4a_5^2a_6^5 - 236a_6a_3^3a_5^4a_6^4 + 2541a_6a_3^2a_4^3a_5a_6^5 \\
& + 579a_6a_3^2a_4^2a_5^3a_6^4 + 434a_6a_3^2a_4a_5^5a_6^3 - 22a_6a_3^2a_5^7a_6^2 - 114a_6a_3a_4^5a_6^5 \\
& + 3278a_6a_3a_4^4a_5^2a_6^4 - 2135a_6a_3a_4^3a_5^4a_6^3 + 5a_6a_3a_4^2a_5^6a_6^2 - 141a_6a_4^6a_5a_6^4 \\
& - 224a_6a_4^5a_5^3a_6^3 - 94a_6a_4^4a_5^5a_6^2 - 2995a_6a_1^3a_2a_6^8 + 130a_6a_1^3a_3a_5a_6^7 \\
& + 976a_6a_1^3a_4a_6^7 - 5578a_6a_1^3a_4a_5^2a_6^6 + 1938a_6a_1^3a_5^4a_6^5 - 828a_6a_1^2a_2^2a_5a_6^7 \\
& + 2252a_6a_1^2a_2a_3a_4a_6^7 - 1544a_6a_1^2a_2a_3a_5^2a_6^6 - 5139a_6a_1^2a_2a_4^2a_5a_6^6 + 9416a_6a_1^2a_2a_4a_5^3a_6^5 \\
& + 182a_6a_1^2a_2a_5^5a_6^4 - 262a_6a_1^2a_3^3a_6^7 + 522a_6a_1^2a_3^2a_4a_5a_6^6 + 3500a_6a_1^2a_3^2a_5^3a_6^5 \\
& - 679a_6a_1^2a_3a_4^3a_6^6 - 2948a_6a_1^2a_3a_4^2a_5^2a_6^5 + 3792a_6a_1^2a_3a_4a_5^4a_6^4 - 318a_6a_1^2a_3a_5^6a_6^3 \\
& + 508a_6a_1^2a_4^4a_5a_6^5 + 1986a_6a_1^2a_4^3a_5^3a_6^4 - 52a_6a_1^2a_4^2a_5^5a_6^3 - 56a_6a_1^2a_4a_5^7a_6^2 \\
& - 1853a_6a_1a_2^3a_4a_6^7 + 4842a_6a_1a_2^3a_5^2a_6^6 + 25a_6a_1a_2^2a_3^2a_6^7 + 5066a_6a_1a_2^2a_3a_4a_5a_6^6 \\
& - 2806a_6a_1a_2^2a_3a_5^3a_6^5 + 1232a_6a_1a_2^2a_4^3a_6^6 + 1795a_6a_1a_2^2a_4^2a_5^2a_6^5 - 5938a_6a_1a_2^2a_4a_5^4a_6^4 \\
& + 212a_6a_1a_2^2a_5^6a_6^3 + 2460a_6a_1a_2a_3^3a_5a_6^6 + 1022a_6a_1a_2a_3^2a_4^2a_6^6 - 1291a_6a_1a_2a_3^2a_4a_5^2a_6^5 \\
& - 478a_6a_1a_2a_3^2a_5^4a_6^4 - 1266a_6a_1a_2a_3a_4^3a_5a_6^5 - 36a_6a_1a_2a_3a_4^2a_5^3a_6^4 - 346a_6a_1a_2a_3a_4a_5^5a_6^3 \\
& - 10a_6a_1a_2a_3a_5^7a_6^2 - 221a_6a_1a_2a_4^5a_6^5 - 2010a_6a_1a_2a_4^4a_5^2a_6^4 + 376a_6a_1a_2a_4^3a_5^4a_6^3 \\
& + 106a_6a_1a_2a_4^2a_5^6a_6^2 + 640a_6a_1a_3^4a_4a_6^6 - 1318a_6a_1a_3^4a_5^2a_6^5 - 1658a_6a_1a_3^3a_4^2a_5a_6^5 \\
& - 496a_6a_1a_3^3a_4a_5^3a_6^4 + 34a_6a_1a_3^3a_5^5a_6^3 - 39a_6a_1a_3^2a_4^4a_6^5 + 29a_6a_1a_3^2a_4^3a_5^2a_6^4 \\
& + 354a_6a_1a_3^2a_4^2a_5^4a_6^3 + 32a_6a_1a_3^2a_4a_5^6a_6^2 - 116a_6a_1a_3a_4^5a_5a_6^4 + 210a_6a_1a_3a_4^4a_5^3a_6^3 \\
& - 80a_6a_1a_3a_4^3a_5^5a_6^2 + 3a_6a_1a_4^7a_6^4 + 210a_6a_1a_4^6a_5^2a_6^3 + 78a_6a_1a_4^5a_5^4a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 1276a_2^4a_3a_6^7 + 914a_2^4a_4a_5a_6^6 - 995a_2^4a_5^3a_6^5 - 2844a_2^3a_3^2a_5a_6^6 \\
& - 470a_2^3a_3a_4^2a_6^6 - 2969a_2^3a_3a_4a_5^2a_6^5 + 1064a_2^3a_3a_5^4a_6^4 - 1616a_2^3a_4^3a_5a_6^5 \\
& + 1679a_2^3a_4^2a_5^3a_6^4 + 295a_2^3a_4a_5^5a_6^3 - 9a_2^3a_5^7a_6^2 - 492a_2^2a_3^3a_4a_6^6 \\
& + 1512a_2^2a_3^3a_5^2a_6^5 + 2237a_2^2a_3^2a_4^2a_5a_6^5 - 319a_2^2a_3^2a_4a_5^3a_6^4 - 116a_2^2a_3^2a_5^5a_6^3 \\
& + 106a_2^2a_3a_4^4a_6^5 - 81a_2^2a_3a_4^3a_5^2a_6^4 - 153a_2^2a_3a_4^2a_5^4a_6^3 + 19a_2^2a_3a_4a_5^6a_6^2 \\
& + 429a_2^2a_4^5a_5a_6^4 + 78a_2^2a_4^4a_5^3a_6^3 - 34a_2^2a_4^3a_5^5a_6^2 - 238a_2a_3^5a_6^6 \\
& - 776a_2a_3^4a_4a_5a_6^5 - 4a_2a_3^4a_5^3a_6^4 - 357a_2a_3^3a_4^3a_6^5 + 955a_2a_3^3a_4^2a_5^2a_6^4 \\
& + 120a_2a_3^3a_4a_5^4a_6^3 + 2a_2a_3^3a_5^6a_6^2 - 175a_2a_3^2a_4^4a_5a_6^4 - 94a_2a_3^2a_4^3a_5^3a_6^3 \\
& - 46a_2a_3^2a_4^2a_5^5a_6^2 - 43a_2a_3a_4^6a_6^4 + 104a_2a_3a_4^5a_5^2a_6^3 + 42a_2a_3a_4^4a_5^4a_6^2 \\
& - 39a_2a_4^7a_5a_6^3 - 23a_2a_4^6a_5^3a_6^2 + 268a_3^6a_5a_6^5 + 36a_3^5a_4^2a_6^5 \\
& - 186a_3^5a_4a_5^2a_6^4 + 18a_3^5a_5^4a_6^3 + 330a_3^4a_4^3a_5a_6^4 - 60a_3^4a_4^2a_5^3a_6^3 \\
& - 6a_3^4a_4a_5^5a_6^2 + 57a_3^3a_4^5a_6^4 - 60a_3^3a_4^4a_5^2a_6^3 + 30a_3^3a_4^3a_5^4a_6^2 \\
& + 90a_3^2a_4^6a_5a_6^3 - 36a_3^2a_4^5a_5^3a_6^2 + 9a_3a_4^8a_6^3 + 9a_3a_4^7a_5^2a_6^2 \\
& + 4a_4^9a_5a_6^2 - 21801a_0^3a_4a_6^7 + 23490a_0^3a_5^2a_6^6 + 16521a_0^2a_1a_3a_6^7 \\
& - 17412a_0^2a_1a_4a_5a_6^6 - 18017a_0^2a_1a_5^3a_6^5 - 23901a_0^2a_2^2a_6^7 - 10152a_0^2a_2a_3a_5a_6^6 \\
& + 12072a_0^2a_2a_4^2a_6^6 - 22027a_0^2a_2a_4a_5^2a_6^5 + 10725a_0^2a_2a_5^4a_6^4 - 4806a_0^2a_3^2a_4a_6^6 \\
& - 1323a_0^2a_3^2a_5^2a_6^5 + 39072a_0^2a_3a_4^2a_5a_6^5 + 724a_0^2a_3a_4a_5^3a_6^4 - 1603a_0^2a_3a_5^5a_6^3 \\
& + 2487a_0^2a_4^4a_6^5 - 20610a_0^2a_4^3a_5^2a_6^4 + 3536a_0^2a_4^2a_5^4a_6^3 - 348a_0^2a_4a_5^6a_6^2 \\
& + 5a_0^2a_5^8a_6 + 4737a_0a_1^2a_2a_6^7 - 10367a_0a_1^2a_3a_5a_6^6 - 7656a_0a_1^2a_4^2a_6^6 \\
& + 10184a_0a_1^2a_4a_5^2a_6^5 - 1750a_0a_1^2a_5^4a_6^4 + 19023a_0a_1a_2^2a_5a_6^6 - 6684a_0a_1a_2a_3a_4a_6^6 \\
& + 5451a_0a_1a_2a_3a_5^2a_6^5 + 8832a_0a_1a_2a_4^2a_5a_6^5 - 10998a_0a_1a_2a_4a_5^3a_6^4 + 1426a_0a_1a_2a_5^5a_6^3 \\
& - 5045a_0a_1a_3^3a_6^6 - 45a_0a_1a_3^2a_4a_5a_6^5 + 10408a_0a_1a_3^2a_5^3a_6^4 + 2892a_0a_1a_3a_4^3a_6^5 \\
& - 1266a_0a_1a_3a_4^2a_5^2a_6^4 + 7308a_0a_1a_3a_4a_5^4a_6^3 - 213a_0a_1a_3a_5^6a_6^2 - 3240a_0a_1a_4^4a_5a_6^4 \\
& - 6722a_0a_1a_4^3a_5^3a_6^3 + 726a_0a_1a_4^2a_5^5a_6^2 - 87a_0a_1a_4a_5^7a_6 - 530a_0a_2^3a_4a_6^6 \\
& + 5497a_0a_2^3a_5^2a_6^5 + 10881a_0a_2^2a_3^2a_6^6 - 22095a_0a_2^2a_3a_4a_5a_6^5 - 2785a_0a_2^2a_3a_5^3a_6^4 \\
& - 2378a_0a_2^2a_4^3a_6^5 + 8573a_0a_2^2a_4^2a_5^2a_6^4 - 1041a_0a_2^2a_4a_5^4a_6^3 - 182a_0a_2^2a_5^6a_6^2 \\
& + 2482a_0a_2a_3^3a_5a_6^5 - 1686a_0a_2a_3^2a_4^2a_6^5 + 6374a_0a_2a_3^2a_4a_5^2a_6^4 - 1244a_0a_2a_3^2a_5^4a_6^3 \\
& + 712a_0a_2a_3a_4^3a_5a_6^4 + 2694a_0a_2a_3a_4^2a_5^3a_6^3 - 18a_0a_2a_3a_4a_5^5a_6^2 - 34a_0a_2a_3a_5^7a_6 \\
& + 48a_0a_2a_4^5a_6^4 + 6546a_0a_2a_4^4a_5^2a_6^3 - 735a_0a_2a_4^3a_5^4a_6^2 + 9a_0a_2a_4^2a_5^6a_6 \\
& + 3150a_0a_3^4a_4a_6^5 + 754a_0a_3^4a_5^2a_6^4 - 9295a_0a_3^3a_4^2a_5a_6^4 - 2132a_0a_3^3a_4a_5^3a_6^3 \\
& + 197a_0a_3^3a_5^5a_6^2 - 1191a_0a_3^2a_4^4a_6^4 + 5167a_0a_3^2a_4^3a_5^2a_6^3 - 345a_0a_3^2a_4^2a_5^4a_6^2 \\
& + 31a_0a_3^2a_4a_5^6a_6 - 1246a_0a_3a_4^5a_5^3a_6^3 + 1749a_0a_3a_4^4a_5^3a_6^2 + 14a_0a_3a_4^3a_5^5a_6 \\
& - 4a_0a_4^7a_6^3 - 44a_0a_4^6a_5^2a_6^2 - 18a_0a_4^5a_5^4a_6 + 877a_1^4a_6^7 \\
& - 3728a_1^3a_2a_5a_6^6 + 3233a_1^3a_3a_4a_6^6 - 5342a_1^3a_3a_5^2a_6^5 + 4365a_1^3a_4^2a_5a_6^5 \\
& - 2470a_1^3a_4a_5^3a_6^4 + 142a_1^3a_5^5a_6^3 + 3567a_1^2a_2^2a_4a_6^6 - 6299a_1^2a_2^2a_5^2a_6^5
\end{aligned}$$

$$\begin{aligned}
& -3715a_1^2a_2a_3^2a_6^6 + 6291a_1^2a_2a_3a_4a_5a_6^5 - 3334a_1^2a_2a_3a_5^3a_6^4 + 609a_1^2a_2a_4^3a_6^5 \\
& - 16164a_1^2a_2a_4^2a_5^2a_6^4 + 1746a_1^2a_2a_4a_5^4a_6^3 + 136a_1^2a_2a_5^6a_6^2 - 1952a_1^2a_3^3a_5a_6^5 \\
& - 2247a_1^2a_3^2a_4^2a_6^5 - 4804a_1^2a_3^2a_4a_5^2a_6^4 + 641a_1^2a_3^2a_5^4a_6^3 + 840a_1^2a_3a_4^3a_5a_6^4 \\
& - 2680a_1^2a_3a_4^2a_5^3a_6^3 + 414a_1^2a_3a_4a_5^5a_6^2 + 14a_1^2a_3a_5^7a_6 - 194a_1^2a_4^5a_6^4 \\
& - 135a_1^2a_4^4a_5^2a_6^3 + 541a_1^2a_4^3a_5^4a_6^2 + 35a_1^2a_4^2a_5^6a_6 - 2441a_1a_2^3a_3a_6^6 \\
& - 8677a_1a_2^3a_4a_5a_6^5 + 4990a_1a_2^3a_5^3a_6^4 + 788a_1a_2^2a_3^2a_5a_6^5 - 1990a_1a_2^2a_3a_4^2a_6^5 \\
& + 12539a_1a_2^2a_3a_4a_5^2a_6^4 - 321a_1a_2^2a_3a_5^4a_6^3 + 3883a_1a_2^2a_4^3a_5a_6^4 + 4099a_1a_2^2a_4^2a_5^3a_6^3 \\
& - 883a_1a_2^2a_4a_5^5a_6^2 - 2a_1a_2^2a_5^7a_6 + 1196a_1a_2a_3^3a_4a_6^5 + 834a_1a_2a_3^3a_5^2a_6^4 \\
& - 32a_1a_2a_3^2a_4^2a_5a_6^4 + 290a_1a_2a_3^2a_4a_5^3a_6^3 + 66a_1a_2a_3^2a_5^5a_6^2 + 840a_1a_2a_3a_4^4a_6^4 \\
& - 1682a_1a_2a_3a_4^3a_5^2a_6^3 + 495a_1a_2a_3a_4^2a_5^4a_6^2 + 8a_1a_2a_3a_4a_5^6a_6 + 98a_1a_2a_4^5a_5a_6^3 \\
& - 866a_1a_2a_4^4a_5^2a_6^2 - 49a_1a_2a_4^3a_5^5a_6 + 48a_1a_3^5a_6^5 + 1862a_1a_4^4a_5a_6^4 \\
& + 88a_1a_3^4a_5^3a_6^3 + 1097a_1a_3^3a_4^3a_6^4 - 478a_1a_3^3a_4^2a_5^2a_6^3 - 247a_1a_3^3a_4a_5^4a_6^2 \\
& - 4a_1a_3^3a_5^6a_6 - 483a_1a_3^2a_4^4a_5a_6^3 - 124a_1a_3^2a_4^3a_5^3a_6^2 - 29a_1a_3^2a_4^2a_5^5a_6 \\
& + 16a_1a_3a_4^6a_6^3 - 133a_1a_3a_4^5a_5^2a_6^2 + 9a_1a_3a_4^4a_5^4a_6 - 99a_1a_4^7a_5a_6^2 \\
& - 13a_1a_4^6a_5^3a_6 - 430a_2^5a_6^6 + 3443a_2^4a_3a_5a_6^5 + 312a_2^4a_4^2a_6^5 \\
& - 773a_2^4a_4a_5^2a_6^4 - 445a_2^4a_5^4a_6^3 + 1648a_2^3a_3^2a_4a_6^5 - 1922a_2^3a_3^2a_5^2a_6^4 \\
& + 127a_2^3a_3a_4^2a_5a_6^4 - 1507a_2^3a_3a_4a_5^3a_6^3 + 41a_2^3a_3a_5^5a_6^2 + 293a_2^3a_4^3a_6^4 \\
& - 1797a_2^3a_4^3a_5^2a_6^3 + 132a_2^3a_4^2a_5^4a_6^2 + 17a_2^3a_4a_5^6a_6 - 154a_2^2a_3^4a_6^5 \\
& - 2166a_2^2a_3^3a_4a_5a_6^4 + 210a_2^2a_3^3a_5^3a_6^3 - 853a_2^2a_3^2a_4^3a_6^4 + 1569a_2^2a_3^2a_4^2a_5^2a_6^3 \\
& + 60a_2^2a_3^2a_4a_5^4a_6^2 + 5a_2^2a_3^2a_5^6a_6 + 62a_2^2a_3a_4^4a_5a_6^3 - 114a_2^2a_3a_4^3a_5^3a_6^2 \\
& - 34a_2^2a_3a_4^2a_5^5a_6 - 118a_2^2a_4^6a_6^3 + 157a_2^2a_4^5a_5^2a_6^2 + 25a_2^2a_4^4a_5^4a_6 \\
& + 254a_2a_3^5a_5a_6^4 - 55a_2a_3^4a_4^2a_6^4 - 254a_2a_3^4a_4a_5^2a_6^3 - 11a_2a_3^4a_5^4a_6^2 \\
& - 441a_2a_3^3a_4^3a_5a_6^3 + 52a_2a_3^3a_4^2a_5^3a_6^2 - 7a_2a_3^3a_4a_5^5a_6 + 40a_2a_3^2a_4^5a_6^3 \\
& + 133a_2a_3^2a_4^4a_5^2a_6^2 + 33a_2a_3^2a_4^3a_5^4a_6 - 75a_2a_3a_4^6a_5a_6^2 - 3a_2a_3a_4^5a_5^3a_6 \\
& + 12a_2a_4^8a_6^2 + 6a_2a_4^7a_5^2a_6 - 136a_3^6a_4a_6^4 - 96a_3^6a_5^2a_6^3 \\
& + 264a_3^5a_4^2a_5a_6^3 + 48a_3^5a_4a_5^3a_6^2 - 159a_3^4a_4^4a_6^3 - 96a_3^4a_4^3a_5^2a_6^2 \\
& + 6a_3^4a_4^2a_5^4a_6 + 138a_3^3a_4^5a_5a_6^2 - 6a_3^3a_4^4a_5^3a_6 - 27a_3^2a_4^7a_6^2 \\
& - 3a_3^2a_4^6a_5^2a_6 + 6a_3a_4^8a_5a_6 - a_4^{10}a_6 + 3783a_0^3a_3a_6^6 \\
& - 66768a_0^3a_4a_5a_6^5 - 11775a_0^3a_5^3a_6^4 + 21711a_0^2a_1a_2a_6^6 + 48864a_0^2a_1a_3a_5a_6^5 \\
& - 882a_0^2a_1a_4^2a_6^5 + 44111a_0^2a_1a_4a_5^2a_6^4 - 7320a_0^2a_1a_5^4a_6^3 + 55077a_0^2a_2^2a_5a_6^5 \\
& - 32208a_0^2a_2a_3a_4a_6^5 - 34463a_0^2a_2a_3a_5^2a_6^4 + 32412a_0^2a_2a_4^2a_5a_6^4 - 10200a_0^2a_2a_4a_5^3a_6^3 \\
& + 633a_0^2a_2a_5^5a_6^2 + 4953a_0^2a_3^3a_6^5 - 8685a_0^2a_3^2a_4a_5a_6^4 + 6798a_0^2a_3^2a_5^3a_6^3 \\
& - 2964a_0^2a_3a_4^3a_6^4 - 4738a_0^2a_3a_4^2a_5^2a_6^3 + 2271a_0^2a_3a_4a_5^4a_6^2 - a_0^2a_3a_5^6a_6 \\
& + 8126a_0^2a_4^4a_5a_6^3 - 3106a_0^2a_4^3a_5^3a_6^2 - 86a_0^2a_4^2a_5^5a_6 - 37a_0^2a_4a_5^7 \\
& - 7570a_0a_1^3a_6^6 - 40995a_0a_1^2a_2a_5a_6^5 + 19092a_0a_1^2a_3a_4a_6^5 + 14118a_0a_1^2a_3a_5^2a_6^4
\end{aligned}$$

$$\begin{aligned}
& -17867a_0a_1^2a_4^2a_5a_6^4 + 1382a_0a_1^2a_4a_5^3a_6^3 + 332a_0a_1^2a_5^5a_6^2 + 8475a_0a_1a_2^2a_4a_5^5 \\
& -14175a_0a_1a_2^2a_5^2a_6^4 + 3411a_0a_1a_2a_3^2a_5^5 - 532a_0a_1a_2a_3a_4a_5a_6^4 - 8032a_0a_1a_2a_3a_5^3a_6^3 \\
& -9370a_0a_1a_2a_4^3a_6^4 + 12626a_0a_1a_2a_4^2a_5^2a_6^3 - 1984a_0a_1a_2a_4a_5^4a_6^2 + 75a_0a_1a_2a_5^6a_6 \\
& -7544a_0a_1a_3^3a_5a_6^4 - 10701a_0a_1a_3^2a_4^2a_6^4 - 17318a_0a_1a_3^2a_4a_5^2a_6^3 - 466a_0a_1a_3^2a_5^4a_6^2 \\
& + 13576a_0a_1a_3a_4^3a_5a_6^3 - 1060a_0a_1a_3a_4^2a_5^3a_6^2 + 640a_0a_1a_3a_4a_5^5a_6 + 2a_0a_1a_3a_5^7 \\
& + 2230a_0a_1a_4^5a_6^3 + 762a_0a_1a_4^4a_5^2a_6^2 - 444a_0a_1a_4^3a_5^4a_6 + 33a_0a_1a_4^2a_5^6 \\
& - 22401a_0a_2^3a_3a_5^5 - 6496a_0a_2^3a_4a_5a_6^4 + 1735a_0a_2^3a_5^3a_6^3 + 7870a_0a_2^2a_3^2a_5a_6^4 \\
& + 17721a_0a_2^2a_3a_4^2a_6^4 + 7164a_0a_2^2a_3a_4a_5^2a_6^3 + 1261a_0a_2^2a_3a_5^4a_6^2 - 10249a_0a_2^2a_4^3a_5a_6^3 \\
& + 2011a_0a_2^2a_4^2a_5^3a_6^2 - 61a_0a_2^2a_4a_5^5a_6 - 6a_0a_2^2a_5^7 + 5674a_0a_2a_3^3a_4a_6^4 \\
& + 1204a_0a_2a_3^3a_5^2a_6^3 - 15127a_0a_2a_3^2a_4^2a_5a_6^3 + 244a_0a_2a_3^2a_4a_5^3a_6^2 + 323a_0a_2a_3^2a_5^5a_6 \\
& - 2150a_0a_2a_3a_4^4a_6^3 - 574a_0a_2a_3a_4^3a_5^2a_6^2 - 33a_0a_2a_3a_4^2a_5^4a_6 + 24a_0a_2a_3a_4a_5^6 \\
& - 1664a_0a_2a_4^5a_5a_6^2 + 777a_0a_2a_4^4a_5^3a_6 - a_0a_2a_4^3a_5^5 - 2076a_0a_3^5a_6^4 \\
& + 3928a_0a_3^4a_4a_5a_6^3 - 568a_0a_3^4a_5^3a_6^2 + 3100a_0a_3^3a_4^3a_6^3 + 1084a_0a_3^3a_4^2a_5^2a_6^2 \\
& - 218a_0a_3^3a_4a_5^4a_6 - 2a_0a_3^3a_5^6 - 4377a_0a_3^2a_4^4a_5a_6^2 - 146a_0a_3^2a_4^3a_5^3a_6 \\
& - 25a_0a_3^2a_4^2a_5^5 + 198a_0a_3a_4^6a_6^2 - 428a_0a_3a_4^5a_5^2a_6 - 21a_0a_3a_4^4a_5^4 \\
& + 27a_0a_4^7a_5a_6 + 11a_0a_4^6a_5^3 + 9772a_1^4a_5a_6^5 - 5467a_1^3a_2a_4a_5^5 \\
& + 4990a_1^3a_2a_5^2a_6^4 + 24a_1^3a_3^2a_6^5 + 1226a_1^3a_3a_4a_5a_6^4 + 164a_1^3a_3a_5^3a_6^3 \\
& + 249a_1^3a_4^3a_6^4 + 3684a_1^3a_4^2a_5^2a_6^3 + 338a_1^3a_4a_5^4a_6^2 + 66a_1^3a_5^6a_6 \\
& + 8328a_1^2a_2^2a_3a_6^5 + 14981a_1^2a_2^2a_4a_5a_6^4 - 1270a_1^2a_2^2a_5^3a_6^3 + 2562a_1^2a_2a_3^2a_5a_6^4 \\
& + 1531a_1^2a_2a_3a_4^2a_6^4 + 2002a_1^2a_2a_3a_4a_5^2a_6^3 - 1020a_1^2a_2a_3a_5^4a_6^2 + 5832a_1^2a_2a_4^3a_5a_6^3 \\
& - 4922a_1^2a_2a_4^2a_5^3a_6^2 - 250a_1^2a_2a_4a_5^5a_6 + 1566a_1^2a_3^3a_4a_6^4 - 96a_1^2a_3^2a_5^2a_6^3 \\
& + 1416a_1^2a_3^2a_4^2a_5a_6^3 - 1108a_1^2a_3^2a_4a_5^3a_6^2 - 188a_1^2a_3^2a_5^5a_6 - 1322a_1^2a_3a_4^4a_6^3 \\
& + 850a_1^2a_3a_4^3a_5^2a_6^2 - 120a_1^2a_3a_4^2a_5^4a_6 - 2a_1^2a_3a_4a_5^6 - 40a_1^2a_4^5a_5a_6^2 \\
& - 124a_1^2a_4^4a_5^3a_6 + 1329a_1a_2^4a_6^5 - 6852a_1a_2^3a_3a_5a_6^4 + 2119a_1a_2^3a_4^2a_6^4 \\
& - 9155a_1a_2^3a_4a_5^2a_6^3 + 925a_1a_2^3a_5^4a_6^2 - 8176a_1a_2^2a_3^2a_4a_6^4 - 420a_1a_2^2a_3^2a_5^2a_6^3 \\
& - 4938a_1a_2^2a_3a_4^2a_5a_6^3 + 2242a_1a_2^2a_3a_4a_5^3a_6^2 + 70a_1a_2^2a_3a_5^5a_6 - 1659a_1a_2^2a_4^4a_6^3 \\
& + 1207a_1a_2^2a_4^3a_5^2a_6^2 + 720a_1a_2^2a_4^2a_5^4a_6 - 2a_1a_2^2a_4a_5^6 - 680a_1a_2a_3^4a_6^4 \\
& + 568a_1a_2a_3^3a_4a_5a_6^3 - 84a_1a_2a_3^3a_5^2a_6^2 + 1361a_1a_2a_3^2a_4^3a_6^3 - 1032a_1a_2a_3^2a_4^2a_5^2a_6^2 \\
& - 39a_1a_2a_3^2a_4a_5^4a_6 + 1134a_1a_2a_3a_4^4a_5a_6^2 - 360a_1a_2a_3a_4^3a_5^3a_6 + 6a_1a_2a_3a_4^2a_5^5 \\
& + 178a_1a_2a_4^6a_6^2 + 215a_1a_2a_4^5a_5^2a_6 - 8a_1a_2a_4^4a_5^4 - 384a_1a_3^5a_5a_6^3 \\
& - 648a_1a_3^4a_4^2a_6^3 + 536a_1a_3^4a_4a_5^2a_6^2 + 48a_1a_3^4a_5^4a_6 + 564a_1a_3^3a_4^3a_5a_6^2 \\
& + 208a_1a_3^3a_4^2a_5^3a_6 + 309a_1a_3^2a_4^5a_6^2 + 80a_1a_3^2a_4^4a_5^2a_6 - a_1a_3^2a_4^3a_5^4 \\
& + 20a_1a_3a_4^5a_5^3 + 15a_1a_4^8a_6 - 3a_1a_4^7a_5^2 - 493a_2^5a_5a_6^4 \\
& - 1565a_2^4a_3a_4a_6^4 + 2322a_2^4a_3a_5^2a_6^3 + 2577a_2^4a_4^2a_5a_6^3 - 133a_2^4a_4a_5^3a_6^2 \\
& - 47a_2^4a_5^5a_6 + 1872a_2^3a_3^3a_6^4 - 122a_2^3a_3^2a_4a_5a_6^3 - 106a_2^3a_3^2a_5^3a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 331a_2^3a_3a_4^3a_6^3 + 621a_2^3a_3a_4^2a_5^2a_6^2 + 3a_2^3a_3a_4a_5^4a_6 + 260a_2^3a_4^4a_5a_6^2 \\
& - 252a_2^3a_3^3a_5^3a_6 - 4a_2^3a_4^2a_5^5 + 232a_2^2a_3^2a_5^4a_6^3 + 1194a_2^2a_3^3a_4^2a_6^3 \\
& - 426a_2^2a_3^3a_4a_5^2a_6^2 - 52a_2^2a_3^3a_5^4a_6 - 1055a_2^2a_3^2a_4^3a_5^2a_6 + 6a_2^2a_3^2a_4^2a_5^3a_6 \\
& - a_2^2a_3^2a_4a_5^5 + 58a_2^2a_3a_4^5a_6^2 + 90a_2^2a_3a_4^4a_5^2a_6 + 4a_2^2a_3a_4^3a_5^3a_6^4 \\
& - 75a_2^2a_4^6a_5a_6 - a_2^2a_4^5a_5^3 - 472a_2a_3^5a_4a_6^3 - 8a_2a_3^5a_5^2a_6^2 \\
& + 372a_2a_3^4a_4^2a_5a_6^2 + 56a_2a_3^4a_4a_5^3a_6 - 141a_2a_3^3a_4^4a_6^2 - 62a_2a_3^3a_4^3a_5^2a_6 \\
& + a_2a_3^3a_4^2a_5^4 + 6a_2a_3^2a_4^5a_5a_6 + 4a_2a_3^2a_4^4a_5^3 + 3a_2a_3a_4^7a_6 \\
& - 9a_2a_3a_4^6a_5^2 + 160a_3^7a_6^3 - 96a_3^6a_4a_5a_6^2 - 48a_3^5a_4^2a_5^2a_6 \\
& + 12a_3^4a_4^4a_5a_6 - 36a_3^3a_4^6a_6 - 6a_3^3a_4^5a_5^2 + 6a_3^2a_4^7a_5 \\
& - 2a_3a_4^9 - 33653a_0^3a_2a_6^5 + 49513a_0^3a_3a_5a_6^4 + 31148a_0^3a_4^2a_6^4 \\
& + 11949a_0^3a_4a_5^2a_6^3 - 1900a_0^3a_5^4a_6^2 + 9270a_0^2a_1^2a_6^5 - 39518a_0^2a_1a_2a_5a_6^4 \\
& - 22593a_0^2a_1a_3a_4a_6^4 + 18320a_0^2a_1a_3a_5^2a_6^3 - 34582a_0^2a_1a_4^2a_5a_6^3 + 14370a_0^2a_1a_4a_5^3a_6^2 \\
& + 292a_0^2a_1a_5^5a_6 - 89782a_0^2a_2^2a_4a_6^4 + 793a_0^2a_2^2a_5^2a_6^3 + 20058a_0^2a_2a_3^2a_6^4 \\
& + 23969a_0^2a_2a_3a_4a_5a_6^3 + 2642a_0^2a_2a_3a_5^3a_6^2 + 17282a_0^2a_2a_4^3a_6^3 - 974a_0^2a_2a_4^2a_5^2a_6^2 \\
& - 697a_0^2a_2a_4a_5^4a_6 - 11a_0^2a_2a_5^6 - 10924a_0^2a_3^3a_5a_6^3 - 2943a_0^2a_3^2a_4^2a_6^3 \\
& - 13092a_0^2a_3^2a_4a_5^2a_6^2 - 346a_0^2a_3^2a_5^4a_6 + 15000a_0^2a_3a_4^3a_5^2a_6 + 2522a_0^2a_3a_4^2a_5^3a_6 \\
& + 368a_0^2a_3a_4a_5^5 - 3678a_0^2a_4^5a_6^2 - 1533a_0^2a_4^4a_5^2a_6 - 421a_0^2a_4^3a_5^4 \\
& + 4180a_0a_1^3a_5a_6^4 + 36998a_0a_1^2a_2a_4a_6^4 + 8878a_0a_1^2a_2a_5^2a_6^3 - 16878a_0a_1^2a_3^2a_6^4 \\
& - 11654a_0a_1^2a_3a_4a_5a_6^3 - 2746a_0a_1^2a_3a_5^3a_6^2 + 1793a_0a_1^2a_4^3a_6^3 + 388a_0a_1^2a_4^2a_5^2a_6^2 \\
& + 468a_0a_1^2a_4a_5^4a_6 + 78a_0a_1^2a_5^6 + 36537a_0a_1a_2^2a_3a_6^4 + 22329a_0a_1a_2^2a_4a_5a_6^3 \\
& - 2436a_0a_1a_2^2a_5^3a_6^2 + 12764a_0a_1a_2a_3^2a_5a_6^3 - 22595a_0a_1a_2a_3a_4^2a_6^3 + 6804a_0a_1a_2a_3a_4a_5^2a_6^2 \\
& - 618a_0a_1a_2a_3a_5^4a_6 - 7656a_0a_1a_2a_4^3a_5a_6^2 - 92a_0a_1a_2a_4^2a_5^3a_6 - 262a_0a_1a_2a_4a_5^5 \\
& + 15830a_0a_1a_3^3a_4a_6^3 + 4214a_0a_1a_3^3a_5^2a_6^2 - 426a_0a_1a_3^2a_4^2a_5^2a_6^2 - 1606a_0a_1a_3^2a_4a_5^3a_6 \\
& - 40a_0a_1a_3^2a_5^5 - 2976a_0a_1a_3a_4^4a_6^2 + 1000a_0a_1a_3a_4^3a_5^2a_6 - 30a_0a_1a_3a_4^2a_5^4 \\
& + 1910a_0a_1a_4^5a_5a_6 + 346a_0a_1a_4^4a_5^3 + 12822a_0a_2^4a_6^4 - 4698a_0a_2^3a_3a_5a_6^3 \\
& - 5219a_0a_2^3a_4^2a_6^3 - 1704a_0a_2^3a_4a_5^2a_6^2 + 123a_0a_2^3a_5^4a_6 - 9934a_0a_2^2a_3^2a_4a_6^3 \\
& - 4638a_0a_2^2a_3^2a_5^2a_6^2 - 3435a_0a_2^2a_3a_4^2a_5^2a_6 + 626a_0a_2^2a_3a_4a_5^3a_6 + 54a_0a_2^2a_3a_5^5 \\
& + 4449a_0a_2^2a_4^4a_6^2 - 1477a_0a_2^2a_4^3a_5^2a_6 + 129a_0a_2^2a_4^2a_5^4 - 1520a_0a_2a_3^4a_6^3 \\
& + 1312a_0a_2a_3^3a_4a_5a_6^2 - 1048a_0a_2a_3^3a_5^3a_6 + 9138a_0a_2a_3^2a_4^3a_6^2 - 720a_0a_2a_3^2a_4^2a_5^2a_6 \\
& - 156a_0a_2a_3^2a_4a_5^4 - 876a_0a_2a_3a_4^4a_5a_6 - 24a_0a_2a_3a_4^3a_5^3 - 46a_0a_2a_4^6a_6 \\
& - 200a_0a_2a_4^5a_5^2 + 528a_0a_3^5a_5a_6^2 - 3000a_0a_3^4a_4^2a_6^2 + 448a_0a_3^4a_4a_5^2a_6 \\
& + 24a_0a_3^4a_5^4 + 1136a_0a_3^3a_4^3a_5a_6 + 152a_0a_3^3a_4^2a_5^3 + 684a_0a_3^2a_4^5a_6 \\
& + 52a_0a_3^2a_4^4a_5^2 - 36a_0a_3a_4^6a_5 - 5376a_1^4a_4a_6^4 + 440a_1^4a_5^2a_6^3 \\
& - 10536a_1^3a_2a_3a_6^4 - 10926a_1^3a_2a_4a_5a_6^3 + 344a_1^3a_2a_5^3a_6^2 - 3224a_1^3a_3^2a_5a_6^3 \\
& + 4132a_1^3a_3a_4^2a_6^3 - 1462a_1^3a_3a_4a_5^2a_6^2 - 520a_1^3a_3a_5^4a_6 - 2918a_1^3a_4^3a_5a_6^2
\end{aligned}$$

$$\begin{aligned}
& -138a_1^3a_4^2a_5^3a_6 - 6a_1^3a_4a_5^5 - 4959a_1^2a_2^3a_6^4 + 860a_1^2a_2^2a_3a_5a_6^3 \\
& - 6565a_1^2a_2^2a_4^3a_6^3 + 6642a_1^2a_2^2a_4a_5^2a_6^2 + 253a_1^2a_2^2a_5^4a_6 + 1790a_1^2a_2a_3^2a_4a_6^3 \\
& + 2242a_1^2a_2a_3^2a_5^2a_6^2 + 4030a_1^2a_2a_3a_4^2a_5a_6^2 + 1566a_1^2a_2a_3a_4a_5^3a_6 + 2a_1^2a_2a_3a_5^5 \\
& + 742a_1^2a_2a_4^4a_6^2 + 1956a_1^2a_2a_4^3a_5^2a_6 + 6a_1^2a_2a_4^2a_5^4 + 208a_1^2a_3^4a_6^3 \\
& + 1424a_1^2a_3^3a_4a_5a_6^2 + 832a_1^2a_3^3a_5^3a_6 - 1380a_1^2a_3^2a_4^3a_6^2 - 264a_1^2a_3^2a_4^2a_5^2a_6 \\
& + 24a_1^2a_3^2a_4a_5^4 - 288a_1^2a_3a_4^4a_5a_6 - 12a_1^2a_3a_4^3a_5^3 - 177a_1^2a_4^6a_6 \\
& - 54a_1^2a_4^5a_5^2 + 2424a_1a_2^4a_5a_6^3 + 8996a_1a_2^3a_3a_4a_6^3 - 2702a_1a_2^3a_3a_5^2a_6^2 \\
& + 1388a_1a_2^3a_4a_5a_6^2 - 1632a_1a_2^3a_4a_5^3a_6 + 2a_1a_2^3a_5^5 + 288a_1a_2^2a_3^3a_6^3 \\
& - 1748a_1a_2^2a_3^2a_4a_5a_6^2 - 464a_1a_2^2a_3^2a_5^3a_6 - 1813a_1a_2^2a_3a_4^3a_6^2 - 810a_1a_2^2a_3a_4^2a_5^2a_6 \\
& - 12a_1a_2^2a_3a_4a_5^4 - 761a_1a_2^2a_4^4a_5a_6 - 70a_1a_2^2a_4^3a_5^3 - 80a_1a_2a_3^4a_5a_6^2 \\
& - 208a_1a_2a_3^3a_4^2a_6^2 - 24a_1a_2a_3^3a_4a_5^2a_6 + 560a_1a_2a_3^2a_4^3a_5a_6 - 36a_1a_2a_3^2a_4^2a_5^3 \\
& - 208a_1a_2a_3a_4^5a_6 + 24a_1a_2a_3a_4^4a_5^2 + 24a_1a_2a_4^6a_5 - 240a_1a_3^5a_4a_6^2 \\
& - 192a_1a_3^5a_5a_6 - 368a_1a_3^4a_4^2a_5a_6 - 72a_1a_3^3a_4^4a_6 + 8a_1a_3^3a_4^3a_5^2 \\
& - 42a_1a_3^2a_4^5a_5 + 30a_1a_3a_4^7 - 103a_2^5a_4a_6^3 - 67a_2^5a_5^2a_6^2 \\
& - 1933a_2^4a_3^2a_6^3 - 718a_2^4a_3a_4a_5a_6^2 + 284a_2^4a_3a_5^3a_6 - 879a_2^4a_4^3a_6^2 \\
& + 642a_2^4a_4^2a_5^2a_6 + 20a_2^4a_4a_5^4 + 284a_2^3a_3^3a_5a_6^2 + 826a_2^3a_3^2a_4^2a_6^2 \\
& - 282a_2^3a_3^2a_4a_5^2a_6 + a_2^3a_3^2a_5^4 - 265a_2^3a_3a_4^3a_5a_6 - 14a_2^3a_3a_4^2a_5^3 \\
& + 91a_2^3a_4^5a_6 + 41a_2^3a_4^4a_5^2 - 136a_2^2a_3^4a_4a_6^2 + 176a_2^2a_3^4a_5^2a_6 \\
& + 424a_2^2a_3^3a_4^2a_5a_6 + 8a_2^2a_3^3a_4a_5^3 + 264a_2^2a_3^2a_4^4a_6 + 6a_2^2a_3^2a_4^3a_5^2 \\
& + 18a_2^2a_3a_4^5a_5 + 6a_2^2a_4^7 + 80a_2a_3^6a_6^2 - 112a_2a_3^5a_4a_5a_6 \\
& - 168a_2a_3^4a_4^3a_6 - 8a_2a_3^4a_4^2a_5^2 - 18a_2a_3^3a_4^4a_5 - 24a_2a_3^2a_4^6 \\
& + 96a_3^6a_4^2a_6 + 24a_3^4a_5^5 + 39662a_0^3a_1a_6^4 - 17099a_0^3a_2a_5a_6^3 \\
& - 71900a_0^3a_3a_4a_6^3 - 96a_0^3a_3a_5^2a_6^2 + 456a_0^3a_4^2a_5a_6^2 + 4990a_0^3a_4a_5^3a_6 \\
& + 62a_0^3a_5^5 + 12074a_0^2a_1^2a_5a_6^3 + 73179a_0^2a_1a_2a_4a_6^3 - 15190a_0^2a_1a_2a_5^2a_6^2 \\
& + 8034a_0^2a_1a_3^2a_6^3 - 21720a_0^2a_1a_3a_4a_5a_6^2 - 3054a_0^2a_1a_3a_5^3a_6 - 3192a_0^2a_1a_4^3a_6^2 \\
& - 6438a_0^2a_1a_4^2a_5^2a_6 + 184a_0^2a_1a_4a_5^4 + 19849a_0^2a_2^2a_3a_6^3 + 22042a_0^2a_2^2a_4a_5a_6^2 \\
& + 2244a_0^2a_2^2a_5^3a_6 - 9906a_0^2a_2a_3^2a_5a_6^2 - 42960a_0^2a_2a_3a_4^2a_6^2 - 5414a_0^2a_2a_3a_4a_5^2a_6 \\
& + 134a_0^2a_2a_3a_5^4 + 12892a_0^2a_2a_4^3a_5a_6 + 1026a_0^2a_2a_4^2a_5^3 + 18048a_0^2a_3^3a_4a_6^2 \\
& + 2016a_0^2a_3^3a_5^2a_6 - 4668a_0^2a_3^2a_4^2a_5a_6 - 1264a_0^2a_3^2a_4a_5^3 + 1578a_0^2a_3a_4^4a_6 \\
& + 1864a_0^2a_3a_4^3a_5^2 - 430a_0^2a_4^5a_5 - 7036a_0a_1^3a_4a_6^3 + 4106a_0a_1^3a_5^2a_6^2 \\
& - 23948a_0a_1^2a_2a_3a_6^3 - 34034a_0a_1^2a_2a_4a_5a_6^2 - 1858a_0a_1^2a_2a_5^3a_6 + 6420a_0a_1^2a_3^2a_5^2a_6^2 \\
& + 15402a_0a_1^2a_3a_4^2a_6^2 - 2882a_0a_1^2a_3a_4a_5^2a_6 - 746a_0a_1^2a_3a_5^4 - 1642a_0a_1^2a_4^3a_5a_6 \\
& - 36a_0a_1^2a_4^2a_5^3 - 28215a_0a_1a_2^3a_6^3 + 2768a_0a_1a_2^2a_3a_5a_6^2 + 17452a_0a_1a_2^2a_4^2a_6^2 \\
& + 6698a_0a_1a_2^2a_4a_5^2a_6 + 272a_0a_1a_2^2a_5^4 + 2310a_0a_1a_2a_3^2a_4a_6^2 + 1900a_0a_1a_2a_3^2a_5^2a_6 \\
& + 2140a_0a_1a_2a_3a_4^2a_5a_6 + 1634a_0a_1a_2a_3a_4a_5^3 - 5284a_0a_1a_2a_4^4a_6 - 1536a_0a_1a_2a_4^3a_5^2
\end{aligned}$$

$$\begin{aligned}
& -6456a_0a_1a_3^4a_6^2 + 2200a_0a_1a_3^3a_4a_5a_6 + 224a_0a_1a_3^3a_5^3 - 2610a_0a_1a_3^2a_4^3a_6 \\
& - 810a_0a_1a_3^2a_4^2a_5^2 - 300a_0a_1a_3a_4^4a_5 - 66a_0a_1a_4^6 + 2744a_0a_2^4a_5a_6^2 \\
& - 3674a_0a_2^3a_3a_4a_6^2 - 1048a_0a_2^3a_3a_5^2a_6 - 1479a_0a_2^3a_4^2a_5a_6 - 326a_0a_2^3a_4^3a_6^3 \\
& + 5952a_0a_2^2a_3^3a_6^2 + 176a_0a_2^2a_3^2a_4a_5a_6 - 192a_0a_2^2a_3^2a_5^3 + 1934a_0a_2^2a_3a_4^3a_6 \\
& - 666a_0a_2^2a_3a_4^2a_5^2 + 761a_0a_2^2a_4^4a_5 + 1200a_0a_2a_3^4a_5a_6 - 1744a_0a_2a_3^3a_4^2a_6 \\
& + 288a_0a_2a_3^3a_4a_5^2 + 832a_0a_2a_3^2a_4^3a_5 + 256a_0a_2a_3a_4^5 - 288a_0a_3^5a_4a_6 \\
& - 96a_0a_3^5a_5^2 - 208a_0a_3^4a_4^2a_5 - 308a_0a_3^3a_4^4 + 4864a_1^4a_3a_6^3 \\
& + 4688a_1^4a_4a_5a_6^2 + 728a_1^4a_5^3a_6 + 8176a_1^3a_2^2a_6^3 - 432a_1^3a_2a_3a_5a_6^2 \\
& + 1468a_1^3a_2a_4^2a_6^2 - 1478a_1^3a_2a_4a_5^2a_6 - 18a_1^3a_2a_5^4 - 1944a_1^3a_3^2a_4a_6^2 \\
& + 800a_1^3a_3^2a_5^2a_6 + 1104a_1^3a_3a_4^2a_5a_6 + 48a_1^3a_3a_4a_5^3 + 1060a_1^3a_4^4a_6 \\
& - 6a_1^3a_4^3a_5^2 - 1654a_1^2a_3^3a_5a_6^2 - 4606a_1^2a_2^2a_3a_4a_6^2 - 860a_1^2a_2^2a_3a_5^2a_6 \\
& - 4236a_1^2a_2^2a_4^2a_5a_6 - 6a_1^2a_2^2a_4a_5^3 - 1272a_1^2a_2a_3^3a_6^2 - 2376a_1^2a_2a_3^2a_4a_5a_6 \\
& - 16a_1^2a_2a_3^2a_5^3 + 38a_1^2a_2a_3a_4^3a_6 - 30a_1^2a_2a_3a_4^2a_5^2 + 174a_1^2a_2a_4^4a_5 \\
& - 1216a_1^2a_3^4a_5a_6 + 704a_1^2a_3^3a_4^2a_6 - 96a_1^2a_3^3a_4a_5^2 + 48a_1^2a_3^2a_4^3a_5 \\
& - 90a_1^2a_3a_5^4 - 1616a_1a_2^4a_4a_6^2 + 654a_1a_2^4a_5^2a_6 + 1656a_1a_2^3a_3^2a_6^2 \\
& + 3296a_1a_2^3a_3a_4a_5a_6 - 4a_1a_2^3a_3a_5^3 + 1271a_1a_2^3a_4^3a_6 + 366a_1a_2^3a_4^2a_5^2 \\
& + 864a_1a_2^2a_3^3a_5a_6 - 616a_1a_2^2a_3^2a_4^2a_6 + 144a_1a_2^2a_3^2a_4a_5^2 - 180a_1a_2^2a_3a_4^3a_5 \\
& - 123a_1a_2^2a_4^5 + 208a_1a_2a_3^4a_4a_6 + 48a_1a_2a_3^3a_4^2a_5 + 96a_1a_2a_3^2a_4^4 \\
& + 256a_1a_3^5a_6 - 16a_1a_3^4a_4^3 + 454a_2^5a_3a_6^2 - 326a_2^5a_4a_5a_6 \\
& - 10a_2^5a_5^3 - 404a_2^4a_3^2a_5a_6 - 346a_2^4a_3a_4^2a_6 - 84a_2^4a_3a_4a_5^2 \\
& - 157a_2^4a_4^3a_5 + 136a_2^3a_3^3a_4a_6 - 8a_2^3a_3^3a_5^2 + 108a_2^3a_3^2a_4^2a_5 \\
& + 41a_2^3a_3a_4^4 - 192a_2^2a_3^5a_6 - 16a_2^2a_3^4a_4a_5 - 72a_2^2a_3^3a_4^3 \\
& + 16a_2a_3^5a_4^2 + 17665a_0^4a_6^3 + 28384a_0^3a_1a_5a_6^2 - 26830a_0^3a_2a_4a_6^2 \\
& - 6652a_0^3a_2a_5^2a_6 + 12474a_0^3a_3^2a_6^2 - 15582a_0^3a_3a_4a_5a_6 - 180a_0^3a_3a_5^3 \\
& + 9160a_0^3a_4^3a_6 + 1982a_0^3a_4^2a_5^2 - 3192a_0^2a_1^2a_4a_6^2 + 1502a_0^2a_1^2a_5^2a_6 \\
& + 20748a_0^2a_1a_2a_3a_6^2 + 4272a_0^2a_1a_2a_4a_5a_6 - 534a_0^2a_1a_2a_5^3 + 7800a_0^2a_1a_3^2a_5a_6 \\
& + 4614a_0^2a_1a_3a_4^2a_6 - 900a_0^2a_1a_3a_4a_5^2 - 1828a_0^2a_1a_4^3a_5 - 9588a_0^2a_2^3a_6^2 \\
& - 4168a_0^2a_2^2a_3a_5a_6 - 23512a_0^2a_2^2a_4^2a_6 - 390a_0^2a_2^2a_4a_5^2 + 14832a_0^2a_2a_3^2a_4a_6 \\
& - 304a_0^2a_2a_3^2a_5^2 - 4202a_0^2a_2a_3a_4^2a_5 + 978a_0^2a_2a_4^4 - 3744a_0^2a_3^4a_6 \\
& + 1536a_0^2a_3^3a_4a_5 - 480a_0^2a_3^2a_4^3 - 10440a_0a_1^3a_3a_6^2 + 5000a_0a_1^3a_4a_5a_6 \\
& + 364a_0a_1^3a_5^3 + 26274a_0a_1^2a_2^2a_6^2 + 9488a_0a_1^2a_2a_3a_5a_6 + 11480a_0a_1^2a_2a_4^2a_6 \\
& - 910a_0a_1^2a_2a_4a_5^2 - 1992a_0a_1^2a_3^2a_4a_6 + 2128a_0a_1^2a_3^2a_5^2 + 1500a_0a_1^2a_3a_4^2a_5 \\
& + 528a_0a_1^2a_4^4 - 7060a_0a_1a_2^3a_5a_6 - 9254a_0a_1a_2^2a_3a_4a_6 - 1834a_0a_1a_2^2a_3a_5^2 \\
& + 2322a_0a_1a_2^2a_4^2a_5 - 3408a_0a_1a_2a_3^3a_6 - 1896a_0a_1a_2a_3^2a_4a_5 - 470a_0a_1a_2a_3a_4^3 \\
& - 384a_0a_1a_3^4a_5 + 1160a_0a_1a_3^3a_4^2 + 3584a_0a_2^4a_4a_6 + 312a_0a_2^4a_5^2
\end{aligned}$$

$$\begin{aligned}
& + 1840a_0a_2^3a_3^2a_6 + 1176a_0a_2^3a_3a_4a_5 - 992a_0a_2^3a_4^3 + 288a_0a_2^2a_3^3a_5 \\
& - 560a_0a_2^2a_3^2a_4^2 - 192a_0a_2a_3^4a_4 + 128a_0a_3^6 - 5472a_1^4a_2a_6^2 \\
& - 3168a_1^4a_3a_5a_6 - 2976a_1^4a_4^2a_6 + 1976a_1^3a_2^2a_5a_6 + 3464a_1^3a_2a_3a_4a_6 \\
& + 144a_1^3a_2a_3a_5^2 + 896a_1^3a_3^3a_6 - 96a_1^3a_3^2a_4a_5 + 8a_1^3a_3a_4^3 \\
& + 690a_1^2a_3^2a_4a_6 - 30a_1^2a_3^2a_5^2 - 352a_1^2a_2^2a_3^2a_6 - 120a_1^2a_2^2a_3a_4a_5 \\
& + 222a_1^2a_2^2a_4^3 + 32a_1^2a_2a_3^3a_5 + 168a_1^2a_2a_3^2a_4^2 + 128a_1^2a_3^4a_4 \\
& - 880a_1a_2^4a_3a_6 - 330a_1a_2^4a_4a_5 - 16a_1a_2^3a_3^2a_5 - 72a_1a_2^3a_3a_4^2 \\
& - 256a_1a_2^2a_3^3a_4 + 12a_2^6a_6 + 38a_2^5a_3a_5 + 129a_2^5a_4^2 \\
& + 32a_2^4a_3^2a_4 + 16a_2^3a_3^4 + 6430a_0^4a_5a_6 - 10200a_0^3a_1a_4a_6 \\
& - 1278a_0^3a_1a_5^2 + 7868a_0^3a_2a_3a_6 - 2602a_0^3a_2a_4a_5 - 400a_0^3a_3^2a_5 \\
& - 3790a_0^3a_3a_4^2 - 2280a_0^2a_1^2a_3a_6 + 1688a_0^2a_1^2a_4a_5 + 21234a_0^2a_1a_2^2a_6 \\
& + 2680a_0^2a_1a_2a_3a_5 + 3564a_0^2a_1a_2a_4^2 - 240a_0^2a_1a_3^2a_4 - 546a_0^2a_2^3a_5 \\
& + 2464a_0^2a_2^2a_3a_4 - 224a_0^2a_2a_3^3 - 19872a_0a_1^3a_2a_6 - 1584a_0a_1^3a_3a_5 \\
& - 1488a_0a_1^3a_4^2 + 988a_0a_1^2a_2^2a_5 - 440a_0a_1^2a_2a_3a_4 - 1568a_0a_1^2a_3^3 \\
& - 342a_0a_1a_2^3a_4 + 2536a_0a_1a_2^2a_3^2 - 836a_0a_2^4a_3 + 3456a_1^5a_6 \\
& - 384a_1^3a_2^2a_4 - 288a_1^3a_2a_3^2 + 264a_1^2a_2^3a_3 + 48a_1a_2^5 \\
& - 2780a_0^4a_4 + 7640a_0^3a_1a_3 + 1874a_0^3a_2^2 - 5760a_0^2a_1^2a_2 \\
& + 1728a_0a_1^4
\end{aligned} \tag{A.31}$$

$$\begin{aligned}
e_{32} = & 240a_0^2a_2a_5a_6^{11} - 120a_0^2a_3a_5^2a_6^{10} + 12a_0^2a_4^2a_5a_6^{10} + 360a_0^2a_4a_5^3a_6^9 \\
& + 82a_0^2a_5^5a_6^8 + 132a_0a_1^2a_5a_6^{11} - 168a_0a_1a_2a_5^2a_6^{10} - 348a_0a_1a_3a_4a_5a_6^{10} \\
& - 648a_0a_1a_3a_5^3a_6^9 + 228a_0a_1a_4^2a_5^2a_6^9 + 284a_0a_1a_4a_5^4a_6^8 - 158a_0a_1a_5^6a_6^7 \\
& + 228a_0a_2^2a_5^3a_6^9 - 12a_0a_2a_3^2a_5a_6^{10} + 132a_0a_2a_3a_4a_5^2a_6^9 + 16a_0a_2a_3a_5^4a_6^8 \\
& - 16a_0a_2a_4^3a_5a_6^9 - 152a_0a_2a_4^2a_5^3a_6^8 + 176a_0a_2a_4a_5^5a_6^7 + 62a_0a_2a_5^7a_6^6 \\
& + 6a_0a_3^3a_5^2a_6^9 + 108a_0a_3^2a_4^2a_5a_6^9 + 336a_0a_3^2a_4a_5^3a_6^8 + 62a_0a_3^2a_5^5a_6^7 \\
& - 136a_0a_3a_4^3a_5^2a_6^8 - 394a_0a_3a_4^2a_5^4a_6^7 - 54a_0a_3a_4a_5^6a_6^6 + 4a_0a_3a_5^8a_6^5 \\
& + 4a_0a_4^5a_5a_6^8 + 62a_0a_4^4a_5^3a_6^7 + 124a_0a_4^3a_5^5a_6^6 + 38a_0a_4^2a_5^7a_6^5 \\
& - 22a_1^3a_5^2a_6^{10} + 72a_1^2a_2a_4a_5a_6^{10} + 230a_1^2a_2a_5^3a_6^9 + 102a_1^2a_3^2a_5a_6^{10} \\
& - 114a_1^2a_3a_4a_5^2a_6^9 + 46a_1^2a_3a_5^4a_6^8 - 20a_1^2a_4^3a_5a_6^9 - 66a_1^2a_4^2a_5^3a_6^8 \\
& - 134a_1^2a_4a_5^5a_6^7 - 44a_1^2a_5^7a_6^6 - 48a_1a_2^2a_3a_5a_6^{10} - 16a_1a_2^2a_4a_5^2a_6^9 \\
& - 186a_1a_2^2a_5^4a_6^8 - 18a_1a_2a_3^2a_5^2a_6^9 - 12a_1a_2a_3a_4^2a_5a_6^9 - 148a_1a_2a_3a_4a_5^3a_6^8 \\
& - 90a_1a_2a_3a_5^5a_6^7 + 24a_1a_2a_4^3a_5^2a_6^8 + 186a_1a_2a_4^2a_5^4a_6^7 + 90a_1a_2a_4a_5^6a_6^6 \\
& + 4a_1a_2a_5^8a_6^5 - 48a_1a_3^3a_4a_5a_6^9 - 66a_1a_3^3a_5^3a_6^8 + 84a_1a_3^2a_4^2a_5^2a_6^8 \\
& + 140a_1a_3^2a_4a_5^4a_6^7 + 19a_1a_3^2a_5^6a_6^6 + 8a_1a_3a_4^4a_5a_6^8 + 16a_1a_3a_4^3a_5^3a_6^7
\end{aligned}$$

$$\begin{aligned}
& -32a_1a_3a_4^2a_5^5a_6^6 - 8a_1a_3a_4a_5^7a_6^5 - 6a_1a_4^5a_5^2a_6^7 - 37a_1a_4^4a_5^4a_6^6 \\
& -18a_1a_4^3a_5^6a_6^5 + 32a_2^4a_5a_6^{10} - 48a_2^3a_3a_5^2a_6^9 - 16a_2^3a_4^2a_5a_6^9 \\
& + 72a_2^3a_4a_5^3a_6^8 + 50a_2^3a_5^5a_6^7 + 24a_2^2a_3^2a_4a_5a_6^9 + 24a_2^2a_3^2a_5^3a_6^8 \\
& - 4a_2^2a_3a_4^2a_5^8a_6^8 - 18a_2^2a_3a_4a_5^4a_6^7 - 4a_2^2a_3a_5^6a_6^6 + 2a_2^2a_4^4a_5a_6^8 \\
& - 36a_2^2a_4^3a_5^3a_6^7 - 38a_2^2a_4^2a_5^5a_6^6 - 4a_2^2a_4a_5^7a_6^5 - 12a_2a_3^4a_5a_6^9 \\
& + 24a_2a_3^3a_4a_5^2a_6^8 + 16a_2a_3^3a_5^4a_6^7 - 8a_2a_3^2a_4^3a_5a_6^8 - 16a_2a_3^2a_4^2a_5^3a_6^7 \\
& - 16a_2a_3^2a_4a_5^5a_6^6 - a_2a_3^2a_5^7a_6^5 + 6a_2a_3a_4^4a_5^2a_6^7 + 4a_2a_3a_4^3a_5^4a_6^6 \\
& + 4a_2a_5^5a_3^3a_6^6 + 3a_2a_4^4a_5^5a_6^5 + 6a_3^5a_5^2a_6^8 + 6a_3^4a_4^2a_5a_6^8 \\
& - 6a_3^4a_4a_5^3a_6^7 - 3a_3^4a_5^5a_6^6 - 12a_3^3a_4^3a_5^2a_6^7 - 12a_3^3a_4^2a_5^4a_6^6 \\
& + 3a_3^3a_4a_5^6a_6^5 + 3a_3^2a_4^4a_5^3a_6^6 + 12a_3^2a_4^3a_5^5a_6^5 + 3a_3a_4^5a_5^4a_6^5 \\
& + 960a_0^3a_6^{11} - 984a_0^2a_1a_5a_6^{10} - 480a_0^2a_2a_4a_6^{10} + 1728a_0^2a_2a_5a_6^9 \\
& - 720a_0^2a_3a_6^{10} + 1644a_0^2a_3a_4a_5a_6^9 - 792a_0^2a_3a_5^3a_6^8 + 48a_0^2a_4^3a_6^9 \\
& - 1668a_0^2a_4^2a_5^2a_6^8 + 296a_0^2a_4a_5^4a_6^7 - 118a_0^2a_5^6a_6^6 + 168a_0a_1^2a_4a_6^{10} \\
& + 882a_0a_1^2a_5^2a_6^9 + 360a_0a_1a_2a_3a_6^{10} + 180a_0a_1a_2a_4a_5a_6^9 - 1228a_0a_1a_2a_5^3a_6^8 \\
& + 216a_0a_1a_3^2a_5a_6^9 - 42a_0a_1a_3a_4^2a_6^9 - 396a_0a_1a_3a_4a_5^2a_6^8 - 305a_0a_1a_3a_5^4a_6^7 \\
& - 160a_0a_1a_4^3a_5a_6^8 + 606a_0a_1a_4^2a_5^3a_6^7 + 848a_0a_1a_4a_5^5a_6^6 - 243a_0a_1a_5^7a_6^5 \\
& + 480a_0a_2^3a_6^{10} - 768a_0a_2^2a_3a_5a_6^9 - 240a_0a_2^2a_4^2a_6^9 - 260a_0a_2^2a_4a_5^2a_6^8 \\
& + 578a_0a_2^2a_5^4a_6^7 - 48a_0a_2a_3^2a_4a_6^9 + 228a_0a_2a_3^2a_5^2a_6^8 + 30a_0a_2a_3a_4^2a_5a_6^8 \\
& + 28a_0a_2a_3a_4a_5^3a_6^7 - 423a_0a_2a_3a_5^5a_6^6 + 46a_0a_2a_4^4a_6^8 + 212a_0a_2a_4^3a_5^2a_6^7 \\
& - 1146a_0a_2a_4^2a_5^4a_6^6 + 78a_0a_2a_4a_5^6a_6^5 + 35a_0a_2a_5^8a_6^4 + 180a_0a_3^4a_6^9 \\
& - 630a_0a_3^3a_4a_5a_6^8 - 174a_0a_3^3a_5^3a_6^7 - 18a_0a_3^2a_4^3a_6^8 + 324a_0a_3^2a_4^2a_5^2a_6^7 \\
& + 455a_0a_3^2a_4a_5^4a_6^6 + a_0a_3^2a_5^6a_6^5 + 109a_0a_3a_4^4a_5a_6^7 + 280a_0a_3a_4^3a_5^3a_6^6 \\
& - 488a_0a_3a_4^2a_5^5a_6^5 - a_0a_3a_4a_5^7a_6^4 - 4a_0a_4^6a_6^7 - 102a_0a_4^5a_5^2a_6^6 \\
& - 212a_0a_4^4a_5^4a_6^5 - 42a_0a_4^3a_5^6a_6^4 - 210a_1^3a_3a_6^{10} + 62a_1^3a_4a_5a_6^9 \\
& - 390a_1^3a_5^3a_6^8 - 120a_1^2a_2^2a_6^{10} + 90a_1^2a_2a_3a_5a_6^9 + 18a_1^2a_2a_4^2a_6^9 \\
& - 274a_1^2a_2a_4a_5^2a_6^8 + 711a_1^2a_2a_5^4a_6^7 + 138a_1^2a_3^2a_4a_6^9 + 597a_1^2a_3^2a_5^2a_6^8 \\
& - 468a_1^2a_3a_4a_5^3a_6^7 + 416a_1^2a_3a_5^5a_6^6 + 5a_1^2a_4^4a_6^8 + 134a_1^2a_4^3a_5^2a_6^7 \\
& + 457a_1^2a_4^2a_5^4a_6^6 - 9a_1^2a_4a_5^6a_6^5 - 20a_1^2a_5^8a_6^4 - 80a_1a_2^3a_5a_6^9 \\
& - 72a_1a_2^2a_3a_4a_6^9 - 74a_1a_2^2a_3a_5^2a_6^8 + 152a_1a_2^2a_4^2a_5a_6^8 + 530a_1a_2^2a_4a_5^3a_6^7 \\
& - 522a_1a_2^2a_5^5a_6^6 - 90a_1a_2a_3^3a_6^9 + 306a_1a_2a_3^2a_4a_5a_6^8 + 130a_1a_2a_3^2a_5^3a_6^7 \\
& + 42a_1a_2a_3a_4^3a_6^8 + 20a_1a_2a_3a_4^2a_5^2a_6^7 - 187a_1a_2a_3a_4a_5^4a_6^6 - 75a_1a_2a_3a_5^6a_6^5 \\
& - 75a_1a_2a_4^4a_5a_6^7 - 460a_1a_2a_4^3a_5^3a_6^6 + 6a_1a_2a_4^2a_5^5a_6^5 + 29a_1a_2a_4a_5^7a_6^4 \\
& + 75a_1a_3^4a_5a_6^8 - 12a_1a_3^3a_4^2a_6^8 - 312a_1a_3^3a_4a_5^2a_6^7 - 198a_1a_3^3a_5^4a_6^6 \\
& - 102a_1a_3^2a_4^3a_5a_6^7 + 11a_1a_3^2a_4^2a_5^3a_6^6 + 136a_1a_3^2a_4a_5^5a_6^5 + 5a_1a_3^2a_5^7a_6^4 \\
& - 8a_1a_3a_4^5a_6^7 - 3a_1a_3a_4^4a_5^2a_6^6 + 62a_1a_3a_4^3a_5^4a_6^5 - 23a_1a_3a_4^2a_5^6a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 12a_1a_4^6a_5a_6^6 + 96a_1a_4^5a_5^3a_6^5 + 37a_1a_4^4a_5^5a_6^4 - 32a_2^4a_4a_6^9 \\
& + 248a_2^4a_5^2a_6^8 + 120a_2^3a_3^2a_6^9 - 40a_2^3a_3a_4a_5a_6^8 - 438a_2^3a_3a_5^3a_6^7 \\
& + 16a_2^3a_4^3a_6^8 - 298a_2^3a_4^2a_5^2a_6^7 + 122a_2^3a_4a_5^4a_6^6 + 69a_2^3a_5^6a_6^5 \\
& - 126a_2^2a_3^3a_5a_6^8 - 54a_2^2a_3^2a_4^2a_6^8 + 278a_2^2a_3^2a_4a_5^2a_6^7 + 138a_2^2a_3^2a_5^4a_6^6 \\
& + 46a_2^2a_3a_4^3a_5a_6^7 + 44a_2^2a_3a_4^2a_5^3a_6^6 - 57a_2^2a_3a_4a_5^5a_6^5 - 5a_2^2a_3a_5^7a_6^4 \\
& - 2a_2^2a_4^5a_6^7 + 111a_2^2a_4^4a_5^2a_6^6 + 48a_2^2a_4^3a_5^4a_6^5 - 9a_2^2a_4^2a_5^6a_6^4 \\
& + 42a_2a_3^4a_4a_6^8 - 81a_2a_3^4a_5^2a_6^7 - 78a_2a_3^3a_4^2a_5a_6^7 + 70a_2a_3^3a_4a_5^3a_6^6 \\
& + 11a_2a_3^3a_5^5a_6^5 + 8a_2a_3^2a_4^4a_6^7 - 6a_2a_3^2a_4^3a_5^2a_6^6 + a_2a_3^2a_4^2a_5^4a_6^5 \\
& - a_2a_3^2a_4a_5^6a_6^4 - 12a_2a_3a_4^5a_5a_6^6 + 21a_2a_3a_4^4a_5^3a_6^5 + 20a_2a_3a_4^3a_5^5a_6^4 \\
& - 12a_2a_3^6a_4^2a_5^5 - 7a_2a_3^5a_4^4a_6^4 - 15a_3^6a_6^8 + 21a_3^5a_4a_5a_6^7 \\
& + 33a_3^5a_4^3a_6^6 - 6a_3^4a_4^3a_6^7 + 36a_3^4a_4^2a_5^2a_6^6 - 39a_3^4a_4a_5^4a_6^5 \\
& + 24a_3^3a_4^4a_5a_6^6 - 15a_3^3a_4^3a_5^3a_6^5 + 3a_3^3a_4^2a_5^5a_6^4 - 9a_3^2a_4^5a_5^2a_6^5 \\
& - 33a_3^2a_4^4a_5^4a_6^4 - 12a_3a_4^6a_5^3a_6^4 + 1492a_0^3a_5a_6^9 + 624a_0^2a_1a_4a_6^9 \\
& - 4562a_0^2a_1a_5^2a_6^8 + 1560a_0^2a_2a_3a_6^9 - 4108a_0^2a_2a_4a_5a_6^8 - 34a_0^2a_2a_5^3a_6^7 \\
& - 2826a_0^2a_3^2a_5a_6^8 - 1302a_0^2a_3a_4^2a_6^8 + 9594a_0^2a_3a_4a_5^2a_6^7 - 493a_0^2a_3a_4^5a_6^6 \\
& + 2176a_0^2a_4^3a_5a_6^7 - 5218a_0^2a_4^2a_5^3a_6^6 + 666a_0^2a_4a_5^5a_6^5 - 145a_0^2a_5^7a_6^4 \\
& + 102a_0a_1^2a_3a_6^9 - 910a_0a_1^2a_4a_5a_6^8 + 42a_0a_1^2a_5^3a_6^7 - 1560a_0a_1a_2^2a_6^9 \\
& + 1260a_0a_1a_2a_3a_5a_6^8 + 678a_0a_1a_2a_4^2a_6^8 + 3380a_0a_1a_2a_4a_5^2a_6^7 - 897a_0a_1a_2a_5^4a_6^6 \\
& - 204a_0a_1a_2^3a_4a_6^8 - 824a_0a_1a_2^3a_5^2a_6^7 + 562a_0a_1a_3a_4^2a_5a_6^7 + 1946a_0a_1a_3a_4a_5^3a_6^6 \\
& + 1612a_0a_1a_3a_5^5a_6^5 - 68a_0a_1a_4^4a_6^7 - 1782a_0a_1a_4^3a_5^2a_6^6 - 1534a_0a_1a_4^2a_5^4a_6^5 \\
& + 631a_0a_1a_4a_5^6a_6^4 - 40a_0a_1a_5^8a_6^3 + 1708a_0a_2^3a_5a_6^8 + 1488a_0a_2^2a_3a_4a_6^8 \\
& - 2388a_0a_2^2a_3a_5^2a_6^7 - 1314a_0a_2^2a_4^2a_5a_6^7 - 286a_0a_2^2a_4a_5^3a_6^6 - 11a_0a_2^2a_5^5a_6^5 \\
& - 582a_0a_2a_3^3a_6^8 + 392a_0a_2a_3^2a_4a_5a_6^7 + 1728a_0a_2a_3^2a_5^3a_6^6 - 508a_0a_2a_3a_4^3a_6^7 \\
& - 218a_0a_2a_3a_4^2a_5^2a_6^6 - 196a_0a_2a_3a_4a_5^4a_6^5 - 279a_0a_2a_3a_5^6a_6^4 + 276a_0a_2a_4^4a_5a_6^6 \\
& + 2628a_0a_2a_4^3a_5^3a_6^5 - 620a_0a_2a_4^2a_5^5a_6^4 - 12a_0a_2a_4a_5^7a_6^3 + 975a_0a_3^4a_5a_6^7 \\
& + 522a_0a_3^3a_4^2a_6^7 - 2190a_0a_3^3a_4a_5^2a_6^6 - 16a_0a_3^3a_4^5a_6^5 - 760a_0a_3^2a_4^3a_5a_6^6 \\
& + 807a_0a_3^2a_4^2a_5^3a_6^5 - 178a_0a_3^2a_4a_5^5a_6^4 + 14a_0a_3^2a_5^7a_6^3 + 31a_0a_3a_4^5a_6^6 \\
& + 345a_0a_3a_4^4a_5^2a_6^5 + 1200a_0a_3a_4^3a_5^4a_6^4 - 6a_0a_3a_4^2a_5^6a_6^3 + 30a_0a_4^6a_5a_6^5 \\
& + 28a_0a_4^5a_5^3a_6^4 - 54a_0a_4^4a_5^5a_6^3 + 450a_1^3a_2a_6^9 - 526a_1^3a_3a_5a_6^8 \\
& - 130a_1^3a_4^2a_6^8 + 894a_1^3a_4a_5^2a_6^7 - 700a_1^3a_5^4a_6^6 - 248a_1^2a_2^2a_5a_6^8 \\
& - 438a_1^2a_2a_3a_4a_6^8 + 94a_1^2a_2a_3a_5^2a_6^7 + 362a_1^2a_2a_4^2a_5a_6^7 - 2934a_1^2a_2a_4a_5^3a_6^6 \\
& + 304a_1^2a_2a_5^5a_6^5 - 48a_1^2a_3^3a_6^8 - 230a_1^2a_3^2a_4a_5a_6^7 - 610a_1^2a_3^2a_5^3a_6^6 \\
& + 124a_1^2a_3a_4^3a_6^7 + 222a_1^2a_3a_4^2a_5^2a_6^6 - 764a_1^2a_3a_4a_5^4a_6^5 + 202a_1^2a_3a_5^6a_6^4 \\
& - 160a_1^2a_4^4a_5a_6^6 - 376a_1^2a_4^3a_5^3a_6^5 + 298a_1^2a_4^2a_5^5a_6^4 + 24a_1^2a_4a_5^7a_6^3 \\
& + 200a_1a_2^3a_4a_6^8 - 836a_1a_2^3a_5^2a_6^7 - 18a_1a_2^2a_3^2a_6^8 - 552a_1a_2^2a_3a_4a_5a_6^7
\end{aligned}$$

$$\begin{aligned}
& + 1164a_1a_2^2a_3a_5^3a_6^6 - 166a_1a_2^2a_4^3a_6^7 + 368a_1a_2^2a_4^2a_5^2a_6^6 + 1787a_1a_2^2a_4a_5^4a_6^5 \\
& - 208a_1a_2^2a_5^6a_6^4 - 438a_1a_2a_3^3a_5a_6^7 - 150a_1a_2a_3^2a_4^2a_6^7 + 1014a_1a_2a_3^2a_4a_5^2a_6^6 \\
& + 114a_1a_2a_3^2a_5^4a_6^5 + 394a_1a_2a_3a_4^3a_5a_6^6 - 240a_1a_2a_3a_4^2a_5^3a_6^5 + 112a_1a_2a_3a_4a_5^5a_6^4 \\
& + 4a_1a_2a_3a_5^3a_6^3 + 51a_1a_2a_4^5a_6^6 + 173a_1a_2a_4^4a_5^2a_6^5 - 506a_1a_2a_4^3a_5^4a_6^4 \\
& - 60a_1a_2a_4^2a_5^6a_6^3 - 33a_1a_3^4a_4a_6^7 + 400a_1a_3^4a_5^2a_6^6 + 434a_1a_3^3a_4^2a_5a_6^6 \\
& - 154a_1a_3^3a_4a_5^3a_6^5 - 40a_1a_3^3a_5^5a_6^4 + 26a_1a_3^2a_4^4a_6^6 - 351a_1a_3^2a_4^3a_5^2a_6^5 \\
& - 139a_1a_3^2a_4^2a_5^4a_6^4 - 14a_1a_3^2a_4a_5^6a_6^3 - 30a_1a_3a_4^5a_5a_6^5 + 2a_1a_3a_4^4a_5^3a_6^4 \\
& + 50a_1a_3a_4^3a_5^5a_6^3 - 6a_1a_4^7a_6^5 - 78a_1a_4^6a_5^2a_6^4 - 6a_1a_4^5a_5^4a_6^3 \\
& - 208a_2^4a_3a_6^8 - 288a_2^4a_4a_5a_6^7 + 262a_2^4a_5^3a_6^6 + 778a_2^3a_3^2a_5a_6^7 \\
& + 132a_2^3a_3a_4^2a_6^7 + 378a_2^3a_3a_4a_5^2a_6^6 - 504a_2^3a_3a_5^4a_6^5 + 302a_2^3a_3^3a_4a_5a_6^6 \\
& - 760a_2^3a_4^2a_5^3a_6^5 - 59a_2^3a_4a_5^5a_6^4 + 10a_2^3a_5^7a_6^3 + 90a_2^2a_3^3a_4a_6^7 \\
& - 608a_2^2a_3^2a_5^2a_6^6 - 698a_2^2a_3^2a_4^2a_5a_6^6 + 366a_2^2a_3^2a_4a_5^3a_6^5 + 77a_2^2a_3^2a_5^5a_6^4 \\
& - 40a_2^2a_3a_4^4a_6^6 + 116a_2^2a_3a_4^3a_5^2a_6^5 + 93a_2^2a_3a_4^2a_5^4a_6^4 - 18a_2^2a_3a_4a_5^6a_6^3 \\
& - 108a_2^2a_4^5a_5a_6^5 + 86a_2^2a_4^4a_5^3a_6^4 + 36a_2^2a_4^3a_5^5a_6^3 + 48a_2a_5^5a_6^7 \\
& + 155a_2a_3^4a_4a_5a_6^6 + 7a_2a_3^4a_5^3a_6^5 + 46a_2a_3^3a_4^3a_6^6 - 284a_2a_3^3a_4^2a_5^2a_6^5 \\
& - 55a_2a_3^3a_4a_5^4a_6^4 - a_2a_3^3a_5^6a_6^3 + 48a_2a_3^2a_4^4a_5a_6^5 + 55a_2a_3^2a_4^3a_5^3a_6^4 \\
& + 20a_2a_3^2a_4^2a_5^5a_6^3 + 6a_2a_3a_4^6a_6^5 - 63a_2a_3a_4^5a_5^2a_6^4 - 41a_2a_3a_4^4a_5^4a_6^3 \\
& + 12a_2a_4^7a_5a_6^4 + 2a_2a_4^6a_5^3a_6^3 - 82a_3^6a_5a_6^6 - 21a_3^5a_4^2a_6^6 \\
& + 93a_3^5a_4a_5^2a_6^5 - 3a_3^5a_5^4a_6^4 - 60a_3^4a_4^3a_5a_6^5 + 24a_3^4a_4^2a_5^3a_6^4 \\
& + 3a_3^4a_4a_5^5a_6^3 - 12a_3^3a_4^5a_6^5 + 102a_3^3a_4^4a_5^2a_6^4 - 8a_3^3a_4^3a_5^4a_6^3 \\
& + 9a_3^2a_4^6a_5a_6^4 + 27a_3^2a_4^5a_5^3a_6^3 + 18a_3a_4^7a_5^2a_6^3 + 1896a_0^3a_4a_6^8 \\
& - 9418a_0^3a_5^2a_6^7 - 1974a_0^2a_1a_3a_6^8 + 6554a_0^2a_1a_4a_5a_6^7 + 3726a_0^2a_1a_5^3a_6^6 \\
& + 7760a_0^2a_2^2a_6^8 + 986a_0^2a_2a_3a_5a_6^7 - 2704a_0^2a_2a_4^2a_6^7 + 5594a_0^2a_2a_4a_5^2a_6^6 \\
& - 3990a_0^2a_2a_5^4a_6^5 + 1314a_0^2a_3^2a_4a_6^7 + 1231a_0^2a_3^2a_5^2a_6^6 - 10988a_0^2a_3a_4^2a_5a_6^6 \\
& + 1722a_0^2a_3a_4a_5^3a_6^5 + 1021a_0^2a_3a_5^5a_6^4 - 190a_0^2a_4^4a_6^6 + 8788a_0^2a_4^3a_5^2a_6^5 \\
& - 1795a_0^2a_4^2a_5^4a_6^4 + 356a_0^2a_4a_5^6a_6^3 - 10a_0^2a_5^8a_6^2 - 2662a_0a_1^2a_2a_6^8 \\
& + 3540a_0a_1^2a_3a_5a_6^7 + 1618a_0a_1^2a_4^2a_6^7 - 3990a_0a_1^2a_4a_5^2a_6^6 + 149a_0a_1^2a_5^4a_6^5 \\
& - 6164a_0a_1a_2^2a_5a_6^7 + 1900a_0a_1a_2a_3a_4a_6^7 - 3112a_0a_1a_2a_3a_5^2a_6^6 - 1694a_0a_1a_2a_4^2a_5a_6^6 \\
& + 4514a_0a_1a_2a_4a_5^3a_6^5 - 474a_0a_1a_2a_5^5a_6^4 + 786a_0a_1a_3^3a_6^7 - 3738a_0a_1a_3^2a_4a_5a_6^6 \\
& - 4118a_0a_1a_3^2a_5^3a_6^5 - 1244a_0a_1a_3a_4^3a_6^6 + 601a_0a_1a_3a_4^2a_5^2a_6^5 - 2996a_0a_1a_3a_4a_5^4a_6^4 \\
& + 281a_0a_1a_3a_5^6a_6^3 + 1620a_0a_1a_4^4a_5a_6^5 + 316a_0a_1a_4^3a_5^3a_6^4 - 676a_0a_1a_4^2a_5^5a_6^3 \\
& + 76a_0a_1a_4a_5^7a_6^2 - 704a_0a_2^3a_4a_6^7 - 1590a_0a_2^3a_5^2a_6^6 - 3766a_0a_2^2a_3^2a_6^7 \\
& + 9886a_0a_2^2a_3a_4a_5a_6^6 + 1542a_0a_2^2a_3a_5^3a_6^5 + 770a_0a_2^2a_4^3a_6^6 - 3190a_0a_2^2a_4^2a_5^2a_6^5 \\
& + 797a_0a_2^2a_4a_5^4a_6^4 + 23a_0a_2^2a_5^6a_6^3 + 390a_0a_2a_3^3a_5a_6^6 + 1108a_0a_2a_3^2a_4^2a_6^6 \\
& - 3508a_0a_2a_3^2a_4a_5^2a_6^5 + 1059a_0a_2a_3^2a_5^4a_6^4 - 2764a_0a_2a_3a_4^3a_5a_6^5 + 969a_0a_2a_3a_4^2a_5^3a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 34a_0a_2a_3a_4a_5^5a_6^3 + 35a_0a_2a_3a_5^7a_6^2 - 236a_0a_2a_4^5a_5^5 - 1828a_0a_2a_4^4a_5^2a_6^4 \\
& + 1380a_0a_2a_4^3a_5^4a_6^3 - 46a_0a_2a_4^2a_5^6a_6^2 - 789a_0a_3^4a_4a_6^6 - 88a_0a_3^4a_5^2a_6^5 \\
& + 3859a_0a_3^3a_4^2a_5a_6^5 + 745a_0a_3^3a_4a_5^3a_6^4 - 117a_0a_3^3a_5^5a_6^3 + 249a_0a_3^2a_4^4a_5^5 \\
& - 3464a_0a_3^2a_4^3a_5^2a_6^4 + 205a_0a_3^2a_4^2a_5^4a_6^3 - 27a_0a_3^2a_4a_5^6a_6^2 - 203a_0a_3a_4^5a_5a_6^4 \\
& - 730a_0a_3a_4^4a_5^3a_6^3 + 28a_0a_3a_4^3a_5^5a_6^2 + 10a_0a_4^7a_6^4 + 92a_0a_4^6a_5^2a_6^3 \\
& + 86a_0a_4^5a_5^4a_6^2 + 125a_1^4a_6^8 + 1542a_1^3a_2a_5a_6^7 - 554a_1^3a_3a_4a_6^7 \\
& + 1875a_1^3a_3a_5^2a_6^6 - 639a_1^3a_4^2a_5a_6^6 + 1445a_1^3a_4a_5^3a_6^5 - 41a_1^3a_5^5a_6^4 \\
& - 180a_1^2a_2^2a_4a_6^7 + 1803a_1^2a_2^2a_5^2a_6^6 + 1288a_1^2a_2a_3^2a_6^7 - 1716a_1^2a_2a_3a_4a_5a_6^6 \\
& + 197a_1^2a_2a_3a_5^3a_6^5 + 14a_1^2a_2a_4^3a_6^6 + 4676a_1^2a_2a_4^2a_5^2a_6^5 - 2104a_1^2a_2a_4a_5^4a_6^4 \\
& - 44a_1^2a_2a_5^6a_6^3 + 197a_1^2a_3^3a_5a_6^6 + 482a_1^2a_3^2a_4^2a_6^6 + 559a_1^2a_3^2a_4a_5^2a_6^5 \\
& - 652a_1^2a_3^2a_5^4a_6^4 - 97a_1^2a_3a_4^3a_5a_6^5 + 467a_1^2a_3a_4^2a_5^3a_6^4 - 297a_1^2a_3a_4a_5^5a_6^3 \\
& - 12a_1^2a_3a_5^7a_6^2 + 40a_1^2a_4^5a_6^5 - 134a_1^2a_4^4a_5^2a_6^4 - 424a_1^2a_4^3a_5^4a_6^3 \\
& + 14a_1^2a_4^2a_5^6a_6^2 + 418a_1a_2^3a_3a_6^7 + 1798a_1a_2^3a_4a_5a_6^6 - 1764a_1a_2^3a_5^3a_6^5 \\
& - 740a_1a_2^2a_3^2a_5a_6^6 + 162a_1a_2^2a_3a_4^2a_6^6 - 4054a_1a_2^2a_3a_4a_5^2a_6^5 + 775a_1a_2^2a_3a_5^4a_6^4 \\
& - 1354a_1a_2^2a_4^3a_5a_6^5 - 888a_1a_2^2a_4^2a_5^3a_6^4 + 737a_1a_2^2a_4a_5^5a_6^3 - 6a_1a_2^2a_5^7a_6^2 \\
& - 736a_1a_2a_3^3a_4a_6^6 - 93a_1a_2a_3^3a_5^2a_6^5 - 15a_1a_2a_3^2a_4^2a_5a_6^5 - 13a_1a_2a_3^2a_4a_5^3a_6^4 \\
& - 43a_1a_2a_3^2a_5^5a_6^3 - 197a_1a_2a_3a_4^4a_6^5 + 1165a_1a_2a_3a_4^3a_5^2a_6^4 - 252a_1a_2a_3a_4^2a_5^4a_6^3 \\
& + 17a_1a_2a_3a_4a_5^6a_6^2 + 229a_1a_2a_4^5a_5a_6^4 + 558a_1a_2a_4^4a_5^3a_6^3 - 6a_1a_2a_4^3a_5^5a_6^2 \\
& - 45a_1a_5^5a_6^6 - 361a_1a_3^4a_4a_5a_6^5 + 66a_1a_3^4a_5^3a_6^4 - 198a_1a_3^3a_4^3a_6^5 \\
& + 550a_1a_3^3a_4^2a_5^2a_6^4 + 170a_1a_3^3a_4a_5^4a_6^3 + 3a_1a_3^3a_5^6a_6^2 + 349a_1a_3^2a_4^4a_5a_6^4 \\
& - 93a_1a_3^2a_4^3a_5^3a_6^3 - 15a_1a_3^2a_4^2a_5^5a_6^2 + 17a_1a_3a_4^6a_6^4 - 112a_1a_3a_4^5a_5^2a_6^3 \\
& + a_1a_3a_4^4a_5^4a_6^2 + 16a_1a_4^7a_5a_6^3 - 30a_1a_4^6a_5^3a_6^2 + 88a_2^5a_6^7 \\
& - 970a_2^4a_3a_5a_6^6 + 26a_2^4a_4^2a_6^6 + 264a_2^4a_4a_5^2a_6^5 + 131a_2^4a_5^4a_6^4 \\
& - 522a_2^3a_3^2a_4a_6^6 + 978a_2^3a_3^2a_5^2a_6^5 + 452a_2^3a_3a_4^2a_5a_6^5 + 686a_2^3a_3a_4a_5^3a_6^4 \\
& - 56a_2^3a_3a_5^5a_6^3 - 78a_2^3a_4^4a_6^5 + 764a_2^3a_4^3a_5^2a_6^4 - 249a_2^3a_4^2a_5^4a_6^3 \\
& - 18a_2^3a_4a_5^6a_6^2 + 236a_2^2a_3^4a_6^6 + 648a_2^2a_3^3a_4a_5a_6^5 - 248a_2^2a_3^3a_5^3a_6^4 \\
& + 338a_2^2a_3^2a_4^3a_6^5 - 1201a_2^2a_3^2a_4^2a_5^2a_6^4 - 6a_2^2a_3^2a_4a_5^4a_6^3 - 4a_2^2a_3^2a_5^6a_6^2 \\
& - 200a_2^2a_3a_4^4a_5a_6^4 + 73a_2^2a_3a_4^3a_5^3a_6^3 + 31a_2^2a_3a_4^2a_5^5a_6^2 + 33a_2^2a_4^6a_6^4 \\
& - 168a_2^2a_4^5a_5^2a_6^3 - 23a_2^2a_4^4a_5^4a_6^2 - 137a_2a_3^5a_5a_6^5 - 6a_2a_3^4a_4^2a_6^5 \\
& + 203a_2a_3^4a_4a_5^2a_6^4 + 6a_2a_3^4a_5^4a_6^3 + 136a_2a_3^3a_4^3a_5a_6^4 - 39a_2a_3^3a_4^2a_5^3a_6^3 \\
& + 3a_2a_3^3a_4a_5^5a_6^2 - 26a_2a_3^2a_4^5a_6^4 - 5a_2a_3^2a_4^4a_5^2a_6^3 - 4a_2a_3^2a_4^3a_5^4a_6^2 \\
& + 47a_2a_3a_4^6a_5a_6^3 + 21a_2a_3a_4^5a_5^3a_6^2 - 4a_2a_4^8a_6^3 + 6a_2a_4^7a_5^2a_6^2 \\
& + 63a_3^6a_4a_6^5 + 27a_3^6a_5^2a_6^4 - 174a_3^5a_4^2a_5a_6^4 - 30a_3^5a_4a_5^3a_6^3 \\
& + 30a_3^4a_4^4a_6^4 + 51a_3^4a_4^3a_5^2a_6^3 - 111a_3^3a_4^5a_5a_6^3 - 9a_3^3a_4^4a_5^3a_6^2 \\
& - 3a_3^2a_4^7a_6^3 - 3a_3^2a_4^6a_5^2a_6^2 - 12a_3a_4^8a_5a_6^2 + 930a_0^3a_3a_6^7
\end{aligned}$$

$$\begin{aligned}
& + 25286a_0^3a_4a_5a_6^6 + 2954a_0^3a_5^3a_6^5 - 7706a_0^2a_1a_2a_6^7 - 11230a_0^2a_1a_3a_5a_6^6 \\
& - 598a_0^2a_1a_4a_6^6 - 11430a_0^2a_1a_4a_5^2a_6^5 + 4500a_0^2a_1a_5^4a_6^4 - 22056a_0^2a_2^2a_5a_6^6 \\
& + 7602a_0^2a_2a_3a_4a_6^6 + 15858a_0^2a_2a_3a_5^2a_6^5 - 6930a_0^2a_2a_4^2a_5a_6^5 + 3228a_0^2a_2a_4a_5^3a_6^4 \\
& - 473a_0^2a_2a_5^3a_6^3 - 1842a_0^2a_3^3a_6^6 - 1236a_0^2a_3^2a_4a_5a_6^5 - 4290a_0^2a_3^2a_5^3a_6^4 \\
& + 1542a_0^2a_3a_4^3a_6^5 + 15a_0^2a_3a_4^2a_5^2a_6^4 - 1853a_0^2a_3a_4a_5^4a_6^3 + 37a_0^2a_3a_5^6a_6^2 \\
& - 4672a_0^2a_4^4a_5a_6^4 + 1840a_0^2a_4^3a_5^3a_6^3 - 412a_0^2a_4^2a_5^5a_6^2 + 16a_0^2a_4a_5^7a_6 \\
& + 2132a_0a_1^3a_6^7 + 15808a_0a_1^2a_2a_5a_6^6 - 5888a_0a_1^2a_3a_4a_6^6 - 3715a_0a_1^2a_3a_5^2a_6^5 \\
& + 6223a_0a_1^2a_4^2a_5a_6^5 + 1159a_0a_1^2a_4a_5^3a_6^4 + 33a_0a_1^2a_5^5a_6^3 - 448a_0a_1a_2^2a_4a_6^6 \\
& + 4740a_0a_1a_2^2a_5^2a_6^5 + 1418a_0a_1a_2a_3^2a_6^6 - 2200a_0a_1a_2a_3a_4a_5a_6^5 + 1722a_0a_1a_2a_3a_5^3a_6^4 \\
& + 2112a_0a_1a_2a_4^3a_6^5 - 5729a_0a_1a_2a_4^2a_5^2a_6^4 + 554a_0a_1a_2a_4a_5^4a_6^3 - 161a_0a_1a_2a_5^6a_6^2 \\
& + 3576a_0a_1a_3^3a_5a_6^5 + 4584a_0a_1a_3^2a_4^2a_6^5 + 6488a_0a_1a_3^2a_4a_5^2a_6^4 - 408a_0a_1a_3^2a_5^4a_6^3 \\
& - 4758a_0a_1a_3a_4^3a_5a_6^4 + 1710a_0a_1a_3a_4^2a_5^3a_6^3 - 520a_0a_1a_3a_4a_5^5a_6^2 - 8a_0a_1a_3a_5^7a_6 \\
& - 724a_0a_1a_4^5a_6^4 + 2388a_0a_1a_4^4a_5^2a_6^3 + 564a_0a_1a_4^3a_5^4a_6^2 - 2a_0a_1a_4^2a_5^6a_6 \\
& + 8350a_0a_2^3a_3a_6^6 + 1392a_0a_2^3a_4a_5a_6^5 - 868a_0a_2^3a_5^3a_6^4 - 4004a_0a_2^2a_3^2a_5a_6^5 \\
& - 8808a_0a_2^2a_3a_4^2a_6^5 - 3174a_0a_2^2a_3a_4a_5^2a_6^4 - 13a_0a_2^2a_3a_5^4a_6^3 + 4534a_0a_2^2a_4^3a_5a_6^4 \\
& - 1949a_0a_2^2a_4^2a_5^3a_6^3 + 287a_0a_2^2a_4a_5^5a_6^2 + 14a_0a_2^2a_5^7a_6 - 2760a_0a_2a_3^3a_4a_6^5 \\
& - 1291a_0a_2a_3^3a_5^2a_6^4 + 8416a_0a_2a_3^2a_4^2a_5a_6^4 - 906a_0a_2a_3^2a_4a_5^3a_6^3 - 310a_0a_2a_3^2a_5^5a_6^2 \\
& + 2145a_0a_2a_3a_4^4a_6^4 - 3310a_0a_2a_3a_4^3a_5^2a_6^3 + 238a_0a_2a_3a_4^2a_5^4a_6^2 - 30a_0a_2a_3a_4a_5^6a_6 \\
& + 92a_0a_2a_4^5a_5a_6^3 - 1178a_0a_2a_4^4a_5^3a_6^2 + 44a_0a_2a_4^3a_5^5a_6 + 663a_0a_3^5a_6^5 \\
& - 1607a_0a_3^4a_4a_5a_6^4 + 330a_0a_3^4a_5^3a_6^3 - 1359a_0a_3^3a_4^3a_6^4 - 223a_0a_3^3a_4^2a_5^2a_6^3 \\
& + 183a_0a_3^3a_4a_5^4a_6^2 + 3a_0a_3^3a_5^6a_6 + 2575a_0a_3^2a_4^4a_5a_6^3 - 217a_0a_3^2a_4^3a_5^3a_6^2 \\
& + 6a_0a_3^2a_4^2a_5^5a_6 - 16a_0a_3a_4^6a_6^3 + 20a_0a_3a_4^5a_5^2a_6^2 - 37a_0a_3a_4^4a_5^4a_6 \\
& - 28a_0a_4^7a_5a_6^2 - 28a_0a_4^6a_5^3a_6 - 3276a_1^4a_5a_6^6 + 502a_1^3a_2a_4a_6^6 \\
& - 1911a_1^3a_2a_5^2a_6^5 - 550a_1^3a_3^2a_6^6 - 478a_1^3a_3a_4a_5a_6^5 - 156a_1^3a_3a_5^3a_6^4 \\
& - 167a_1^3a_4^3a_6^5 - 2279a_1^3a_4^2a_5^2a_6^4 + 55a_1^3a_4a_5^4a_6^3 - 24a_1^3a_5^6a_6^2 \\
& - 2792a_1^2a_2^2a_3a_6^6 - 4230a_1^2a_2^2a_4a_5a_6^5 + 1541a_1^2a_2^2a_5^3a_6^4 + 17a_1^2a_2a_3^2a_5a_6^5 \\
& + 168a_1^2a_2a_3a_4^2a_6^5 + 2101a_1^2a_2a_3a_4a_5^2a_6^4 + 524a_1^2a_2a_3a_5^4a_6^3 - 1613a_1^2a_2a_4^3a_5a_6^4 \\
& + 3639a_1^2a_2a_4^2a_5^3a_6^3 - 47a_1^2a_2a_4a_5^5a_6^2 - 4a_1^2a_2a_5^7a_6 + 255a_1^2a_3^3a_4a_6^5 \\
& + 854a_1^2a_3^2a_5^2a_6^4 - 238a_1^2a_3^2a_4^2a_5a_6^4 + 968a_1^2a_3^2a_4a_5^3a_6^3 + 152a_1^2a_3^2a_5^5a_6^2 \\
& + 347a_1^2a_3a_4^4a_6^4 - 177a_1^2a_3a_4^3a_5^2a_6^3 + 57a_1^2a_3a_4^2a_5^4a_6^2 + 4a_1^2a_3a_4a_5^6a_6 \\
& + 44a_1^2a_4^5a_5a_6^3 + 144a_1^2a_4^4a_5^3a_6^2 - 42a_1^2a_4^3a_5^5a_6 - 278a_1a_2^4a_6^6 \\
& + 2494a_1a_2^3a_3a_5a_6^5 - 592a_1a_2^3a_4^2a_6^5 + 3340a_1a_2^3a_4a_5^2a_6^4 - 839a_1a_2^3a_5^4a_6^3 \\
& + 3318a_1a_2^2a_3^2a_4a_6^5 - 1123a_1a_2^2a_3^2a_5^2a_6^4 + 666a_1a_2^2a_3a_4^2a_5a_6^4 - 2308a_1a_2^2a_3a_4a_5^3a_6^3 \\
& - 28a_1a_2^2a_3a_5^5a_6^2 + 567a_1a_2^2a_4^4a_6^4 - 941a_1a_2^2a_4^3a_5^2a_6^3 - 566a_1a_2^2a_4^2a_5^4a_6^2 \\
& + 22a_1a_2^2a_4a_5^6a_6 + 141a_1a_2a_3^4a_6^5 - 644a_1a_2a_3^3a_4a_5a_6^4 + 60a_1a_2a_3^3a_5^3a_6^3
\end{aligned}$$

$$\begin{aligned}
& -773a_1a_2a_3^2a_4^3a_6^4 + 731a_1a_2a_3^2a_4^2a_5^2a_6^3 - 11a_1a_2a_3^2a_4a_5^4a_6^2 + a_1a_2a_3^2a_5^6a_6 \\
& - 656a_1a_2a_3a_4^4a_5a_6^3 + 250a_1a_2a_3a_4^3a_5^3a_6^2 - 46a_1a_2a_3a_4^2a_5^5a_6 - 128a_1a_2a_4^6a_6^3 \\
& - 116a_1a_2a_4^5a_5^2a_6^2 + 49a_1a_2a_4^4a_5^4a_6 + 38a_1a_3^5a_5a_6^4 - 81a_1a_3^4a_4^2a_6^4 \\
& - 510a_1a_3^4a_4a_5^2a_6^3 - 36a_1a_3^4a_5^4a_6^2 - 210a_1a_3^3a_4^3a_5a_6^3 - 18a_1a_3^3a_4^2a_5^3a_6^2 \\
& - 131a_1a_3^2a_4^5a_6^3 + 101a_1a_3^2a_4^4a_5^2a_6^2 + 46a_1a_3^2a_4^3a_5^4a_6 + 130a_1a_3a_4^6a_5a_6^2 \\
& - 24a_1a_3a_4^5a_5^3a_6 + 3a_1a_4^8a_6^2 + 20a_1a_4^7a_5^2a_6 + 234a_2^5a_5a_6^5 \\
& + 542a_2^4a_3a_4a_6^5 - 982a_2^4a_3a_5^2a_6^4 - 970a_2^4a_4^2a_5a_6^4 + 388a_2^4a_4a_5^3a_6^3 \\
& + 47a_2^4a_5^5a_6^2 - 972a_2^3a_3^3a_6^5 + 112a_2^3a_3^2a_4a_5a_6^4 + 128a_2^3a_3^2a_5^3a_6^3 \\
& - 300a_2^3a_3a_4^3a_6^4 - 286a_2^3a_3a_4^2a_5^2a_6^3 + 28a_2^3a_3a_4a_5^4a_6^2 + a_2^3a_3a_5^6a_6 \\
& - 120a_2^3a_4^4a_5a_6^3 + 311a_2^3a_4^3a_5^3a_6^2 + 67a_2^2a_3^4a_5a_6^4 - 228a_2^2a_3^3a_4^2a_6^4 \\
& + 400a_2^2a_3^3a_4a_5^2a_6^3 + 48a_2^2a_3^3a_5^4a_6^2 + 898a_2^2a_3^2a_4^3a_5a_6^3 - 131a_2^2a_3^2a_4^2a_5^3a_6^2 \\
& - 3a_2^2a_3^2a_4a_5^5a_6 + 64a_2^2a_3a_4^5a_6^3 - 128a_2^2a_3a_4^4a_5^2a_6^2 + 6a_2^2a_3a_4^3a_5^4a_6 \\
& + 82a_2^2a_4^6a_5a_6^2 - 4a_2^2a_4^5a_5^3a_6 + 225a_2a_3^5a_4a_6^4 + 12a_2a_3^5a_5^2a_6^3 \\
& - 350a_2a_3^4a_4^2a_5a_6^3 - 42a_2a_3^4a_4a_5^3a_6^2 + 44a_2a_3^3a_4^4a_6^3 + 103a_2a_3^3a_4^3a_5^2a_6^2 \\
& + 6a_2a_3^3a_4^2a_5^4a_6 - 67a_2a_3^2a_4^5a_5a_6^2 - 27a_2a_3^2a_4^4a_5^3a_6 - 9a_2a_3a_4^7a_6^2 \\
& + a_2a_3a_4^6a_5^2a_6 - 5a_2a_4^8a_5a_6 - 60a_3^7a_6^4 + 72a_3^6a_4a_5a_6^3 \\
& + 54a_3^5a_4^3a_6^3 + 18a_3^5a_4^2a_5^2a_6^2 - 24a_3^4a_4^4a_5a_6^2 - 6a_3^4a_4^3a_5^3a_6 \\
& + 36a_3^3a_4^6a_6^2 + 15a_3^3a_4^5a_5^2a_6 - 3a_3^2a_4^7a_5a_6 + 3a_3a_4^9a_6 \\
& + 9662a_0^3a_2a_6^6 - 19992a_0^3a_3a_5a_6^5 - 14538a_0^3a_4^2a_6^5 - 2954a_0^3a_4a_5^2a_6^4 \\
& + 2195a_0^3a_5^4a_6^3 - 1546a_0^2a_1^2a_6^6 + 15710a_0^2a_1a_2a_5a_6^5 + 6222a_0^2a_1a_3a_4a_5^5a_6 \\
& - 11081a_0^2a_1a_3a_5^2a_6^4 + 9071a_0^2a_1a_4^2a_5a_6^4 - 9267a_0^2a_1a_4a_5^3a_6^3 + 53a_0^2a_1a_5^5a_6^2 \\
& + 37612a_0^2a_2^2a_4a_6^5 + 3829a_0^2a_2^2a_5^2a_6^4 - 6546a_0^2a_2a_3^2a_6^5 - 19756a_0^2a_2a_3a_4a_5a_6^4 \\
& - 2417a_0^2a_2a_3a_5^3a_6^3 - 9468a_0^2a_2a_4^3a_6^4 + 4709a_0^2a_2a_4^2a_5^2a_6^3 + 547a_0^2a_2a_4a_5^4a_6^2 \\
& - 14a_0^2a_2a_5^6a_6 + 6261a_0^2a_3^3a_5a_6^4 + 3438a_0^2a_3^2a_4^2a_6^4 + 8103a_0^2a_3^2a_4a_5^2a_6^3 \\
& + 220a_0^2a_3^2a_5^4a_6^2 - 2917a_0^2a_3a_4^3a_5a_6^3 + 1180a_0^2a_3a_4^2a_5^3a_6^2 - 118a_0^2a_3a_4a_5^5a_6 \\
& + 3a_0^2a_3a_5^7a_6 + 2180a_0^2a_4^5a_6^3 - 794a_0^2a_4^4a_5^2a_6^2 + 408a_0^2a_4^3a_5^4a_6 \\
& + 32a_0^2a_4^2a_5^6a_6 - 1756a_0a_1^3a_5a_6^5 - 17444a_0a_1^2a_2a_4a_6^5 - 8805a_0a_1^2a_2a_5^2a_6^4 \\
& + 4848a_0a_1^2a_3^3a_6^5 + 3216a_0a_1^2a_3a_4a_5a_6^4 - 383a_0a_1^2a_3a_5^3a_6^3 - 389a_0a_1^2a_4^3a_6^4 \\
& - 2381a_0a_1^2a_4^2a_5^2a_6^3 - 215a_0a_1^2a_4a_5^4a_6^2 - 103a_0a_1^2a_5^6a_6 - 15714a_0a_1a_2^2a_3a_6^5 \\
& - 4246a_0a_1a_2^2a_4a_5a_6^4 + 2944a_0a_1a_2^2a_5^3a_6^3 - 2084a_0a_1a_2a_3^2a_5a_6^4 + 13886a_0a_1a_2a_3a_4^2a_6^4 \\
& + 1110a_0a_1a_2a_3a_4a_5^2a_6^3 + 1305a_0a_1a_2a_3a_5^4a_6^2 + 578a_0a_1a_2a_4^3a_5a_6^3 - 228a_0a_1a_2a_4^2a_5^3a_6^2 \\
& + 300a_0a_1a_2a_4a_5^5a_6 + 7a_0a_1a_2a_5^7a_6 - 6378a_0a_1a_3^3a_4a_6^4 - 751a_0a_1a_3^3a_5^2a_6^3 \\
& - 671a_0a_1a_3^2a_4^2a_5a_6^3 + 1241a_0a_1a_3^2a_4a_5^3a_6^2 + 120a_0a_1a_3^2a_5^5a_6 + 183a_0a_1a_3a_4^4a_6^3 \\
& - 1990a_0a_1a_3a_4^3a_5^2a_6^2 - 206a_0a_1a_3a_4^2a_5^4a_6 + a_0a_1a_3a_4a_5^6a_6 - 1930a_0a_1a_4^5a_5a_6^2 \\
& - 83a_0a_1a_4^4a_5^3a_6 - 30a_0a_1a_4^3a_5^5a_6 - 4918a_0a_2^4a_6^5 + 1426a_0a_2^3a_3a_5a_6^4
\end{aligned}$$

$$\begin{aligned}
& + 2362a_0a_2^3a_4^2a_6^4 + 700a_0a_2^3a_4a_5^2a_6^3 - 246a_0a_2^3a_5^4a_6^2 + 4006a_0a_2^2a_3^2a_4a_6^4 \\
& + 1417a_0a_2^2a_3^2a_5^2a_6^3 + 2425a_0a_2^2a_3a_4^2a_5a_6^3 - 2289a_0a_2^2a_3a_4a_5^3a_6^2 - 127a_0a_2^2a_3a_5^5a_6 \\
& - 2094a_0a_2^2a_4^4a_6^3 + 2255a_0a_2^2a_4^3a_5^2a_6^2 - 309a_0a_2^2a_4^2a_5^4a_6 - 8a_0a_2^2a_4a_5^6 \\
& + 1257a_0a_2a_3^4a_6^4 - 480a_0a_2a_3^3a_4a_5a_6^3 + 950a_0a_2a_3^3a_5^3a_6^2 - 5016a_0a_2a_3^2a_4^3a_6^3 \\
& + 1139a_0a_2a_3^2a_4^2a_5^2a_6^2 + 190a_0a_2a_3^2a_4a_5^4a_6 + a_0a_2a_3^2a_5^6 + 3011a_0a_2a_3a_4^4a_5a_6^2 \\
& - 138a_0a_2a_3a_4^3a_5^3a_6 - 4a_0a_2a_3a_4^2a_5^5 + 198a_0a_2a_4^6a_6^2 + 326a_0a_2a_4^5a_5^2a_6 \\
& - 11a_0a_2a_4^4a_5^4 - 344a_0a_3^5a_5a_6^3 + 1122a_0a_3^4a_4^2a_6^3 - 420a_0a_3^4a_4a_5^2a_6^2 \\
& - 36a_0a_3^4a_5^4a_6 - 342a_0a_3^3a_4^3a_5a_6^2 + 6a_0a_3^3a_4^2a_5^3a_6 - 372a_0a_3^2a_4^5a_6^2 \\
& + 235a_0a_3^2a_4^4a_5^2a_6 + 18a_0a_3^2a_4^3a_5^4 + 57a_0a_3a_4^6a_5a_6 + 16a_0a_3a_4^5a_5^3 \\
& - 4a_0a_4^8a_6 + 2684a_1^4a_4a_6^5 + 722a_1^4a_5^2a_6^4 + 4240a_1^3a_2a_3a_6^5 \\
& + 3922a_1^3a_2a_4a_5a_6^4 - 532a_1^3a_2a_5^3a_6^3 + 874a_1^3a_3^2a_5a_6^4 - 2148a_1^3a_3a_4^2a_6^4 \\
& + 127a_1^3a_3a_4a_5^2a_6^3 + 156a_1^3a_3a_5^4a_6^2 + 1973a_1^3a_4^3a_5a_6^3 - 451a_1^3a_4^2a_5^3a_6^2 \\
& - 37a_1^3a_4a_5^5a_6 - 4a_1^3a_5^7 + 1720a_1^2a_2^3a_6^5 - 1313a_1^2a_2^2a_3a_5a_6^4 \\
& + 2034a_1^2a_2^2a_4^2a_6^4 - 5508a_1^2a_2^2a_4a_5^2a_6^3 - 5a_1^2a_2^2a_5^4a_6^2 - 2173a_1^2a_2a_3^2a_4a_6^4 \\
& - 1313a_1^2a_2a_3^2a_5^2a_6^3 - 3345a_1^2a_2a_3a_4^2a_5a_6^3 - 429a_1^2a_2a_3a_4a_5^3a_6^2 + 39a_1^2a_2a_3a_5^5a_6 \\
& - 387a_1^2a_2a_4^4a_6^3 - 1343a_1^2a_2a_4^3a_5^2a_6^2 + 310a_1^2a_2a_4^2a_5^4a_6 + 9a_1^2a_2a_4a_5^6 \\
& - 435a_1^2a_3^4a_6^4 - 1310a_1^2a_3^3a_4a_5a_6^3 - 640a_1^2a_3^3a_5^3a_6^2 + 962a_1^2a_3^2a_4^3a_6^3 \\
& + 139a_1^2a_3^2a_4^2a_5^2a_6^2 - 29a_1^2a_3^2a_4a_5^4a_6 + a_1^2a_3^2a_5^6 - 95a_1^2a_3a_4^4a_5a_6^2 \\
& + 18a_1^2a_3a_4^3a_5^3a_6 - 4a_1^2a_3a_4^2a_5^5 + 77a_1^2a_4^6a_6^2 - 37a_1^2a_4^5a_5^2a_6 \\
& + 17a_1^2a_4^4a_5^4 - 1039a_1a_2^4a_5a_6^4 - 3728a_1a_2^3a_3a_4a_6^4 + 2489a_1a_2^3a_3a_5^2a_6^3 \\
& - 49a_1a_2^3a_4^2a_5a_6^3 + 1567a_1a_2^3a_4a_5^3a_6^2 - 35a_1a_2^3a_5^5a_6 + 498a_1a_2^2a_3^3a_6^4 \\
& + 2374a_1a_2^2a_3^2a_4a_5a_6^3 + 336a_1a_2^2a_3^2a_5^3a_6^2 + 1464a_1a_2^2a_3a_4^3a_6^3 + 921a_1a_2^2a_3a_4^2a_5^2a_6^2 \\
& - 18a_1a_2^2a_3a_4a_5^4a_6 - 2a_1a_2^2a_3a_5^6 + 303a_1a_2^2a_4^4a_5a_6^2 - 75a_1a_2^2a_4^3a_5^3a_6 \\
& - 15a_1a_2^2a_4^2a_5^5 + 120a_1a_2a_3^4a_5a_6^3 + 166a_1a_2a_3^3a_4^2a_6^3 - 74a_1a_2a_3^3a_4a_5^2a_6^2 \\
& - 12a_1a_2a_3^3a_5^4a_6 - 716a_1a_2a_3^2a_4^3a_5a_6^2 + 80a_1a_2a_3^2a_4^2a_5^3a_6 + 55a_1a_2a_3a_4^5a_6^2 \\
& + 6a_1a_2a_3a_4^4a_5^2a_6 + 24a_1a_2a_3a_4^3a_5^4 - 29a_1a_2a_4^6a_5a_6 - 16a_1a_2a_4^5a_5^3 \\
& + 360a_1a_3^5a_4a_6^3 + 144a_1a_3^5a_5^2a_6^2 + 204a_1a_3^4a_4^2a_5a_6^2 - 57a_1a_3^3a_4^4a_6^2 \\
& - 166a_1a_3^3a_4^3a_5^2a_6 - 11a_1a_3^2a_4^5a_5a_6 - 20a_1a_3^2a_4^4a_5^3 - 50a_1a_3a_4^7a_6 \\
& + 4a_1a_3a_4^6a_5^2 - 3a_1a_4^8a_5 - 56a_2^5a_4a_6^4 + 7a_2^5a_5^2a_6^3 \\
& + 1029a_2^4a_3^2a_6^4 - 21a_2^4a_3a_4a_5a_6^3 - 297a_2^4a_3a_5^3a_6^2 + 418a_2^4a_4^3a_6^3 \\
& - 821a_2^4a_4^2a_5^2a_6^2 - 12a_2^4a_4a_5^4a_6 + a_2^4a_5^6 - 298a_2^3a_3^3a_5a_6^3 \\
& - 648a_2^3a_3^2a_4^2a_6^3 + 324a_2^3a_3^2a_4a_5^2a_6^2 - 6a_2^3a_3^2a_5^4a_6 + 278a_2^3a_3a_4^3a_5a_6^2 \\
& - 11a_2^3a_3a_4^2a_5^3a_6 - 3a_2^3a_3a_4a_5^5 - 62a_2^3a_4^5a_6^2 - 64a_2^3a_4^4a_5^2a_6 \\
& + 6a_2^3a_4^3a_5^4 - 26a_2^2a_3^4a_4a_6^3 - 174a_2^2a_3^4a_5^2a_6^2 - 338a_2^2a_3^3a_4^2a_5a_6^2 \\
& + 6a_2^2a_3^3a_4a_5^3a_6 - 220a_2^2a_3^2a_4^4a_6^2 + 99a_2^2a_3^2a_4^3a_5^2a_6 + 6a_2^2a_3^2a_4^2a_5^4
\end{aligned}$$

$$\begin{aligned}
& + 28a_2^2a_3a_4^5a_5a_6 - 12a_2^2a_3a_4^4a_5^3 - 10a_2^2a_4^7a_6 + 4a_2^2a_4^6a_5^2 \\
& - 80a_2a_3^6a_6^3 + 120a_2a_3^5a_4a_5a_6^2 + 150a_2a_3^4a_4^3a_6^2 - 12a_2a_3^4a_4^2a_5^2a_6 \\
& - 33a_2a_3^3a_4^4a_5a_6 - 6a_2a_3^3a_4^3a_5^3 + 32a_2a_3^2a_4^6a_6 + 13a_2a_3^2a_4^5a_5^2 \\
& - a_2a_3a_4^7a_5 + a_2a_4^9 - 72a_3^6a_4^2a_6^2 + 24a_3^5a_4^3a_5a_6 \\
& - 9a_3^4a_4^5a_6 + 3a_3^4a_4^4a_5^2 - 4a_3^3a_4^6a_5 - 9948a_0^3a_1a_6^5 \\
& + 2988a_0^3a_2a_5a_6^4 + 28572a_0^3a_3a_4a_6^4 - 3855a_0^3a_3a_5^2a_6^3 + 3461a_0^3a_4^2a_5a_6^3 \\
& - 4437a_0^3a_4a_5^3a_6^2 - 37a_0^3a_5^5a_6 - 2744a_0^2a_1^2a_5a_6^4 - 32458a_0^2a_1a_2a_4a_6^4 \\
& + 7389a_0^2a_1a_2a_5^2a_6^3 - 1494a_0^2a_1a_3^2a_6^4 + 12826a_0^2a_1a_3a_4a_5a_6^3 + 872a_0^2a_1a_3a_5^3a_6^2 \\
& + 3303a_0^2a_1a_4^3a_6^3 + 4147a_0^2a_1a_4^2a_5^2a_6^2 - 395a_0^2a_1a_4a_5^4a_6 - 60a_0^2a_1a_6^5 \\
& - 10758a_0^2a_2^2a_3a_6^4 - 21848a_0^2a_2^2a_4a_5a_6^3 - 1237a_0^2a_2^2a_5^3a_6^2 + 9325a_0^2a_2a_3^2a_5a_6^3 \\
& + 24042a_0^2a_2a_3a_4^2a_6^3 + 4663a_0^2a_2a_3a_4a_5^2a_6^2 + 59a_0^2a_2a_3a_5^4a_6 - 7509a_0^2a_2a_4^3a_5a_6^2 \\
& - 84a_0^2a_2a_4^2a_5^3a_6 + 70a_0^2a_2a_4a_5^5 - 10419a_0^2a_3^3a_4a_6^3 - 1675a_0^2a_3^3a_5^2a_6^2 \\
& - 1341a_0^2a_3^2a_4^2a_5a_6^2 + 359a_0^2a_3^2a_4a_5^3a_6 - 30a_0^2a_3^2a_5^5 - 2640a_0^2a_3a_4^4a_6^2 \\
& - 2724a_0^2a_3a_4^3a_5^2a_6 - 356a_0^2a_3a_4^2a_5^4 + 1486a_0^2a_4^5a_5a_6 + 181a_0^2a_4^4a_5^3 \\
& + 3972a_0a_1^3a_4a_6^4 - 256a_0a_1^3a_5^2a_6^3 + 15012a_0a_1^2a_2a_3a_6^4 + 24600a_0a_1^2a_2a_4a_5a_6^3 \\
& + 209a_0a_1^2a_2a_5^3a_6^2 - 1532a_0a_1^2a_3^2a_5a_6^3 - 7618a_0a_1^2a_3a_4^2a_6^3 + 3232a_0a_1^2a_3a_4a_5^2a_6^2 \\
& + 936a_0a_1^2a_3a_5^4a_6 + 2207a_0a_1^2a_4^3a_5a_6^2 - 499a_0a_1^2a_4^2a_5^3a_6 - 19a_0a_1^2a_4a_5^5 \\
& + 12694a_0a_1a_2^3a_6^4 - 6872a_0a_1a_2^2a_3a_5a_6^3 - 11414a_0a_1a_2^2a_4^2a_6^3 - 5178a_0a_1a_2^2a_4a_5^2a_6^2 \\
& - 211a_0a_1a_2^2a_5^4a_6 - 5942a_0a_1a_2a_3^2a_4a_6^3 - 3033a_0a_1a_2a_3^2a_5^2a_6^2 - 3210a_0a_1a_2a_3a_4^2a_5a_6^2 \\
& - 2210a_0a_1a_2a_3a_4a_5^3a_6 - 80a_0a_1a_2a_3a_5^5 + 4451a_0a_1a_2a_4^4a_6^2 + 812a_0a_1a_2a_4^3a_5^2a_6 \\
& + 130a_0a_1a_2a_4^2a_5^4 + 2604a_0a_1a_3^3a_6^3 - 2396a_0a_1a_3^3a_4a_5a_6^2 - 576a_0a_1a_3^3a_5^3a_6 \\
& + 2817a_0a_1a_3^2a_4^3a_6^2 + 1501a_0a_1a_3^2a_4^2a_5^2a_6 - 12a_0a_1a_3^2a_4a_5^4 + 210a_0a_1a_3a_4^4a_5a_6 \\
& + 148a_0a_1a_3a_4^3a_5^3 + 82a_0a_1a_4^6a_6 - 181a_0a_1a_4^5a_5^2 - 1187a_0a_2^4a_5a_6^3 \\
& + 4560a_0a_2^3a_3a_4a_6^3 + 1803a_0a_2^3a_3a_5^2a_6^2 + 180a_0a_2^3a_4^2a_5a_6^2 + 394a_0a_2^3a_4a_5^3a_6 \\
& + 14a_0a_2^3a_5^5 - 2614a_0a_2^2a_3^3a_6^3 + 1542a_0a_2^2a_3^2a_4a_5a_6^2 + 446a_0a_2^2a_3^2a_5^3a_6 \\
& - 2791a_0a_2^2a_3a_4^3a_6^2 + 2052a_0a_2^2a_3a_4^2a_5^2a_6 + 55a_0a_2^2a_3a_4a_5^4 - 1192a_0a_2^2a_4^4a_5a_6 \\
& + 9a_0a_2^2a_4^3a_5^3 - 1080a_0a_2a_3^4a_5a_6^2 + 1186a_0a_2a_3^3a_4^2a_6^2 - 392a_0a_2a_3^3a_4a_5^2a_6 \\
& - 12a_0a_2a_3^3a_5^4 - 1240a_0a_2a_3^2a_4^3a_5a_6 + 38a_0a_2a_3^2a_4^2a_5^3 - 614a_0a_2a_3a_4^5a_6 \\
& + 30a_0a_2a_3a_4^4a_5^2 + 12a_0a_2a_4^6a_5 + 480a_0a_3^5a_4a_6^2 + 144a_0a_3^5a_5^2a_6 \\
& - 120a_0a_3^4a_4^2a_5a_6 - 81a_0a_3^3a_4^4a_6 - 132a_0a_3^3a_4^3a_5^2 - 31a_0a_3^2a_4^5a_5 \\
& + 7a_0a_3a_4^7 - 2992a_1^4a_3a_6^4 - 3572a_1^4a_4a_5a_6^3 - 410a_1^4a_5^3a_6^2 \\
& - 3490a_1^3a_2^2a_6^4 + 1316a_1^3a_2a_3a_5a_6^3 - 254a_1^3a_2a_4^2a_6^3 + 1879a_1^3a_2a_4a_5^2a_6^2 \\
& + 30a_1^3a_2a_5^4a_6 + 2336a_1^3a_3^2a_4a_6^3 + 10a_1^3a_3^2a_5^2a_6^2 + 42a_1^3a_3a_4^2a_5a_6^2 \\
& + 306a_1^3a_3a_4a_5^3a_6 + 50a_1^3a_3a_5^5 - 715a_1^3a_4^4a_6^2 + 459a_1^3a_4^3a_5^2a_6 \\
& + 15a_1^3a_4^2a_5^4 + 1259a_1^2a_2^3a_5a_6^3 + 2885a_1^2a_2^2a_3a_4a_6^3 - 28a_1^2a_2^2a_3a_5^2a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 3871a_1^2a_2^2a_4^2a_5a_6^2 - 485a_1^2a_2^2a_4a_5^3a_6 - 17a_1^2a_2^2a_5^5 + 568a_1^2a_2a_3^3a_6^3 \\
& + 1194a_1^2a_2a_3^2a_4a_5a_6^2 - 120a_1^2a_2a_3^2a_5^3a_6 - 5a_1^2a_2a_3a_4^3a_6^2 - 655a_1^2a_2a_3a_4^2a_5^2a_6 \\
& - 83a_1^2a_2a_3a_4a_5^4 - 153a_1^2a_2a_4^4a_5a_6 - 142a_1^2a_2a_4^3a_5^3 + 896a_1^2a_3^4a_5a_6^2 \\
& - 558a_1^2a_3^3a_4^2a_6^2 + 40a_1^2a_3^3a_4a_5^2a_6 - 12a_1^2a_3^3a_5^4 + 42a_1^2a_3^2a_4^3a_5a_6 \\
& + 32a_1^2a_3^2a_4^2a_5^3 + 187a_1^2a_3a_4^5a_6 + 2a_1^2a_3a_4^4a_5^2 + 36a_1^2a_4^6a_5 \\
& + 923a_1a_2^4a_4a_6^3 - 705a_1a_2^4a_5^2a_6^2 - 1482a_1a_2^3a_3^2a_6^3 - 3148a_1a_2^3a_3a_4a_5a_6^2 \\
& + 148a_1a_2^3a_3a_5^3a_6 - 1029a_1a_2^3a_4^3a_6^2 - 272a_1a_2^3a_4^2a_5^2a_6 + 55a_1a_2^3a_4a_5^4 \\
& - 620a_1a_2^2a_3^3a_5a_6^2 + 300a_1a_2^2a_3^2a_4^2a_6^2 - 202a_1a_2^2a_3^2a_4a_5^2a_6 + 24a_1a_2^2a_3^2a_5^4 \\
& + 114a_1a_2^2a_3a_4^3a_5a_6 - 14a_1a_2^2a_3a_4^2a_5^3 + 202a_1a_2^2a_4^5a_6 + 99a_1a_2^2a_4^4a_5^2 \\
& - 184a_1a_2a_3^4a_4a_6^2 + 48a_1a_2a_3^4a_5^2a_6 + 200a_1a_2a_3^3a_4^2a_5a_6 + 73a_1a_2a_3^2a_4^4a_6 \\
& - 14a_1a_2a_3^2a_4^3a_5^2 - 34a_1a_2a_3a_4^5a_5 - 3a_1a_2a_4^7 - 192a_1a_3^6a_6^2 \\
& + 72a_1a_3^4a_4^3a_6 + 62a_1a_3^3a_4^4a_5 - a_1a_3^2a_4^6 - 332a_2^5a_3a_6^3 \\
& + 381a_2^5a_4a_5a_6^2 + 9a_2^5a_5^3a_6 + 388a_2^4a_3^2a_5a_6^2 + 525a_2^4a_3a_4^2a_6^2 \\
& + 107a_2^4a_3a_4a_5^2a_6 - 9a_2^4a_3a_5^4 + 288a_2^4a_4^3a_5a_6 - 23a_2^4a_4^2a_5^3 \\
& - 142a_2^3a_3^3a_4a_6^2 + 12a_2^3a_3^3a_5^2a_6 - 250a_2^3a_3^2a_4^2a_5a_6 + 18a_2^3a_3^2a_4a_5^3 \\
& - 136a_2^3a_3a_4^4a_6 + 27a_2^3a_3a_4^3a_5^2 - 14a_2^3a_4^5a_5 + 184a_2^2a_3^5a_6^2 \\
& + 24a_2^2a_3^4a_4a_5a_6 + 98a_2^2a_3^3a_4^3a_6 - 30a_2^2a_3^3a_4^2a_5^2 - 31a_2^2a_3^2a_4^4a_5 \\
& + a_2^2a_3a_4^6 - 48a_2a_3^5a_4^2a_6 + 24a_2a_3^4a_4^3a_5 + 5a_2a_3^3a_4^5 \\
& - 12a_3^5a_4^4 - 5739a_0^4a_6^4 - 17468a_0^3a_1a_5a_6^3 + 15800a_0^3a_2a_4a_6^3 \\
& + 6455a_0^3a_2a_5^2a_6^2 - 6276a_0^3a_3^2a_6^3 + 10932a_0^3a_3a_4a_5a_6^2 + 449a_0^3a_3a_5^3a_6 \\
& - 8347a_0^3a_4^3a_6^2 + 277a_0^3a_4^2a_5^2a_6 - 97a_0^3a_4a_5^4 + 340a_0^2a_1^2a_4a_6^3 \\
& - 1446a_0^2a_1^2a_5^2a_6^2 - 5232a_0^2a_1a_2a_3a_6^3 + 474a_0^2a_1a_2a_4a_5a_6^2 + 620a_0^2a_1a_2a_5^3a_6 \\
& - 3342a_0^2a_1a_3^2a_5a_6^2 - 3132a_0^2a_1a_3a_4^2a_6^2 + 1261a_0^2a_1a_3a_4a_5^2a_6 + 622a_0^2a_1a_3a_5^4 \\
& + 1085a_0^2a_1a_4^3a_5a_6 - 164a_0^2a_1a_4^2a_5^3 + 7026a_0^2a_2^3a_6^3 + 1711a_0^2a_2^2a_3a_5a_6^2 \\
& + 20276a_0^2a_2^2a_4^2a_6^2 - 1354a_0^2a_2^2a_4a_5^2a_6 - 170a_0^2a_2^2a_5^4 - 14499a_0^2a_2a_3^2a_4a_6^2 \\
& - 670a_0^2a_2a_3^2a_5^2a_6 + 1873a_0^2a_2a_3a_4^2a_5a_6 - 449a_0^2a_2a_3a_4a_5^3 - 2578a_0^2a_2a_4^4a_6 \\
& - 640a_0^2a_2a_4^3a_5^2 + 2892a_0^2a_3^4a_6^2 - 92a_0^2a_3^3a_4a_5a_6 + 96a_0^2a_3^3a_5^3 \\
& + 3657a_0^2a_3^2a_4^3a_6 + 1285a_0^2a_3^2a_4^2a_5^2 - 583a_0^2a_3a_4^4a_5 - 97a_0^2a_4^6 \\
& + 2584a_0a_1^3a_3a_6^3 - 6300a_0a_1^3a_4a_5a_6^2 - 528a_0a_1^3a_5^3a_6 - 17092a_0a_1^2a_2^2a_6^3 \\
& - 5908a_0a_1^2a_2a_3a_5a_6^2 - 9122a_0a_1^2a_2a_4^2a_6^2 + 3626a_0a_1^2a_2a_4a_5^2a_6 + 132a_0a_1^2a_2a_5^4 \\
& + 1668a_0a_1^2a_3a_4a_6^2 - 2400a_0a_1^2a_3^2a_5^2a_6 - 760a_0a_1^2a_3a_4^2a_5a_6 + 152a_0a_1^2a_3a_4a_5^3 \\
& - 662a_0a_1^2a_4^4a_6 + 243a_0a_1^2a_4^3a_5^2 + 6244a_0a_1a_2^3a_5a_6^2 + 9754a_0a_1a_2^2a_3a_4a_6^2 \\
& + 2195a_0a_1a_2^2a_3a_5^2a_6 - 2135a_0a_1a_2^2a_4^2a_5a_6 - 395a_0a_1a_2^2a_4a_5^3 + 3876a_0a_1a_2a_3^3a_6^2 \\
& + 2392a_0a_1a_2a_3^2a_4a_5a_6 + 304a_0a_1a_2a_3^2a_5^3 + 1128a_0a_1a_2a_3a_4^3a_6 - 391a_0a_1a_2a_3a_4^2a_5^2 \\
& + 731a_0a_1a_2a_4^4a_5 + 896a_0a_1a_3^4a_5a_6 - 1476a_0a_1a_3^3a_4^2a_6 + 48a_0a_1a_3^3a_4a_5^2
\end{aligned}$$

$$\begin{aligned}
& -184a_0a_1a_3^2a_4^3a_5 + 103a_0a_1a_3a_4^5 - 3289a_0a_2^4a_4a_6^2 - 337a_0a_2^4a_5^2a_6 \\
& - 2266a_0a_2^3a_3^2a_6^2 - 2264a_0a_2^3a_3a_4a_5a_6 - 94a_0a_2^3a_3a_5^3 + 1718a_0a_2^3a_4^3a_6 \\
& + 232a_0a_2^3a_4^2a_5^2 - 648a_0a_2^2a_3^3a_5a_6 + 106a_0a_2^2a_3^2a_4^2a_6 - 116a_0a_2^2a_3^2a_4a_5^2 \\
& - 450a_0a_2^2a_3a_4^3a_5 - 18a_0a_2^2a_4^5 + 448a_0a_2a_3^4a_4a_6 + 48a_0a_2a_3^4a_5^2 \\
& - 88a_0a_2a_3^3a_4^2a_5 + 141a_0a_2a_3^2a_4^4 - 192a_0a_3^6a_6 + 240a_0a_3^4a_4^3 \\
& + 3496a_1^4a_2a_6^3 + 2056a_1^4a_3a_5a_6^2 + 2120a_1^4a_4^2a_6^2 - 364a_1^4a_4a_5^2a_6 \\
& - 27a_1^4a_5^4 - 1878a_1^3a_2^3a_5a_6^2 - 3740a_1^3a_2a_3a_4a_6^2 - 252a_1^3a_2a_3a_5^2a_6 \\
& - 860a_1^3a_2a_4^2a_5a_6 + 18a_1^3a_2a_4a_5^3 - 1064a_1^3a_3^3a_6^2 - 568a_1^3a_3^2a_4a_5a_6 \\
& - 208a_1^3a_3^2a_5^3 - 296a_1^3a_3a_4^3a_6 - 126a_1^3a_3a_4^2a_5^2 - 144a_1^3a_4^4a_5 \\
& - 743a_1^2a_2^3a_4a_6^2 + 179a_1^2a_2^3a_5^2a_6 + 862a_1^2a_2^2a_3^2a_6^2 + 1194a_1^2a_2^2a_3a_4a_5a_6 \\
& + 136a_1^2a_2^2a_3a_5^3 - 313a_1^2a_2^2a_4^3a_6 + 321a_1^2a_2^2a_4^2a_5^2 + 112a_1^2a_2a_3^3a_5a_6 \\
& + 152a_1^2a_2a_3^2a_4^2a_6 + 232a_1^2a_2a_3^2a_4a_5^2 + 154a_1^2a_2a_3a_4^3a_5 - 60a_1^2a_2a_4^5 \\
& + 48a_1^2a_3^4a_4a_6 + 48a_1^2a_3^4a_5^2 - 64a_1^2a_3^3a_4^2a_5 - 19a_1^2a_3^3a_4^4 \\
& + 891a_1a_2^4a_3a_6^2 + 451a_1a_2^4a_4a_5a_6 - 34a_1a_2^4a_5^3 - 104a_1a_2^3a_3^2a_5a_6 \\
& - 10a_1a_2^3a_3a_4^2a_6 - 206a_1a_2^3a_3a_4a_5^2 - 212a_1a_2^3a_4^3a_5 + 120a_1a_2^2a_3^3a_4a_6 \\
& - 96a_1a_2^2a_3^3a_5^2 + 188a_1a_2^2a_3^2a_4^2a_5 + 80a_1a_2^2a_3a_4^4 - 64a_1a_2a_3^5a_6 \\
& - 184a_1a_2a_3^3a_4^3 + 6a_2^6a_6^2 - 61a_2^5a_3a_5a_6 - 238a_2^5a_4^2a_6 \\
& + 25a_2^5a_4a_5^2 + 43a_2^4a_3^2a_4a_6 + 27a_2^4a_3^2a_5^2 + 8a_2^4a_3a_4^2a_5 \\
& + 13a_2^4a_4^4 - 16a_2^3a_3^4a_6 - 24a_2^3a_3^3a_4a_5 + 18a_2^3a_3^2a_4^3 \\
& + 24a_2^2a_3^4a_4^2 - 8548a_0^4a_5a_6^2 + 9404a_0^3a_1a_4a_6^2 + 1116a_0^3a_1a_5^2a_6 \\
& - 4900a_0^3a_2a_3a_6^2 - 1508a_0^3a_2a_4a_5a_6 - 291a_0^3a_2a_5^3 - 532a_0^3a_3^2a_5a_6 \\
& + 4298a_0^3a_3a_4^2a_6 - 22a_0^3a_3a_4a_5^2 - 1709a_0^3a_4^3a_5 + 2448a_0^2a_1^2a_3a_6^2 \\
& - 1140a_0^2a_1^2a_4a_5a_6 + 254a_0^2a_1^2a_5^3 - 20562a_0^2a_1a_2^2a_6^2 - 1804a_0^2a_1a_2a_3a_5a_6 \\
& - 4622a_0^2a_1a_2a_4^2a_6 + 385a_0^2a_1a_2a_4a_5^2 - 2700a_0^2a_1a_3^2a_4a_6 - 2192a_0^2a_1a_3^2a_5^2 \\
& + 1842a_0^2a_1a_3a_4^2a_5 + 605a_0^2a_1a_4^4 + 1439a_0^2a_2^3a_5a_6 + 567a_0^2a_2^2a_3a_4a_6 \\
& + 1157a_0^2a_2^2a_3a_5^2 + 876a_0^2a_2^2a_4^2a_5 + 1672a_0^2a_2a_3^3a_6 + 1156a_0^2a_2a_3^2a_4a_5 \\
& + 1275a_0^2a_2a_3a_4^3 - 96a_0^2a_3^4a_5 - 1972a_0^2a_3^3a_4^2 + 17304a_0a_1^3a_2a_6^2 \\
& + 2368a_0a_1^3a_3a_5a_6 + 2384a_0a_1^3a_4^2a_6 - 236a_0a_1^3a_4a_5^2 - 3176a_0a_1^2a_2^2a_5a_6 \\
& - 2272a_0a_1^2a_2a_3a_4a_6 - 944a_0a_1^2a_2a_3a_5^2 - 1004a_0a_1^2a_2a_4^2a_5 + 1216a_0a_1^2a_3^3a_6 \\
& - 304a_0a_1^2a_3^2a_4a_5 - 728a_0a_1^2a_3a_4^3 + 358a_0a_1a_2^3a_4a_6 + 445a_0a_1a_2^3a_5^2 \\
& - 3212a_0a_1a_2^2a_3^2a_6 + 820a_0a_1a_2^2a_3a_4a_5 - 929a_0a_1a_2^2a_4^3 - 384a_0a_1a_2a_3^3a_5 \\
& + 460a_0a_1a_2a_3^2a_4^2 - 64a_0a_1a_3^4a_4 + 1263a_0a_2^4a_3a_6 - 455a_0a_2^4a_4a_5 \\
& + 152a_0a_2^3a_3^2a_5 + 434a_0a_2^3a_3a_4^2 + 96a_0a_2^2a_3^3a_4 - 64a_0a_2a_3^5 \\
& - 2736a_1^5a_6^2 + 384a_1^4a_2a_5a_6 + 624a_1^4a_3a_4a_6 + 216a_1^4a_3a_5^2 \\
& + 192a_1^4a_4^2a_5 + 820a_1^3a_2^2a_4a_6 - 118a_1^3a_2^2a_5^2 + 400a_1^3a_2a_3^2a_6
\end{aligned}$$

$$\begin{aligned}
& -8a_1^3a_2a_3a_4a_5 + 368a_1^3a_2a_4^3 + 288a_1^3a_3^3a_5 + 200a_1^3a_3^2a_4^2 \\
& -600a_1^2a_2^3a_3a_6 - 126a_1^2a_2^3a_4a_5 - 272a_1^2a_2^2a_3^2a_5 - 438a_1^2a_2^2a_3a_4^2 \\
& -176a_1^2a_2a_3^3a_4 - 64a_1^2a_3^5 - 99a_1a_2^5a_6 + 118a_1a_2^4a_3a_5 \\
& +129a_1a_2^4a_4^2 + 88a_1a_2^3a_3^2a_4 + 128a_1a_2^2a_3^4 - 5a_2^6a_5 \\
& -35a_2^5a_3a_4 - 28a_2^4a_3^3 + 6640a_0^4a_4a_6 + 776a_0^4a_5^2 \\
& -10280a_0^3a_1a_3a_6 + 884a_0^3a_1a_4a_5 - 2584a_0^3a_2^2a_6 + 1708a_0^3a_2a_3a_5 \\
& +3318a_0^3a_2a_4^2 + 680a_0^3a_2^3a_4 + 10200a_0^2a_1^2a_2a_6 - 1112a_0^2a_1^2a_3a_5 \\
& -1048a_0^2a_1^2a_4^2 + 2a_0^2a_1a_2^2a_5 - 4044a_0^2a_1a_2a_3a_4 + 2656a_0^2a_1a_3^3 \\
& -705a_0^2a_2^3a_4 - 2388a_0^2a_2^2a_3^2 - 3168a_0a_1^4a_6 + 192a_0a_1^3a_2a_5 \\
& +816a_0a_1^3a_3a_4 + 1844a_0a_1^2a_2^2a_4 + 1664a_0a_1^2a_2a_3^2 - 1092a_0a_1a_2^3a_3 \\
& +225a_0a_2^5 - 576a_1^4a_2a_4 - 432a_1^4a_3^2 + 408a_1^3a_2^2a_3 \\
& -27a_1^2a_2^4 - 3280a_0^4a_3 - 2200a_0^3a_1a_2 + 720a_0^2a_1^3
\end{aligned} \tag{A.32}$$

$$\begin{aligned}
e_{33} = & -64a_0^3a_6^{12} + 32a_0^2a_1a_5a_6^{11} + 16a_0^2a_2a_4a_6^{11} - 192a_0^2a_2a_5^2a_6^{10} \\
& + 48a_0^2a_3^2a_6^{11} - 72a_0^2a_3a_4a_5a_6^{10} + 112a_0^2a_3a_5^3a_6^9 - 4a_0^2a_4^3a_6^{10} \\
& + 20a_0^2a_4^2a_5^2a_6^9 - 162a_0^2a_4a_5^4a_6^8 + 2a_0^2a_5^6a_6^7 - 20a_0a_1^2a_4a_6^{11} \\
& - 96a_0a_1^2a_5^2a_6^{10} - 24a_0a_1a_2a_3a_6^{11} + 24a_0a_1a_2a_4a_5a_6^{10} + 120a_0a_1a_2a_5^3a_6^9 \\
& - 4a_0a_1a_2^3a_5a_6^{10} + 26a_0a_1a_3a_4^2a_6^{10} + 248a_0a_1a_3a_4a_5^2a_6^9 + 167a_0a_1a_3a_5^4a_6^8 \\
& - 16a_0a_1a_3^3a_5a_6^9 - 158a_0a_1a_4^2a_5^3a_6^8 - 44a_0a_1a_4a_5^5a_6^7 + 39a_0a_1a_5^7a_6^6 \\
& - 32a_0a_2^3a_6^{11} + 56a_0a_2^2a_3a_5a_6^{10} + 16a_0a_2^2a_4^2a_6^{10} - 36a_0a_2^2a_4a_5^2a_6^9 \\
& - 88a_0a_2^2a_5^4a_6^8 + 4a_0a_2a_3^2a_4a_6^{10} - 20a_0a_2a_3^2a_5^2a_6^9 - 30a_0a_2a_3a_4^2a_5a_6^9 \\
& - 56a_0a_2a_3a_4a_5^3a_6^8 + 11a_0a_2a_3a_5^5a_6^7 - 2a_0a_2a_4^4a_6^9 + 28a_0a_2a_4^3a_5^2a_6^8 \\
& + 62a_0a_2a_4^2a_5^4a_6^7 - 60a_0a_2a_4a_5^6a_6^6 - 7a_0a_2a_5^8a_6^5 - 12a_0a_3^4a_6^{10} \\
& + 30a_0a_3^3a_4a_5a_6^9 - 8a_0a_3^3a_5^3a_6^8 - 6a_0a_3^2a_4^3a_6^9 - 96a_0a_3^2a_4^2a_5^2a_6^8 \\
& - 75a_0a_3^2a_4a_5^4a_6^7 - 6a_0a_3^2a_5^6a_6^6 + 9a_0a_3a_4^4a_5a_6^8 + 96a_0a_3a_4^3a_5^3a_6^7 \\
& + 84a_0a_3a_4^2a_5^5a_6^6 - 3a_0a_3a_4a_5^7a_6^5 - 6a_0a_4^5a_5^2a_6^7 - 33a_0a_4^4a_5^4a_6^6 \\
& - 22a_0a_4^3a_5^6a_6^5 + 14a_1^3a_3a_6^{11} - 6a_1^3a_4a_5a_6^{10} + 18a_1^3a_5^3a_6^9 \\
& + 8a_1^2a_2^2a_6^{11} - 18a_1^2a_2a_3a_5a_6^{10} - 6a_1^2a_2a_4^2a_6^{10} - 42a_1^2a_2a_4a_5^2a_6^9 \\
& - 87a_1^2a_2a_5^4a_6^8 - 16a_1^2a_3^2a_4a_6^{10} - 65a_1^2a_3^2a_5^2a_6^9 + 20a_1^2a_3a_4^2a_5a_6^9 \\
& + 68a_1^2a_3a_4a_5^3a_6^8 - 22a_1^2a_3a_5^5a_6^7 + a_1^2a_4^4a_6^9 + 8a_1^2a_4^3a_5^2a_6^8 \\
& + 15a_1^2a_4^2a_5^4a_6^7 + 33a_1^2a_4a_5^6a_6^6 + 4a_1^2a_5^8a_6^5 + 8a_1a_2^2a_3a_4a_6^{10} \\
& + 34a_1a_2^2a_3a_5^2a_6^9 - 4a_1a_2^2a_4^2a_5a_6^9 + 14a_1a_2^2a_4a_5^3a_6^8 + 68a_1a_2^2a_5^5a_6^7 \\
& + 6a_1a_2a_3^3a_6^{10} - 10a_1a_2a_3^2a_4a_5a_6^9 + 16a_1a_2a_3^2a_5^3a_6^8 - 2a_1a_2a_3a_4^3a_6^9 \\
& + 16a_1a_2a_3a_4^2a_5^2a_6^8 + 31a_1a_2a_3a_4a_5^4a_6^7 + 9a_1a_2a_3a_5^6a_6^6 + a_1a_2a_4^4a_5a_6^8
\end{aligned}$$

$$\begin{aligned}
& -20a_1a_2a_4^3a_5^3a_6^7 - 54a_1a_2a_4^2a_5^5a_6^6 - 9a_1a_2a_4a_5^7a_6^5 - a_1a_3^4a_5a_6^9 \\
& + 4a_1a_3^3a_4^2a_6^9 + 34a_1a_3^3a_4a_5^2a_6^8 + 16a_1a_3^3a_5^4a_6^7 - 6a_1a_3^2a_4^3a_5a_6^8 \\
& - 57a_1a_3^2a_4^2a_5^3a_6^7 - 30a_1a_3^2a_4a_5^5a_6^6 - a_1a_3^2a_5^7a_6^5 - 3a_1a_3a_4^4a_5^2a_6^7 \\
& + 14a_1a_3a_4^3a_5^4a_6^6 + 11a_1a_3a_4^2a_5^6a_6^5 + 4a_1a_4^5a_5^3a_6^6 + 7a_1a_4^4a_5^5a_6^5 \\
& - 24a_2^4a_5^2a_6^9 - 8a_2^3a_3^2a_6^{10} + 8a_2^3a_3a_4a_5a_6^9 + 34a_2^3a_3a_5^3a_6^8 \\
& + 10a_2^3a_4^2a_5^2a_6^8 - 38a_2^3a_4a_5^4a_6^7 - 11a_2^3a_5^6a_6^6 + 10a_2^2a_3^3a_5a_6^9 \\
& + 2a_2^2a_3^2a_4^2a_6^9 - 34a_2^2a_3^2a_4a_5^2a_6^8 - 16a_2^2a_3^2a_5^4a_6^7 - 2a_2^2a_3a_4^3a_5a_6^8 \\
& + 12a_2^2a_3a_4^2a_5^3a_6^7 + 15a_2^2a_3a_4a_5^5a_6^6 + a_2^2a_3a_5^7a_6^5 - a_2^2a_4^4a_5^2a_6^7 \\
& + 14a_2^2a_3^4a_5^4a_6^6 + 5a_2^2a_4^2a_5^6a_6^5 - 2a_2a_3^4a_4a_6^9 + 5a_2a_3^4a_5^2a_6^8 \\
& + 2a_2a_3^3a_4^2a_5a_6^8 - 10a_2a_3^3a_4a_5^3a_6^7 - a_2a_3^3a_5^5a_6^6 + 6a_2a_3^2a_4^3a_5^2a_6^7 \\
& + 9a_2a_3^2a_4^2a_5^4a_6^6 + a_2a_3^2a_4a_5^6a_6^5 - 5a_2a_3a_4^4a_5^3a_6^6 - 4a_2a_3a_4^3a_5^5a_6^5 \\
& - a_2a_4^5a_5^4a_6^5 + a_3^6a_6^9 - 3a_3^5a_4a_5a_6^8 - 3a_3^5a_5^3a_6^7 \\
& + 6a_3^4a_4a_5^4a_6^6 + 5a_3^3a_4^3a_5^3a_6^6 - 3a_3^3a_4^2a_5^5a_6^5 - 3a_3^2a_4^4a_5^4a_6^5 \\
& - 544a_0^3a_5a_6^{10} - 8a_0^2a_1a_4a_6^{10} + 672a_0^2a_1a_5^2a_6^9 - 88a_0^2a_2a_3a_6^{10} \\
& + 576a_0^2a_2a_4a_5a_6^9 - 352a_0^2a_2a_5^3a_6^8 + 508a_0^2a_3^2a_5a_6^9 + 58a_0^2a_3a_4^2a_6^9 \\
& - 1344a_0^2a_3a_4a_5^2a_6^8 + 327a_0^2a_3a_5^4a_6^7 - 64a_0^2a_4^3a_5a_6^8 + 1116a_0^2a_4^2a_5^3a_6^7 \\
& - 202a_0^2a_4a_5^5a_6^6 + 47a_0^2a_5^7a_6^5 + 2a_0a_1^2a_3a_6^{10} + 6a_0a_1^2a_4a_5a_6^9 \\
& - 144a_0a_1^2a_5^3a_6^8 + 104a_0a_1a_2^2a_6^{10} - 308a_0a_1a_2a_3a_5a_6^9 - 70a_0a_1a_2a_4^2a_6^9 \\
& - 212a_0a_1a_2a_4a_5^2a_6^8 + 243a_0a_1a_2a_5^4a_6^7 - 20a_0a_1a_3^2a_4a_6^9 - 138a_0a_1a_3^2a_5^2a_6^8 \\
& - 198a_0a_1a_3a_4^2a_5a_6^8 - 314a_0a_1a_3a_4a_5^3a_6^7 - 214a_0a_1a_3a_5^5a_6^6 + 16a_0a_1a_4^4a_6^8 \\
& + 254a_0a_1a_4^3a_5^2a_6^7 - 58a_0a_1a_4^2a_5^4a_6^6 - 101a_0a_1a_4a_5^6a_6^5 + 20a_0a_1a_5^8a_6^4 \\
& - 344a_0a_2^3a_5a_6^9 - 104a_0a_2^2a_3a_4a_6^9 + 602a_0a_2^2a_3a_5^2a_6^8 + 266a_0a_2^2a_4^2a_5a_6^8 \\
& - 2a_0a_2^2a_4a_5^3a_6^7 - 25a_0a_2^2a_5^5a_6^6 + 38a_0a_2a_3^3a_6^9 + 60a_0a_2a_3^2a_4a_5a_6^8 \\
& - 246a_0a_2a_3^2a_5^3a_6^7 + 52a_0a_2a_3a_4^3a_6^8 - 70a_0a_2a_3a_4^2a_5^2a_6^7 + 258a_0a_2a_3a_4a_5^4a_6^6 \\
& + 71a_0a_2a_3a_5^6a_6^5 - 88a_0a_2a_4^4a_5a_6^7 - 108a_0a_2a_4^3a_5^3a_6^6 + 266a_0a_2a_4^2a_5^5a_6^5 \\
& - 22a_0a_2a_4a_5^7a_6^4 - 149a_0a_3^4a_5a_6^8 - 16a_0a_3^3a_4^2a_6^8 + 512a_0a_3^3a_4a_5^2a_6^7 \\
& + 8a_0a_3^3a_5^4a_6^6 + 96a_0a_3^2a_4^3a_5a_6^7 - 273a_0a_3^2a_4^2a_5^3a_6^6 + 48a_0a_3^2a_4a_5^5a_6^5 \\
& - 5a_0a_3^2a_5^7a_6^4 - 9a_0a_3a_4^5a_6^7 - 147a_0a_3a_4^4a_5^2a_6^6 - 74a_0a_3a_4^3a_5^4a_6^5 \\
& + 22a_0a_3a_4^2a_5^6a_6^4 + 12a_0a_4^6a_5a_6^6 + 80a_0a_4^5a_5^3a_6^5 + 53a_0a_4^4a_5^5a_6^4 \\
& - 30a_1^3a_2a_6^{10} + 150a_1^3a_3a_5a_6^9 + 12a_1^3a_4^2a_6^9 - 72a_1^3a_4a_5^2a_6^8 \\
& + 120a_1^3a_5^4a_6^7 + 90a_1^2a_2^2a_5a_6^9 + 46a_1^2a_2a_3a_4a_6^9 - 76a_1^2a_2a_3a_5^2a_6^8 \\
& + 12a_1^2a_2a_4^2a_5a_6^8 + 330a_1^2a_2a_4a_5^3a_6^7 - 138a_1^2a_2a_5^5a_6^6 + 10a_1^2a_3^3a_6^9 \\
& - 60a_1^2a_3^2a_4a_5a_6^8 - 34a_1^2a_3^2a_5^3a_6^7 - 22a_1^2a_3a_4^3a_6^8 - 24a_1^2a_3a_4^2a_5^2a_6^7 \\
& + 10a_1^2a_3a_4a_5^4a_6^6 - 58a_1^2a_3a_5^6a_6^5 - 6a_1^2a_4^4a_5a_6^7 - 94a_1^2a_4^3a_5^3a_6^6 \\
& - 114a_1^2a_4^2a_5^5a_6^5 + 4a_1^2a_4a_5^7a_6^4 - 8a_1a_2^3a_4a_6^9 + 62a_1a_2^3a_5^2a_6^8
\end{aligned}$$

$$\begin{aligned}
& -2a_1a_2^2a_3^2a_6^9 + 16a_1a_2^2a_3a_4a_5a_6^8 - 134a_1a_2^2a_3a_5^3a_6^7 + 6a_1a_2^2a_4^3a_6^8 \\
& - 132a_1a_2^2a_4^2a_5^2a_6^7 - 209a_1a_2^2a_4a_5^4a_6^6 + 83a_1a_2^2a_5^6a_6^5 + 54a_1a_2a_3^3a_5a_6^8 \\
& - 224a_1a_2a_3^2a_4a_5^2a_6^7 + 10a_1a_2a_3^2a_5^4a_6^6 - 42a_1a_2a_3a_4^3a_5a_6^7 + 128a_1a_2a_3a_4^2a_5^3a_6^6 \\
& - 8a_1a_2a_3a_4a_5^5a_6^5 - a_1a_2a_4^5a_6^7 + 77a_1a_2a_4^4a_5^2a_6^6 + 148a_1a_2a_4^3a_5^4a_6^5 \\
& + 6a_1a_2a_4^2a_5^6a_6^4 - 5a_1a_3^4a_4a_6^8 - 50a_1a_3^4a_5^2a_6^7 - 8a_1a_3^3a_4^2a_5a_6^7 \\
& + 108a_1a_3^3a_4a_5^3a_6^6 + 14a_1a_3^3a_5^5a_6^5 + 6a_1a_3^2a_4^4a_6^7 + 105a_1a_3^2a_4^3a_5^2a_6^6 \\
& - 12a_1a_3^2a_4^2a_5^4a_6^5 - a_1a_3^2a_4a_5^6a_6^4 + 6a_1a_3a_4^5a_5a_6^6 - 50a_1a_3a_4^4a_5^3a_6^5 \\
& - 14a_1a_3a_4^3a_5^5a_6^4 - 12a_1a_4^6a_5^2a_6^5 - 23a_1a_4^5a_5^4a_6^4 + 16a_2^4a_3a_6^9 \\
& + 40a_2^4a_4a_5a_6^8 - 58a_2^4a_5^3a_6^7 - 118a_2^3a_3^2a_5a_6^8 - 12a_2^3a_3a_4^2a_6^8 \\
& + 34a_2^3a_3a_4a_5^2a_6^7 + 118a_2^3a_3a_5^4a_6^6 - 18a_2^3a_4^3a_5a_6^7 + 140a_2^3a_4^2a_5^3a_6^6 \\
& - 23a_2^3a_4a_5^5a_6^5 - 5a_2^3a_5^7a_6^4 - 6a_2^2a_3^3a_4a_6^8 + 124a_2^2a_3^3a_5^2a_6^7 \\
& + 90a_2^2a_3^2a_4^2a_5a_6^7 - 158a_2^2a_3^2a_4a_5^3a_6^6 - 25a_2^2a_3^2a_5^5a_6^5 + 2a_2^2a_3a_4^4a_6^7 \\
& - 68a_2^2a_3a_4^3a_5^2a_6^6 - 9a_2^2a_3a_4^2a_5^4a_6^5 + 11a_2^2a_3a_4a_5^6a_6^4 + 2a_2^2a_4^5a_5a_6^6 \\
& - 49a_2^2a_4^4a_5^3a_6^5 - 10a_2^2a_4^3a_5^5a_6^4 - 4a_2a_3^5a_6^8 - 21a_2a_3^4a_4a_5a_6^7 \\
& - 5a_2a_3^4a_5^3a_6^6 - 2a_2a_3^3a_4^3a_6^7 + 36a_2a_3^3a_4^2a_5^2a_6^6 + 13a_2a_3^3a_4a_5^4a_6^5 \\
& - 12a_2a_3^2a_4^4a_5a_6^6 - 13a_2a_3^2a_4^3a_5^3a_6^5 - 9a_2a_3^2a_4^2a_5^5a_6^4 + 15a_2a_3a_4^5a_5^2a_6^5 \\
& + 6a_2a_3a_4^4a_5^4a_6^4 + 4a_2a_4^6a_5^3a_6^4 + 14a_3^6a_5a_6^7 + 3a_3^5a_4^2a_6^7 \\
& - 27a_3^5a_4a_5^2a_6^6 + 6a_3^4a_4^2a_5^3a_6^5 - 15a_3^3a_4^4a_5^2a_6^5 + 7a_3^3a_4^3a_5^4a_6^4 \\
& + 12a_3^2a_4^5a_5^3a_6^4 + 284a_0^3a_4a_6^9 + 1936a_0^3a_5^2a_6^8 + 98a_0^2a_1a_3a_6^9 \\
& - 1050a_0^2a_1a_4a_5a_6^8 - 102a_0^2a_1a_5^3a_6^7 - 1728a_0^2a_2^2a_6^9 + 382a_0^2a_2a_3a_5a_6^8 \\
& + 560a_0^2a_2a_4^2a_6^8 - 362a_0^2a_2a_4a_5^2a_6^7 + 918a_0^2a_2a_5^4a_6^6 - 360a_0^2a_3^2a_4a_6^8 \\
& - 493a_0^2a_3^2a_5^2a_6^7 + 1544a_0^2a_3a_4^2a_5a_6^7 - 1194a_0^2a_3a_4a_5^3a_6^6 - 279a_0^2a_3a_5^5a_6^5 \\
& - 78a_0^2a_4^4a_6^7 - 1718a_0^2a_4^3a_5^2a_6^6 + 951a_0^2a_4^2a_5^4a_6^5 - 164a_0^2a_4a_5^6a_6^4 \\
& + 10a_0^2a_5^8a_6^3 + 766a_0a_1^2a_2a_6^9 - 592a_0a_1^2a_3a_5a_6^8 - 176a_0a_1^2a_4^2a_6^8 \\
& + 1170a_0a_1^2a_4a_5^2a_6^7 + 85a_0a_1^2a_5^4a_6^6 + 1192a_0a_1a_2^2a_5a_6^8 - 476a_0a_1a_2a_3a_4a_6^8 \\
& + 408a_0a_1a_2a_3a_5^2a_6^7 - 34a_0a_1a_2a_4^2a_5a_6^7 - 1322a_0a_1a_2a_4a_5^3a_6^6 + 20a_0a_1a_2a_5^5a_6^5 \\
& - 42a_0a_1a_3^3a_6^8 + 958a_0a_1a_3^2a_4a_5a_6^7 + 562a_0a_1a_3^2a_5^3a_6^6 + 232a_0a_1a_3a_4^3a_6^7 \\
& - 675a_0a_1a_3a_4^2a_5^2a_6^6 + 270a_0a_1a_3a_4a_5^4a_6^5 - 177a_0a_1a_3a_5^6a_6^4 - 176a_0a_1a_4^4a_5a_6^6 \\
& + 736a_0a_1a_4^3a_5^3a_6^5 + 74a_0a_1a_4^2a_5^5a_6^4 - 44a_0a_1a_4a_5^7a_6^3 + 260a_0a_2^3a_4a_6^8 \\
& + 218a_0a_2^3a_5^2a_6^7 + 854a_0a_2^2a_3^2a_6^8 - 2510a_0a_2^2a_3a_4a_5a_6^7 - 136a_0a_2^2a_3a_5^3a_6^6 \\
& - 182a_0a_2^2a_4^3a_6^7 + 814a_0a_2^2a_4^2a_5^2a_6^6 - 95a_0a_2^2a_4a_5^4a_6^5 + 44a_0a_2^2a_5^6a_6^4 \\
& - 366a_0a_2a_3^3a_5a_6^7 - 320a_0a_2a_3^2a_4^2a_6^7 + 1154a_0a_2a_3^2a_4a_5^2a_6^6 - 399a_0a_2a_3^2a_5^4a_6^5 \\
& + 808a_0a_2a_3a_4^3a_5a_6^6 - 1083a_0a_2a_3a_4^2a_5^3a_6^5 + 166a_0a_2a_3a_4a_5^5a_6^4 - 17a_0a_2a_3a_5^7a_6^3 \\
& + 52a_0a_2a_4^5a_6^6 - 196a_0a_2a_4^4a_5^2a_6^5 - 514a_0a_2a_4^3a_5^4a_6^4 + 88a_0a_2a_4^2a_5^6a_6^3 \\
& + 119a_0a_3^4a_4a_6^7 + 6a_0a_3^4a_5^2a_6^6 - 791a_0a_3^3a_4^2a_5a_6^6 - 57a_0a_3^3a_4a_5^3a_6^5
\end{aligned}$$

$$\begin{aligned}
& + 43a_0a_3^3a_5^5a_6^4 - 15a_0a_3^2a_4^4a_6^6 + 792a_0a_3^2a_4^3a_5^2a_6^5 - 156a_0a_3^2a_4^2a_5^4a_6^4 \\
& + 13a_0a_3^2a_4a_5^6a_6^3 + 21a_0a_3a_4^5a_5a_6^5 - 178a_0a_3a_4^4a_5^3a_6^4 - 62a_0a_3a_4^3a_5^5a_6^3 \\
& - 6a_0a_4^7a_6^5 - 54a_0a_4^6a_5^2a_6^4 - 30a_0a_4^5a_5^4a_6^3 - 83a_1^4a_6^9 \\
& - 342a_1^3a_2a_5a_6^8 + 74a_1^3a_3a_4a_6^8 - 289a_1^3a_3a_5^2a_6^7 + 49a_1^3a_4^2a_5a_6^7 \\
& - 251a_1^3a_4a_5^3a_6^6 + 53a_1^3a_5^5a_6^5 - 62a_1^2a_2^2a_4a_6^8 - 189a_1^2a_2^2a_5^2a_6^7 \\
& - 254a_1^2a_2a_3^2a_6^8 + 580a_1^2a_2a_3a_4a_5a_6^7 + 165a_1^2a_2a_3a_5^3a_6^6 + 6a_1^2a_2a_4^3a_6^7 \\
& - 550a_1^2a_2a_4^2a_5^2a_6^6 + 746a_1^2a_2a_4a_5^4a_6^5 - 28a_1^2a_2a_5^6a_6^4 + 101a_1^2a_3^3a_5a_6^7 \\
& - 10a_1^2a_3^2a_4^2a_6^7 + 135a_1^2a_3^2a_4a_5^2a_6^6 + 226a_1^2a_3^2a_5^4a_6^5 - 9a_1^2a_3a_4^3a_5a_6^6 \\
& + 31a_1^2a_3a_4^2a_5^3a_6^5 + 61a_1^2a_3a_4a_5^5a_6^4 + 4a_1^2a_3a_5^7a_6^3 + 2a_1^2a_5^5a_6^6 \\
& + 158a_1^2a_4^4a_5^2a_6^5 + 138a_1^2a_4^3a_5^4a_6^4 - 30a_1^2a_4^2a_5^6a_6^3 - 30a_1a_2^3a_3a_6^8 \\
& - 198a_1a_2^3a_4a_5a_6^7 + 310a_1a_2^3a_5^3a_6^6 + 120a_1a_2^2a_3^2a_5a_6^7 - 10a_1a_2^2a_3a_4^2a_6^7 \\
& + 432a_1a_2^2a_3a_4a_5^2a_6^6 - 391a_1a_2^2a_3a_5^4a_6^5 + 226a_1a_2^2a_4^3a_5a_6^6 - 32a_1a_2^2a_4^2a_5^3a_6^5 \\
& - 277a_1a_2^2a_4a_5^5a_6^4 + 10a_1a_2^2a_5^7a_6^3 + 188a_1a_2a_3^3a_4a_6^7 - 65a_1a_2a_3^2a_5^2a_6^6 \\
& - 81a_1a_2a_3^2a_4^2a_5a_6^6 - 47a_1a_2a_3^2a_4a_5^3a_6^5 + 21a_1a_2a_3^2a_5^5a_6^4 + 19a_1a_2a_3a_4^4a_6^6 \\
& - 279a_1a_2a_3a_4^3a_5^2a_6^5 + 104a_1a_2a_3a_4^2a_5^4a_6^4 - 19a_1a_2a_3a_4a_5^6a_6^3 - 91a_1a_2a_4^5a_5a_6^5 \\
& - 102a_1a_2a_4^4a_5^3a_6^4 + 32a_1a_2a_4^3a_5^5a_6^3 + 7a_1a_5^5a_6^7 - 41a_1a_4^4a_4a_5a_6^6 \\
& - 42a_1a_3^4a_5^3a_6^5 - 16a_1a_3^3a_4^3a_6^6 - 132a_1a_3^3a_4^2a_5^2a_6^5 - 40a_1a_3^3a_4a_5^4a_6^4 \\
& - a_1a_3^3a_5^6a_6^3 - 63a_1a_3^2a_4^4a_5a_6^5 + 91a_1a_3^2a_4^3a_5^3a_6^4 + 21a_1a_3^2a_4^2a_5^5a_6^3 \\
& - 3a_1a_3a_4^6a_6^5 + 72a_1a_3a_4^5a_5^2a_6^4 - 13a_1a_3a_4^4a_5^4a_6^3 + 12a_1a_4^7a_5a_6^6 \\
& + 26a_1a_4^6a_5^3a_6^3 - 8a_2^5a_6^8 + 174a_2^4a_3a_5a_6^7 - 14a_2^4a_4^2a_6^7 \\
& + 16a_2^4a_4a_5^2a_6^6 - 9a_2^4a_5^4a_6^5 + 94a_2^3a_3^2a_4a_6^7 - 266a_2^3a_3^2a_5^2a_6^6 \\
& - 180a_2^3a_3a_4^2a_5a_6^6 - 122a_2^3a_3a_4a_5^3a_6^5 + 36a_2^3a_3a_5^5a_6^4 + 8a_2^3a_4^4a_6^6 \\
& - 144a_2^3a_4^3a_5^2a_6^5 + 153a_2^3a_4^2a_5^4a_6^4 + 6a_2^3a_4a_5^6a_6^3 - 88a_2^2a_3^4a_6^7 \\
& - 48a_2^2a_3^3a_4a_5a_6^6 + 120a_2^2a_3^3a_5^3a_6^5 - 50a_2^2a_3^2a_4^3a_6^6 + 415a_2^2a_3^2a_4^2a_5^2a_6^5 \\
& - 44a_2^2a_3^2a_4a_5^4a_6^4 + a_2^2a_3^2a_5^6a_6^3 + 92a_2^2a_3a_4^4a_5a_6^5 - 83a_2^2a_3a_4^3a_5^3a_6^4 \\
& - 17a_2^2a_3a_4^2a_5^5a_6^3 - a_2^2a_4^6a_6^5 + 61a_2^2a_4^5a_5^2a_6^4 - 2a_2^2a_4^4a_5^4a_6^3 \\
& + 51a_2a_3^5a_5a_6^6 + 10a_2a_3^4a_4^2a_6^6 - 93a_2a_3^4a_4a_5^2a_6^5 - a_2a_3^4a_5^4a_6^4 \\
& - 24a_2a_3^3a_4^3a_5a_6^5 + 37a_2a_3^3a_4^2a_5^3a_6^4 + a_2a_3^3a_4a_5^5a_6^3 + 6a_2a_3^2a_4^5a_6^5 \\
& - 18a_2a_3^2a_4^4a_5^2a_6^4 + 8a_2a_3^2a_4^3a_5^4a_6^3 - 15a_2a_3a_4^6a_5a_6^4 + 7a_2a_3a_4^5a_5^3a_6^3 \\
& - 6a_2a_4^7a_5^2a_6^3 - 13a_3^6a_4a_6^6 - 3a_3^6a_5^2a_6^5 + 54a_3^5a_4^2a_5a_6^5 \\
& + 6a_3^5a_4a_5^3a_6^4 - 39a_3^4a_4^3a_5^2a_6^4 - 3a_3^4a_4^2a_5^4a_6^3 + 15a_3^3a_4^5a_5a_6^4 \\
& - 5a_3^3a_4^4a_5^3a_6^3 - 18a_3^2a_4^6a_5^2a_6^3 - 498a_3^2a_4^3a_6^8 - 5494a_3^2a_4a_5a_6^7 \\
& + 212a_3^2a_5^3a_6^6 + 1758a_3^2a_4a_5a_6^8 + 1302a_3^2a_4a_5a_6^7 + 924a_3^2a_4a_5^2a_6^6 \\
& - 1408a_3^2a_4a_5^4a_6^5 + 5918a_3^2a_4^2a_5a_6^7 - 1474a_3^2a_4a_5a_6^7 - 5084a_3^2a_4a_5^2a_6^6 \\
& - 188a_3^2a_4^2a_5a_6^6 - 680a_3^2a_4a_5^3a_6^5 + 119a_3^2a_4a_5^5a_6^4 + 444a_3^2a_5^3a_6^7
\end{aligned}$$

$$\begin{aligned}
& + 1238a_0^2a_3^2a_4a_5a_6^6 + 1230a_0^2a_3^2a_5^3a_6^5 - 156a_0^2a_3a_4^3a_6^6 + 1353a_0^2a_3a_4^2a_5^2a_6^5 \\
& + 555a_0^2a_3a_4a_5^4a_6^4 - 71a_0^2a_3a_5^6a_6^3 + 1026a_0^2a_4^4a_5a_6^5 - 1826a_0^2a_4^3a_5^3a_6^4 \\
& + 268a_0^2a_4^2a_5^5a_6^3 - 24a_0^2a_4a_5^7a_6^2 - 432a_0a_1^3a_6^8 - 4204a_0a_1^2a_2a_5a_6^7 \\
& + 1048a_0a_1^2a_3a_4a_6^7 + 505a_0a_1^2a_3a_5^2a_6^6 - 1483a_0a_1^2a_4^2a_5a_6^6 - 581a_0a_1^2a_4a_5^3a_6^5 \\
& + 27a_0a_1^2a_5^5a_6^4 - 112a_0a_1a_2^2a_4a_6^7 - 478a_0a_1a_2^2a_5^2a_6^6 - 614a_0a_1a_2a_3^2a_6^7 \\
& + 2316a_0a_1a_2a_3a_4a_5a_6^6 + 414a_0a_1a_2a_3a_5^3a_6^5 - 224a_0a_1a_2a_4^3a_6^6 + 1443a_0a_1a_2a_4^2a_5^2a_6^5 \\
& + 104a_0a_1a_2a_4a_5^4a_6^4 + 75a_0a_1a_2a_5^6a_6^3 - 628a_0a_1a_3^3a_5a_6^6 - 842a_0a_1a_3^2a_4^2a_6^6 \\
& - 564a_0a_1a_3^2a_4a_5^2a_6^5 + 524a_0a_1a_3^2a_5^4a_6^4 + 1258a_0a_1a_3a_4^3a_5a_6^5 + 86a_0a_1a_3a_4^2a_5^3a_6^4 \\
& + 368a_0a_1a_3a_4a_5^5a_6^3 + 12a_0a_1a_3a_5^7a_6^2 + 80a_0a_1a_4^5a_6^5 - 1356a_0a_1a_4^4a_5^2a_6^4 \\
& + 76a_0a_1a_4^3a_5^4a_6^3 + 24a_0a_1a_4^2a_5^6a_6^2 - 1982a_0a_2^3a_3a_6^7 - 12a_0a_2^3a_4a_5a_6^6 \\
& + 258a_0a_2^3a_5^3a_6^5 + 856a_0a_2^2a_3^2a_5a_6^6 + 2394a_0a_2^2a_3a_4^2a_6^6 - 108a_0a_2^2a_3a_4a_5^2a_6^5 \\
& - 449a_0a_2^2a_3a_5^4a_6^4 - 1190a_0a_2^2a_4^3a_5a_6^5 + 551a_0a_2^2a_4^2a_5^3a_6^4 - 259a_0a_2^2a_4a_5^5a_6^3 \\
& - 7a_0a_2^2a_5^7a_6^2 + 772a_0a_2a_3^3a_4a_6^6 + 557a_0a_2a_3^3a_5^2a_6^5 - 2706a_0a_2a_3^2a_4^2a_5a_6^5 \\
& + 292a_0a_2a_3^2a_4a_5^3a_6^4 + 154a_0a_2a_3^2a_5^5a_6^3 - 631a_0a_2a_3a_4^4a_6^5 + 2486a_0a_2a_3a_4^3a_5^2a_6^4 \\
& - 580a_0a_2a_3a_4^2a_5^4a_6^3 + 12a_0a_2a_3a_4a_5^6a_6^2 + 384a_0a_2a_4^5a_5a_6^4 + 360a_0a_2a_4^4a_5^3a_6^3 \\
& - 102a_0a_2a_4^3a_5^5a_6^2 - 127a_0a_3^5a_6^6 + 249a_0a_3^4a_4a_5a_6^5 - 132a_0a_3^4a_5^3a_6^4 \\
& + 259a_0a_3^3a_4^3a_6^5 - 105a_0a_3^3a_4^2a_5^2a_6^4 - 101a_0a_3^3a_4a_5^4a_6^3 - 3a_0a_3^3a_5^6a_6^2 \\
& - 471a_0a_3^2a_4^4a_5a_6^4 + 371a_0a_3^2a_4^3a_5^3a_6^3 - 3a_0a_3^2a_4^2a_5^5a_6^2 + 30a_0a_3a_4^6a_6^4 \\
& + 234a_0a_3a_4^5a_5^2a_6^3 + 69a_0a_3a_4^4a_5^4a_6^2 - 14a_0a_4^6a_5^3a_6^2 + 756a_1^4a_5a_6^7 \\
& + 50a_1^3a_2a_4a_6^7 + 265a_1^3a_2a_5^2a_6^6 + 158a_1^3a_3^2a_6^7 - 270a_1^3a_3a_4a_5a_6^6 \\
& - 148a_1^3a_3a_5^3a_6^5 + 27a_1^3a_4^3a_6^6 + 459a_1^3a_4^2a_5^2a_6^5 - 245a_1^3a_4a_5^4a_6^4 \\
& - 8a_1^3a_5^6a_6^3 + 566a_1^2a_2^2a_3a_6^7 + 440a_1^2a_2^2a_4a_5a_6^6 - 571a_1^2a_2^2a_5^3a_6^5 \\
& - 183a_1^2a_2a_3^2a_5a_6^6 - 286a_1^2a_2a_3a_4^2a_6^6 - 999a_1^2a_2a_3a_4a_5^2a_6^5 + 8a_1^2a_2a_3a_5^4a_6^4 \\
& + 131a_1^2a_2a_3^2a_5a_6^5 - 1175a_1^2a_2a_4^2a_5^3a_6^4 + 197a_1^2a_2a_4a_5^5a_6^3 + 4a_1^2a_2a_5^7a_6^2 \\
& - 245a_1^2a_3^3a_4a_6^6 - 360a_1^2a_3^2a_5^2a_6^5 + 132a_1^2a_3^2a_4^2a_5a_6^5 - 230a_1^2a_3^2a_4a_5^3a_6^4 \\
& - 48a_1^2a_3^2a_5^5a_6^3 - 35a_1^2a_3a_4^4a_6^5 - 63a_1^2a_3a_4^3a_5^2a_6^4 + 39a_1^2a_3a_4^2a_5^4a_6^3 \\
& - 64a_1^2a_4^5a_5a_6^4 - 62a_1^2a_4^4a_5^3a_6^3 + 32a_1^2a_4^3a_5^5a_6^2 + 26a_1a_2^4a_6^7 \\
& - 446a_1a_2^3a_3a_5a_6^6 + 102a_1a_2^3a_4^2a_6^6 - 622a_1a_2^3a_4a_5^2a_6^5 + 333a_1a_2^3a_5^4a_6^4 \\
& - 710a_1a_2^2a_3^2a_4a_6^6 + 685a_1a_2^2a_3^2a_5^2a_6^5 + 424a_1a_2^2a_3a_4^2a_5a_6^5 + 1018a_1a_2^2a_3a_4a_5^3a_6^4 \\
& - 32a_1a_2^2a_3a_5^5a_6^3 - 101a_1a_2^2a_4^4a_6^5 + 307a_1a_2^2a_4^3a_5^2a_6^4 + 191a_1a_2^2a_4^2a_5^4a_6^3 \\
& - 33a_1a_2^2a_4a_5^6a_6^2 + 11a_1a_2a_3^4a_6^6 + 284a_1a_2a_3^3a_4a_5a_6^5 - 60a_1a_2a_3^3a_5^3a_6^4 \\
& + 249a_1a_2a_3^2a_4^3a_6^5 - 331a_1a_2a_3^2a_4^2a_5^2a_6^4 + 13a_1a_2a_3^2a_4a_5^4a_6^3 - a_1a_2a_3^2a_5^6a_6^2 \\
& + 60a_1a_2a_3a_4^4a_5a_6^4 - 186a_1a_2a_3a_4^3a_5^3a_6^3 + 44a_1a_2a_3a_4^2a_5^5a_6^2 + 34a_1a_2a_4^6a_6^4 \\
& - 22a_1a_2a_4^5a_5^2a_6^3 - 41a_1a_2a_4^4a_5^4a_6^2 + 26a_1a_3^5a_5a_6^5 + 119a_1a_3^4a_4^2a_6^5 \\
& + 180a_1a_3^4a_4a_5^2a_6^4 + 12a_1a_3^4a_5^4a_6^3 - 76a_1a_3^3a_4^3a_5a_6^4 - 54a_1a_3^3a_4^2a_5^3a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 15a_1a_3^2a_4^5a_6^4 - 9a_1a_3^2a_4^4a_5^2a_6^3 - 29a_1a_3^2a_4^3a_5^4a_6^2 - 50a_1a_3a_4^6a_5a_6^3 \\
& + 22a_1a_3a_4^5a_5^3a_6^2 - 4a_1a_4^8a_6^3 - 10a_1a_4^7a_5^2a_6^2 - 66a_2^5a_5a_6^6 \\
& - 130a_2^4a_3a_4a_6^6 + 228a_2^4a_3a_5^2a_6^5 + 128a_2^4a_4^2a_5a_6^5 - 222a_2^4a_4a_5^3a_6^4 \\
& - 14a_2^4a_5^5a_6^3 + 300a_2^3a_3^3a_6^6 - 88a_2^3a_3^2a_4a_5a_6^5 - 72a_2^3a_3^2a_5^3a_6^4 \\
& + 100a_2^3a_3a_4^3a_6^5 + 26a_2^3a_3a_4^2a_5^2a_6^4 - 40a_2^3a_3a_4a_5^4a_6^3 - 2a_2^3a_3a_5^6a_6^2 \\
& + 28a_2^3a_4^4a_5a_6^4 - 173a_2^3a_4^3a_5^3a_6^3 + 11a_2^3a_4^2a_5^5a_6^2 - 89a_2^2a_3^4a_5a_6^5 \\
& - 44a_2^2a_3^3a_4^2a_5^6 - 144a_2^2a_3^3a_4a_5^2a_6^4 - 20a_2^2a_3^3a_5^4a_6^3 - 302a_2^2a_3^2a_4^3a_5a_6^4 \\
& + 165a_2^2a_3^2a_4^2a_5^3a_6^3 + 7a_2^2a_3^2a_4a_5^5a_6^2 - 36a_2^2a_3a_4^5a_6^4 + 127a_2^2a_3a_4^4a_5^2a_6^3 \\
& - 11a_2^2a_3a_4^3a_5^4a_6^2 - 31a_2^2a_4^6a_5a_6^3 + 15a_2^2a_4^5a_5^3a_6^2 - 71a_2a_3^5a_4a_5^6 \\
& - 6a_2a_3^5a_5^2a_6^4 + 180a_2a_3^4a_4^2a_5a_6^4 + 14a_2a_3^4a_4a_5^3a_6^3 - 2a_2a_3^3a_4^4a_6^4 \\
& - 105a_2a_3^3a_4^3a_5^2a_6^3 - 9a_2a_3^3a_4^2a_5^4a_6^2 + 39a_2a_3^2a_4^5a_5a_6^3 + 5a_2a_3^2a_4^4a_5^3a_6^2 \\
& + 5a_2a_3a_4^7a_6^3 - 17a_2a_3a_4^6a_5^2a_6^2 + 4a_2a_4^8a_5a_6^2 + 12a_3^7a_6^5 \\
& - 24a_3^6a_4a_5a_6^4 - 24a_3^5a_4^3a_6^4 + 6a_3^5a_4^2a_5^2a_6^3 + 36a_3^4a_4^4a_5a_6^3 \\
& + 6a_3^4a_4^3a_5^3a_6^2 - 5a_3^3a_4^6a_6^3 + 3a_3^3a_4^5a_5^2a_6^2 + 12a_3^2a_4^7a_5a_6^2 \\
& - 1982a_0^3a_2a_6^7 + 4244a_0^3a_3a_5a_6^6 + 3812a_0^3a_4^2a_6^6 - 1122a_0^3a_4a_5^2a_6^5 \\
& - 1245a_0^3a_5^4a_6^4 + 210a_0^2a_1^2a_6^7 - 3998a_0^2a_1a_2a_5a_6^6 - 918a_0^2a_1a_3a_4a_6^6 \\
& + 3427a_0^2a_1a_3a_5^2a_6^5 - 397a_0^2a_1a_4^2a_5a_6^5 + 3205a_0^2a_1a_4a_5^3a_6^4 - 117a_0^2a_1a_5^5a_6^3 \\
& - 10310a_0^2a_2^2a_4a_6^6 - 2661a_0^2a_2^2a_5^2a_6^5 + 1664a_0^2a_2a_3^2a_6^6 + 9032a_0^2a_2a_3a_4a_5a_6^5 \\
& + 1931a_0^2a_2a_3a_5^3a_6^4 + 3440a_0^2a_2a_4^3a_6^5 - 1389a_0^2a_2a_4^2a_5^2a_6^4 - 229a_0^2a_2a_4a_5^4a_6^3 \\
& + 64a_0^2a_2a_5^6a_6^2 - 1695a_0^2a_3^3a_5a_6^5 - 1512a_0^2a_3^2a_4^2a_6^5 - 2709a_0^2a_3^2a_4a_5^2a_6^4 \\
& + 138a_0^2a_3^2a_5^4a_6^3 - 925a_0^2a_3a_4^3a_5a_6^4 - 432a_0^2a_3a_4^2a_5^3a_6^3 + 168a_0^2a_3a_4a_5^5a_6^2 \\
& + 3a_0^2a_3a_5^7a_6 - 602a_0^2a_4^5a_6^4 + 2074a_0^2a_4^4a_5^2a_6^3 - 216a_0^2a_4^3a_5^4a_6^2 \\
& + 18a_0^2a_4^2a_5^6a_6 + 764a_0a_1^3a_5a_6^6 + 5260a_0a_1^2a_2a_4a_6^6 + 3403a_0a_1^2a_2a_5^2a_6^5 \\
& - 788a_0a_1^2a_3^2a_6^6 - 1000a_0a_1^2a_3a_4a_5a_6^5 + 9a_0a_1^2a_3a_5^3a_6^4 - 5a_0a_1^2a_4^3a_6^5 \\
& + 929a_0a_1^2a_4^2a_5^4a_6^4 - 243a_0a_1^2a_4a_5^4a_6^3 + 11a_0a_1^2a_5^6a_6^2 + 3826a_0a_1a_2^2a_3a_6^6 \\
& - 1034a_0a_1a_2^2a_4a_5a_6^5 - 1428a_0a_1a_2^2a_5^3a_6^4 - 968a_0a_1a_2a_3^2a_5a_6^5 - 5294a_0a_1a_2a_3a_4^2a_6^5 \\
& - 2054a_0a_1a_2a_3a_4a_5^2a_6^4 - 627a_0a_1a_2a_3a_5^4a_6^3 + 786a_0a_1a_2a_4^3a_5a_6^4 - 192a_0a_1a_2a_4^2a_5^3a_6^3 \\
& - 68a_0a_1a_2a_4a_5^5a_6^2 + a_0a_1a_2a_5^7a_6 + 1170a_0a_1a_3^3a_4a_6^5 - 447a_0a_1a_3^3a_5^2a_6^4 \\
& - 171a_0a_1a_3^2a_4^2a_5a_6^4 - 1111a_0a_1a_3^2a_4a_5^3a_6^3 - 128a_0a_1a_3^2a_5^5a_6^2 + 147a_0a_1a_3a_4^4a_6^4 \\
& - 274a_0a_1a_3a_4^3a_5^2a_6^3 - 140a_0a_1a_3a_4^2a_5^4a_6^2 - 21a_0a_1a_3a_4a_5^6a_6 + 722a_0a_1a_4^5a_5a_6^3 \\
& - 257a_0a_1a_4^4a_5^3a_6^2 - 18a_0a_1a_4^3a_5^5a_6 + 1166a_0a_2^4a_6^6 + 54a_0a_2^3a_3a_5a_6^5 \\
& - 650a_0a_2^3a_4^2a_5^5a_6 - 306a_0a_2^3a_4a_5^2a_6^4 + 132a_0a_2^3a_5^4a_6^3 - 454a_0a_2^2a_3^2a_4a_6^5 \\
& + 317a_0a_2^2a_3^2a_5^4a_6^4 - 201a_0a_2^2a_3a_4^2a_5a_6^4 + 2149a_0a_2^2a_3a_4a_5^3a_6^3 + 67a_0a_2^2a_3a_5^5a_6^2 \\
& + 530a_0a_2^2a_4^4a_6^4 - 877a_0a_2^2a_4^3a_5^2a_6^3 + 294a_0a_2^2a_4^2a_5^4a_6^2 - 6a_0a_2^2a_4a_5^6a_6 \\
& - 483a_0a_2a_3^4a_6^5 + 88a_0a_2a_3^3a_4a_5a_6^4 - 518a_0a_2a_3^3a_5^3a_6^3 + 1498a_0a_2a_3^2a_4^3a_6^4
\end{aligned}$$

$$\begin{aligned}
& -251a_0a_2a_3^2a_4^2a_5^2a_6^3 - 82a_0a_2a_3^2a_4a_5^4a_6^2 - 5a_0a_2a_3^2a_5^6a_6 - 2113a_0a_2a_3a_4^4a_5a_6^3 \\
& + 570a_0a_2a_3a_4^3a_5^3a_6^2 + 22a_0a_2a_3a_4^2a_5^5a_6 - 154a_0a_2a_4^6a_6^3 + 4a_0a_2a_4^5a_5^2a_6^2 \\
& + 49a_0a_2a_4^4a_5^4a_6 + 136a_0a_3^5a_5a_6^4 - 98a_0a_3^4a_4^2a_6^4 + 312a_0a_3^4a_4a_5^2a_6^3 \\
& + 30a_0a_3^4a_5^4a_6^2 + 40a_0a_3^3a_4^3a_5a_6^3 + 6a_0a_3^3a_4^2a_5^3a_6^2 + 6a_0a_3^3a_4a_5^5a_6 \\
& + 12a_0a_3^2a_4^5a_6^3 - 369a_0a_3^2a_4^4a_5^2a_6^2 - 10a_0a_3^2a_4^3a_5^4a_6 - 55a_0a_3a_4^6a_5a_6^2 \\
& - 28a_0a_3a_4^5a_5^3a_6 + 7a_0a_4^8a_6^2 + 16a_0a_4^7a_5^2a_6 - 796a_1^4a_4a_6^6 \\
& - 362a_1^4a_5^2a_6^5 - 952a_1^3a_2a_3a_6^6 - 342a_1^3a_2a_4a_5a_6^5 + 404a_1^3a_2a_5^3a_6^4 \\
& + 142a_1^3a_3a_5a_6^5 + 896a_1^3a_3a_4^2a_6^5 + 631a_1^3a_3a_4a_5^2a_6^4 + 92a_1^3a_3a_5^4a_6^3 \\
& - 535a_1^3a_4^3a_5a_6^4 + 501a_1^3a_4^2a_5^3a_6^3 + 41a_1^3a_4a_5^5a_6^2 + 4a_1^3a_5^7a_6 \\
& - 346a_1^2a_2^3a_6^6 + 399a_1^2a_2^2a_3a_5a_6^5 - 244a_1^2a_2^2a_4^2a_6^5 + 1960a_1^2a_2^2a_4a_5^2a_6^4 \\
& - 195a_1^2a_2^2a_5^4a_6^3 + 751a_1^2a_2a_3^2a_4a_6^5 + 125a_1^2a_2a_3^2a_5^2a_6^4 + 979a_1^2a_2a_3a_4^2a_5a_6^4 \\
& - 457a_1^2a_2a_3a_4a_5^3a_6^3 - 47a_1^2a_2a_3a_5^5a_6^2 + 131a_1^2a_2a_4^4a_6^4 + 485a_1^2a_2a_4^3a_5^2a_6^3 \\
& - 338a_1^2a_2a_4^2a_5^4a_6^2 - 9a_1^2a_2a_4a_5^6a_6 + 171a_1^2a_3^4a_6^5 + 306a_1^2a_3^3a_4a_5a_6^4 \\
& + 192a_1^2a_3^3a_5^3a_6^3 - 448a_1^2a_3^2a_4^3a_6^4 - 183a_1^2a_3^2a_4^2a_5^2a_6^3 - 11a_1^2a_3^2a_4a_5^4a_6^2 \\
& - a_1^2a_3^2a_5^6a_6 + 191a_1^2a_3a_4^4a_5a_6^3 - 10a_1^2a_3a_4^3a_5^3a_6^2 + 2a_1^2a_3a_4^2a_5^5a_6 \\
& - 9a_1^2a_4^6a_6^3 + 15a_1^2a_4^5a_5^2a_6^2 - 3a_1^2a_4^4a_5^4a_6 + 241a_1^2a_4^4a_5a_6^5 \\
& + 852a_1a_2^3a_3a_4a_6^5 - 1043a_1a_2^3a_3a_5^2a_6^4 - 123a_1a_2^3a_4^2a_5a_6^4 - 579a_1a_2^3a_4a_5^3a_6^3 \\
& + 59a_1a_2^3a_5^5a_6^2 - 318a_1a_2^2a_3^3a_6^5 - 1074a_1a_2^2a_3^2a_4a_5a_6^4 + 4a_1a_2^2a_3^2a_5^3a_6^3 \\
& - 618a_1a_2^2a_3a_4^3a_6^4 - 465a_1a_2^2a_3a_4^2a_5^2a_6^3 + 74a_1a_2^2a_3a_4a_5^4a_6^2 + 4a_1a_2^2a_3a_5^6a_6 \\
& - 25a_1a_2^2a_4^4a_5a_6^3 + 65a_1a_2^2a_4^3a_5^3a_6^2 + 29a_1a_2^2a_4^2a_5^5a_6 - 4a_1a_2^2a_4a_5^4a_6^4 \\
& - 22a_1a_2^2a_3^3a_4^2a_6^4 + 106a_1a_2^2a_3^3a_4a_5^2a_6^3 + 14a_1a_2^2a_3^3a_5^4a_6^2 + 418a_1a_2^2a_3^2a_4^3a_5a_6^3 \\
& - 60a_1a_2^2a_3^2a_4^2a_5^3a_6^2 - 2a_1a_2^2a_3^2a_4a_5^5a_6 + 51a_1a_2^2a_3a_4^5a_6^3 + 92a_1a_2^2a_3a_4^4a_5^2a_6^2 \\
& - 26a_1a_2^2a_3a_4^3a_5^4a_6 + 31a_1a_2^2a_4^6a_5a_6^2 + 8a_1a_2^2a_4^5a_5^3a_6 - 160a_1a_3^5a_4a_6^4 \\
& - 48a_1a_3^5a_5^2a_6^3 - 12a_1a_3^4a_4^2a_5a_6^3 + 87a_1a_3^3a_4^4a_6^3 + 98a_1a_3^3a_4^3a_5^2a_6^2 \\
& - 75a_1a_3^3a_4^5a_5a_6^2 + 4a_1a_3^2a_4^4a_5^3a_6 + 14a_1a_3a_4^7a_6^2 - 4a_1a_3a_4^6a_5^2a_6 \\
& - a_1a_4^8a_5a_6 + 44a_2^5a_4a_6^5 - 3a_2^5a_5^2a_6^4 - 339a_2^4a_3^2a_6^5 \\
& + 155a_2^4a_3a_4a_5a_6^4 + 123a_2^4a_3a_5^3a_6^3 - 80a_2^4a_4^3a_6^4 + 423a_2^4a_4^2a_5^2a_6^3 \\
& - 35a_2^4a_4a_5^4a_6^2 - 2a_2^4a_5^6a_6 + 142a_2^3a_3^3a_5a_6^4 + 272a_2^3a_3^2a_4^2a_6^4 \\
& - 172a_2^3a_3^2a_4a_5^2a_6^3 + 10a_2^3a_3^2a_5^4a_6^2 - 130a_2^3a_3a_4^3a_5a_6^3 + 57a_2^3a_3a_4^2a_5^3a_6^2 \\
& + 7a_2^3a_3a_4a_5^5a_6 + 14a_2^3a_4^5a_6^3 + 43a_2^3a_4^4a_5^2a_6^2 - 16a_2^3a_4^3a_5^4a_6 \\
& + 42a_2^2a_3^4a_4a_6^4 + 86a_2^2a_3^4a_5^2a_6^3 + 86a_2^2a_3^3a_4^2a_5a_6^3 - 26a_2^2a_3^3a_4a_5^3a_6^2 \\
& + 66a_2^2a_3^2a_4^4a_6^3 - 137a_2^2a_3^2a_4^3a_5^2a_6^2 - 13a_2^2a_3^2a_4^2a_5^4a_6 - 54a_2^2a_3a_4^5a_5a_6^2 \\
& + 20a_2^2a_3a_4^4a_5^3a_6 + 5a_2^2a_4^7a_6^2 - 9a_2^2a_4^6a_5^2a_6 + 40a_2^2a_3^6a_6^4 \\
& - 72a_2^2a_3^5a_4a_5a_6^3 - 82a_2^2a_3^4a_4^3a_6^3 + 24a_2^2a_3^4a_4^2a_5^2a_6^2 + 67a_2^2a_3^3a_4^4a_5a_6^2 \\
& + 10a_2^2a_3^3a_4^3a_5^3a_6 - 17a_2^2a_3^2a_4^6a_6^2 - 3a_2^2a_3^2a_4^5a_5^2a_6 + 9a_2^2a_3a_4^7a_5a_6
\end{aligned}$$

$$\begin{aligned}
& -a_2a_4^9a_6 + 24a_3^6a_4^2a_6^3 - 24a_3^5a_4^3a_5a_6^2 - 9a_3^4a_4^5a_6^2 \\
& - 3a_3^4a_4^4a_5^2a_6 - 4a_3^3a_4^6a_5a_6 - 3a_3^2a_4^8a_6 + 1336a_0^3a_1a_6^6 \\
& + 1248a_0^3a_2a_5a_6^5 - 6444a_0^3a_3a_4a_6^5 + 2513a_0^3a_3a_5^2a_6^4 - 581a_0^3a_4^2a_5a_6^4 \\
& + 3423a_0^3a_4a_5^3a_6^3 - 67a_0^3a_5^2a_6^2 - 64a_0^2a_1^2a_5a_6^5 + 9050a_0^2a_1a_2a_4a_6^5 \\
& - 1851a_0^2a_1a_2a_5^2a_6^4 + 162a_0^2a_1a_3^2a_6^5 - 4426a_0^2a_1a_3a_4a_5a_6^4 - 4a_0^2a_1a_3a_5^3a_6^3 \\
& - 1691a_0^2a_1a_4^3a_6^4 - 1911a_0^2a_1a_4^2a_5^2a_6^3 + 105a_0^2a_1a_4a_5^4a_6^2 + 46a_0^2a_1a_5^6a_6 \\
& + 3452a_0^2a_2^2a_3a_6^5 + 10434a_0^2a_2^2a_4a_5a_6^4 + 567a_0^2a_2^2a_5^3a_6^3 - 5335a_0^2a_2a_3^2a_5a_6^4 \\
& - 9024a_0^2a_2a_3a_4^2a_6^4 - 4929a_0^2a_2a_3a_4a_5^2a_6^3 - 461a_0^2a_2a_3a_5^4a_6^2 + 715a_0^2a_2a_4^3a_5a_6^3 \\
& + 518a_0^2a_2a_4^2a_5^3a_6^2 - 122a_0^2a_2a_4a_5^5a_6 - 6a_0^2a_2a_5^7 + 2973a_0^2a_3^3a_4a_6^4 \\
& + 343a_0^2a_3^3a_5^2a_6^3 + 2229a_0^2a_3^2a_4^2a_5a_6^3 - 483a_0^2a_3^2a_4a_5^3a_6^2 - 36a_0^2a_3^2a_5^5a_6 \\
& + 1538a_0^2a_3a_4^4a_6^3 + 118a_0^2a_3a_4^3a_5^2a_6^2 - 42a_0^2a_3a_4^2a_5^4a_6 - 6a_0^2a_3a_4a_6^5 \\
& - 1486a_0^2a_4^5a_5a_6^2 + 15a_0^2a_4^4a_5^3a_6 - 26a_0^2a_4^3a_5^5 - 1628a_0a_1^3a_4a_6^5 \\
& - 668a_0a_1^3a_5^2a_6^4 - 5044a_0a_1^2a_2a_3a_6^5 - 8680a_0a_1^2a_2a_4a_5a_6^4 + 625a_0a_1^2a_2a_5^3a_6^3 \\
& + 712a_0a_1^2a_2^2a_5a_6^4 + 2930a_0a_1^2a_3a_4^2a_6^4 + 496a_0a_1^2a_3a_4a_5^2a_6^3 - 48a_0a_1^2a_3a_5^4a_6^2 \\
& - 777a_0a_1^2a_4^3a_5a_6^3 + 495a_0a_1^2a_4^2a_5^3a_6^2 + 69a_0a_1^2a_4a_5^5a_6 + 8a_0a_1^2a_5^7 \\
& - 3282a_0a_1a_2^3a_6^5 + 3924a_0a_1a_2^2a_3a_5a_6^4 + 4436a_0a_1a_2^2a_4^2a_6^4 + 2350a_0a_1a_2^2a_4a_5^2a_6^3 \\
& - 79a_0a_1a_2^2a_5^4a_6^2 + 3574a_0a_1a_2a_3^2a_4a_6^4 + 1587a_0a_1a_2a_3^2a_5^2a_6^3 + 1906a_0a_1a_2a_3a_4^2a_5a_6^3 \\
& + 530a_0a_1a_2a_3a_4a_5^3a_6^2 + 12a_0a_1a_2a_3a_5^5a_6 - 1809a_0a_1a_2a_4^4a_6^3 + 252a_0a_1a_2a_4^3a_5^2a_6^2 \\
& - 118a_0a_1a_2a_4^2a_5^4a_6 - 18a_0a_1a_2a_4a_5^6 - 462a_0a_1a_3^3a_6^4 + 1196a_0a_1a_3^3a_4a_5a_6^3 \\
& + 436a_0a_1a_3^3a_5^3a_6^2 - 1103a_0a_1a_3^2a_4^3a_6^3 + 237a_0a_1a_3^2a_4^2a_5^2a_6^2 + 164a_0a_1a_3^2a_4a_5^4a_6 \\
& - 2a_0a_1a_3^2a_5^6 + 902a_0a_1a_3a_4^4a_5a_6^2 + 152a_0a_1a_3a_4^3a_5^3a_6 + 22a_0a_1a_3a_4^2a_5^5 \\
& + 131a_0a_1a_4^5a_5^2a_6 + 18a_0a_1a_4^4a_5^4 + 309a_0a_2^4a_5a_6^4 - 2460a_0a_2^3a_3a_4a_6^4 \\
& - 1165a_0a_2^3a_3a_5^2a_6^3 + 410a_0a_2^3a_4^2a_5a_6^3 - 104a_0a_2^3a_4a_5^3a_6^2 + 4a_0a_2^3a_5^5a_6 \\
& + 486a_0a_2^2a_3^3a_6^4 - 1834a_0a_2^2a_3^2a_4a_5a_6^3 - 286a_0a_2^2a_3^2a_5^3a_6^2 + 1243a_0a_2^2a_3a_4^3a_6^3 \\
& - 2306a_0a_2^2a_3a_4^2a_5^2a_6^2 + 29a_0a_2^2a_3a_4a_5^4a_6 + 442a_0a_2^2a_4^4a_5a_6^2 - 111a_0a_2^2a_4^3a_5^3a_6 \\
& + 15a_0a_2^2a_4^2a_5^5 + 600a_0a_2a_3^4a_5a_6^3 - 398a_0a_2a_3^3a_4^2a_6^3 + 404a_0a_2a_3^3a_4a_5^2a_6^2 \\
& + 48a_0a_2a_3^3a_5^4a_6 + 454a_0a_2a_3^2a_4^3a_5a_6^2 - 182a_0a_2a_3^2a_4^2a_5^3a_6 + 6a_0a_2a_3^2a_4a_5^5 \\
& + 474a_0a_2a_3a_4^5a_6^2 - 222a_0a_2a_3a_4^4a_5^2a_6 - 20a_0a_2a_3a_4^3a_5^4 - 74a_0a_2a_4^6a_5a_6 \\
& - 8a_0a_2a_4^5a_5^3 - 272a_0a_3^5a_4a_6^3 - 96a_0a_3^5a_5^2a_6^2 - 120a_0a_3^4a_4^2a_5a_6^2 \\
& - 48a_0a_3^4a_4a_5^3a_6 + 87a_0a_3^3a_4^4a_6^2 + 88a_0a_3^3a_4^3a_5^2a_6 - 6a_0a_3^3a_4^2a_5^4 \\
& + 105a_0a_3^2a_4^5a_5a_6 + 2a_0a_3^2a_4^4a_5^3 - 11a_0a_3a_4^7a_6 + 2a_0a_3a_4^6a_5^2 \\
& - 3a_0a_4^8a_5 + 944a_1^4a_3a_6^5 + 1188a_1^4a_4a_5a_6^4 + 50a_1^4a_5^3a_6^3 \\
& + 834a_1^3a_2^2a_6^5 - 948a_1^3a_2a_3a_5a_6^4 - 374a_1^3a_2a_4^2a_6^4 - 1385a_1^3a_2a_4a_5^2a_6^3 \\
& - 26a_1^3a_2a_5^4a_6^2 - 1272a_1^3a_3^2a_4a_6^4 - 362a_1^3a_3^2a_5^2a_6^3 - 610a_1^3a_3a_4^2a_5a_6^3 \\
& - 342a_1^3a_3a_4a_5^3a_6^2 - 50a_1^3a_3a_5^5a_6 + 235a_1^3a_4^4a_6^3 - 355a_1^3a_4^3a_5^2a_6^2
\end{aligned}$$

$$\begin{aligned}
& -11a_1^3a_4^2a_5^4a_6 - 445a_1^2a_2^3a_5a_6^4 - 759a_1^2a_2^2a_3a_4a_6^4 + 618a_1^2a_2^2a_3a_5^2a_6^3 \\
& - 1487a_1^2a_2^2a_4^2a_5a_6^3 + 725a_1^2a_2^2a_4a_5^3a_6^2 + 25a_1^2a_2^2a_5^5a_6 - 24a_1^2a_2a_3^3a_6^4 \\
& + 362a_1^2a_2a_3^2a_4a_5a_6^3 + 188a_1^2a_2a_3^2a_5^3a_6^2 + 139a_1^2a_2a_3a_4^3a_6^3 + 699a_1^2a_2a_3a_4^2a_5^2a_6^2 \\
& + 89a_1^2a_2a_3a_4a_5^4a_6 - 57a_1^2a_2a_4^4a_5a_6^2 + 118a_1^2a_2a_4^3a_5^3a_6 - 256a_1^2a_3^4a_5a_6^3 \\
& + 398a_1^2a_3^3a_4^2a_6^3 + 88a_1^2a_3^3a_4a_5^2a_6^2 + 12a_1^2a_3^3a_5^4a_6 - 34a_1^2a_3^2a_4^3a_5a_6^2 \\
& - 16a_1^2a_3^2a_4^2a_5^3a_6 - 135a_1^2a_3a_4^5a_6^2 - 20a_1^2a_3a_4^4a_5^2a_6 + 6a_1^2a_4^6a_5a_6 \\
& - 6a_1^2a_4^5a_6^3 - 293a_1a_2^4a_4a_6^4 + 321a_1a_2^4a_5^2a_6^3 + 666a_1a_2^3a_3^2a_6^4 \\
& + 1236a_1a_2^3a_3a_4a_5a_6^3 - 252a_1a_2^3a_3a_5^3a_6^2 + 361a_1a_2^3a_4^3a_6^3 - 78a_1a_2^3a_4^2a_5^2a_6^2 \\
& - 123a_1a_2^3a_4a_5^4a_6 + 44a_1a_2^2a_3^3a_5a_6^3 - 196a_1a_2^2a_3^2a_4^2a_6^3 - 50a_1a_2^2a_3^2a_4a_5^2a_6^2 \\
& - 44a_1a_2^2a_3^2a_5^4a_6 + 108a_1a_2^2a_3a_4^3a_5a_6^2 + 26a_1a_2^2a_3a_4^2a_5^3a_6 - 2a_1a_2^2a_3a_4a_5^5 \\
& - 100a_1a_2^2a_4^5a_6^2 - 29a_1a_2^2a_4^4a_5^2a_6 - 5a_1a_2^2a_4^3a_5^4 - 8a_1a_2a_3^4a_4a_6^3 \\
& - 64a_1a_2a_3^4a_5^2a_6^2 - 128a_1a_2a_3^3a_4^2a_5a_6^2 + 16a_1a_2a_3^3a_4a_5^3a_6 - 87a_1a_2a_3^2a_4^4a_6^2 \\
& - 2a_1a_2a_3^2a_4^3a_5^2a_6 + 2a_1a_2a_3^2a_4^2a_5^4 - 34a_1a_2a_3a_4^5a_5a_6 - a_1a_2a_4^7a_6 \\
& + 4a_1a_2a_4^6a_5^2 + 64a_1a_3^6a_6^3 - 72a_1a_3^4a_4^3a_6^2 + 2a_1a_3^3a_4^4a_5a_6 \\
& + 35a_1a_3^2a_4^6a_6 + 6a_1a_3^2a_4^5a_5^2 - 2a_1a_3a_4^7a_5 + a_1a_4^9 \\
& + 144a_2^5a_3a_6^4 - 171a_2^5a_4a_5a_6^3 + 13a_2^5a_5^3a_6^2 - 148a_2^4a_3^2a_5a_6^3 \\
& - 291a_2^4a_3a_4^2a_6^3 + 3a_2^4a_3a_4a_5^2a_6^2 + 16a_2^4a_3a_5^4a_6 - 180a_2^4a_4^3a_5a_6^2 \\
& + 75a_2^4a_4^2a_5^3a_6 + a_2^4a_4a_5^5 + 98a_2^3a_3^3a_4a_6^3 - 8a_2^3a_3^3a_5^2a_6^2 \\
& + 154a_2^3a_3^2a_4^2a_5a_6^2 - 38a_2^3a_3^2a_4a_5^3a_6 + 98a_2^3a_3a_4^4a_6^2 - 85a_2^3a_3a_4^3a_5^2a_6 \\
& - 3a_2^3a_3a_4^2a_5^4 + 12a_2^3a_4^5a_5a_6 + 4a_2^3a_4^4a_5^3 - 88a_2^2a_3^5a_6^3 \\
& - 8a_2^2a_3^4a_4a_5a_6^2 - 34a_2^2a_3^3a_4^3a_6^2 + 62a_2^2a_3^3a_4^2a_5^2a_6 + 49a_2^2a_3^2a_4^4a_5a_6 \\
& + 4a_2^2a_3^2a_4^3a_5^3 + 4a_2^2a_3a_4^6a_6 - 4a_2^2a_3a_4^5a_5^2 + a_2^2a_4^7a_5 \\
& + 48a_2a_3^5a_4^2a_6^2 - 40a_2a_3^4a_4^3a_5a_6 - 11a_2a_3^3a_4^5a_6 - 2a_2a_3^3a_4^4a_5^2 \\
& - 2a_2a_3^2a_4^6a_5 - a_2a_3a_4^8 + 12a_3^5a_4^4a_6 + 2a_3^3a_4^7 \\
& + 633a_0^4a_6^5 + 5148a_0^3a_1a_5a_6^4 - 6216a_0^3a_2a_4a_6^4 - 4149a_0^3a_2a_5^2a_6^3 \\
& + 1812a_0^3a_3^2a_6^4 - 4576a_0^3a_3a_4a_5a_6^3 - 19a_0^3a_3a_5^3a_6^2 + 3045a_0^3a_4^3a_6^3 \\
& - 3521a_0^3a_4^2a_5^2a_6^2 + 139a_0^3a_4a_5^4a_6 + 16a_0^3a_5^6 + 412a_0^2a_1^2a_4a_6^4 \\
& + 786a_0^2a_1^2a_5^2a_6^3 - 80a_0^2a_1a_2a_3a_6^4 - 1218a_0^2a_1a_2a_4a_5a_6^3 + 40a_0^2a_1a_2a_5^3a_6^2 \\
& + 430a_0^2a_1a_3^2a_5a_6^3 + 1484a_0^2a_1a_3a_4^2a_6^3 + 633a_0^2a_1a_3a_4a_5^2a_6^2 - 458a_0^2a_1a_3a_5^4a_6 \\
& + 425a_0^2a_1a_4^3a_5a_6^2 + 248a_0^2a_1a_4^2a_5^3a_6 + 34a_0^2a_1a_4a_5^5 - 2832a_0^2a_2^3a_6^4 \\
& + 275a_0^2a_2^2a_3a_5a_6^3 - 8960a_0^2a_2^2a_4^2a_6^3 - 382a_0^2a_2^2a_4a_5^2a_6^2 + 44a_0^2a_2^2a_5^4a_6 \\
& + 8413a_0^2a_2a_3^2a_4a_6^3 + 1740a_0^2a_2a_3^2a_5^2a_6^2 + 3297a_0^2a_2a_3a_4^2a_5a_6^2 + 871a_0^2a_2a_3a_4a_5^3a_6 \\
& + 74a_0^2a_2a_3a_5^5 + 2332a_0^2a_2a_4^4a_6^2 - 676a_0^2a_2a_4^3a_5^2a_6 - 14a_0^2a_2a_4^2a_5^4 \\
& - 924a_0^2a_3^4a_6^3 + 12a_0^2a_3^3a_4a_5a_6^2 + 144a_0^2a_3^3a_5^3a_6 - 2805a_0^2a_3^2a_4^3a_6^2 \\
& - 225a_0^2a_3^2a_4^2a_5^2a_6 + 60a_0^2a_3^2a_4a_5^4 + 1151a_0^2a_3a_4^4a_5a_6 + 204a_0^2a_3a_4^3a_5^3
\end{aligned}$$

$$\begin{aligned}
& + 135a_0^2a_4^6a_6 - 102a_0^2a_4^5a_5^2 + 200a_0a_1^3a_3a_6^4 + 3268a_0a_1^3a_4a_5a_6^3 \\
& + 172a_0a_1^3a_5^3a_6^2 + 5784a_0a_1^2a_2^2a_6^4 + 948a_0a_1^2a_2a_3a_5a_6^3 + 3406a_0a_1^2a_2a_4^2a_6^3 \\
& - 2782a_0a_1^2a_2a_4a_5^2a_6^2 - 92a_0a_1^2a_2a_5^4a_6 - 1884a_0a_1^2a_3^2a_4a_6^3 - 36a_0a_1^2a_3^2a_5^2a_6^2 \\
& - 1932a_0a_1^2a_3a_4^2a_5a_6^2 - 712a_0a_1^2a_3a_4a_5^3a_6 - 100a_0a_1^2a_3a_5^5 + 172a_0a_1^2a_4^4a_6^2 \\
& - 9a_0a_1^2a_4^3a_5^2a_6 + 2a_0a_1^2a_4^2a_5^4 - 2808a_0a_1a_2^3a_5a_6^3 - 4562a_0a_1a_2^2a_3a_4a_6^3 \\
& - 357a_0a_1a_2^2a_3a_5^2a_6^2 + 177a_0a_1a_2^2a_4^2a_5a_6^2 + 489a_0a_1a_2^2a_4a_5^3a_6 + 28a_0a_1a_2^2a_5^5 \\
& - 1956a_0a_1a_2a_3^3a_6^3 - 840a_0a_1a_2a_3^2a_4a_5a_6^2 - 168a_0a_1a_2a_3^2a_5^3a_6 - 972a_0a_1a_2a_3a_4^3a_6^2 \\
& + 853a_0a_1a_2a_3a_4^2a_5^2a_6 + 144a_0a_1a_2a_3a_4a_5^4 - 589a_0a_1a_2a_4^4a_5a_6 - 60a_0a_1a_2a_4^3a_5^3 \\
& - 464a_0a_1a_3^4a_5a_6^2 + 944a_0a_1a_3^3a_4^2a_6^2 - 256a_0a_1a_3^3a_4a_5^2a_6 + 24a_0a_1a_3^3a_5^4 \\
& - 832a_0a_1a_3^2a_4^3a_5a_6 - 188a_0a_1a_3^2a_4^2a_5^3 - 309a_0a_1a_3a_4^5a_6 - 56a_0a_1a_3a_4^4a_5^2 \\
& + 38a_0a_1a_4^6a_5 + 1383a_0a_2^4a_4a_6^3 + 61a_0a_2^4a_5^2a_6^2 + 1286a_0a_2^3a_3^2a_6^3 \\
& + 1680a_0a_2^3a_3a_4a_5a_6^2 - 10a_0a_2^3a_3a_5^3a_6 - 992a_0a_2^3a_4^3a_6^2 - 202a_0a_2^3a_4^2a_5^2a_6 \\
& - 42a_0a_2^3a_4a_5^4 + 376a_0a_2^2a_3^3a_5a_6^2 + 534a_0a_2^2a_3^2a_4^2a_6^2 + 112a_0a_2^2a_3^2a_4a_5^2a_6 \\
& - 6a_0a_2^2a_3^2a_5^4 + 936a_0a_2^2a_3a_4^3a_5a_6 - 56a_0a_2^2a_3a_4^2a_5^3 + 54a_0a_2^2a_4^5a_6 \\
& + 52a_0a_2^2a_4^4a_5^2 - 496a_0a_2a_3^4a_4a_6^2 - 144a_0a_2a_3^4a_5^2a_6 + 88a_0a_2a_3^3a_4^2a_5a_6 \\
& - 48a_0a_2a_3^3a_4a_5^3 - 99a_0a_2a_3^2a_4^4a_6 + 148a_0a_2a_3^2a_4^3a_5^2 - 4a_0a_2a_3a_4^5a_5 \\
& + 10a_0a_2a_4^7 + 96a_0a_3^6a_6^2 + 96a_0a_3^5a_4a_5a_6 + 48a_0a_3^4a_4^2a_5^2 \\
& - 32a_0a_3^3a_4^4a_5 - 2a_0a_3^2a_4^6 - 1128a_1^4a_2a_6^4 - 488a_1^4a_3a_5a_6^3 \\
& - 680a_1^4a_4^3a_6^3 + 364a_1^4a_4a_5^2a_6^2 + 27a_1^4a_5^4a_6 + 910a_1^3a_2^2a_5a_6^3 \\
& + 1988a_1^3a_2a_3a_4a_6^3 + 188a_1^3a_2a_3a_5^2a_6^2 + 1140a_1^3a_2a_4^2a_5a_6^2 - 6a_1^3a_2a_4a_5^3a_6 \\
& + 552a_1^3a_3^3a_6^3 + 648a_1^3a_3^2a_4a_5a_6^2 + 208a_1^3a_3^2a_5^3a_6 + 296a_1^3a_3a_4^3a_6^2 \\
& + 110a_1^3a_3a_4^2a_5^2a_6 + 24a_1^3a_4^4a_5a_6 - 4a_1^3a_4^3a_5^3 + 317a_1^2a_2^3a_4a_6^3 \\
& - 259a_1^2a_2^3a_5^2a_6^2 - 710a_1^2a_2^2a_3^3a_6^3 - 1542a_1^2a_2^2a_3a_4a_5a_6^2 - 192a_1^2a_2^2a_3a_5^3a_6 \\
& + 121a_1^2a_2^2a_4^3a_6^2 - 523a_1^2a_2^2a_4^2a_5^2a_6 - 10a_1^2a_2^2a_4a_5^4 - 256a_1^2a_2a_3^3a_5a_6^2 \\
& - 552a_1^2a_2a_3^2a_4^2a_6^2 - 296a_1^2a_2a_3^2a_4a_5^2a_6 + 2a_1^2a_2a_3a_4^3a_5a_6 + 4a_1^2a_2a_3a_4^2a_5^3 \\
& + 92a_1^2a_2a_4^5a_6 + 26a_1^2a_2a_4^4a_5^2 - 176a_1^2a_3^4a_4a_6^2 - 48a_1^2a_3^4a_5^2a_6 \\
& + 32a_1^2a_3^3a_4^3a_5a_6 + 27a_1^2a_3^2a_4^4a_6 + 4a_1^2a_3a_4^5a_5 - 12a_1^2a_4^7 \\
& - 441a_1a_2^4a_3a_6^3 - 165a_1a_2^4a_4a_5a_6^2 + 78a_1a_2^4a_5^3a_6 + 256a_1a_2^3a_3^2a_5a_6^2 \\
& + 202a_1a_2^3a_3a_4^2a_6^2 + 398a_1a_2^3a_3a_4a_5^2a_6 + 2a_1a_2^3a_3a_5^4 + 230a_1a_2^3a_4^3a_5a_6 \\
& + 44a_1a_2^3a_4^2a_5^3 + 216a_1a_2^2a_3^3a_4a_6^2 + 160a_1a_2^2a_3^3a_5^2a_6 - 220a_1a_2^2a_3^2a_4^2a_5a_6 \\
& + 16a_1a_2^2a_3^2a_4a_5^3 - 168a_1a_2^2a_3a_4^4a_6 - 36a_1a_2^2a_3a_4^3a_5^2 - 28a_1a_2^2a_4^5a_5 \\
& + 96a_1a_2a_3^5a_6^2 - 32a_1a_2a_3^4a_4a_5a_6 + 120a_1a_2a_3^3a_4^3a_6 - 16a_1a_2a_3^3a_4^2a_5^2 \\
& + 20a_1a_2a_3^2a_4^4a_5 + 18a_1a_2a_3a_4^6 - 24a_1a_3^3a_4^5 - 14a_2^6a_6^3 \\
& + 15a_2^5a_3a_5a_6^2 + 138a_2^5a_4^2a_6^2 - 79a_2^5a_4a_5^2a_6 - a_2^5a_5^4 \\
& - 121a_2^4a_3^2a_4a_6^2 - 43a_2^4a_3^2a_5^2a_6 - 20a_2^4a_3a_4^2a_5a_6 - 4a_2^4a_3a_4a_5^3
\end{aligned}$$

$$\begin{aligned}
& -7a_2^4a_4^4a_6 - 26a_2^4a_4^3a_5^2 + 40a_2^3a_3^3a_4a_5a_6 + 10a_2^3a_3^2a_4^3a_6 \\
& + 12a_2^3a_3^2a_4^2a_5^2 + 32a_2^3a_3a_4^4a_5 - a_2^3a_4^6 - 40a_2^2a_3^4a_4^2a_6 \\
& - 16a_2^2a_3^3a_4^3a_5 - 8a_2^2a_3^2a_4^5 + 8a_2a_3^4a_4^4 + 4916a_0^4a_5a_6^3 \\
& - 3172a_0^3a_1a_4a_6^3 + 772a_0^3a_1a_5^2a_6^2 + 1868a_0^3a_2a_3a_6^3 + 5120a_0^3a_2a_4a_5a_6^2 \\
& + 105a_0^3a_2a_5^3a_6 + 24a_0^3a_3^2a_5a_6^2 - 1782a_0^3a_3a_4^2a_6^2 - 2a_0^3a_3a_4a_5^2a_6 \\
& - 170a_0^3a_3a_5^4 + 3051a_0^3a_4^3a_5a_6 + 58a_0^3a_4^2a_5^3 - 1072a_0^2a_1^2a_3a_6^3 \\
& - 132a_0^2a_1^2a_4a_5a_6^2 - 286a_0^2a_1^2a_5^3a_6 + 9990a_0^2a_1a_2^2a_6^3 - 1988a_0^2a_1a_2a_3a_5a_6^2 \\
& + 1946a_0^2a_1a_2a_4^2a_6^2 - 419a_0^2a_1a_2a_4a_5^2a_6 - 134a_0^2a_1a_2a_5^4 + 1548a_0^2a_1a_3^2a_4a_6^2 \\
& + 1600a_0^2a_1a_3^2a_5^2a_6 - 2642a_0^2a_1a_3a_4^2a_5a_6 - 296a_0^2a_1a_3a_4a_5^3 - 745a_0^2a_1a_4^4a_6 \\
& + 50a_0^2a_1a_4^3a_5^2 - 653a_0^2a_2^3a_5a_6^2 - 1657a_0^2a_2^2a_3a_4a_6^2 - 253a_0^2a_2^2a_3a_5^2a_6 \\
& + 1402a_0^2a_2^2a_4^2a_5a_6 + 298a_0^2a_2^2a_4a_5^3 - 1824a_0^2a_2a_3^3a_6^2 - 2908a_0^2a_2a_3^2a_4a_5a_6 \\
& - 304a_0^2a_2a_3^2a_5^3 - 2793a_0^2a_2a_3a_4^3a_6 - 82a_0^2a_2a_3a_4^2a_5^2 + 536a_0^2a_2a_4^4a_5 \\
& - 192a_0^2a_3^4a_5a_6 + 1476a_0^2a_3^3a_4^2a_6 - 192a_0^2a_3^3a_4a_5^2 - 496a_0^2a_3^2a_4^3a_5 \\
& - 18a_0^2a_3a_4^5 - 7528a_0a_1^3a_2a_6^3 - 688a_0a_1^3a_3a_5a_6^2 - 1280a_0a_1^3a_4^2a_6^2 \\
& + 324a_0a_1^3a_4a_5^2a_6 + 54a_0a_1^3a_5^4 + 2844a_0a_1^2a_2^2a_5a_6^2 + 2960a_0a_1^2a_2a_3a_4a_6^2 \\
& + 792a_0a_1^2a_2a_3a_5^2a_6 + 420a_0a_1^2a_2a_4^2a_5a_6 - 120a_0a_1^2a_2a_4a_5^3 + 144a_0a_1^2a_3^3a_6^2 \\
& + 1808a_0a_1^2a_3^2a_4a_5a_6 + 416a_0a_1^2a_3^2a_5^3 + 1464a_0a_1^2a_3a_4^3a_6 + 124a_0a_1^2a_3a_4^2a_5^2 \\
& - 136a_0a_1^2a_4^4a_5 - 22a_0a_1a_2^3a_4a_6^2 - 599a_0a_1a_2^3a_5^2a_6 + 1128a_0a_1a_2^2a_3^2a_6^2 \\
& - 1556a_0a_1a_2^2a_3a_4a_5a_6 - 236a_0a_1a_2^2a_3a_5^3 + 1155a_0a_1a_2^2a_4^3a_6 + 96a_0a_1a_2^2a_4^2a_5^2 \\
& + 416a_0a_1a_2a_3^3a_5a_6 - 396a_0a_1a_2a_3^2a_4^2a_6 - 240a_0a_1a_2a_3^2a_4a_5^2 - 96a_0a_1a_2a_3a_4^3a_5 \\
& - 104a_0a_1a_2a_4^5 - 256a_0a_1a_3^4a_4a_6 - 96a_0a_1a_3^4a_5^2 + 400a_0a_1a_3^3a_4^2a_5 \\
& + 198a_0a_1a_3^2a_4^4 - 679a_0a_2^4a_3a_6^2 + 553a_0a_2^4a_4a_5a_6 + 34a_0a_2^4a_5^3 \\
& - 120a_0a_2^3a_3^2a_5a_6 - 806a_0a_2^3a_3a_4^2a_6 + 196a_0a_2^3a_3a_4a_5^2 - 110a_0a_2^3a_4^3a_5 \\
& + 48a_0a_2^2a_3^3a_4a_6 + 48a_0a_2^2a_3^3a_5^2 - 160a_0a_2^2a_3^2a_4^2a_5 - 2a_0a_2^2a_3a_4^4 \\
& + 128a_0a_2a_3^5a_6 + 96a_0a_2a_3^4a_4a_5 - 80a_0a_2a_3^3a_4^3 - 96a_0a_3^5a_4^2 \\
& + 1072a_1^5a_6^3 - 512a_1^4a_2a_5a_6^2 - 688a_1^4a_3a_4a_6^2 - 216a_1^4a_3a_5^2a_6 \\
& - 128a_1^4a_4^2a_5a_6 - 628a_1^3a_2^2a_4a_6^2 + 182a_1^3a_2^2a_5^2a_6 - 144a_1^3a_2a_3^2a_6^2 \\
& - 104a_1^3a_2a_3a_4a_5a_6 - 480a_1^3a_2a_4^3a_6 - 288a_1^3a_3^3a_5a_6 - 200a_1^3a_3^2a_4^2a_6 \\
& + 16a_1^3a_3a_4^3a_5 + 48a_1^3a_4^5 + 520a_1^2a_2^3a_3a_6^2 + 290a_1^2a_2^3a_4a_5a_6 \\
& + 4a_1^2a_2^3a_5^3 + 368a_1^2a_2^2a_3^2a_5a_6 + 670a_1^2a_2^2a_3a_4^2a_6 + 64a_1^2a_2^2a_3a_4a_5^2 \\
& + 28a_1^2a_2^2a_4^3a_5 + 336a_1^2a_2a_3^3a_4a_6 - 16a_1^2a_2a_3^2a_4^2a_5 - 72a_1^2a_2a_3a_4^4 \\
& + 64a_1^2a_5^3a_6 + 75a_1a_2^5a_6^2 - 214a_1a_2^4a_3a_5a_6 - 191a_1a_2^4a_4^2a_6 \\
& - 58a_1a_2^4a_4a_5^2 - 248a_1a_2^3a_3^2a_4a_6 - 16a_1a_2^3a_3^2a_5^2 - 56a_1a_2^3a_3a_4^2a_5 \\
& + 26a_1a_2^3a_4^4 - 192a_1a_2^2a_3^4a_6 - 32a_1a_2^2a_3^3a_4a_5 + 64a_1a_2^2a_3^2a_4^3 \\
& + 32a_1a_2a_3^4a_4^2 + 15a_2^6a_5a_6 + 85a_2^5a_3a_4a_6 + 6a_2^5a_3a_5^2
\end{aligned}$$

$$\begin{aligned}
& + 42a_2^5a_4^2a_5 + 44a_2^4a_3^3a_6 - 30a_2^4a_3a_4^3 - 4800a_0^4a_4a_6^2 \\
& - 192a_0^4a_5^2a_6 + 4440a_0^3a_1a_3a_6^2 - 2764a_0^3a_1a_4a_5a_6 - 348a_0^3a_1a_5^3 \\
& + 1680a_0^3a_2^2a_6^2 - 2348a_0^3a_2a_3a_5a_6 - 4938a_0^3a_2a_4^2a_6 + 516a_0^3a_2a_4a_5^2 \\
& + 648a_0^3a_3^2a_4a_6 + 688a_0^3a_3^2a_5^2 - 684a_0^3a_3a_4^2a_5 - 162a_0^3a_4^4 \\
& - 6888a_0^2a_1^2a_2a_6^2 + 1304a_0^2a_1^2a_3a_5a_6 + 1336a_0^2a_1^2a_4^2a_6 + 152a_0^2a_1^2a_4a_5^2 \\
& - 370a_0^2a_1a_2^2a_5a_6 + 7012a_0^2a_1a_2a_3a_4a_6 + 1088a_0^2a_1a_2a_3a_5^2 - 704a_0^2a_1a_2a_4^2a_5 \\
& - 2016a_0^2a_1a_3^3a_6 + 640a_0^2a_1a_3^2a_4a_5 + 64a_0^2a_1a_3a_4^3 - 129a_0^2a_2^3a_4a_6 \\
& - 202a_0^2a_2^3a_5^2 + 2004a_0^2a_2^2a_3^2a_6 - 1800a_0^2a_2^2a_3a_4a_5 - 606a_0^2a_2^2a_4^3 \\
& + 416a_0^2a_2a_3^3a_5 + 1448a_0^2a_2a_3^2a_4^2 + 192a_0^2a_3^4a_4 + 2112a_0a_1^4a_6^2 \\
& - 128a_0a_1^3a_2a_5a_6 - 1744a_0a_1^3a_3a_4a_6 - 432a_0a_1^3a_3a_5^2 + 128a_0a_1^3a_4^2a_5 \\
& - 2204a_0a_1^2a_2^2a_4a_6 + 108a_0a_1^2a_2^2a_5^2 - 1824a_0a_1^2a_2a_3^2a_6 + 544a_0a_1^2a_2a_3a_4a_5 \\
& + 304a_0a_1^2a_2a_4^3 - 576a_0a_1^2a_3^3a_5 - 592a_0a_1^2a_3^2a_4^2 + 1492a_0a_1a_2^3a_3a_6 \\
& + 32a_0a_1a_2^3a_4a_5 + 496a_0a_1a_2^2a_3^2a_5 + 68a_0a_1a_2^2a_3a_4^2 - 192a_0a_1a_2a_3^3a_4 \\
& + 128a_0a_1a_3^5 - 259a_0a_2^5a_6 - 160a_0a_2^4a_3a_5 + 80a_0a_2^4a_4^2 \\
& + 80a_0a_2^3a_3^2a_4 - 96a_0a_2^2a_3^4 + 768a_1^4a_2a_4a_6 + 432a_1^4a_3^2a_6 \\
& - 64a_1^4a_4^3 - 536a_1^3a_2^2a_3a_6 - 64a_1^3a_2^2a_4a_5 + 64a_1^3a_2a_3a_4^2 \\
& - 5a_1^2a_2^4a_6 - 16a_1^2a_2^3a_3a_5 - 48a_1^2a_2^3a_4^2 - 96a_1^2a_2^2a_3^2a_4 \\
& + 20a_1a_2^5a_5 + 72a_1a_2^4a_3a_4 + 32a_1a_2^3a_3^3 - 20a_2^6a_4 \\
& - 8a_2^5a_3^2 + 3552a_0^4a_3a_6 - 1128a_0^4a_4a_5 + 2472a_0^3a_1a_2a_6 \\
& + 1456a_0^3a_1a_3a_5 + 1008a_0^3a_1a_4^2 - 868a_0^3a_2^2a_5 + 672a_0^3a_2a_3a_4 \\
& - 1056a_0^3a_3^3 - 528a_0^2a_1^3a_6 + 64a_0^2a_1^2a_2a_5 - 736a_0^2a_1^2a_3a_4 \\
& + 984a_0^2a_1a_2^2a_4 - 2208a_0^2a_1a_2a_3^2 + 1320a_0^2a_2^3a_3 - 192a_0a_1^3a_2a_4 \\
& + 864a_0a_1^3a_3^2 - 496a_0a_1^2a_2^2a_3 - 106a_0a_1a_2^4 + 64a_1^3a_2^3 \\
& + 1552a_0^4a_2 - 928a_0^3a_1^2
\end{aligned} \tag{A.33}$$

$$\begin{aligned}
e_{34} = & 64a_0^3a_5a_6^{11} - 32a_0^2a_1a_5^2a_6^{10} - 16a_0^2a_2a_4a_5a_6^{10} + 72a_0^2a_2a_5^3a_6^9 \\
& - 48a_0^2a_3^2a_5a_6^{10} + 72a_0^2a_3a_4a_5^2a_6^9 - 52a_0^2a_3a_5^4a_6^8 + 4a_0^2a_4^3a_5a_6^9 \\
& - 26a_0^2a_4^2a_5^3a_6^8 + 48a_0^2a_4a_5^5a_6^7 - 6a_0^2a_5^7a_6^6 + 20a_0a_1^2a_4a_5a_6^{10} \\
& + 30a_0a_1^2a_5^3a_6^9 + 24a_0a_1a_2a_3a_5a_6^{10} - 24a_0a_1a_2a_4a_5^2a_6^9 - 36a_0a_1a_2a_5^4a_6^8 \\
& + 4a_0a_1a_3^2a_5^2a_6^9 - 26a_0a_1a_3a_4^2a_5a_6^9 - 74a_0a_1a_3a_4a_5^3a_6^8 - 14a_0a_1a_3a_5^5a_6^7 \\
& + 16a_0a_1a_4^3a_5^2a_6^8 + 44a_0a_1a_4^2a_5^4a_6^7 - 6a_0a_1a_4a_5^6a_6^6 - 4a_0a_1a_5^8a_6^5 \\
& + 32a_0a_2^3a_5a_6^{10} - 56a_0a_2^2a_3a_5^2a_6^9 - 16a_0a_2^2a_4^2a_5a_6^9 + 36a_0a_2^2a_4a_5^3a_6^8 \\
& + 14a_0a_2^2a_5^5a_6^7 - 4a_0a_2^2a_3^2a_4a_5a_6^9 + 26a_0a_2a_3^2a_5^3a_6^8 + 30a_0a_2a_3a_4^2a_5^2a_6^8 \\
& - 10a_0a_2a_3a_4a_5^4a_6^7 - 2a_0a_2a_3a_5^6a_6^6 + 2a_0a_2a_4^4a_5a_6^8 - 20a_0a_2a_4^3a_5^3a_6^7
\end{aligned}$$

$$\begin{aligned}
& -4a_0a_2a_4^2a_5^5a_6^6 + 10a_0a_2a_4a_5^7a_6^5 + 12a_0a_3^4a_5a_6^9 - 30a_0a_3^3a_4a_5^2a_6^8 \\
& + 5a_0a_3^3a_5^4a_6^7 + 6a_0a_3^2a_4^3a_5a_6^8 + 42a_0a_3^2a_4^2a_5^3a_6^7 + a_0a_3^2a_5^7a_6^5 \\
& - 9a_0a_3a_4^4a_5^2a_6^7 - 28a_0a_3a_4^3a_5^4a_6^6 - 2a_0a_3a_4^2a_5^6a_6^5 + 4a_0a_4^5a_5^3a_6^6 \\
& + 7a_0a_4^4a_5^5a_6^5 - 14a_1^3a_3a_5a_6^{10} + 6a_1^3a_4a_5^2a_6^9 - 7a_1^3a_5^4a_6^8 \\
& - 8a_1^2a_2^2a_5a_6^{10} + 18a_1^2a_2a_3a_5^2a_6^9 + 6a_1^2a_2a_4^2a_5a_6^9 + 6a_1^2a_2a_4a_5^3a_6^8 \\
& + 19a_1^2a_2a_5^5a_6^7 + 16a_1^2a_3^2a_4a_5a_6^9 + 14a_1^2a_3^2a_5^3a_6^8 - 20a_1^2a_3a_4^2a_5^2a_6^8 \\
& - 11a_1^2a_3a_4a_5^4a_6^7 + 4a_1^2a_3a_5^6a_6^6 - a_1^2a_4^4a_5a_6^8 + 2a_1^2a_4^3a_5^3a_6^7 \\
& - 4a_1^2a_4^2a_5^5a_6^6 - 4a_1^2a_4a_5^7a_6^5 - 8a_1a_2^2a_3a_4a_5a_6^9 - 10a_1a_2^2a_3a_5^3a_6^8 \\
& + 4a_1a_2^2a_4^2a_5^2a_6^8 - 6a_1a_2^2a_4a_5^4a_6^7 - 13a_1a_2^2a_5^6a_6^6 - 6a_1a_2a_3^3a_5a_6^9 \\
& + 10a_1a_2a_3^2a_4a_5^2a_6^8 - 7a_1a_2a_3^2a_5^4a_6^7 + 2a_1a_2a_3a_4^3a_5a_6^8 - 10a_1a_2a_3a_4^2a_5^3a_6^7 \\
& + 2a_1a_2a_3a_4a_5^5a_6^6 - a_1a_2a_4^4a_5^2a_6^7 + 8a_1a_2a_4^3a_5^4a_6^6 + 6a_1a_2a_4^2a_5^6a_6^5 \\
& + a_1a_3^4a_5^2a_6^8 - 4a_1a_3^3a_4^2a_5a_6^8 - 10a_1a_3^3a_4a_5^3a_6^7 - a_1a_3^3a_5^5a_6^6 \\
& + 6a_1a_3^2a_4^3a_5^2a_6^7 + 15a_1a_3^2a_4^2a_5^4a_6^6 + a_1a_3^2a_4a_5^6a_6^5 - a_1a_3a_4^4a_5^3a_6^6 \\
& - 6a_1a_3a_4^3a_5^5a_6^5 - a_1a_4^5a_5^4a_6^5 + 8a_2^4a_5^3a_6^8 + 8a_2^3a_5^2a_6^9 \\
& - 8a_2^3a_3a_4a_5^2a_6^8 - 10a_2^3a_3a_5^4a_6^7 - 2a_2^3a_4^2a_5^3a_6^7 + 10a_2^3a_4a_5^5a_6^6 \\
& + a_2^3a_5^7a_6^5 - 10a_2^2a_3^3a_5^2a_6^8 - 2a_2^2a_3^2a_4^2a_5a_6^8 + 22a_2^2a_3^2a_4a_5^3a_6^7 \\
& + 3a_2^2a_3^2a_5^5a_6^6 + 2a_2^2a_3a_4^3a_5^2a_6^7 - 10a_2^2a_3a_4^2a_5^4a_6^6 - 3a_2^2a_3a_4a_5^6a_6^5 \\
& - 2a_2^2a_4^3a_5^5a_6^5 + 2a_2a_3^4a_4a_5a_6^8 + a_2a_3^4a_5^3a_6^7 - 2a_2a_3^3a_4^2a_5^2a_6^7 \\
& - 2a_2a_3^3a_4a_5^4a_6^6 - 2a_2a_3^2a_4^3a_5^3a_6^6 + a_2a_3^2a_4^2a_5^5a_6^5 + 2a_2a_3a_4^4a_5^4a_6^5 \\
& - a_3^6a_5a_6^8 + 3a_3^5a_4a_5^2a_6^7 - 3a_3^4a_4^2a_5^3a_6^6 + a_3^3a_4^3a_5^4a_6^5 \\
& - 64a_0^3a_4a_6^{10} - 112a_0^3a_5^2a_6^9 + 40a_0^2a_1a_4a_5a_6^9 - 92a_0^2a_1a_5^3a_6^8 \\
& + 240a_0^2a_2^2a_6^{10} - 152a_0^2a_2a_3a_5a_6^9 - 104a_0^2a_2a_4^2a_6^9 - 64a_0^2a_2a_4a_5^2a_6^8 \\
& - 80a_0^2a_2a_5^4a_6^7 + 48a_0^2a_3^2a_4a_6^9 + 44a_0^2a_3^2a_5^2a_6^8 - 70a_0^2a_3a_4^2a_5a_6^8 \\
& + 226a_0^2a_3a_4a_5^3a_6^7 + 2a_0^2a_3a_5^5a_6^6 + 11a_0^2a_4^4a_6^8 + 16a_0^2a_4^3a_5^2a_6^7 \\
& - 266a_0^2a_4^2a_5^4a_6^6 + 42a_0^2a_4a_5^6a_6^5 - 5a_0^2a_5^8a_6^4 - 120a_0a_1^2a_2a_6^{10} \\
& + 58a_0a_1^2a_3a_5a_6^9 + 10a_0a_1^2a_4^2a_6^9 - 154a_0a_1^2a_4a_5^2a_6^8 - 3a_0a_1^2a_5^4a_6^7 \\
& - 104a_0a_1a_2^2a_5a_6^9 + 96a_0a_1a_2a_3a_4a_6^9 + 32a_0a_1a_2a_3a_5^2a_6^8 + 34a_0a_1a_2a_4^2a_5a_6^8 \\
& + 170a_0a_1a_2a_4a_5^3a_6^7 - 4a_0a_1a_2a_5^5a_6^6 - 44a_0a_1a_3^2a_4a_5a_6^8 + 34a_0a_1a_3^2a_5^3a_6^7 \\
& - 4a_0a_1a_3a_4^3a_6^8 + 273a_0a_1a_3a_4^2a_5^2a_6^7 + 86a_0a_1a_3a_4a_5^4a_6^6 + 43a_0a_1a_3a_5^6a_6^5 \\
& - 17a_0a_1a_4^4a_5a_6^7 - 178a_0a_1a_4^3a_5^3a_6^6 + 35a_0a_1a_4^2a_5^5a_6^5 + 6a_0a_1a_4a_5^7a_6^4 \\
& - 32a_0a_2^3a_4a_6^9 + 16a_0a_2^3a_5^2a_6^8 - 120a_0a_2^2a_3^2a_6^9 + 280a_0a_2^2a_3a_4a_5a_6^8 \\
& - 94a_0a_2^2a_3a_5^3a_6^7 + 16a_0a_2^2a_4^3a_6^8 - 168a_0a_2^2a_4^2a_5^2a_6^7 - 12a_0a_2^2a_4a_5^4a_6^6 \\
& - 13a_0a_2^2a_5^6a_6^5 + 82a_0a_2a_3^3a_5a_6^8 + 34a_0a_2a_3^2a_4^2a_6^8 - 252a_0a_2a_3^2a_4a_5^2a_6^7 \\
& + 74a_0a_2a_3^2a_5^4a_6^6 - 112a_0a_2a_3a_4^3a_5a_6^7 + 161a_0a_2a_3a_4^2a_5^3a_6^6 - 112a_0a_2a_3a_4a_5^5a_6^5 \\
& + 5a_0a_2a_3a_5^7a_6^4 - 2a_0a_2a_4^5a_6^7 + 79a_0a_2a_4^4a_5^2a_6^6 - 10a_0a_2a_4^3a_5^4a_6^5
\end{aligned}$$

$$\begin{aligned}
& -25a_0a_2a_4^2a_5^6a_6^4 - 12a_0a_3^4a_4a_6^8 - 4a_0a_3^4a_5^2a_6^7 + 31a_0a_3^3a_4^2a_5a_6^7 \\
& - 49a_0a_3^3a_4a_5^3a_6^6 - 9a_0a_3^3a_5^5a_6^5 - 6a_0a_3^2a_4^4a_6^7 - 69a_0a_3^2a_4^3a_5^2a_6^6 \\
& + 30a_0a_3^2a_4^2a_5^4a_6^5 - 4a_0a_3^2a_4a_5^6a_6^4 + 18a_0a_3a_4^5a_5a_6^6 + 66a_0a_3a_4^4a_5^3a_6^5 \\
& + 3a_0a_3a_4^3a_5^5a_6^4 - 12a_0a_4^6a_5^2a_6^5 - 23a_0a_4^5a_5^4a_6^4 + 15a_1^4a_6^{10} \\
& + 30a_1^3a_2a_5a_6^9 - 16a_1^3a_3a_4a_6^9 + a_1^3a_3a_5^2a_6^8 - 3a_1^3a_4^2a_5a_6^8 \\
& + 13a_1^3a_4a_5^3a_6^7 - 13a_1^3a_5^5a_6^6 + 8a_1^2a_2^2a_4a_6^9 - 8a_1^2a_2^2a_5^2a_6^8 \\
& + 30a_1^2a_2a_3a_6^9 - 94a_1^2a_2a_3a_4a_5a_6^8 - 11a_1^2a_2a_3a_5^3a_6^7 - 6a_1^2a_2a_4^3a_6^8 \\
& - 105a_1^2a_2a_4a_5^4a_6^6 + 14a_1^2a_2a_5^6a_6^5 - 25a_1^2a_3^3a_5a_6^8 - a_1^2a_3^2a_4^2a_6^8 \\
& - a_1^2a_3^2a_4a_5^2a_6^7 - 24a_1^2a_3^2a_5^4a_6^6 + 27a_1^2a_3a_4^3a_5a_6^7 + 33a_1^2a_3a_4^2a_5^3a_6^6 \\
& + 19a_1^2a_3a_4a_5^5a_6^5 + a_1^2a_4^5a_6^7 - 2a_1^2a_4^4a_5^2a_6^6 + 23a_1^2a_4^3a_5^4a_6^5 \\
& + 11a_1^2a_4^2a_5^6a_6^4 + 8a_1a_2^3a_4a_5a_6^8 - 22a_1a_2^3a_5^3a_6^7 + 2a_1a_2^2a_3^2a_5a_6^8 \\
& + 8a_1a_2^2a_3a_4^2a_6^8 + 28a_1a_2^2a_3a_4a_5^2a_6^7 + 65a_1a_2^2a_3a_5^4a_6^6 - 10a_1a_2^2a_4^3a_5a_6^7 \\
& + 43a_1a_2^2a_4^2a_5^3a_6^6 + 43a_1a_2^2a_4a_5^5a_6^5 - 5a_1a_2^2a_5^7a_6^4 - 24a_1a_2a_3^3a_4a_6^8 \\
& + 15a_1a_2a_3^3a_5^2a_6^7 + 35a_1a_2a_3^2a_4^2a_5a_6^7 + 19a_1a_2a_3^2a_4a_5^3a_6^6 - 6a_1a_2a_3^2a_5^5a_6^5 \\
& - 2a_1a_2a_3a_4^4a_6^7 - 3a_1a_2a_3a_4^3a_5^2a_6^6 - 39a_1a_2a_3a_4^2a_5^4a_6^5 + 5a_1a_2a_3a_4a_5^6a_6^4 \\
& + 2a_1a_2a_4^5a_5a_6^6 - 29a_1a_2a_4^4a_5^3a_6^5 - 19a_1a_2a_4^3a_5^5a_6^4 + 19a_1a_4^3a_4a_5a_6^7 \\
& + 6a_1a_4^3a_5^3a_6^6 + 4a_1a_3^3a_4^3a_6^7 - 19a_1a_3^3a_4^2a_5^2a_6^6 - 3a_1a_3^3a_4a_5^4a_6^5 \\
& - 12a_1a_3^2a_4^4a_5a_6^6 - 21a_1a_3^2a_4^3a_5^3a_6^5 - 4a_1a_3^2a_4^2a_5^5a_6^4 + 3a_1a_3a_5^5a_5^2a_6^5 \\
& + 19a_1a_3a_4^4a_5^4a_6^4 + 4a_1a_4^6a_5^3a_6^4 - 16a_2^4a_3a_5a_6^8 - 16a_2^4a_4a_5^2a_6^7 \\
& + a_2^4a_5^4a_6^6 - 8a_2^3a_3^2a_4a_6^8 + 36a_2^3a_3^2a_5^2a_6^7 + 20a_2^3a_3a_4^2a_5a_6^7 \\
& - 6a_2^3a_3a_4a_5^3a_6^6 - 11a_2^3a_3a_5^5a_6^5 + 4a_2^3a_4^3a_5^2a_6^6 - 36a_2^3a_4^2a_5^4a_6^5 \\
& + a_2^3a_4a_5^6a_6^4 + 15a_2^2a_3^4a_6^8 - 14a_2^2a_3^3a_4a_5a_6^7 - 26a_2^2a_3^3a_5^3a_6^6 \\
& + 2a_2^2a_3^2a_4^3a_6^7 - 45a_2^2a_3^2a_4^2a_5^2a_6^6 + 27a_2^2a_3^2a_4a_5^4a_6^5 - 4a_2^2a_3a_4^4a_5a_6^6 \\
& + 40a_2^2a_3a_4^3a_5^3a_6^5 + 2a_2^2a_3a_4^2a_5^5a_6^4 + 8a_2^2a_4^4a_5^4a_6^4 - 11a_2a_3^5a_5a_6^7 \\
& - 2a_2a_3^4a_4^2a_6^6 + 25a_2a_3^4a_4a_5^2a_6^6 + 4a_2a_3^3a_4^3a_5a_6^6 - 15a_2a_3^3a_4^2a_5^3a_6^5 \\
& + 6a_2a_3^2a_4^4a_5^2a_6^5 + a_2a_3^2a_4^3a_5^4a_6^4 - 8a_2a_3a_4^5a_5^3a_6^4 + a_3^6a_4a_6^7 \\
& - 6a_3^5a_4^2a_5a_6^6 + 9a_3^4a_4^3a_5^2a_6^5 - 4a_3^3a_4^4a_5^3a_6^4 + 64a_0^3a_3a_6^9 \\
& + 420a_0^3a_4a_5a_6^8 - 370a_0^3a_5^3a_6^7 - 240a_0^2a_1a_2a_6^9 - 10a_0^2a_1a_3a_5a_6^8 \\
& + 52a_0^2a_1a_4^2a_6^8 + 170a_0^2a_1a_4a_5^2a_6^7 + 251a_0^2a_1a_5^4a_6^6 - 992a_0^2a_2^2a_5a_6^8 \\
& + 256a_0^2a_2a_3a_4a_6^8 + 998a_0^2a_2a_3a_5^2a_6^7 + 344a_0^2a_2a_4^2a_5a_6^7 - 112a_0^2a_2a_4a_5^3a_6^6 \\
& - 51a_0^2a_2a_5^5a_6^5 - 48a_0^2a_3^3a_6^8 - 176a_0^2a_3^2a_4a_5a_6^7 - 90a_0^2a_3^2a_5^3a_6^6 \\
& - 28a_0^2a_3a_4^3a_6^7 - 379a_0^2a_3a_4^2a_5^2a_6^6 + 39a_0^2a_3a_4a_5^4a_6^5 + 43a_0^2a_3a_5^6a_6^4 \\
& - 6a_0^2a_4^4a_5a_6^6 + 620a_0^2a_4^3a_5^3a_6^5 - 133a_0^2a_4^2a_5^5a_6^4 + 16a_0^2a_4a_5^7a_6^3 \\
& + 60a_0a_1^3a_6^9 + 654a_0a_1^2a_2a_5a_6^8 - 128a_0a_1^2a_3a_4a_6^8 - 117a_0a_1^2a_3a_5^2a_6^7 \\
& + 139a_0a_1^2a_4^2a_5a_6^7 + 19a_0a_1^2a_4a_5^3a_6^6 - 32a_0a_1^2a_5^5a_6^5 - 16a_0a_1a_2^2a_4a_6^8
\end{aligned}$$

$$\begin{aligned}
& -96a_0a_1a_2^2a_5^2a_6^7 + 84a_0a_1a_2a_3^2a_6^8 - 628a_0a_1a_2a_3a_4a_5a_6^7 - 114a_0a_1a_2a_3a_5^3a_6^6 \\
& + 20a_0a_1a_2a_4^3a_6^7 - 61a_0a_1a_2a_4^2a_5^2a_6^6 + 34a_0a_1a_2a_4a_5^4a_6^5 + 3a_0a_1a_2a_5^6a_6^4 \\
& + 16a_0a_1a_3^3a_5a_6^7 + 44a_0a_1a_3^2a_4^2a_6^7 - 140a_0a_1a_3^2a_4a_5^2a_6^6 - 159a_0a_1a_3^2a_5^4a_6^5 \\
& - 254a_0a_1a_3a_4^3a_5a_6^6 - 192a_0a_1a_3a_4^2a_5^3a_6^5 - 74a_0a_1a_3a_4a_5^5a_6^4 - 4a_0a_1a_3a_5^7a_6^3 \\
& + a_0a_1a_4^5a_6^6 + 241a_0a_1a_4^4a_5^2a_6^5 - 93a_0a_1a_4^3a_5^4a_6^4 + 8a_0a_1a_4^2a_5^6a_6^3 \\
& + 272a_0a_2^3a_3a_6^8 - 100a_0a_2^3a_4a_5a_6^7 - 84a_0a_2^3a_5^3a_6^6 - 30a_0a_2^2a_3^2a_5a_6^7 \\
& - 300a_0a_2^2a_3a_4^2a_6^7 + 398a_0a_2^2a_3a_4a_5^2a_6^6 + 184a_0a_2^2a_3a_5^4a_6^5 + 226a_0a_2^2a_4^3a_5a_6^6 \\
& - 84a_0a_2^2a_4^2a_5^3a_6^5 + 37a_0a_2^2a_4a_5^5a_6^4 - 116a_0a_2a_3^3a_4a_6^7 - 139a_0a_2a_3^3a_5^2a_6^6 \\
& + 520a_0a_2a_3^2a_4^2a_5a_6^6 - 12a_0a_2a_3^2a_4a_5^3a_6^5 - 48a_0a_2a_3^2a_5^5a_6^4 + 84a_0a_2a_3a_4^4a_6^6 \\
& - 504a_0a_2a_3a_4^3a_5^2a_6^5 + 345a_0a_2a_3a_4^2a_5^4a_6^4 - 6a_0a_2a_3a_4a_5^6a_6^3 - 92a_0a_2a_4^5a_5a_6^5 \\
& + 94a_0a_2a_4^4a_5^3a_6^4 + 12a_0a_2a_4^3a_5^5a_6^3 + 12a_0a_3^5a_6^7 + 11a_0a_3^4a_4a_5a_6^6 \\
& + 29a_0a_3^4a_5^3a_6^5 + 5a_0a_3^3a_4^3a_6^6 + 118a_0a_3^3a_4^2a_5^2a_6^5 + 33a_0a_3^3a_4a_5^4a_6^4 \\
& + a_0a_3^3a_5^6a_6^3 + 12a_0a_3^2a_4^4a_5a_6^5 - 111a_0a_3^2a_4^3a_5^3a_6^4 + 4a_0a_3^2a_4^2a_5^5a_6^3 \\
& - 9a_0a_3a_4^6a_6^5 - 36a_0a_3a_4^5a_5^2a_6^4 + 11a_0a_3a_4^4a_5^4a_6^3 + 12a_0a_4^7a_5a_6^4 \\
& + 26a_0a_4^6a_5^3a_6^3 - 102a_1^4a_5a_6^8 + 27a_1^3a_2a_5^2a_6^7 - 14a_1^3a_3^2a_6^8 \\
& + 134a_1^3a_3a_4a_5a_6^7 + 58a_1^3a_3a_5^3a_6^6 - 3a_1^3a_4^3a_6^7 - 63a_1^3a_4^2a_5^2a_6^6 \\
& + 52a_1^3a_4a_5^4a_6^5 - 68a_1^2a_2^2a_3a_6^8 + 26a_1^2a_2^2a_4a_5a_6^7 + 77a_1^2a_2^2a_5^3a_6^6 \\
& + 7a_1^2a_2a_3^2a_5a_6^7 + 52a_1^2a_2a_3a_4^2a_6^7 + 73a_1^2a_2a_3a_4a_5^2a_6^6 - 63a_1^2a_2a_3a_5^4a_6^5 \\
& + 15a_1^2a_2a_4^3a_5a_6^6 + 179a_1^2a_2a_4^2a_5^3a_6^5 - 65a_1^2a_2a_4a_5^5a_6^4 + 41a_1^2a_3^3a_4a_6^7 \\
& + 46a_1^2a_3^2a_5^2a_6^6 - 96a_1^2a_3^2a_4^2a_5a_6^6 - 24a_1^2a_3^2a_4a_5^3a_6^5 + 4a_1^2a_3^2a_5^5a_6^4 \\
& - 8a_1^2a_3a_4^4a_6^6 - 11a_1^2a_3a_4^3a_5^2a_6^5 - 65a_1^2a_3a_4^2a_5^4a_6^4 - 4a_1^2a_3a_4a_5^6a_6^3 \\
& - 5a_1^2a_4^5a_5a_6^5 - 52a_1^2a_4^4a_5^3a_6^4 - 8a_1^2a_4^3a_5^5a_6^3 + 30a_1a_2^3a_3a_5a_6^7 \\
& - 8a_1a_2^3a_4^2a_6^7 + 58a_1a_2^3a_4a_5^2a_6^6 - 54a_1a_2^3a_5^4a_6^5 + 80a_1a_2^2a_3^2a_4a_6^7 \\
& - 125a_1a_2^2a_3^2a_5^2a_6^6 - 144a_1a_2^2a_3a_4^2a_5a_6^6 - 152a_1a_2^2a_3a_4a_5^3a_6^5 + 31a_1a_2^2a_3a_5^5a_6^4 \\
& + 6a_1a_2^2a_4^4a_6^6 - 58a_1a_2^2a_4^3a_5^2a_6^5 - 32a_1a_2^2a_4^2a_5^4a_6^4 + 16a_1a_2^2a_4a_5^6a_6^3 \\
& - 6a_1a_2a_3^4a_6^7 - 36a_1a_2a_3^3a_4a_5a_6^6 + 30a_1a_2a_3^3a_5^3a_6^5 - 43a_1a_2a_3^2a_4^3a_6^6 \\
& + 48a_1a_2a_3^2a_4^2a_5^2a_6^5 - 14a_1a_2a_3^2a_4a_5^4a_6^4 + 40a_1a_2a_3a_4^4a_5a_6^5 + 84a_1a_2a_3a_4^3a_5^3a_6^4 \\
& - 10a_1a_2a_3a_4^2a_5^5a_6^3 - a_1a_2a_4^6a_6^5 + 37a_1a_2a_4^5a_5^2a_6^4 + 20a_1a_2a_4^4a_5^4a_6^3 \\
& - 6a_1a_3^5a_5a_6^6 - 24a_1a_3^4a_4^2a_6^6 - 19a_1a_3^4a_4a_5^2a_6^5 - a_1a_3^4a_5^4a_6^4 \\
& + 68a_1a_3^3a_4^3a_5a_6^5 + 34a_1a_3^3a_4^2a_5^3a_6^4 + a_1a_3^3a_4a_5^5a_6^3 + 6a_1a_3^2a_4^5a_6^5 \\
& - 18a_1a_3^2a_4^4a_5^2a_6^4 - 3a_1a_3a_4^6a_5a_6^4 - 21a_1a_3a_4^5a_5^3a_6^3 - 6a_1a_4^7a_5^2a_6^3 \\
& + 8a_2^5a_5a_6^7 + 16a_2^4a_3a_4a_6^7 - 34a_2^4a_3a_5^2a_6^6 + 6a_2^4a_4^2a_5a_6^6 \\
& + 36a_2^4a_4a_5^3a_6^5 - 2a_2^4a_5^5a_6^4 - 52a_2^3a_3^3a_6^7 + 26a_2^3a_3^2a_4a_5a_6^6 \\
& + 20a_2^3a_3^2a_5^3a_6^5 - 12a_2^3a_3a_4^3a_6^6 + 24a_2^3a_3a_4^2a_5^2a_6^5 + 19a_2^3a_3a_4a_5^4a_6^4 \\
& + a_2^3a_3a_5^6a_6^3 - 2a_2^3a_4^4a_5a_6^5 + 42a_2^3a_4^3a_5^3a_6^4 - 10a_2^3a_4^2a_5^5a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 28a_2^2a_3^4a_5a_6^6 + 22a_2^2a_3^3a_4^2a_6^6 + 12a_2^2a_3^3a_4a_5^2a_6^5 + 3a_2^2a_3^3a_5^4a_6^4 \\
& + 22a_2^2a_3^2a_4^3a_5a_6^5 - 77a_2^2a_3^2a_4^2a_5^3a_6^4 - 3a_2^2a_3^2a_4a_5^5a_6^3 + 2a_2^2a_3a_4^5a_6^5 \\
& - 56a_2^2a_3a_4^4a_5^2a_6^4 + 12a_2^2a_3a_4^3a_5^4a_6^3 - 12a_2^2a_4^5a_5^3a_6^3 + 13a_2a_3^5a_4a_6^6 \\
& + a_2a_3^5a_5^2a_6^5 - 51a_2a_3^4a_4^2a_5a_6^5 - 2a_2a_3^4a_4a_5^3a_6^4 - 2a_2a_3^3a_4^4a_6^5 \\
& + 47a_2a_3^3a_4^3a_5^2a_6^4 + a_2a_3^3a_4^2a_5^4a_6^3 - 6a_2a_3^2a_4^5a_5a_6^4 - 8a_2a_3^2a_4^4a_5^3a_6^3 \\
& + 12a_2a_3a_4^6a_5^2a_6^3 - a_3^7a_6^6 + 3a_3^6a_4a_5a_6^5 + 3a_3^5a_4^3a_6^5 \\
& - 3a_3^5a_4^2a_5^2a_6^4 - 9a_3^4a_4^4a_5a_6^4 + a_3^4a_4^3a_5^3a_6^3 + 6a_3^3a_4^5a_5^2a_6^3 \\
& + 296a_0^3a_2a_6^8 - 322a_0^3a_3a_5a_6^7 - 398a_0^3a_4^2a_6^7 + 1226a_0^3a_4a_5^2a_6^6 \\
& + 447a_0^3a_5^4a_6^5 - 30a_0^2a_1^2a_6^8 + 634a_0^2a_1a_2a_5a_6^7 + 16a_0^2a_1a_3a_4a_6^7 \\
& - 689a_0^2a_1a_3a_5^2a_6^6 - 297a_0^2a_1a_4^2a_5a_6^6 - 657a_0^2a_1a_4a_5^3a_6^5 - 5a_0^2a_1a_5^5a_6^4 \\
& + 1720a_0^2a_2^2a_4a_6^7 + 848a_0^2a_2^2a_5^2a_6^6 - 314a_0^2a_2a_3^2a_6^7 - 2222a_0^2a_2a_3a_4a_5a_6^6 \\
& - 611a_0^2a_2a_3a_5^3a_6^5 - 742a_0^2a_2a_4^3a_6^6 + 407a_0^2a_2a_4^2a_5^2a_6^5 + 353a_0^2a_2a_4a_5^4a_6^4 \\
& - 19a_0^2a_2a_5^6a_6^3 + 161a_0^2a_3^3a_5a_6^6 + 251a_0^2a_3^2a_4^2a_6^6 + 285a_0^2a_3^2a_4a_5^2a_6^5 \\
& - 133a_0^2a_3^2a_5^4a_6^4 + 421a_0^2a_3a_4^3a_5a_6^5 - 112a_0^2a_3a_4^2a_5^3a_6^4 - 119a_0^2a_3a_4a_5^5a_6^3 \\
& - 3a_0^2a_3a_5^7a_6^2 + 68a_0^2a_4^5a_6^5 - 774a_0^2a_4^4a_5^2a_6^4 + 216a_0^2a_4^3a_5^4a_6^3 \\
& - 18a_0^2a_4^2a_5^6a_6^2 - 188a_0a_1^3a_5a_6^7 - 880a_0a_1^2a_2a_4a_6^7 - 595a_0a_1^2a_2a_5^2a_6^6 \\
& + 88a_0a_1^2a_3^2a_6^7 + 464a_0a_1^2a_3a_4a_5a_6^6 + 197a_0a_1^2a_3a_5^3a_6^5 + 59a_0a_1^2a_4^3a_6^6 \\
& - 5a_0a_1^2a_4^2a_5^2a_6^5 + 136a_0a_1^2a_4a_5^4a_6^4 + 8a_0a_1^2a_5^6a_6^3 - 488a_0a_1a_2^2a_3a_6^7 \\
& + 600a_0a_1a_2^2a_4a_5a_6^6 + 292a_0a_1a_2^2a_5^3a_6^5 + 364a_0a_1a_2a_3^2a_5a_6^6 + 1024a_0a_1a_2a_3a_4^2a_6^6 \\
& + 226a_0a_1a_2a_3a_4a_5^2a_6^5 - 6a_0a_1a_2a_3a_5^4a_6^4 - 414a_0a_1a_2a_4^3a_5a_6^5 - 88a_0a_1a_2a_4^2a_5^3a_6^4 \\
& - 72a_0a_1a_2a_4a_5^5a_6^3 - 4a_0a_1a_2a_5^7a_6^2 - 82a_0a_1a_3^3a_4a_6^6 + 165a_0a_1a_3^3a_5^2a_6^5 \\
& + 51a_0a_1a_3^2a_4^2a_5a_6^5 + 307a_0a_1a_3^2a_4a_5^3a_6^4 + 39a_0a_1a_3^2a_5^5a_6^3 - 34a_0a_1a_3a_4^4a_6^5 \\
& + 316a_0a_1a_3a_4^3a_5^2a_6^4 - 21a_0a_1a_3a_4^2a_5^4a_6^3 + 6a_0a_1a_3a_4a_5^6a_6^2 - 87a_0a_1a_4^5a_5a_6^4 \\
& + 120a_0a_1a_4^4a_5^3a_6^3 - 20a_0a_1a_4^3a_5^5a_6^2 - 152a_0a_2^4a_6^7 - 90a_0a_2^3a_3a_5a_6^6 \\
& + 130a_0a_2^3a_4^2a_6^6 + 236a_0a_2^3a_4a_5^2a_6^5 + 9a_0a_2^3a_5^4a_6^4 - 124a_0a_2^2a_3^2a_4a_6^6 \\
& - 247a_0a_2^2a_3^2a_5^2a_6^5 - 315a_0a_2^2a_3a_4^2a_5a_6^5 - 751a_0a_2^2a_3a_4a_5^3a_6^4 - 2a_0a_2^2a_3a_5^5a_6^3 \\
& - 96a_0a_2^2a_4^4a_6^5 + 138a_0a_2^2a_4^3a_5^2a_6^4 + 31a_0a_2^2a_4^2a_5^4a_6^3 + 10a_0a_2^2a_4a_5^6a_6^2 \\
& + 100a_0a_2a_3^4a_6^6 + 60a_0a_2a_3^3a_4a_5a_6^5 + 186a_0a_2a_3^3a_5^3a_6^4 - 248a_0a_2a_3^2a_4^3a_6^5 \\
& - 68a_0a_2a_3^2a_4^2a_5^2a_6^4 + 42a_0a_2a_3^2a_4a_5^4a_6^3 + 4a_0a_2a_3^2a_5^6a_6^2 + 481a_0a_2a_3a_4^4a_5a_6^4 \\
& - 434a_0a_2a_3a_4^3a_5^3a_6^3 - 5a_0a_2a_3a_4^2a_5^5a_6^2 + 33a_0a_2a_4^6a_6^4 - 152a_0a_2a_4^5a_5^2a_6^3 \\
& + 21a_0a_2a_4^4a_5^4a_6^2 - 21a_0a_3^5a_5a_6^5 - 36a_0a_3^4a_4^2a_6^5 - 113a_0a_3^4a_4a_5^2a_6^4 \\
& - 9a_0a_3^4a_5^4a_6^3 - 74a_0a_3^3a_4^3a_5a_6^4 - 21a_0a_3^3a_4^2a_5^3a_6^3 - 3a_0a_3^3a_4a_5^5a_6^2 \\
& + 15a_0a_3^2a_5^5a_6^4 + 123a_0a_3^2a_4^4a_5^2a_6^3 - a_0a_3^2a_4^3a_5^4a_6^2 - 14a_0a_3a_4^6a_5a_6^3 \\
& - 28a_0a_3a_4^5a_5^3a_6^2 - 4a_0a_4^8a_6^3 - 10a_0a_4^7a_5^2a_6^2 + 124a_1^4a_4a_6^7 \\
& + 62a_1^4a_5^2a_6^6 + 104a_1^3a_2a_3a_6^7 - 138a_1^3a_2a_4a_5a_6^6 - 118a_1^3a_2a_5^3a_6^5
\end{aligned}$$

$$\begin{aligned}
& -94a_1^3a_3^2a_5a_6^6 - 200a_1^3a_3a_4^2a_6^6 - 187a_1^3a_3a_4a_5^2a_6^5 - 19a_1^3a_3a_5^4a_6^4 \\
& + 121a_1^3a_4^3a_5a_6^5 - 65a_1^3a_4^2a_5^3a_6^4 + 8a_1^3a_4a_5^5a_6^3 + 38a_1^2a_2^3a_6^7 \\
& - 11a_1^2a_2^2a_3a_5a_6^6 - 9a_1^2a_2^2a_4^2a_6^6 - 296a_1^2a_2^2a_4a_5^2a_6^5 + 87a_1^2a_2^2a_5^4a_6^4 \\
& - 67a_1^2a_2a_3^2a_4a_6^6 + 113a_1^2a_2a_3^2a_5^2a_6^5 - 11a_1^2a_2a_3a_4^2a_5a_6^5 + 267a_1^2a_2a_3a_4a_5^3a_6^4 \\
& + 14a_1^2a_2a_3a_5^5a_6^3 - 20a_1^2a_2a_4^4a_6^5 - 107a_1^2a_2a_4^3a_5^2a_6^4 + 75a_1^2a_2a_4^2a_5^4a_6^3 \\
& - 4a_1^2a_2a_4a_5^6a_6^2 - 25a_1^2a_3^4a_6^6 + 24a_1^2a_3^3a_4a_5a_6^5 - 22a_1^2a_3^3a_5^3a_6^4 \\
& + 122a_1^2a_3^2a_4^3a_6^5 + 92a_1^2a_3^2a_4^2a_5^4a_6^4 + 19a_1^2a_3^2a_4a_5^4a_6^3 - 56a_1^2a_3a_4^4a_5a_6^4 \\
& + 64a_1^2a_3a_4^3a_5^3a_6^3 + 9a_1^2a_3a_4^2a_5^5a_6^2 + 5a_1^2a_4^6a_6^4 + 60a_1^2a_4^5a_5^2a_6^3 \\
& - 2a_1^2a_4^4a_5^4a_6^2 - 26a_1a_2^4a_5a_6^6 - 104a_1a_2^3a_3a_4a_6^6 + 181a_1a_2^3a_3a_5^2a_6^5 \\
& + 17a_1a_2^3a_4^2a_5a_6^5 + 75a_1a_2^3a_4a_5^3a_6^4 - 28a_1a_2^3a_5^5a_6^3 + 68a_1a_2^2a_3^3a_6^6 \\
& + 142a_1a_2^2a_3^2a_4a_5a_6^5 - 78a_1a_2^2a_3^2a_5^3a_6^4 + 122a_1a_2^2a_3a_4^3a_6^5 + 62a_1a_2^2a_3a_4^2a_5^2a_6^4 \\
& - 74a_1a_2^2a_3a_4a_5^4a_6^3 - 3a_1a_2^2a_3a_5^6a_6^2 + 7a_1a_2^2a_4^4a_5a_6^4 + 5a_1a_2^2a_4^3a_5^3a_6^3 \\
& - 12a_1a_2^2a_4^2a_5^5a_6^2 - 30a_1a_2a_3^4a_5a_6^5 - 24a_1a_2a_3^3a_4^2a_6^5 - 40a_1a_2a_3^3a_4a_5^2a_6^4 \\
& - 5a_1a_2a_3^3a_5^4a_6^3 - 62a_1a_2a_3^2a_4^3a_5^4a_6^4 + 58a_1a_2a_3^2a_4^2a_5^3a_6^3 + 4a_1a_2a_3^2a_4a_5^5a_6^2 \\
& - 27a_1a_2a_3a_5^4a_6^4 - 74a_1a_2a_3a_4^4a_5^2a_6^3 - 3a_1a_2a_3a_4^3a_5^4a_6^2 - 19a_1a_2a_4^6a_5a_6^3 \\
& - 7a_1a_2a_4^5a_5^3a_6^2 + 27a_1a_3^5a_4a_6^5 + 6a_1a_3^5a_5^2a_6^4 - 23a_1a_3^4a_4^2a_5a_6^4 \\
& - 4a_1a_3^4a_4a_5^3a_6^3 - 39a_1a_3^3a_4^4a_6^4 - 43a_1a_3^3a_4^3a_5^2a_6^3 - 3a_1a_3^3a_4^2a_5^4a_6^2 \\
& + 39a_1a_3^2a_4^5a_5a_6^3 + 11a_1a_3^2a_4^4a_5^3a_6^2 + a_1a_3a_4^7a_6^3 + 9a_1a_3a_4^6a_5^2a_6^2 \\
& + 4a_1a_4^8a_5a_6^2 - 8a_2^5a_4a_6^6 + 8a_2^5a_5^2a_6^5 + 66a_2^4a_3^2a_6^6 \\
& - 34a_2^4a_3a_4a_5a_6^5 - 14a_2^4a_3a_5^3a_6^4 + 2a_2^4a_4^3a_6^5 - 79a_2^4a_4^2a_5^2a_6^4 \\
& + 32a_2^4a_4a_5^4a_6^3 + a_2^4a_5^6a_6^2 - 30a_2^3a_3^3a_5a_6^5 - 56a_2^3a_3^2a_4^2a_6^5 \\
& + 40a_2^3a_3^2a_4a_5^2a_6^4 - 6a_2^3a_3^2a_5^4a_6^3 + 14a_2^3a_3a_4^3a_5a_6^4 - 39a_2^3a_3a_4^2a_5^3a_6^3 \\
& - 3a_2^3a_3a_4a_5^5a_6^2 - 14a_2^3a_4^4a_5^2a_6^3 + 12a_2^3a_4^3a_5^4a_6^2 - 12a_2^2a_3^4a_4a_6^5 \\
& - 21a_2^2a_3^4a_5^2a_6^4 + 14a_2^2a_3^3a_4^2a_5a_6^4 + 20a_2^2a_3^3a_4a_5^3a_6^3 + a_2^2a_3^2a_4^4a_6^4 \\
& + 71a_2^2a_3^2a_4^3a_5^2a_6^3 + 4a_2^2a_3^2a_4^2a_5^4a_6^2 + 32a_2^2a_3a_4^5a_5a_6^3 - 16a_2^2a_3a_4^4a_5^3a_6^2 \\
& + 8a_2^2a_4^6a_5^2a_6^2 - 10a_2a_3^6a_6^5 + 23a_2a_3^5a_4a_5a_6^4 + 25a_2a_3^4a_4^3a_6^4 \\
& - 14a_2a_3^4a_4^2a_5^3a_6^3 - 41a_2a_3^3a_4^4a_5a_6^3 + a_2a_3^3a_4^3a_5^3a_6^2 + 2a_2a_3^2a_4^6a_6^3 \\
& + 8a_2a_3^2a_4^5a_5^2a_6^2 - 8a_2a_3a_4^7a_5a_6^2 - 3a_3^6a_4^4a_6^4 + 6a_3^5a_4^3a_5a_6^3 \\
& + 3a_3^4a_4^5a_6^3 - 3a_3^4a_4^4a_5^2a_6^2 - 4a_3^3a_4^6a_5a_6^2 - 116a_0^3a_1a_6^7 \\
& - 742a_0^3a_2a_5a_6^6 + 608a_0^3a_3a_4a_6^6 - 875a_0^3a_3a_5^2a_6^5 - 967a_0^3a_4^2a_5a_6^5 \\
& - 1755a_0^3a_4a_5^3a_6^4 + 20a_0^3a_5^5a_6^3 + 128a_0^2a_1^2a_5a_6^6 - 1488a_0^2a_1a_2a_4a_6^6 \\
& + 173a_0^2a_1a_2a_5^2a_6^5 + 34a_0^2a_1a_3^2a_6^6 + 1182a_0^2a_1a_3a_4a_5a_6^5 + 254a_0^2a_1a_3a_5^3a_6^4 \\
& + 431a_0^2a_1a_4^3a_6^5 + 547a_0^2a_1a_4^2a_5^2a_6^4 + 106a_0^2a_1a_4a_5^4a_6^3 + 7a_0^2a_1a_5^6a_6^2 \\
& - 628a_0^2a_2^2a_3a_6^6 - 2822a_0^2a_2^2a_4a_5a_6^5 - 327a_0^2a_2^2a_5^3a_6^4 + 1517a_0^2a_2a_3^2a_5a_6^5 \\
& + 2072a_0^2a_2a_3a_4^2a_6^5 + 1737a_0^2a_2a_3a_4a_5^2a_6^4 + 107a_0^2a_2a_3a_5^4a_6^3 + 229a_0^2a_2a_4^3a_5a_6^4
\end{aligned}$$

$$\begin{aligned}
& -966a_0^2a_2a_4^2a_5^3a_6^3 + 17a_0^2a_2a_4a_5^5a_6^2 - 2a_0^2a_2a_5^7a_6 - 351a_0^2a_3^3a_4a_5^5 \\
& + 51a_0^2a_3^3a_5^2a_6^4 - 579a_0^2a_3^2a_4^2a_5a_6^4 + 347a_0^2a_3^2a_4a_5^3a_6^3 + 28a_0^2a_3^2a_5^5a_6^2 \\
& - 400a_0^2a_3a_4^4a_6^4 + 230a_0^2a_3a_4^3a_5^2a_6^3 + 100a_0^2a_3a_4^2a_5^4a_6^2 + 6a_0^2a_3a_4a_5^6a_6 \\
& + 472a_0^2a_4^5a_5a_6^3 - 194a_0^2a_4^4a_5^3a_6^2 + 8a_0^2a_4^3a_5^5a_6 + 356a_0a_1^3a_4a_6^6 \\
& + 216a_0a_1^3a_5^2a_6^5 + 844a_0a_1^2a_2a_3a_6^6 + 1504a_0a_1^2a_2a_4a_5a_6^5 - 255a_0a_1^2a_2a_5^3a_6^4 \\
& - 364a_0a_1^2a_3^2a_5a_6^5 - 790a_0a_1^2a_3a_4^2a_6^5 - 796a_0a_1^2a_3a_4a_5^2a_6^4 - 113a_0a_1^2a_3a_5^4a_6^3 \\
& - 25a_0a_1^2a_4^3a_5a_6^4 - 163a_0a_1^2a_4^2a_5^3a_6^3 - 16a_0a_1^2a_4a_5^5a_6^2 - 4a_0a_1^2a_5^7a_6 \\
& + 436a_0a_1a_2^3a_6^6 - 848a_0a_1a_2^2a_3a_5a_6^5 - 952a_0a_1a_2^2a_4^2a_6^5 - 538a_0a_1a_2^2a_4a_5^2a_6^4 \\
& + 113a_0a_1a_2^2a_5^4a_6^3 - 798a_0a_1a_2a_3^2a_4a_6^5 - 165a_0a_1a_2a_3^2a_5^2a_6^4 - 22a_0a_1a_2a_3a_4^2a_5a_6^4 \\
& + 494a_0a_1a_2a_3a_4a_5^3a_6^3 + 34a_0a_1a_2a_3a_5^5a_6^2 + 432a_0a_1a_2a_4^4a_6^4 + 72a_0a_1a_2a_4^3a_5^2a_6^3 \\
& + 131a_0a_1a_2a_4^2a_5^4a_6^2 + 14a_0a_1a_2a_4a_5^6a_6 + 24a_0a_1a_3^4a_6^5 - 144a_0a_1a_3^3a_4a_5a_6^4 \\
& - 110a_0a_1a_3^3a_5^3a_6^3 + 275a_0a_1a_3^2a_4^3a_6^4 - 161a_0a_1a_3^2a_4^2a_5^2a_6^3 - 49a_0a_1a_3^2a_4a_5^4a_6^2 \\
& + a_0a_1a_3^2a_5^6a_6 - 398a_0a_1a_3a_4^4a_5a_6^3 + 76a_0a_1a_3a_4^3a_5^3a_6^2 - 2a_0a_1a_3a_4^2a_5^5a_6 \\
& - 20a_0a_1a_4^6a_6^3 - 54a_0a_1a_4^5a_5^2a_6^2 + 19a_0a_1a_4^4a_5^4a_6 - 70a_0a_2^4a_5a_6^5 \\
& + 596a_0a_2^3a_3a_4a_6^5 + 251a_0a_2^3a_3a_5^2a_6^4 - 340a_0a_2^3a_4^2a_5a_6^4 - 164a_0a_2^3a_4a_5^3a_6^3 \\
& - 15a_0a_2^3a_5^5a_6^2 - 12a_0a_2^2a_3^3a_6^5 + 718a_0a_2^2a_3^2a_4a_5a_6^4 + 68a_0a_2^2a_3^2a_5^3a_6^3 \\
& - 147a_0a_2^2a_3a_4^3a_6^4 + 983a_0a_2^2a_3a_4^2a_5^2a_6^3 - 73a_0a_2^2a_3a_4a_5^4a_6^2 + 2a_0a_2^2a_3a_5^6a_6 \\
& - 16a_0a_2^2a_4^4a_5a_6^3 - 131a_0a_2^2a_4^3a_5^3a_6^2 - 14a_0a_2^2a_4^2a_5^5a_6 - 213a_0a_2^2a_4^2a_5^4a_6^4 \\
& + 12a_0a_2^2a_4^3a_5^2a_6^4 - 280a_0a_2^2a_4^3a_5^2a_6^3 - 37a_0a_2^2a_4^3a_5^2a_6^2 - 20a_0a_2^2a_4^3a_5^2a_6^3 \\
& + 83a_0a_2^2a_4^2a_5^3a_6^2 - 8a_0a_2^2a_4^2a_5^5a_6 - 122a_0a_2^2a_4^2a_5^4a_6^3 + 276a_0a_2^2a_4^2a_5^4a_6^2 \\
& + 6a_0a_2^2a_4^2a_5^4a_6^2 + 88a_0a_2^2a_4^2a_5^4a_6^2 - 26a_0a_2^2a_4^2a_5^4a_6^2 + 54a_0a_2^2a_4^2a_5^4a_6^2 \\
& + 24a_0a_2^2a_4^2a_5^4a_6^2 + 122a_0a_2^2a_4^2a_5^4a_6^2 + 24a_0a_2^2a_4^2a_5^4a_6^2 + 9a_0a_2^2a_4^2a_5^4a_6^2 \\
& - 20a_0a_2^2a_4^2a_5^4a_6^2 + 3a_0a_2^2a_4^2a_5^4a_6^2 - 39a_0a_2^2a_4^2a_5^4a_6^2 + a_0a_2^2a_4^2a_5^4a_6^2 \\
& + 12a_0a_2^2a_4^2a_5^4a_6^2 + 21a_0a_2^2a_4^2a_5^4a_6^2 - a_0a_2^2a_4^2a_5^4a_6^2 - 144a_1^4a_3a_6^6 \\
& - 180a_1^4a_4a_5a_6^5 + 14a_1^4a_5^3a_6^4 - 98a_1^3a_2^2a_6^6 + 252a_1^3a_2a_3a_5a_6^5 \\
& + 182a_1^3a_2a_4^2a_6^5 + 413a_1^3a_2a_4a_5^2a_6^4 - 9a_1^3a_2a_5^4a_6^3 + 296a_1^3a_3^2a_4a_6^5 \\
& + 102a_1^3a_3^2a_5^2a_6^4 + 150a_1^3a_3a_4^2a_5a_6^4 + 4a_1^3a_3a_5^5a_6^2 - 65a_1^3a_4^4a_6^4 \\
& - 25a_1^3a_4^3a_5^2a_6^3 - 27a_1^3a_4^2a_5^4a_6^2 - 4a_1^3a_4a_5^6a_6 + 61a_1^2a_2^3a_5a_6^5 \\
& + 43a_1^2a_2^2a_3a_4a_6^5 - 280a_1^2a_2^2a_3a_5^2a_6^4 + 255a_1^2a_2^2a_4^2a_5a_6^4 - 265a_1^2a_2^2a_4a_5^3a_6^3 \\
& - 48a_1^2a_2^2a_3^3a_6^5 - 392a_1^2a_2^2a_3^2a_4a_5a_6^4 - 84a_1^2a_2^2a_3^2a_5^3a_6^3 - 107a_1^2a_2^2a_3a_4^3a_6^4 \\
& - 263a_1^2a_2^2a_3a_4^2a_5a_6^3 - 14a_1^2a_2^2a_3a_4a_5^4a_6^2 + 50a_1^2a_2^2a_4^4a_5a_6^3 + 12a_1^2a_2^2a_4^3a_5^2a_6^2 \\
& + 12a_1^2a_2^2a_4^2a_5^5a_6 + 24a_1^2a_2^2a_4^2a_5^4a_6^4 - 138a_1^2a_2^2a_4^2a_5^4a_6^4 - 30a_1^2a_2^2a_4^2a_5^4a_6^3 \\
& - a_1^2a_2^2a_4^2a_5^4a_6^2 + 20a_1^2a_2^2a_4^2a_5^4a_6^2 - 26a_1^2a_2^2a_4^2a_5^4a_6^2 + a_1^2a_2^2a_4^2a_5^4a_6^2 \\
& + 42a_1^2a_2^2a_4^2a_5^4a_6^2 - 45a_1^2a_2^2a_4^2a_5^4a_6^2 - 12a_1^2a_2^2a_4^2a_5^4a_6^2 - 39a_1^2a_2^2a_4^2a_5^4a_6^2 \\
& + 3a_1^2a_2^2a_4^2a_5^4a_6^2 + 48a_1^2a_2^2a_4^2a_5^4a_6^2 - 64a_1^2a_2^2a_4^2a_5^4a_6^2 - 144a_1^2a_2^2a_4^2a_5^4a_6^2
\end{aligned}$$

$$\begin{aligned}
& -156a_1a_2^3a_3a_4a_5a_6^4 + 130a_1a_2^3a_3a_5^3a_6^3 - 51a_1a_2^3a_4^3a_6^4 + 71a_1a_2^3a_4^2a_5^2a_6^3 \\
& + 57a_1a_2^3a_4a_5^4a_6^2 - a_1a_2^3a_5^6a_6 + 92a_1a_2^2a_3^3a_5a_6^4 + 116a_1a_2^2a_3^2a_4^2a_6^4 \\
& + 122a_1a_2^2a_3^2a_4a_5^2a_6^3 + 27a_1a_2^2a_3^2a_5^4a_6^2 - 76a_1a_2^2a_3a_4^3a_5a_6^3 + 36a_1a_2^2a_3a_4^2a_5^3a_6^2 \\
& + 5a_1a_2^2a_3a_4a_5^5a_6 + 14a_1a_2^2a_4^5a_6^3 - 23a_1a_2^2a_4^4a_5^2a_6^2 - 4a_1a_2^2a_4^3a_5^4a_6 \\
& + 51a_1a_2a_3^4a_4a_6^4 + 32a_1a_2a_3^4a_5^2a_6^3 - 20a_1a_2a_3^3a_4a_5^3a_6^2 + 7a_1a_2a_3^2a_4^4a_6^3 \\
& - 43a_1a_2a_3^2a_4^3a_5^2a_6^2 - 7a_1a_2a_3^2a_4^2a_5^4a_6 + 44a_1a_2a_3a_4^5a_5a_6^2 + 16a_1a_2a_3a_4^4a_5^3a_6 \\
& + 3a_1a_2a_4^7a_6^2 - 8a_1a_3^6a_6^4 + 30a_1a_3^4a_4^3a_6^3 + 12a_1a_3^4a_4^2a_5^2a_6^2 \\
& + 10a_1a_3^3a_4^4a_5a_6^2 + 4a_1a_3^3a_4^3a_5^3a_6 - 15a_1a_3^2a_4^6a_6^2 - 11a_1a_3^2a_4^5a_5^2a_6 \\
& - a_1a_3a_4^7a_5a_6 - a_1a_4^9a_6 - 36a_2^5a_3a_6^5 + 20a_2^5a_4a_5a_6^4 \\
& - 13a_2^5a_5^3a_6^3 + 15a_2^4a_3^2a_5a_6^4 + 62a_2^4a_3a_4^2a_6^4 - 39a_2^4a_3a_4a_5^2a_6^3 \\
& - 9a_2^4a_3a_4^4a_6^2 + 44a_2^4a_3^3a_5a_6^3 - 54a_2^4a_4^2a_5^3a_6^2 - 38a_2^3a_3^3a_4a_6^4 \\
& + 2a_2^3a_3^3a_5^2a_6^3 - 32a_2^3a_3^2a_4^2a_5a_6^3 + 24a_2^3a_3^2a_4a_5^3a_6^2 - 22a_2^3a_3a_4^4a_6^3 \\
& + 59a_2^3a_3a_4^3a_5^2a_6^2 - a_2^3a_3a_4^2a_5^4a_6 - 4a_2^3a_4^5a_5a_6^2 - 3a_2^3a_4^4a_5^3a_6 \\
& + 21a_2^2a_3^5a_6^4 - 2a_2^2a_3^4a_4a_5a_6^3 - 4a_2^2a_3^3a_4^3a_6^3 - 44a_2^2a_3^3a_4^2a_5^2a_6^2 \\
& - 32a_2^2a_3^2a_4^4a_5a_6^2 + 3a_2^2a_3^2a_4^3a_5^3a_6 - 6a_2^2a_3a_4^6a_6^2 + 3a_2^2a_3a_4^5a_5^2a_6 \\
& - 2a_2^2a_4^7a_5a_6 - 20a_2a_3^5a_4^2a_6^3 + 26a_2a_3^4a_4^3a_5a_6^2 + 11a_2a_3^3a_4^5a_6^2 \\
& - 5a_2a_3^3a_4^4a_5^2a_6 - a_2a_3^2a_4^6a_5a_6 + 2a_2a_3a_4^8a_6 - 3a_3^5a_4^4a_6^2 \\
& + 3a_3^4a_4^5a_5a_6 + a_3^3a_4^7a_6 + 71a_0^4a_6^6 - 436a_0^3a_1a_5a_6^5 \\
& + 1600a_0^3a_2a_4a_6^5 + 1379a_0^3a_2a_5^2a_6^4 - 252a_0^3a_3^2a_6^5 + 1724a_0^3a_3a_4a_5a_6^4 \\
& + 155a_0^3a_3a_5^3a_6^3 - 175a_0^3a_4^3a_6^4 + 2755a_0^3a_4^2a_5^2a_6^3 + 18a_0^3a_4a_5^4a_6^2 \\
& - 3a_0^3a_5^6a_6 - 156a_0^2a_1^2a_4a_6^5 - 242a_0^2a_1^2a_5^2a_6^4 + 192a_0^2a_1a_2a_3a_6^5 \\
& + 542a_0^2a_1a_2a_4a_5a_6^4 - 46a_0^2a_1a_2a_3^3a_6^3 - 370a_0^2a_1a_3^2a_5a_6^4 - 604a_0^2a_1a_3a_4^2a_6^4 \\
& - 1051a_0^2a_1a_3a_4a_5^2a_6^3 - 90a_0^2a_1a_3a_5^4a_6^2 - 333a_0^2a_1a_4^3a_5a_6^3 - 162a_0^2a_1a_4^2a_5^3a_6^2 \\
& - 31a_0^2a_1a_4a_5^5a_6 - 5a_0^2a_1a_5^7 + 606a_0^2a_2^3a_6^5 - 29a_0^2a_2^2a_3a_5a_6^4 \\
& + 2255a_0^2a_2^2a_4^2a_6^4 + 1054a_0^2a_2^2a_4a_5^2a_6^3 + 24a_0^2a_2^2a_5^4a_6^2 - 2431a_0^2a_2a_3^2a_4a_6^4 \\
& - 668a_0^2a_2a_3^2a_5^2a_6^3 - 1875a_0^2a_2a_3a_4^2a_5a_6^3 + 45a_0^2a_2a_3a_4a_5^3a_6^2 + 11a_0^2a_2a_3a_5^5a_6 \\
& - 812a_0^2a_2a_4^4a_6^3 + 1142a_0^2a_2a_4^3a_5^2a_6^2 + 8a_0^2a_2a_4^2a_5^4a_6 + 12a_0^2a_2a_4a_5^6 \\
& + 120a_0^2a_3^4a_6^4 - 30a_0^2a_3^3a_4a_5a_6^3 - 74a_0^2a_3^3a_5^3a_6^2 + 785a_0^2a_3^2a_4^3a_6^3 \\
& - 246a_0^2a_3^2a_4^2a_5^2a_6^2 - 49a_0^2a_3^2a_4a_5^4a_6 + a_0^2a_3^2a_5^6 - 312a_0^2a_3a_4^4a_5a_6^2 \\
& - 42a_0^2a_3a_4^3a_5^3a_6 - 5a_0^2a_3a_4^2a_5^5 - 34a_0^2a_4^6a_6^2 + 106a_0^2a_4^5a_5^2a_6 \\
& + 6a_0^2a_4^4a_5^4 - 168a_0a_1^3a_3a_6^5 - 700a_0a_1^3a_4a_5a_6^4 + 24a_0a_1^3a_5^3a_6^3 \\
& - 948a_0a_1^2a_2^2a_6^5 + 188a_0a_1^2a_2a_3a_5a_6^4 - 638a_0a_1^2a_2a_4^2a_6^4 + 682a_0a_1^2a_2a_4a_5^2a_6^3 \\
& - 3a_0a_1^2a_2a_5^4a_6^2 + 788a_0a_1^2a_3^2a_4a_6^4 + 392a_0a_1^2a_3^2a_5^2a_6^3 + 1148a_0a_1^2a_3a_4^2a_5^3a_6^3 \\
& + 264a_0a_1^2a_3a_4a_5^3a_6^2 + 51a_0a_1^2a_3a_5^5a_6 + 58a_0a_1^2a_4^4a_6^3 + 23a_0a_1^2a_4^3a_5^2a_6^2 \\
& - 26a_0a_1^2a_4^2a_5^4a_6 - 2a_0a_1^2a_4a_5^6 + 660a_0a_1a_2^3a_5a_6^4 + 882a_0a_1a_2^2a_3a_4a_6^4
\end{aligned}$$

$$\begin{aligned}
& -409a_0a_1a_2^2a_3a_5^2a_6^3 + 203a_0a_1a_2^2a_4^2a_5a_6^3 - 237a_0a_1a_2^2a_4a_5^3a_6^2 - 25a_0a_1a_2^2a_5^5a_6 \\
& + 420a_0a_1a_2a_3^3a_6^4 - 356a_0a_1a_2a_3^2a_4a_5a_6^3 - 50a_0a_1a_2a_3^2a_5^3a_6^2 + 108a_0a_1a_2a_3a_4^3a_6^3 \\
& - 971a_0a_1a_2a_3a_4^2a_5^2a_6^2 - 132a_0a_1a_2a_3a_4a_5^4a_6 + a_0a_1a_2a_3a_5^6 - 47a_0a_1a_2a_4^4a_5a_6^2 \\
& - 32a_0a_1a_2a_4^3a_5^3a_6 - a_0a_1a_2a_4^2a_5^5 + 72a_0a_1a_3^4a_5a_6^3 - 384a_0a_1a_3^3a_4^2a_6^3 \\
& + 56a_0a_1a_3^3a_4a_5^2a_6^2 - 12a_0a_1a_3^3a_5^4a_6 + 272a_0a_1a_3^2a_4^3a_5a_6^2 + 34a_0a_1a_3^2a_4^2a_5^3a_6 \\
& + 107a_0a_1a_3a_4^5a_6^2 - 95a_0a_1a_3a_4^4a_5^2a_6 - 7a_0a_1a_3a_4^3a_5^4 + 10a_0a_1a_4^6a_5a_6 \\
& - 9a_0a_1a_4^5a_5^3 - 248a_0a_2^4a_4a_6^4 + 43a_0a_2^4a_5^2a_6^3 - 332a_0a_2^3a_3^2a_6^4 \\
& - 372a_0a_2^3a_3a_4a_5a_6^3 + 118a_0a_2^3a_3a_5^3a_6^2 + 294a_0a_2^3a_4^3a_6^3 + 311a_0a_2^3a_4^2a_5^2a_6^2 \\
& + 40a_0a_2^3a_4a_5^4a_6 + a_0a_2^3a_5^6 - 58a_0a_2^2a_3^3a_5a_6^3 - 336a_0a_2^2a_3^2a_4^2a_6^3 \\
& - 82a_0a_2^2a_3^2a_4a_5^2a_6^2 - 11a_0a_2^2a_3^2a_5^4a_6 - 586a_0a_2^2a_3a_4^3a_5a_6^2 + 99a_0a_2^2a_3a_4^2a_5^3a_6 \\
& - 5a_0a_2^2a_3a_4a_5^5 - 46a_0a_2^2a_4^5a_6^2 + 106a_0a_2^2a_4^4a_5^2a_6 + 2a_0a_2^2a_4^3a_5^4 \\
& + 264a_0a_2a_3^4a_4a_6^3 + 104a_0a_2a_3^4a_5^2a_6^2 + 102a_0a_2a_3^3a_4^2a_5a_6^2 + 64a_0a_2a_3^3a_4a_5^3a_6 \\
& + 37a_0a_2a_3^2a_4^4a_6^2 - 108a_0a_2a_3^2a_4^3a_5^2a_6 + 5a_0a_2a_3^2a_4^2a_5^4 - 86a_0a_2a_3a_4^5a_5a_6 \\
& + 2a_0a_2a_3a_4^4a_5^3 - 16a_0a_2a_4^7a_6 + 8a_0a_2a_4^6a_5^2 - 16a_0a_3^6a_6^3 \\
& - 48a_0a_3^5a_4a_5a_6^2 - 60a_0a_3^4a_4^3a_6^2 - 24a_0a_3^4a_4^2a_5^2a_6 + 23a_0a_3^3a_4^4a_5a_6 \\
& - 3a_0a_3^2a_4^6a_6 - a_0a_3^2a_4^5a_5^2 - 5a_0a_3a_4^7a_5 + a_0a_4^9 \\
& + 168a_1^4a_2a_6^5 + 8a_1^4a_3a_5a_6^4 + 104a_1^4a_4^2a_6^4 - 100a_1^4a_4a_5^2a_6^3 \\
& - 9a_1^4a_5^4a_6^2 - 214a_1^3a_2^2a_5a_6^4 - 468a_1^3a_2a_3a_4a_6^4 + 20a_1^3a_2a_3a_5^2a_6^3 \\
& - 408a_1^3a_2a_4^2a_5a_6^3 + 48a_1^3a_2a_4a_5^3a_6^2 + 6a_1^3a_2a_5^5a_6 - 120a_1^3a_3^3a_6^4 \\
& - 112a_1^3a_3^2a_4a_5a_6^3 - 32a_1^3a_3^2a_5^3a_6^2 - 56a_1^3a_3a_4^3a_6^3 + 106a_1^3a_3a_4^2a_5^2a_6^2 \\
& + 35a_1^3a_3a_4a_5^4a_6 + 96a_1^3a_4^4a_5a_6^2 + 16a_1^3a_4^3a_5^3a_6 + 3a_1^3a_4^2a_5^5 \\
& - 49a_1^2a_2^3a_4a_6^4 + 117a_1^2a_2^3a_5^2a_6^3 + 242a_1^2a_2^2a_3^2a_6^4 + 580a_1^2a_2^2a_3a_4a_5a_6^3 \\
& + 26a_1^2a_2^2a_3a_5^3a_6^2 - 19a_1^2a_2^2a_4^3a_6^3 + 166a_1^2a_2^2a_4^2a_5^2a_6^2 - 25a_1^2a_2^2a_4a_5^4a_6 \\
& - a_1^2a_2^2a_5^6 + 112a_1^2a_2a_3^3a_5a_6^3 + 316a_1^2a_2a_3^2a_4^2a_6^3 + 154a_1^2a_2a_3^2a_4a_5^2a_6^2 \\
& - 2a_1^2a_2a_3^2a_5^4a_6 - 4a_1^2a_2a_3a_4^3a_5a_6^2 - 28a_1^2a_2a_3a_4^2a_5^3a_6 + a_1^2a_2a_3a_4a_5^5 \\
& - 33a_1^2a_2a_4^5a_6^2 - 33a_1^2a_2a_4^4a_5^2a_6 - 7a_1^2a_2a_4^3a_5^4 + 72a_1^2a_3^4a_4a_6^3 \\
& + 8a_1^2a_3^4a_5^2a_6^2 - 40a_1^2a_3^3a_4^2a_5a_6^2 - 8a_1^2a_3^3a_4a_5^3a_6 - 25a_1^2a_3^2a_4^4a_6^2 \\
& + 42a_1^2a_3^2a_4^3a_5^2a_6 - a_1^2a_3^2a_4^2a_5^4 + 20a_1^2a_3a_4^5a_5a_6 + 6a_1^2a_3a_4^4a_5^3 \\
& + 12a_1^2a_4^7a_6 + 108a_1a_2^4a_3a_6^4 + 15a_1a_2^4a_4a_5a_6^3 - 40a_1a_2^4a_5^3a_6^2 \\
& - 148a_1a_2^3a_3^2a_5a_6^3 - 140a_1a_2^3a_3a_4^2a_6^3 - 216a_1a_2^3a_3a_4a_5^2a_6^2 + 5a_1a_2^3a_3a_5^4a_6 \\
& - 66a_1a_2^3a_3^3a_5a_6^2 - 8a_1a_2^3a_4^2a_5^3a_6 + 2a_1a_2^3a_4a_5^5 - 178a_1a_2^2a_3^3a_4a_6^3 \\
& - 84a_1a_2^2a_3^3a_5^2a_6^2 + 66a_1a_2^2a_3^2a_4^2a_5a_6^2 - 28a_1a_2^2a_3^2a_4a_5^3a_6 + 91a_1a_2^2a_3a_4^4a_6^2 \\
& - 17a_1a_2^2a_3a_4^3a_5^2a_6 - 4a_1a_2^2a_3a_4^2a_5^4 + 20a_1a_2^2a_4^5a_5a_6 + 5a_1a_2^2a_4^4a_5^3 \\
& - 48a_1a_2a_3^5a_6^3 + 16a_1a_2a_3^4a_4a_5a_6^2 - 14a_1a_2a_3^3a_4^3a_6^2 + 32a_1a_2a_3^3a_4^2a_5^2a_6 \\
& + 10a_1a_2a_3^2a_4^4a_5a_6 + 4a_1a_2a_3^2a_4^3a_5^3 - 17a_1a_2a_3a_4^6a_6 - 8a_1a_2a_3a_4^5a_5^2
\end{aligned}$$

$$\begin{aligned}
& -16a_1a_3^4a_5a_6 + 3a_1a_3^3a_4^5a_6 - 2a_1a_3^3a_4^4a_5^2 + 3a_1a_3^2a_4^6a_5 \\
& + 7a_2^6a_6^4 + 9a_2^5a_3a_5a_6^3 - 26a_2^5a_4^2a_6^3 + 55a_2^5a_4a_5^2a_6^2 \\
& + 73a_2^4a_3^2a_4a_6^3 + 22a_2^4a_3^2a_5^2a_6^2 + 16a_2^4a_3a_4^2a_5a_6^2 + 4a_2^4a_3a_4a_5^3a_6 \\
& - 2a_2^4a_4^4a_6^2 + 22a_2^4a_4^3a_5^2a_6 - a_2^4a_4^2a_5^4 + 4a_2^3a_3^4a_6^3 \\
& - 18a_2^3a_3^3a_4a_5a_6^2 - 16a_2^3a_3^2a_4^3a_6^2 - 10a_2^3a_3^2a_4^2a_5^2a_6 - 26a_2^3a_3a_4^4a_5a_6 \\
& + 3a_2^3a_3a_4^3a_5^3 + 2a_2^3a_4^6a_6 - a_2^3a_4^5a_5^2 + 22a_2^2a_3^4a_4^2a_6^2 \\
& + 18a_2^2a_3^3a_4^3a_5a_6 + 8a_2^2a_3^2a_4^5a_6 - 4a_2^2a_3^2a_4^4a_5^2 + 2a_2^2a_3a_4^6a_5 \\
& - 10a_2a_3^4a_4^4a_6 + 3a_2a_3^3a_4^5a_5 - a_2a_3^2a_4^7 - a_3^4a_4^6 \\
& - 1130a_0^4a_5a_6^4 + 92a_0^3a_1a_4a_6^4 - 964a_0^3a_1a_5^2a_6^3 - 564a_0^3a_2a_3a_6^4 \\
& - 2772a_0^3a_2a_4a_5a_6^3 + 115a_0^3a_2a_5^3a_6^2 - 60a_0^3a_3^2a_5a_6^3 - 158a_0^3a_3a_4^2a_6^3 \\
& - 870a_0^3a_3a_4a_5^2a_6^2 + 36a_0^3a_3a_5^4a_6 - 2123a_0^3a_4^3a_5a_6^2 - 78a_0^3a_4^2a_5^3a_6 \\
& - 15a_0^3a_4a_5^5 + 240a_0^2a_1^2a_3a_6^4 + 292a_0^2a_1^2a_4a_5a_6^3 + 194a_0^2a_1^2a_5^3a_6^2 \\
& - 2442a_0^2a_1a_2^2a_6^4 + 1524a_0^2a_1a_2a_3a_5a_6^3 - 414a_0^2a_1a_2a_4^2a_6^3 - 223a_0^2a_1a_2a_4a_5^2a_6^2 \\
& + 30a_0^2a_1a_2a_5^4a_6 + 236a_0^2a_1a_3^2a_4a_6^3 + 184a_0^2a_1a_3^2a_5^2a_6^2 + 1306a_0^2a_1a_3a_4^2a_5a_6^2 \\
& + 390a_0^2a_1a_3a_4a_5^3a_6 + 62a_0^2a_1a_3a_5^5 + 283a_0^2a_1a_4^4a_6^2 - 6a_0^2a_1a_4^3a_5^2a_6 \\
& - 19a_0^2a_1a_4^2a_5^4 + 111a_0^2a_2^3a_5a_6^3 + 331a_0^2a_2^2a_3a_4a_6^3 - 545a_0^2a_2^2a_3a_5^2a_6^2 \\
& - 1646a_0^2a_2^2a_4^2a_5a_6^2 - 42a_0^2a_2^2a_4a_5^3a_6 - 15a_0^2a_2^2a_5^5 + 648a_0^2a_2a_3^3a_6^3 \\
& + 1226a_0^2a_2a_3^2a_4a_5a_6^2 + 20a_0^2a_2a_3^2a_5^3a_6 + 1447a_0^2a_2a_3a_4^3a_6^2 - 388a_0^2a_2a_3a_4^2a_5^2a_6 \\
& - 111a_0^2a_2a_3a_4a_5^4 - 549a_0^2a_2a_4^4a_5a_6 + 34a_0^2a_2a_4^3a_5^3 + 40a_0^2a_3^4a_5a_6^2 \\
& - 472a_0^2a_3^3a_4^2a_6^2 + 80a_0^2a_3^3a_4a_5^2a_6 - 12a_0^2a_3^3a_5^4 + 278a_0^2a_3^2a_4^3a_5a_6 \\
& + 46a_0^2a_3^2a_4^2a_5^3 - 91a_0^2a_3a_4^5a_6 - 70a_0^2a_3a_4^4a_5^2 + 5a_0^2a_4^6a_5 \\
& + 1624a_0a_1^3a_2a_6^4 - 112a_0a_1^3a_3a_5a_6^3 + 240a_0a_1^3a_4^2a_6^3 - 212a_0a_1^3a_4a_5^2a_6^2 \\
& - 36a_0a_1^3a_5^4a_6 - 1064a_0a_1^2a_2^2a_5a_6^3 - 1120a_0a_1^2a_2a_3a_4a_6^3 - 112a_0a_1^2a_2a_3a_5^2a_6^2 \\
& + 216a_0a_1^2a_2a_4^2a_5a_6^2 + 132a_0a_1^2a_2a_4a_5^3a_6 + 3a_0a_1^2a_2a_5^5 - 240a_0a_1^2a_3^3a_6^3 \\
& - 832a_0a_1^2a_3^2a_4a_5a_6^2 - 216a_0a_1^2a_3^2a_5^3a_6 - 656a_0a_1^2a_3a_4^3a_6^2 + 60a_0a_1^2a_3a_4^2a_5^2a_6 \\
& + 22a_0a_1^2a_3a_4a_5^4 - 32a_0a_1^2a_4^4a_5a_6 + 8a_0a_1^2a_4^3a_5^3 - 98a_0a_1a_2^3a_4a_6^3 \\
& + 241a_0a_1a_2^3a_5^2a_6^2 + 28a_0a_1a_2^2a_3^3a_6^3 + 1192a_0a_1a_2^2a_3a_4a_5a_6^2 + 230a_0a_1a_2^2a_3a_5^3a_6 \\
& - 423a_0a_1a_2^2a_4^2a_6^2 - 110a_0a_1a_2^2a_4^2a_5^2a_6 + 5a_0a_1a_2^2a_4a_5^4 - 88a_0a_1a_2a_3^3a_5a_6^2 \\
& + 384a_0a_1a_2a_3^2a_4^2a_6^2 + 288a_0a_1a_2a_3^2a_4a_5^2a_6 - 12a_0a_1a_2a_3^2a_5^4 + 460a_0a_1a_2a_3a_4^3a_5a_6 \\
& + 26a_0a_1a_2a_3a_4^2a_5^3 + 134a_0a_1a_2a_4^5a_6 + 19a_0a_1a_2a_4^4a_5^2 + 176a_0a_1a_3^4a_4a_6^2 \\
& + 48a_0a_1a_3^4a_5^2a_6 - 104a_0a_1a_3^3a_4^2a_5a_6 - 4a_0a_1a_3^2a_4^4a_6 + 48a_0a_1a_3^2a_4^3a_5^2 \\
& + 28a_0a_1a_3a_4^5a_5 - 12a_0a_1a_4^7 + 144a_0a_2^4a_3a_6^3 - 289a_0a_2^4a_4a_5a_6^2 \\
& - 17a_0a_2^4a_5^3a_6 - 34a_0a_2^3a_3^2a_5a_6^2 + 312a_0a_2^3a_3a_4^2a_6^2 - 336a_0a_2^3a_3a_4a_5^2a_6 \\
& - 7a_0a_2^3a_3a_5^4 - 104a_0a_2^3a_4^3a_5a_6 - a_0a_2^3a_4^2a_5^3 - 92a_0a_2^2a_3^3a_4a_6^2 \\
& - 8a_0a_2^2a_3^3a_5^2a_6 + 230a_0a_2^2a_3^2a_4^2a_5a_6 + 40a_0a_2^2a_3^2a_4a_5^3 + 67a_0a_2^2a_3a_4^4a_6
\end{aligned}$$

$$\begin{aligned}
& -48a_0a_2^2a_3a_4^3a_5^2 - 36a_0a_2^2a_4^5a_5 - 80a_0a_2a_3^5a_6^2 - 128a_0a_2a_3^4a_4a_5a_6 \\
& -40a_0a_2a_3^3a_4^3a_6 - 40a_0a_2a_3^3a_4^2a_5^2 + 27a_0a_2a_3^2a_4^4a_5 + 19a_0a_2a_3a_4^6 \\
& + 48a_0a_3^5a_4^2a_6 - 2a_0a_3^3a_4^5 - 208a_1^5a_6^4 + 224a_1^4a_2a_5a_6^3 \\
& + 256a_1^4a_3a_4a_6^3 + 72a_1^4a_3a_5^2a_6^2 - 16a_1^4a_4^2a_5a_6^2 + 196a_1^3a_2^2a_4a_6^3 \\
& - 98a_1^3a_2^2a_5^2a_6^2 - 32a_1^3a_2a_3^2a_6^3 - 112a_1^3a_2a_3a_4a_5a_6^2 - 48a_1^3a_2a_3a_5^3a_6 \\
& + 168a_1^3a_2a_4^3a_6^2 - 36a_1^3a_2a_4^2a_5^2a_6 - 9a_1^3a_2a_4a_5^4 + 64a_1^3a_3^3a_5a_6^2 \\
& - 8a_1^3a_3^2a_4^2a_6^2 - 88a_1^3a_3^2a_4^2a_5^2a_6 - 64a_1^3a_3a_4^3a_5a_6 - 24a_1^3a_3a_4^2a_5^3 \\
& - 48a_1^3a_4^5a_6 - 192a_1^2a_2^3a_3a_6^3 - 140a_1^2a_2^3a_4a_5a_6^2 + 8a_1^2a_2^3a_5^3a_6 \\
& - 104a_1^2a_2^2a_3^2a_5a_6^2 - 258a_1^2a_2^2a_3a_4^2a_6^2 + 102a_1^2a_2^2a_3a_4a_5^2a_6 + 11a_1^2a_2^2a_3a_5^4 \\
& + 20a_1^2a_2^2a_4^3a_5a_6 + 24a_1^2a_2^2a_4^2a_5^3 - 136a_1^2a_2a_3^3a_4a_6^2 + 16a_1^2a_2a_3^3a_5^2a_6 \\
& - 80a_1^2a_2a_3^2a_4^2a_5a_6 - 8a_1^2a_2a_3^2a_4a_5^3 + 16a_1^2a_2a_3a_4^4a_6 + 14a_1^2a_2a_3a_4^3a_5^2 \\
& - 16a_1^2a_3^5a_6^2 + 16a_1^2a_3^4a_4a_5a_6 + 24a_1^2a_3^3a_4^3a_6 + 8a_1^2a_3^3a_4^2a_5^2 \\
& - 24a_1^2a_3^2a_4^4a_5 - 26a_1a_2^5a_6^3 + 106a_1a_2^4a_3a_5a_6^2 + 80a_1a_2^4a_4^2a_6^2 \\
& + 25a_1a_2^4a_4a_5^2a_6 - 2a_1a_2^4a_5^4 + 150a_1a_2^3a_3^2a_4a_6^2 + 48a_1a_2^3a_3a_4^2a_5a_6 \\
& - 12a_1a_2^3a_3a_4a_5^3 - 21a_1a_2^3a_4^4a_6 - 17a_1a_2^3a_4^3a_5^2 + 96a_1a_2^2a_3^4a_6^2 \\
& + 32a_1a_2^2a_3^3a_4a_5a_6 - 38a_1a_2^2a_3^2a_4^3a_6 + 20a_1a_2^2a_3^2a_4^2a_5^2 + 16a_1a_2^2a_3a_4^4a_5 \\
& - 16a_1a_2a_3^4a_4^2a_6 - 16a_1a_2a_3^3a_4^3a_5 - a_1a_2a_3^2a_4^5 + 8a_1a_3^4a_4^4 \\
& - 10a_2^6a_5a_6^2 - 55a_2^5a_3a_4a_6^2 - 3a_2^5a_3a_5^2a_6 - 37a_2^5a_4^2a_5a_6 \\
& + 2a_2^5a_4a_5^3 - 23a_2^4a_3^3a_6^2 - a_2^4a_3^2a_4a_5a_6 + 25a_2^4a_3a_4^3a_6 \\
& - a_2^4a_3a_4^2a_5^2 + 2a_2^4a_4^4a_5 - 4a_2^3a_3^3a_4^2a_6 - 2a_2^3a_3^2a_4^3a_5 \\
& - 2a_2^3a_3a_4^5 + a_2^2a_3^3a_4^4 + 1312a_0^4a_4a_6^3 - 368a_0^4a_5^2a_6^2 \\
& - 552a_0^3a_1a_3a_6^3 + 2236a_0^3a_1a_4a_5a_6^2 + 328a_0^3a_1a_3^3a_6 - 688a_0^3a_2^2a_6^3 \\
& + 1636a_0^3a_2a_3a_5a_6^2 + 2550a_0^3a_2a_4^2a_6^2 - 752a_0^3a_2a_4a_5^2a_6 + 36a_0^3a_2a_5^4 \\
& - 240a_0^3a_3^2a_4a_6^2 - 216a_0^3a_3^2a_5^2a_6 + 1380a_0^3a_3a_4^2a_5a_6 + 132a_0^3a_3a_4a_5^3 \\
& + 308a_0^3a_4^4a_6 - 22a_0^3a_4^3a_5^2 + 2136a_0^2a_1^2a_2a_6^3 - 840a_0^2a_1^2a_3a_5a_6^2 \\
& - 600a_0^2a_1^2a_4^2a_6^2 - 224a_0^2a_1^2a_4a_5^2a_6 - 36a_0^2a_1^2a_5^4 + 326a_0^2a_1a_2^2a_5a_6^2 \\
& - 3372a_0^2a_1a_2a_3a_4a_6^2 - 560a_0^2a_1a_2a_3a_5^2a_6 + 876a_0^2a_1a_2a_4^2a_5a_6 + 126a_0^2a_1a_2a_4a_5^3 \\
& + 240a_0^2a_1a_3^3a_6^2 - 1048a_0^2a_1a_3^2a_4a_5a_6 - 256a_0^2a_1a_3^2a_5^3 - 80a_0^2a_1a_3a_4^3a_6 \\
& + 32a_0^2a_1a_3a_4^2a_5^2 - 16a_0^2a_1a_4^4a_5 + 307a_0^2a_2^3a_4a_6^2 - 50a_0^2a_2^3a_5^2a_6 \\
& - 396a_0^2a_2^2a_3^2a_6^2 + 1690a_0^2a_2^2a_3a_4a_5a_6 + 126a_0^2a_2^2a_3a_5^3 + 588a_0^2a_2^2a_4^3a_6 \\
& - 225a_0^2a_2^2a_4^2a_5^2 - 128a_0^2a_2a_3^3a_5a_6 - 968a_0^2a_2a_3^2a_4^2a_6 + 288a_0^2a_2a_3^2a_4a_5^2 \\
& + 146a_0^2a_2a_3a_4^3a_5 + 3a_0^2a_2a_4^5 + 80a_0^2a_3^4a_4a_6 + 48a_0^2a_3^4a_5^2 \\
& - 104a_0^2a_3^3a_4^2a_5 + 92a_0^2a_3^2a_4^4 - 608a_0a_1^4a_6^3 + 48a_0a_1^3a_2a_5a_6^2 \\
& + 912a_0a_1^3a_3a_4a_6^2 + 288a_0a_1^3a_3a_5^2a_6 + 32a_0a_1^3a_4^2a_5a_6 + 964a_0a_1^2a_2^2a_4a_6^2 \\
& - 40a_0a_1^2a_2^2a_5^2a_6 + 624a_0a_1^2a_2a_3^2a_6^2 - 624a_0a_1^2a_2a_3a_4a_5a_6 - 24a_0a_1^2a_2a_3a_5^3
\end{aligned}$$

$$\begin{aligned}
& -304a_0a_1^2a_2a_4^3a_6 - 36a_0a_1^2a_2a_4^2a_5^2 + 304a_0a_1^2a_3^3a_5a_6 + 208a_0a_1^2a_3^2a_4^2a_6 \\
& - 80a_0a_1^2a_3^2a_4a_5^2 - 32a_0a_1^2a_3a_4^3a_5 + 48a_0a_1^2a_4^5 - 660a_0a_1a_2^3a_3a_6^2 \\
& + 60a_0a_1a_2^3a_4a_5a_6 + 10a_0a_1a_2^3a_5^3 - 520a_0a_1a_2^2a_3^2a_5a_6 - 356a_0a_1a_2^2a_3a_4^2a_6 \\
& - 88a_0a_1a_2^2a_3a_4a_5^2 + 76a_0a_1a_2^2a_4^3a_5 + 64a_0a_1a_2a_3^3a_4a_6 + 48a_0a_1a_2a_3^3a_5^2 \\
& - 88a_0a_1a_2a_3^2a_4^2a_5 - 128a_0a_1a_2a_3a_4^4 - 64a_0a_1a_3^5a_6 - 80a_0a_1a_3^3a_4^3 \\
& + 88a_0a_2^5a_6^2 + 175a_0a_2^4a_3a_5a_6 + 16a_0a_2^4a_4^2a_6 - 21a_0a_2^4a_4a_5^2 \\
& + 8a_0a_2^3a_3^2a_4a_6 + 8a_0a_2^3a_3^2a_5^2 + 154a_0a_2^3a_3a_4^2a_5 + 31a_0a_2^3a_4^4 \\
& + 80a_0a_2^2a_3^4a_6 - 80a_0a_2^2a_3^3a_4a_5 - 60a_0a_2^2a_3^2a_4^3 + 80a_0a_2a_3^4a_4^2 \\
& - 336a_1^4a_2a_4a_6^2 - 144a_1^4a_3^2a_6^2 + 64a_1^4a_4^3a_6 + 248a_1^3a_2^2a_3a_6^2 \\
& + 64a_1^3a_2^2a_4a_5a_6 + 96a_1^3a_2a_3^2a_5a_6 + 80a_1^3a_2a_3a_4^2a_6 + 72a_1^3a_2a_3a_4a_5^2 \\
& + 48a_1^3a_3^3a_4a_6 + 48a_1^3a_3^2a_4^2a_5 + 23a_1^2a_2^4a_6^2 - 32a_1^2a_2^3a_3a_5a_6 \\
& + 12a_1^2a_2^3a_4^2a_6 - 18a_1^2a_2^3a_4a_5^2 - 8a_1^2a_2^2a_3^2a_4a_6 - 40a_1^2a_2^2a_3^2a_5^2 \\
& - 96a_1^2a_2^2a_3a_4^2a_5 - 32a_1^2a_2a_3^4a_6 + 16a_1^2a_2a_3^3a_4a_5 + 56a_1^2a_2a_3^2a_4^3 \\
& - 16a_1^2a_3^4a_4^2 - 14a_1a_2^5a_5a_6 - 41a_1a_2^4a_3a_4a_6 + 14a_1a_2^4a_3a_5^2 \\
& + 21a_1a_2^4a_4^2a_5 - 16a_1a_2^3a_3^3a_6 + 16a_1a_2^3a_3^2a_4a_5 - 10a_1a_2^3a_3a_4^3 \\
& - 16a_1a_2^2a_3^3a_4^2 + 18a_2^6a_4a_6 - a_2^6a_5^2 + 6a_2^5a_3^2a_6 \\
& - 5a_2^5a_3a_4a_5 - a_2^5a_4^3 + 5a_2^4a_3^2a_4^2 - 1104a_0^4a_3a_6^2 \\
& + 944a_0^4a_4a_5a_6 + 88a_0^4a_5^3 - 888a_0^3a_1a_2a_6^2 - 1328a_0^3a_1a_3a_5a_6 \\
& - 1184a_0^3a_1a_4^2a_6 - 112a_0^3a_1a_4a_5^2 + 1024a_0^3a_2^2a_5a_6 - 1520a_0^3a_2a_3a_4a_6 \\
& - 392a_0^3a_2a_3a_5^2 + 396a_0^3a_2a_4^2a_5 + 480a_0^3a_3^3a_6 - 288a_0^3a_3^2a_4a_5 \\
& - 296a_0^3a_3a_4^3 + 48a_0^2a_1^3a_6^2 - 192a_0^2a_1^2a_2a_5a_6 + 1008a_0^2a_1^2a_3a_4a_6 \\
& + 288a_0^2a_1^2a_3a_5^2 - 16a_0^2a_1^2a_4^2a_5 - 872a_0^2a_1a_2^2a_4a_6 - 24a_0^2a_1a_2^2a_5^2 \\
& + 1728a_0^2a_1a_2a_3^2a_6 - 488a_0^2a_1a_2a_3a_4a_5 - 72a_0^2a_1a_2a_4^3 + 352a_0^2a_1a_3^3a_5 \\
& + 160a_0^2a_1a_3^2a_4^2 - 928a_0^2a_2^3a_3a_6 + 322a_0^2a_2^3a_4a_5 - 264a_0^2a_2^2a_3^2a_5 \\
& - 84a_0^2a_2^2a_3a_4^2 - 144a_0^2a_2a_3^3a_4 - 64a_0^2a_3^5 + 96a_0a_1^3a_2a_4a_6 \\
& - 576a_0a_1^3a_3^2a_6 - 64a_0a_1^3a_4^3 + 320a_0a_1^2a_2^2a_3a_6 - 64a_0a_1^2a_2^2a_4a_5 \\
& + 48a_0a_1^2a_2a_3^2a_5 + 208a_0a_1^2a_2a_3a_4^2 + 96a_0a_1^2a_3^3a_4 + 92a_0a_1a_2^4a_6 \\
& - 40a_0a_1a_2^3a_3a_5 - 84a_0a_1a_2^3a_4^2 + 272a_0a_1a_2^2a_3^2a_4 - 64a_0a_1a_2a_3^4 \\
& + 23a_0a_2^5a_5 - 106a_0a_2^4a_3a_4 + 16a_0a_2^3a_3^3 - 64a_1^3a_2^3a_6 \\
& - 144a_1^3a_2a_3^2a_4 + 72a_1^2a_2^3a_3a_4 + 48a_1^2a_2^2a_3^3 - 9a_1a_2^5a_4 \\
& - 24a_1a_2^4a_3^2 + 3a_2^6a_3 - 1312a_0^4a_2a_6 - 320a_0^4a_3a_5 \\
& + 144a_0^4a_4^2 + 832a_0^3a_1^2a_6 - 80a_0^3a_1a_2a_5 + 416a_0^3a_1a_3a_4 \\
& - 360a_0^3a_2^2a_4 + 992a_0^3a_2a_3^2 + 240a_0^2a_1^2a_2a_4 - 576a_0^2a_1^2a_3^2 \\
& + 80a_0^2a_1a_2^2a_3 - 132a_0^2a_2^4 + 64a_0a_1^2a_3^2 - 64a_0^4a_1
\end{aligned} \tag{A.34}$$

$$\begin{aligned}
e_{35} = & -16a_0^3a_5^2a_6^{10} + 8a_0^2a_1a_5^3a_6^9 - 16a_0^2a_2^2a_6^{11} + 16a_0^2a_2a_3a_5a_6^{10} \\
& + 8a_0^2a_2a_4^2a_6^{10} - 8a_0^2a_2a_4a_5^2a_6^9 - 8a_0^2a_2a_5^4a_6^8 + 8a_0^2a_3^2a_5^2a_6^9 \\
& - 4a_0^2a_3a_4^2a_5a_6^9 - 12a_0^2a_3a_4a_5^3a_6^8 + 8a_0^2a_3a_5^5a_6^7 - a_0^2a_4^4a_6^9 \\
& + 2a_0^2a_4^3a_5^2a_6^8 + 4a_0^2a_4^2a_5^4a_6^7 - 6a_0^2a_4a_5^6a_6^6 + a_0^2a_5^8a_6^5 \\
& + 8a_0a_1^2a_2a_6^{11} - 4a_0a_1^2a_3a_5a_6^{10} - 2a_0a_1^2a_4^2a_6^{10} - 2a_0a_1^2a_4a_5^2a_6^9 \\
& - 4a_0a_1^2a_5^4a_6^8 - 8a_0a_1a_2a_3a_4a_6^{10} - 4a_0a_1a_2a_3a_5^2a_6^9 + 4a_0a_1a_2a_4^2a_5a_6^9 \\
& + 4a_0a_1a_2a_4a_5^3a_6^8 + 4a_0a_1a_2a_5^5a_6^7 + 4a_0a_1a_3^2a_4a_5a_6^9 - 2a_0a_1a_3^2a_5^3a_6^8 \\
& + 2a_0a_1a_3a_4^3a_6^9 + a_0a_1a_3a_4^2a_5^2a_6^8 + 10a_0a_1a_3a_4a_5^4a_6^7 - a_0a_1a_3a_5^6a_6^6 \\
& - a_0a_1a_4^4a_5a_6^8 - 2a_0a_1a_4^3a_5^3a_6^7 - 5a_0a_1a_4^2a_5^5a_6^6 + 2a_0a_1a_4a_5^7a_6^5 \\
& - 8a_0a_2^3a_5^2a_6^9 + 8a_0a_2^2a_3^2a_6^{10} - 8a_0a_2^2a_3a_4a_5a_6^9 + 16a_0a_2^2a_3a_5^3a_6^8 \\
& + 6a_0a_2^2a_4^2a_5^2a_6^8 - 10a_0a_2^2a_4a_5^4a_6^7 - 8a_0a_2a_3^3a_5a_6^9 - 2a_0a_2a_3^2a_4^2a_6^9 \\
& + 14a_0a_2a_3^2a_4a_5^2a_6^8 - 10a_0a_2a_3^2a_5^4a_6^7 + 2a_0a_2a_3a_4^3a_5a_6^8 - 15a_0a_2a_3a_4^2a_5^3a_6^7 \\
& + 10a_0a_2a_3a_4a_5^5a_6^6 - a_0a_2a_3a_5^7a_6^5 - a_0a_2a_4^4a_5^2a_6^7 + 6a_0a_2a_4^3a_5^4a_6^6 \\
& - 3a_0a_2a_4^2a_5^6a_6^5 - a_0a_3^4a_5^2a_6^8 + a_0a_3^3a_4^2a_5a_6^8 + 3a_0a_3^3a_4a_5^3a_6^7 \\
& - 3a_0a_3^2a_4^3a_5^2a_6^7 - 3a_0a_3^2a_4^2a_5^4a_6^6 + 3a_0a_3a_4^4a_5^3a_6^6 + a_0a_3a_4^3a_5^5a_6^5 \\
& - a_0a_4^5a_5^4a_6^5 - a_1^4a_6^{11} + 2a_1^3a_3a_4a_6^{10} + 3a_1^3a_3a_5^2a_6^9 \\
& - a_1^3a_4^2a_5a_6^9 - a_1^3a_4a_5^3a_6^8 + a_1^3a_5^5a_6^7 + 2a_1^2a_2^2a_5^2a_6^9 \\
& - 2a_1^2a_2a_3^2a_6^{10} + 2a_1^2a_2a_3a_4a_5a_6^9 - 5a_1^2a_2a_3a_5^3a_6^8 - 2a_1^2a_2a_4^2a_5^2a_6^8 \\
& + a_1^2a_2a_4a_5^4a_6^7 - 2a_1^2a_2a_5^6a_6^6 + a_1^2a_3^3a_5a_6^9 - a_1^2a_3^2a_4^2a_6^9 \\
& - 5a_1^2a_3^2a_4a_5^2a_6^8 + a_1^2a_3a_4^3a_5a_6^8 + 5a_1^2a_3a_4^2a_5^3a_6^7 - a_1^2a_3a_4a_5^5a_6^6 \\
& - a_1^2a_4^3a_5^4a_6^6 + a_1^2a_4^2a_5^6a_6^5 + 2a_1a_2^2a_3a_4a_5^2a_6^8 + a_1a_2^2a_3a_5^4a_6^7 \\
& - a_1a_2^2a_4^2a_5^3a_6^7 + a_1a_2^2a_4a_5^5a_6^6 + a_1a_2^2a_5^7a_6^5 + 2a_1a_2a_3^3a_4a_6^9 \\
& + a_1a_2a_3^3a_5^2a_6^8 - 3a_1a_2a_3^2a_4^2a_5a_6^8 - a_1a_2a_3^2a_4a_5^3a_6^7 + a_1a_2a_3^2a_5^5a_6^6 \\
& + a_1a_2a_3a_4^3a_5^2a_6^7 + a_1a_2a_3a_4^2a_5^4a_6^6 - a_1a_2a_3a_4a_5^6a_6^5 - a_1a_2a_4^3a_5^5a_6^5 \\
& - a_1a_3^4a_4a_5a_6^8 + 3a_1a_3^3a_4^2a_5^2a_6^7 - 3a_1a_3^2a_4^3a_5^3a_6^6 + a_1a_3a_4^4a_5^4a_6^5 \\
& - a_2^4a_5^4a_6^7 - 2a_2^3a_3^2a_5^2a_6^8 + 2a_2^3a_3a_4a_5^3a_6^7 + a_2^3a_3a_5^5a_6^6 \\
& - a_2^3a_4a_5^6a_6^5 - a_2^2a_3^4a_6^9 + 2a_2^2a_3^3a_4a_5a_6^8 + 2a_2^2a_3^3a_5^3a_6^7 \\
& - a_2^2a_3^2a_4^2a_5^2a_6^7 - 4a_2^2a_3^2a_4a_5^4a_6^6 + 2a_2^2a_3a_4^2a_5^5a_6^5 + a_2a_3^5a_5a_6^8 \\
& - 3a_2a_3^4a_4a_5^2a_6^7 + 3a_2a_3^3a_4^2a_5^3a_6^6 - a_2a_3^2a_4^3a_5^4a_6^5 + 32a_0^3a_4a_5a_6^9 \\
& + 72a_0^3a_5^3a_6^8 + 16a_0^2a_1a_2a_6^{10} - 8a_0^2a_1a_3a_5a_6^9 - 4a_0^2a_1a_4^2a_6^9 \\
& - 12a_0^2a_1a_4a_5^2a_6^8 - 16a_0^2a_1a_5^4a_6^7 + 80a_0^2a_2^2a_5a_6^9 - 24a_0^2a_2a_3a_4a_6^9 \\
& - 84a_0^2a_2a_3a_5^2a_6^8 - 24a_0^2a_2a_4^2a_5a_6^8 + 72a_0^2a_2a_4a_5^3a_6^7 + 12a_0^2a_2a_5^5a_6^6 \\
& - 12a_0^2a_3^2a_4a_5a_6^8 - 18a_0^2a_3^2a_5^3a_6^7 + 6a_0^2a_3a_4^3a_6^8 + 45a_0^2a_3a_4^2a_5^2a_6^7 \\
& - 22a_0^2a_3a_4a_5^4a_6^6 - 9a_0^2a_3a_5^6a_6^5 + a_0^2a_4^4a_5a_6^7 - 20a_0^2a_4^3a_5^3a_6^6
\end{aligned}$$

$$\begin{aligned}
& + 33a_0^2a_4^2a_5^5a_6^5 - 4a_0^2a_4a_5^7a_6^4 - 4a_0a_1^3a_6^{10} - 44a_0a_1^2a_2a_5a_6^9 \\
& + 10a_0a_1^2a_3a_4a_6^9 + 17a_0a_1^2a_3a_5^2a_6^8 + 13a_0a_1^2a_4^2a_5a_6^8 + 23a_0a_1^2a_4a_5^3a_6^7 \\
& + 5a_0a_1^2a_5^5a_6^6 + 8a_0a_1a_2^2a_4a_6^9 + 24a_0a_1a_2^2a_5^2a_6^8 - 4a_0a_1a_2a_3^2a_6^9 \\
& + 44a_0a_1a_2a_3a_4a_5a_6^8 - 6a_0a_1a_2a_3a_5^3a_6^7 - 6a_0a_1a_2a_4^3a_6^8 - 37a_0a_1a_2a_4^2a_5^2a_6^7 \\
& - 24a_0a_1a_2a_4a_5^4a_6^6 - 3a_0a_1a_2a_5^6a_6^5 + 2a_0a_1a_3^3a_5a_6^8 - 6a_0a_1a_3^2a_4^2a_6^8 \\
& - 16a_0a_1a_3^2a_4a_5^2a_6^7 + 6a_0a_1a_3^2a_5^4a_6^6 - 10a_0a_1a_3a_4^3a_5a_6^7 - 32a_0a_1a_3a_4^2a_5^3a_6^6 \\
& - 12a_0a_1a_3a_4a_5^5a_6^5 + a_0a_1a_4^5a_6^7 + 10a_0a_1a_4^4a_5^2a_6^6 + 25a_0a_1a_4^3a_5^4a_6^5 \\
& - 8a_0a_1a_4^2a_5^6a_6^4 - 16a_0a_2^3a_3a_6^9 + 24a_0a_2^3a_4a_5a_6^8 + 16a_0a_2^3a_5^3a_6^7 \\
& - 8a_0a_2^2a_3^2a_5a_6^8 + 12a_0a_2^2a_3a_4^2a_6^8 - 58a_0a_2^2a_3a_4a_5^2a_6^7 - 19a_0a_2^2a_3a_5^4a_6^6 \\
& - 14a_0a_2^2a_4^3a_5a_6^7 + 38a_0a_2^2a_4^2a_5^3a_6^6 + 9a_0a_2^2a_4a_5^5a_6^5 + 10a_0a_2a_3^3a_4a_6^8 \\
& + 21a_0a_2a_3^3a_5^2a_6^7 - 26a_0a_2a_3^2a_4^2a_5a_6^7 + 20a_0a_2a_3^2a_4a_5^3a_6^6 + 9a_0a_2a_3^2a_5^5a_6^5 \\
& - 2a_0a_2a_3a_4^4a_6^7 + 48a_0a_2a_3a_4^3a_5^2a_6^6 - 33a_0a_2a_3a_4^2a_5^4a_6^5 + 4a_0a_2a_3a_4a_5^6a_6^4 \\
& + 2a_0a_2a_4^5a_5a_6^6 - 25a_0a_2a_4^4a_5^3a_6^5 + 12a_0a_2a_4^3a_5^5a_6^4 + a_0a_3^4a_4a_5a_6^7 \\
& - a_0a_3^3a_4^3a_6^7 - 6a_0a_3^3a_4^2a_5^2a_6^6 + 6a_0a_3^2a_4^4a_5a_6^6 + 9a_0a_3^2a_4^3a_5^3a_6^5 \\
& - 9a_0a_3a_4^5a_5^2a_6^5 - 4a_0a_3a_4^4a_5^4a_6^4 + 4a_0a_4^6a_5^3a_6^4 + 6a_1^4a_5a_6^9 \\
& - 2a_1^3a_2a_4a_6^9 - 7a_1^3a_2a_5^2a_6^8 - 16a_1^3a_3a_4a_5a_6^8 - 6a_1^3a_3a_5^3a_6^7 \\
& + a_1^3a_4^3a_6^8 + 9a_1^3a_4^2a_5^2a_6^7 - 3a_1^3a_4a_5^4a_6^6 + 4a_1^2a_2^2a_3a_6^9 \\
& - 6a_1^2a_2^2a_4a_5a_6^8 - 3a_1^2a_2^2a_5^3a_6^7 + 5a_1^2a_2a_3^2a_5a_6^8 + 17a_1^2a_2a_3a_4a_5^2a_6^7 \\
& + 12a_1^2a_2a_3a_5^4a_6^6 + 3a_1^2a_2a_4^3a_5a_6^7 - a_1^2a_2a_4^2a_5^3a_6^6 + 9a_1^2a_2a_4a_5^5a_6^5 \\
& - a_1^2a_3^3a_4a_6^8 + 14a_1^2a_3^2a_4^2a_5a_6^7 + 6a_1^2a_3^2a_4a_5^3a_6^6 - a_1^2a_3a_4^4a_6^7 \\
& - 19a_1^2a_3a_4^3a_5^2a_6^6 - 3a_1^2a_3a_4^2a_5^4a_6^5 + 4a_1^2a_4^4a_5^3a_6^5 - 4a_1^2a_4^3a_5^5a_6^4 \\
& - 2a_1a_2^3a_4a_5^2a_6^7 + 3a_1a_2^3a_5^4a_6^6 - 6a_1a_2^2a_3^2a_4a_6^8 + a_1a_2^2a_3^2a_5^2a_6^7 \\
& + 2a_1a_2^2a_3a_4^2a_5a_6^7 - 12a_1a_2^2a_3a_4a_5^3a_6^6 - 8a_1a_2^2a_3a_5^5a_6^5 + 2a_1a_2^2a_4^3a_5^2a_6^6 \\
& - 4a_1a_2^2a_4^2a_5^4a_6^5 - 4a_1a_2^2a_4a_5^6a_6^4 - 4a_1a_2a_3^3a_4a_5a_6^7 - 6a_1a_2a_3^3a_5^3a_6^6 \\
& + 3a_1a_2a_3^2a_4^3a_6^7 + 2a_1a_2a_3^2a_4^2a_5^2a_6^6 + 3a_1a_2a_3^2a_4a_5^4a_6^5 - 2a_1a_2a_3a_4^4a_5a_6^6 \\
& + 4a_1a_2a_3a_4^2a_5^5a_6^4 + 4a_1a_2a_4^4a_5^4a_6^4 + a_1a_3^4a_4^2a_6^7 - 6a_1a_3^3a_4^3a_5a_6^6 \\
& + 9a_1a_3^2a_4^4a_5^2a_6^5 - 4a_1a_3a_4^5a_5^3a_6^4 + 4a_2^4a_3a_5^2a_6^7 + 2a_2^4a_4a_5^3a_6^6 \\
& + a_2^4a_5^5a_6^5 + 4a_2^3a_3^3a_6^8 - 2a_2^3a_3^2a_4a_5a_6^7 - 2a_2^3a_3^2a_5^3a_6^6 \\
& - 4a_2^3a_3a_4^2a_5^2a_6^6 - a_2^3a_3a_4a_5^4a_6^5 + 4a_2^3a_4^2a_5^5a_6^4 - 3a_2^2a_3^4a_5a_6^7 \\
& - 2a_2^2a_3^3a_4^2a_6^7 + 2a_2^2a_3^3a_4a_5^2a_6^6 + 2a_2^2a_3^2a_4^3a_5a_6^6 + 9a_2^2a_3^2a_4^2a_5^3a_6^5 \\
& - 8a_2^2a_3a_4^3a_5^4a_6^4 - a_2a_3^5a_4a_6^7 + 6a_2a_3^4a_4^2a_5a_6^6 - 9a_2a_3^3a_4^3a_5^2a_6^5 \\
& + 4a_2a_3^3a_4^4a_5^3a_6^4 - 24a_0^3a_2a_6^9 - 20a_0^3a_3a_5a_6^8 - 10a_0^3a_4^2a_6^8 \\
& - 258a_0^3a_4a_5^2a_6^7 - 68a_0^3a_5^4a_6^6 + 2a_0^2a_1^2a_6^9 - 56a_0^2a_1a_2a_5a_6^8 \\
& + 6a_0^2a_1a_3a_4a_6^8 + 61a_0^2a_1a_3a_5^2a_6^7 + 17a_0^2a_1a_4^2a_5a_6^7 + 29a_0^2a_1a_4a_5^3a_6^6 \\
& + 3a_0^2a_1a_5^5a_6^5 - 136a_0^2a_2^2a_4a_6^8 - 106a_0^2a_2^2a_5^2a_6^7 + 30a_0^2a_2a_3^2a_6^8
\end{aligned}$$

$$\begin{aligned}
& + 218a_0^2a_2a_3a_4a_5a_6^7 + 63a_0^2a_2a_3a_5^3a_6^6 + 62a_0^2a_2a_4^3a_6^7 - 127a_0^2a_2a_4^2a_5^2a_6^6 \\
& - 93a_0^2a_2a_4a_5^4a_6^5 + 3a_0^2a_2a_5^6a_6^4 + 9a_0^2a_3^3a_5a_6^7 - 3a_0^2a_3^2a_4^2a_6^7 \\
& + 39a_0^2a_3^2a_4a_5^2a_6^6 + 32a_0^2a_3^2a_5^4a_6^5 - 69a_0^2a_3a_4^3a_5a_6^6 + 2a_0^2a_3a_4^2a_5^3a_6^5 \\
& + 27a_0^2a_3a_4a_5^5a_6^4 + a_0^2a_3a_5^7a_6^3 - 7a_0^2a_4^5a_6^6 + 35a_0^2a_4^4a_5^2a_6^5 \\
& - 72a_0^2a_4^3a_5^4a_6^4 + 6a_0^2a_4^2a_5^6a_6^3 + 16a_0a_1^3a_5a_6^8 + 62a_0a_1^2a_2a_4a_6^8 \\
& + 43a_0a_1^2a_2a_5^2a_6^7 - 6a_0a_1^2a_3^2a_6^8 - 66a_0a_1^2a_3a_4a_5a_6^7 - 29a_0a_1^2a_3a_5^3a_6^6 \\
& - 17a_0a_1^2a_4^3a_6^7 - 55a_0a_1^2a_4^2a_5^2a_6^6 - 24a_0a_1^2a_4a_5^4a_6^5 + 24a_0a_1a_2^2a_3a_6^8 \\
& - 88a_0a_1a_2^2a_4a_5a_6^7 - 32a_0a_1a_2^2a_5^3a_6^6 - 30a_0a_1a_2a_3^2a_5a_6^7 - 68a_0a_1a_2a_3a_4^2a_6^7 \\
& + 58a_0a_1a_2a_3a_4a_5^2a_6^6 + 22a_0a_1a_2a_3a_5^4a_6^5 + 82a_0a_1a_2a_4^3a_5a_6^6 + 56a_0a_1a_2a_4^2a_5^3a_6^5 \\
& + 18a_0a_1a_2a_4a_5^5a_6^4 + 4a_0a_1a_3^3a_4a_6^7 - 9a_0a_1a_3^3a_5^2a_6^6 + 61a_0a_1a_3^2a_4^2a_5a_6^6 \\
& + 25a_0a_1a_3^2a_4a_5^3a_6^5 - a_0a_1a_3^2a_5^5a_6^4 + 16a_0a_1a_3a_4^4a_6^6 + 28a_0a_1a_3a_4^3a_5^2a_6^5 \\
& + 47a_0a_1a_3a_4^2a_5^4a_6^4 + 2a_0a_1a_3a_4a_5^6a_6^3 - 17a_0a_1a_4^5a_5a_6^5 - 50a_0a_1a_4^4a_5^3a_6^4 \\
& + 12a_0a_1a_4^3a_5^5a_6^3 + 8a_0a_4^4a_6^8 + 8a_0a_2^3a_3a_5a_6^7 - 18a_0a_2^3a_4^2a_6^7 \\
& - 60a_0a_2^3a_4a_5^2a_6^6 - 7a_0a_2^3a_5^4a_6^5 + 26a_0a_2^2a_3^2a_4a_6^7 + 33a_0a_2^2a_3^2a_5^2a_6^6 \\
& + 53a_0a_2^2a_3a_4^2a_5a_6^6 + 71a_0a_2^2a_3a_4a_5^3a_6^5 - 3a_0a_2^2a_3a_5^5a_6^4 + 8a_0a_2^2a_4^4a_6^6 \\
& - 44a_0a_2^2a_4^3a_5^2a_6^5 - 45a_0a_2^2a_4^2a_5^4a_6^4 - 10a_0a_2a_3^4a_6^7 - 36a_0a_2a_3^3a_4a_5a_6^6 \\
& - 34a_0a_2a_3^3a_5^3a_6^5 + 6a_0a_2a_3^2a_4^3a_6^6 - 32a_0a_2a_3^2a_4^2a_5^2a_6^5 - 23a_0a_2a_3^2a_4a_5^4a_6^4 \\
& - a_0a_2a_3^2a_5^6a_6^3 - 47a_0a_2a_3a_4^4a_5a_6^5 + 50a_0a_2a_3a_4^3a_5^3a_6^4 - 9a_0a_2a_3a_4^2a_5^5a_6^3 \\
& - a_0a_2a_4^6a_6^5 + 37a_0a_2a_4^5a_5^2a_6^4 - 18a_0a_2a_4^4a_5^4a_6^3 - a_0a_3^5a_5a_6^6 \\
& + a_0a_3^4a_4^2a_6^6 + 3a_0a_3^4a_4a_5^2a_6^5 - 3a_0a_3^3a_4^2a_5^3a_6^4 - 3a_0a_3^2a_4^5a_6^5 \\
& - 6a_0a_3^2a_4^4a_5^2a_6^4 + a_0a_3^2a_4^3a_5^4a_6^3 + 9a_0a_3a_4^6a_5a_6^4 + 5a_0a_3a_4^5a_5^3a_6^3 \\
& - 6a_0a_4^7a_5^2a_6^3 - 8a_1^4a_4a_6^8 - 4a_1^4a_5^2a_6^7 - 4a_1^3a_2a_3a_6^8 \\
& + 24a_1^3a_2a_4a_5a_6^7 + 10a_1^3a_2a_5^3a_6^6 + 10a_1^3a_3^2a_5a_6^7 + 18a_1^3a_3a_4^2a_6^7 \\
& + 19a_1^3a_3a_4a_5^2a_6^6 + a_1^3a_3a_5^4a_6^5 - 17a_1^3a_4^3a_5a_6^6 - a_1^3a_4^2a_5^3a_6^5 \\
& - 2a_1^2a_2^3a_6^8 - 5a_1^2a_2^2a_3a_5a_6^7 + 3a_1^2a_2^2a_4^2a_6^7 + 16a_1^2a_2^2a_4a_5^2a_6^6 \\
& - 8a_1^2a_2^2a_5^4a_6^5 - 5a_1^2a_2a_3^2a_4a_6^7 - 23a_1^2a_2a_3^2a_5^2a_6^6 - 29a_1^2a_2a_3a_4^2a_5a_6^6 \\
& - 47a_1^2a_2a_3a_4a_5^3a_6^5 - 2a_1^2a_2a_3a_5^5a_6^4 - a_1^2a_2a_4^4a_6^6 - 3a_1^2a_2a_4^3a_5^2a_6^5 \\
& - 13a_1^2a_2a_4^2a_5^4a_6^4 + a_1^2a_3^4a_6^7 - 12a_1^2a_3^3a_4a_5a_6^6 - 12a_1^2a_3^2a_4^3a_6^6 \\
& - 4a_1^2a_3^2a_4^2a_5^2a_6^5 - a_1^2a_3^2a_4a_5^4a_6^4 + 26a_1^2a_3a_4^4a_5a_6^5 + 18a_1^2a_3a_4^3a_5^3a_6^4 \\
& + a_1^2a_3a_4^2a_5^5a_6^3 - 6a_1^2a_4^5a_5^2a_6^4 + 6a_1^2a_4^4a_5^4a_6^3 + 6a_1a_2^3a_3a_4a_6^7 \\
& - 9a_1a_2^3a_3a_5^2a_6^6 + a_1a_2^3a_4^2a_5a_6^6 - 3a_1a_2^3a_4a_5^3a_6^5 + 3a_1a_2^3a_5^5a_6^4 \\
& - 4a_1a_2^2a_3^3a_6^7 + 14a_1a_2^2a_3^2a_4a_5a_6^6 + 22a_1a_2^2a_3^2a_5^3a_6^5 - 4a_1a_2^2a_3a_4^3a_6^6 \\
& + 28a_1a_2^2a_3a_4^2a_5^2a_6^5 + 27a_1a_2^2a_3a_4a_5^4a_6^4 + a_1a_2^2a_3a_5^6a_6^3 - a_1a_2^2a_4^4a_5a_6^5 \\
& + 3a_1a_2^2a_4^3a_5^3a_6^4 + 6a_1a_2^2a_4^2a_5^5a_6^3 + 8a_1a_2a_3^4a_5a_6^6 + 8a_1a_2a_3^3a_4^2a_6^6 \\
& + 8a_1a_2a_3^3a_4a_5^2a_6^5 + a_1a_2a_3^3a_5^4a_6^4 - 8a_1a_2a_3^2a_4^3a_5a_6^5 - 18a_1a_2a_3^2a_4^2a_5^3a_6^4
\end{aligned}$$

$$\begin{aligned}
& -a_1a_2a_3^2a_4a_5^5a_6^3 + a_1a_2a_3a_4^5a_6^5 - 4a_1a_2a_3a_4^4a_5^2a_6^4 - 7a_1a_2a_3a_4^3a_5^4a_6^3 \\
& - 6a_1a_2a_4^5a_5^3a_6^3 - a_1a_3^5a_4a_6^6 + 3a_1a_3^4a_4^2a_5a_6^5 + 3a_1a_3^3a_4^4a_6^5 \\
& - 3a_1a_3^3a_4^3a_5^2a_6^4 - 9a_1a_3^2a_4^5a_5a_6^4 + a_1a_3^2a_4^4a_5^3a_6^3 + 6a_1a_3a_4^6a_5^2a_6^3 \\
& - 2a_2^5a_5^2a_6^6 - 6a_2^4a_3^2a_6^7 - 2a_2^4a_3a_4a_5a_6^6 - 2a_2^4a_3a_5^3a_6^5 \\
& - a_2^4a_4^2a_5^2a_6^5 - 7a_2^4a_4a_5^4a_6^4 + 2a_2^3a_3^3a_5a_6^6 + 4a_2^3a_3^2a_4^2a_6^6 \\
& - 4a_2^3a_3^2a_4a_5^2a_6^5 + a_2^3a_3^2a_5^4a_6^4 + 2a_2^3a_3a_4^3a_5a_6^5 + 3a_2^3a_3a_4^2a_5^3a_6^4 \\
& - a_2^3a_3a_4a_5^5a_6^3 - 6a_2^3a_4^3a_5^4a_6^3 + a_2^2a_3^4a_4a_6^6 + 2a_2^2a_3^4a_5^2a_6^5 \\
& - 6a_2^2a_3^3a_4^2a_5a_6^5 - 4a_2^2a_3^3a_4a_5^3a_6^4 - a_2^2a_3^2a_4^4a_6^5 - 5a_2^2a_3^2a_4^3a_5^2a_6^4 \\
& + 2a_2^2a_3^2a_4^2a_5^4a_6^3 + 12a_2^2a_3a_4^4a_5^3a_6^3 + a_2a_3^6a_6^6 - 3a_2a_3^5a_4a_5a_6^5 \\
& - 3a_2a_3^4a_4^3a_6^5 + 3a_2a_3^4a_4^2a_5^2a_6^4 + 9a_2a_3^3a_4^4a_5a_6^4 - a_2a_3^3a_4^3a_5^3a_6^3 \\
& - 6a_2a_3^2a_4^5a_5^2a_6^3 + 12a_0^3a_1a_6^8 + 100a_0^3a_2a_5a_6^7 + 14a_0^3a_3a_4a_6^7 \\
& + 143a_0^3a_3a_5^2a_6^6 + 283a_0^3a_4^2a_5a_6^6 + 309a_0^3a_4a_5^3a_6^5 - a_0^3a_5^5a_6^4 \\
& - 12a_0^2a_1^2a_5a_6^7 + 122a_0^2a_1a_2a_4a_6^7 + 31a_0^2a_1a_2a_5^2a_6^6 - 12a_0^2a_1a_3^2a_6^7 \\
& - 116a_0^2a_1a_3a_4a_5a_6^6 - 48a_0^2a_1a_3a_5^3a_6^5 - 29a_0^2a_1a_4^3a_6^6 + 9a_0^2a_1a_4^2a_5^2a_6^5 \\
& - 3a_0^2a_1a_4a_5^4a_6^4 - 2a_0^2a_1a_5^6a_6^3 + 52a_0^2a_2^2a_3a_6^7 + 334a_0^2a_2^2a_4a_5a_6^6 \\
& + 51a_0^2a_2^2a_5^3a_6^5 - 171a_0^2a_2a_3^2a_5a_6^6 - 208a_0^2a_2a_3a_4^2a_6^6 - 195a_0^2a_2a_3a_4a_5^2a_6^5 \\
& - 2a_0^2a_2a_3a_5^4a_6^4 + 13a_0^2a_2a_4^3a_5a_6^5 + 260a_0^2a_2a_4^2a_5^3a_6^4 - 13a_0^2a_2a_4a_5^5a_6^3 \\
& + a_0^2a_2a_5^7a_6^2 - 3a_0^2a_3^3a_4a_6^6 - 33a_0^2a_3^3a_5^2a_6^5 + 9a_0^2a_3^2a_4^2a_5a_6^5 \\
& - 89a_0^2a_3^2a_4a_5^3a_6^4 - 9a_0^2a_3^2a_5^5a_6^3 + 51a_0^2a_3a_4^4a_6^5 + 42a_0^2a_3a_4^3a_5^2a_6^4 \\
& - 18a_0^2a_3a_4^2a_5^4a_6^3 - 3a_0^2a_3a_4a_5^6a_6^2 - 19a_0^2a_4^5a_5a_6^4 + 78a_0^2a_4^4a_5^3a_6^3 \\
& - 4a_0^2a_4^3a_5^5a_6^2 - 28a_0a_1^3a_4a_6^7 - 14a_0a_1^3a_5^2a_6^6 - 56a_0a_1^2a_2a_3a_6^7 \\
& - 114a_0a_1^2a_2a_4a_5a_6^6 + 15a_0a_1^2a_2a_5^3a_6^5 + 44a_0a_1^2a_3^2a_5a_6^6 + 84a_0a_1^2a_3a_4^2a_6^6 \\
& + 102a_0a_1^2a_3a_4a_5^2a_6^5 + 7a_0a_1^2a_3a_5^4a_6^4 + 63a_0a_1^2a_4^3a_5a_6^5 + 53a_0a_1^2a_4^2a_5^3a_6^4 \\
& - 4a_0a_1^2a_4a_5^5a_6^3 - 20a_0a_1a_2^3a_6^7 + 70a_0a_1a_2^2a_3a_5a_6^6 + 94a_0a_1a_2^2a_4^2a_6^6 \\
& + 90a_0a_1a_2^2a_4a_5^2a_6^5 - 6a_0a_1a_2^2a_5^4a_6^4 + 56a_0a_1a_2a_3^2a_4a_6^6 - 13a_0a_1a_2a_3^2a_5^2a_6^5 \\
& - 112a_0a_1a_2a_3a_4^2a_5a_6^5 - 136a_0a_1a_2a_3a_4a_5^3a_6^4 - 2a_0a_1a_2a_3a_5^5a_6^3 - 56a_0a_1a_2a_4^4a_6^5 \\
& - 76a_0a_1a_2a_4^3a_5^2a_6^4 - 31a_0a_1a_2a_4^2a_5^4a_6^3 + 2a_0a_1a_2a_4a_5^6a_6^2 + 2a_0a_1a_3^4a_6^6 \\
& - 24a_0a_1a_3^3a_4a_5a_6^5 + 6a_0a_1a_3^3a_5^3a_6^4 - 63a_0a_1a_3^2a_4^3a_6^5 - 97a_0a_1a_3^2a_4^2a_5^2a_6^4 \\
& - 13a_0a_1a_3^2a_4a_5^4a_6^3 - 56a_0a_1a_3a_4^3a_5^3a_6^3 - 6a_0a_1a_3a_4^2a_5^5a_6^2 + 9a_0a_1a_4^6a_6^4 \\
& + 52a_0a_1a_4^5a_5^2a_6^3 - 8a_0a_1a_4^4a_5^4a_6^2 + 8a_0a_2^4a_5a_6^6 - 50a_0a_2^3a_3a_4a_6^6 \\
& - 9a_0a_2^3a_3a_5^2a_6^5 + 84a_0a_2^3a_4^2a_5a_6^5 + 46a_0a_2^3a_4a_5^3a_6^4 - 4a_0a_2^2a_3^3a_6^6 \\
& - 86a_0a_2^2a_3^2a_4a_5a_6^5 - 6a_0a_2^2a_3^2a_5^3a_6^4 + a_0a_2^2a_3a_4^3a_6^5 - 95a_0a_2^2a_3a_4^2a_5^2a_6^4 \\
& + 26a_0a_2^2a_3a_4a_5^4a_6^3 - a_0a_2^2a_3a_5^6a_6^2 + 10a_0a_2^2a_4^4a_5a_6^4 + 81a_0a_2^2a_4^3a_5^3a_6^3 \\
& - 3a_0a_2^2a_4^2a_5^5a_6^2 + 35a_0a_2a_4^3a_5a_6^5 + 28a_0a_2a_3^3a_4^2a_6^5 + 88a_0a_2a_3^3a_4a_5^2a_6^4 \\
& + 9a_0a_2a_3^3a_5^4a_6^3 + 46a_0a_2a_3^2a_4^3a_5a_6^4 + 19a_0a_2a_3^2a_4^2a_5^3a_6^3 + 3a_0a_2a_3^2a_4a_5^5a_6^2
\end{aligned}$$

$$\begin{aligned}
& + 14a_0a_2a_3a_4^5a_6^4 - 48a_0a_2a_3a_4^4a_5^2a_6^3 + 14a_0a_2a_3a_4^3a_5^4a_6^2 - 23a_0a_2a_4^6a_5a_6^3 \\
& + 11a_0a_2a_4^5a_5^3a_6^2 - 3a_0a_3^4a_4^2a_5a_6^4 + 3a_0a_3^3a_4^4a_6^4 + 6a_0a_3^3a_4^3a_5^2a_6^3 \\
& - 3a_0a_3^2a_4^5a_5a_6^3 - 3a_0a_3^2a_4^4a_5^3a_6^2 - 3a_0a_3a_4^7a_6^3 - a_0a_3a_4^6a_5^2a_6^2 \\
& + 4a_0a_4^8a_5a_6^2 + 8a_1^4a_3a_6^7 + 12a_1^4a_4a_5a_6^6 - 2a_1^4a_5^3a_6^5 \\
& + 4a_1^3a_2^7a_6^7 - 20a_1^3a_2a_3a_5a_6^6 - 22a_1^3a_2a_4^2a_6^6 - 37a_1^3a_2a_4a_5^2a_6^5 \\
& + 5a_1^3a_2a_5^4a_6^4 - 24a_1^3a_2^2a_4a_6^6 - 6a_1^3a_2^2a_5^2a_6^5 - 18a_1^3a_3a_4^2a_5a_6^5 \\
& + 9a_1^3a_4^4a_6^5 + 15a_1^3a_4^3a_5^2a_6^4 - 2a_1^3a_4^2a_5^4a_6^3 - a_1^2a_3^2a_5a_6^6 \\
& + 5a_1^2a_2^2a_3a_4a_6^6 + 26a_1^2a_2^2a_3a_5^2a_6^5 - 17a_1^2a_2^2a_4^2a_5a_6^5 + 23a_1^2a_2^2a_4a_5^3a_6^4 \\
& - 4a_1^2a_2^2a_5^5a_6^3 + 8a_1^2a_2a_3^3a_6^6 + 68a_1^2a_2a_3^2a_4a_5a_6^5 + 10a_1^2a_2a_3^2a_5^3a_6^4 \\
& + 23a_1^2a_2a_3a_4^3a_6^5 + 57a_1^2a_2a_3a_4^2a_5^2a_6^4 + 8a_1^2a_2a_3a_4a_5^4a_6^3 + 2a_1^2a_2a_4^4a_5a_6^4 \\
& + 8a_1^2a_2a_4^3a_5^3a_6^3 + a_1^2a_2a_4^2a_5^5a_6^2 + 18a_1^2a_3^3a_4^2a_6^5 + 6a_1^2a_3^3a_4a_5^2a_6^4 \\
& - 14a_1^2a_3^2a_4^3a_5a_6^4 - 4a_1^2a_3^2a_4^2a_5^3a_6^3 - 12a_1^2a_3a_4^5a_6^4 - 28a_1^2a_3a_4^4a_5^2a_6^3 \\
& - 3a_1^2a_3a_4^3a_5^4a_6^2 + 4a_1^2a_4^6a_5a_6^3 - 4a_1^2a_4^5a_5^3a_6^2 - 2a_1a_2^4a_4a_6^6 \\
& + 7a_1a_2^4a_5^5a_6^5 + 12a_1a_2^3a_3^2a_6^6 - 4a_1a_2^3a_3a_4a_5a_6^5 - 18a_1a_2^3a_3a_5^3a_6^4 \\
& + a_1a_2^3a_4^3a_6^5 - 9a_1a_2^3a_4^2a_5^2a_6^4 - 4a_1a_2^3a_4a_5^4a_6^3 + a_1a_2^3a_5^6a_6^2 \\
& - 24a_1a_2^2a_3^3a_5a_6^5 - 30a_1a_2^2a_3^2a_4^2a_6^5 - 48a_1a_2^2a_3^2a_4a_5^2a_6^4 - 7a_1a_2^2a_3^2a_5^4a_6^3 \\
& - 10a_1a_2^2a_3a_4^3a_5a_6^4 - 38a_1a_2^2a_3a_4^2a_5^3a_6^3 - 4a_1a_2^2a_3a_4a_5^5a_6^2 + 3a_1a_2^2a_4^4a_5^2a_6^2 \\
& - 5a_1a_2^2a_4^3a_5^4a_6^2 - 13a_1a_2a_3^4a_4a_6^5 - 6a_1a_2a_3^4a_5^2a_6^4 + 4a_1a_2a_3^3a_4^2a_5a_6^4 \\
& + 4a_1a_2a_3^3a_4a_5^3a_6^3 + 7a_1a_2a_3^2a_4^4a_6^4 + 29a_1a_2a_3^2a_4^3a_5^2a_6^3 + 3a_1a_2a_3^2a_4^2a_5^4a_6^2 \\
& + 4a_1a_2a_3a_4^5a_5a_6^3 + 8a_1a_2a_3a_4^4a_5^3a_6^2 + 4a_1a_2a_4^6a_5^2a_6^2 - 3a_1a_3^4a_4^3a_6^4 \\
& + 6a_1a_3^3a_4^4a_5a_6^3 + 3a_1a_3^2a_4^6a_6^3 - 3a_1a_3^2a_4^5a_5^2a_6^2 - 4a_1a_3a_4^7a_5a_6^2 \\
& + 4a_2^5a_3a_6^6 + 2a_2^5a_4a_5a_6^5 + 2a_2^5a_5^3a_6^4 + 2a_2^4a_3^2a_5a_6^5 \\
& - 2a_2^4a_3a_4^2a_6^5 + 14a_2^4a_3a_4a_5^2a_6^4 + 2a_2^4a_3a_5^4a_6^3 + 13a_2^4a_4^2a_5^3a_6^3 \\
& - a_2^4a_4a_5^5a_6^2 + 6a_2^3a_3^3a_4a_6^5 + 2a_2^3a_3^2a_4^2a_5a_6^4 - 4a_2^3a_3^2a_4a_5^3a_6^3 \\
& - 9a_2^3a_3a_4^3a_5^2a_6^3 + 5a_2^3a_3a_4^2a_5^4a_6^2 + 4a_2^3a_4^4a_5^3a_6^2 - 2a_2^2a_3^5a_6^5 \\
& + a_2^2a_3^4a_4a_5a_6^4 + 2a_2^2a_3^3a_4^3a_6^4 + 8a_2^2a_3^3a_4^2a_5^3a_6^3 - a_2^2a_3^2a_4^4a_5a_6^3 \\
& - 7a_2^2a_3^2a_4^3a_5^2a_6^2 - 8a_2^2a_3a_4^5a_5^2a_6^2 + 3a_2a_3^5a_4^2a_6^4 - 6a_2a_3^4a_4^3a_5a_6^3 \\
& - 3a_2a_3^3a_4^5a_6^3 + 3a_2a_3^3a_4^4a_5^2a_6^2 + 4a_2a_3^2a_4^6a_5a_6^2 - 9a_0^4a_6^7 \\
& - 48a_0^3a_1a_5a_6^6 - 182a_0^3a_2a_4a_6^6 - 139a_0^3a_2a_5^2a_6^5 + 2a_0^3a_3^2a_6^6 \\
& - 330a_0^3a_3a_4a_5a_6^5 - 73a_0^3a_3a_5^3a_6^4 - 75a_0^3a_4^3a_6^5 - 545a_0^3a_4^2a_5^2a_6^4 \\
& + 8a_0^3a_4a_5^4a_6^3 - 2a_0^3a_5^6a_6^2 + 12a_0^2a_1^2a_4a_6^6 + 24a_0^2a_1^2a_5^2a_6^5 \\
& - 20a_0^2a_1a_2a_3a_6^6 - 156a_0^2a_1a_2a_4a_5a_6^5 - 40a_0^2a_1a_2a_5^3a_6^4 + 90a_0^2a_1a_3^2a_5a_6^5 \\
& + 66a_0^2a_1a_3a_4^2a_6^5 + 117a_0^2a_1a_3a_4a_5^2a_6^4 + 28a_0^2a_1a_3a_5^4a_6^3 - 19a_0^2a_1a_4^3a_5a_6^4 \\
& - 28a_0^2a_1a_4^2a_5^3a_6^3 - 3a_0^2a_1a_4a_5^5a_6^2 + a_0^2a_1a_5^7a_6 - 50a_0^2a_2^3a_6^6 \\
& - 29a_0^2a_2^2a_3a_5a_6^5 - 259a_0^2a_2^2a_4^2a_6^5 - 212a_0^2a_2^2a_4a_5^2a_6^4 + 14a_0^2a_2^2a_5^4a_6^3
\end{aligned}$$

$$\begin{aligned}
& + 285a_0^2a_2a_3^2a_4a_6^5 + 100a_0^2a_2a_3^2a_5^2a_6^4 + 277a_0^2a_2a_3a_4^2a_5a_6^4 - 51a_0^2a_2a_3a_4a_5^3a_6^3 \\
& - 5a_0^2a_2a_3a_5^5a_6^2 + 85a_0^2a_2a_4^4a_6^4 - 326a_0^2a_2a_4^3a_5^2a_6^3 + 32a_0^2a_2a_4^2a_5^4a_6^2 \\
& - 2a_0^2a_2a_4a_5^6a_6 + 54a_0^2a_3^3a_4a_5a_6^4 + 24a_0^2a_3^3a_5^3a_6^3 - 63a_0^2a_3^2a_4^3a_6^4 \\
& + 74a_0^2a_3^2a_4^2a_5^2a_6^3 + 24a_0^2a_3^2a_4a_5^4a_6^2 - 56a_0^2a_3a_4^4a_5a_6^3 - 20a_0^2a_3a_4^3a_5^3a_6^2 \\
& + 3a_0^2a_3a_4^2a_5^5a_6 - 6a_0^2a_4^6a_6^3 - 40a_0^2a_4^5a_5^2a_6^2 + a_0^2a_4^4a_5^4a_6 \\
& + 16a_0a_1^3a_3a_6^6 + 36a_0a_1^3a_4a_5a_6^5 - 16a_0a_1^3a_5^3a_6^4 + 54a_0a_1^2a_2^2a_6^6 \\
& - 28a_0a_1^2a_2a_3a_5a_6^5 + 58a_0a_1^2a_2a_4^2a_6^5 - 8a_0a_1^2a_2a_4a_5^2a_6^4 + 19a_0a_1^2a_2a_5^4a_6^3 \\
& - 84a_0a_1^2a_3^2a_4a_6^5 - 36a_0a_1^2a_3^2a_5^2a_6^4 - 132a_0a_1^2a_3a_4^2a_5a_6^4 + 12a_0a_1^2a_3a_4a_5^3a_6^3 \\
& - a_0a_1^2a_3a_5^5a_6^2 - 32a_0a_1^2a_4^4a_6^4 - 77a_0a_1^2a_4^3a_5^2a_6^3 + 7a_0a_1^2a_4^2a_5^4a_6^2 \\
& + 2a_0a_1^2a_4a_5^6a_6 - 74a_0a_1a_2^3a_5a_6^5 - 68a_0a_1a_2^2a_3a_4a_6^5 + 65a_0a_1a_2^2a_3a_5^2a_6^4 \\
& - 89a_0a_1a_2^2a_4^2a_5a_6^4 - 25a_0a_1a_2^2a_4a_5^3a_6^3 - 5a_0a_1a_2^2a_5^5a_6^2 - 36a_0a_1a_2a_3^5a_6^5 \\
& + 108a_0a_1a_2a_3^2a_4a_5a_6^4 - 2a_0a_1a_2a_3^2a_5^3a_6^3 + 46a_0a_1a_2a_3a_4^3a_6^4 + 245a_0a_1a_2a_3a_4^2a_5^2a_6^3 \\
& + 4a_0a_1a_2a_3a_4a_5^4a_6^2 - a_0a_1a_2a_3a_5^6a_6 + 73a_0a_1a_2a_4^4a_5a_6^3 + 24a_0a_1a_2a_4^3a_5^3a_6^2 \\
& - 7a_0a_1a_2a_4^2a_5^5a_6 - 8a_0a_1a_3^4a_5a_6^4 + 68a_0a_1a_3^3a_4^2a_6^4 + 24a_0a_1a_3^3a_4a_5^2a_6^3 \\
& + 92a_0a_1a_3^2a_4^3a_5a_6^3 + 36a_0a_1a_3^2a_4^2a_5^3a_6^2 + 3a_0a_1a_3a_4^5a_6^3 + 23a_0a_1a_3a_4^4a_5^2a_6^2 \\
& + 7a_0a_1a_3a_4^3a_5^4a_6 - 32a_0a_1a_4^6a_5a_6^2 + a_0a_1a_4^5a_5^3a_6 + 14a_0a_2^4a_4a_6^5 \\
& - 4a_0a_2^4a_5^2a_6^4 + 32a_0a_2^3a_3^2a_6^5 + 4a_0a_2^3a_3a_4a_5a_6^4 - 30a_0a_2^3a_3a_5^3a_6^3 \\
& - 46a_0a_2^3a_4^3a_6^4 - 89a_0a_2^3a_4^2a_5^2a_6^3 + 11a_0a_2^3a_4a_5^4a_6^2 - 2a_0a_2^2a_3^3a_5a_6^4 \\
& + 46a_0a_2^2a_3^2a_4^2a_6^4 + 14a_0a_2^2a_3^2a_4a_5^2a_6^3 + 7a_0a_2^2a_3^2a_5^4a_6^2 + 60a_0a_2^2a_3a_4^3a_5a_6^3 \\
& - 55a_0a_2^2a_3a_4^2a_5^3a_6^2 + 2a_0a_2^2a_3a_4a_5^5a_6 + 6a_0a_2^2a_4^5a_6^3 - 64a_0a_2^2a_4^4a_5^2a_6^2 \\
& + 6a_0a_2^2a_4^3a_5^4a_6 - 54a_0a_2a_3^4a_4a_6^4 - 24a_0a_2a_3^3a_5^2a_6^3 - 94a_0a_2a_3^3a_4^2a_5a_6^3 \\
& - 24a_0a_2a_3^3a_4a_5^3a_6^2 - 31a_0a_2a_3^2a_4^4a_6^3 - 2a_0a_2a_3^2a_4^3a_5^2a_6^2 - 3a_0a_2a_3^2a_4^2a_5^4a_6 \\
& + 30a_0a_2a_3a_4^5a_6^2 - 12a_0a_2a_3a_4^4a_5^3a_6 + 5a_0a_2a_4^7a_6^2 - a_0a_2a_4^6a_5^2a_6 \\
& - 3a_0a_3^3a_4^4a_5a_6^2 + 3a_0a_3^2a_4^6a_6^2 + 3a_0a_3^2a_4^5a_5^2a_6 - 2a_0a_3a_4^7a_5a_6 \\
& - a_0a_4^9a_6 - 8a_1^4a_2a_6^6 + 8a_1^4a_3a_5a_6^5 - 8a_1^4a_4^2a_6^5 \\
& + 4a_1^4a_4a_5^2a_6^4 + a_1^4a_5^4a_6^3 + 18a_1^3a_2^2a_5a_6^5 + 36a_1^3a_2a_3a_4a_6^5 \\
& - 20a_1^3a_2a_3a_5^2a_6^4 + 40a_1^3a_2a_4^2a_5a_6^4 - 16a_1^3a_2a_4a_5^3a_6^3 - 2a_1^3a_2a_5^5a_6^2 \\
& + 8a_1^3a_3^3a_6^5 + 8a_1^3a_3a_4^3a_6^4 + 6a_1^3a_3a_4^2a_5^2a_6^3 - a_1^3a_3a_4a_5^4a_6^2 \\
& - 20a_1^3a_4^4a_5a_6^3 + 6a_1^3a_4^3a_5^3a_6^2 + a_1^3a_4^2a_5^5a_6 + a_1^2a_2^3a_4a_6^5 \\
& - 15a_1^2a_2^3a_5^2a_6^4 - 26a_1^2a_2^2a_3^2a_6^5 - 48a_1^2a_2^2a_3a_4a_5a_6^4 + 20a_1^2a_2^2a_3a_5^3a_6^3 \\
& + a_1^2a_2^2a_4^3a_6^4 - 12a_1^2a_2^2a_4^2a_5^2a_6^3 + 13a_1^2a_2^2a_4a_5^4a_6^2 + a_1^2a_2^2a_5^6a_6 \\
& - 8a_1^2a_2a_3^3a_5a_6^4 - 52a_1^2a_2a_3^2a_4^2a_6^4 - 34a_1^2a_2a_3^2a_4a_5^2a_6^3 + a_1^2a_2a_3^2a_5^4a_6^2 \\
& - 28a_1^2a_2a_3a_4^3a_5a_6^3 - 18a_1^2a_2a_3a_4^2a_5^3a_6^2 - a_1^2a_2a_3a_4a_5^5a_6 + a_1^2a_2a_4^5a_6^3 \\
& - 6a_1^2a_2a_4^4a_5^2a_6^2 - 3a_1^2a_2a_4^3a_5^4a_6 - 8a_1^2a_3^4a_4a_6^4 + 9a_1^2a_3^3a_4^4a_6^3 \\
& + 12a_1^2a_3^2a_4^3a_5^2a_6^2 + 22a_1^2a_3a_4^5a_5a_6^2 + 4a_1^2a_3a_4^4a_5^3a_6 - a_1^2a_4^7a_6^2
\end{aligned}$$

$$\begin{aligned}
& + a_1^2 a_4^6 a_5^2 a_6 - 12 a_1 a_2^4 a_3 a_6^5 - 5 a_1 a_2^4 a_4 a_5 a_6^4 + 2 a_1 a_2^4 a_5^3 a_6^3 \\
& + 24 a_1 a_2^3 a_3^2 a_5 a_6^4 + 28 a_1 a_2^3 a_3 a_4^2 a_6^4 + 32 a_1 a_2^3 a_3 a_4 a_5^2 a_6^3 - 5 a_1 a_2^3 a_3 a_5^4 a_6^2 \\
& + 8 a_1 a_2^3 a_4^3 a_5 a_6^3 - 4 a_1 a_2^3 a_4^2 a_5^3 a_6^2 - 3 a_1 a_2^3 a_4 a_5^5 a_6 + 38 a_1 a_2^2 a_3^3 a_4 a_6^4 \\
& + 16 a_1 a_2^2 a_3^3 a_5^2 a_6^3 + 26 a_1 a_2^2 a_3^2 a_4^2 a_5 a_6^3 + 20 a_1 a_2^2 a_3^2 a_4 a_5^3 a_6^2 - 7 a_1 a_2^2 a_3 a_4^4 a_6^3 \\
& + 35 a_1 a_2^2 a_3 a_4^3 a_5^2 a_6^2 + 7 a_1 a_2^2 a_3 a_4^2 a_5^4 a_6 - 4 a_1 a_2^2 a_4^5 a_5 a_6^2 + 3 a_1 a_2^2 a_4^4 a_5^3 a_6 \\
& + 8 a_1 a_2 a_3^5 a_6^4 - 2 a_1 a_2 a_3^3 a_4^3 a_6^3 - 12 a_1 a_2 a_3^3 a_4^2 a_5^2 a_6^2 - 26 a_1 a_2 a_3^2 a_4^4 a_5 a_6^2 \\
& - 4 a_1 a_2 a_3^2 a_4^3 a_5^3 a_6 - a_1 a_2 a_3 a_4^6 a_6^2 - 6 a_1 a_2 a_3 a_4^5 a_5^2 a_6 - a_1 a_2 a_4^7 a_5 a_6 \\
& - 3 a_1 a_3^3 a_4^5 a_6^2 + 3 a_1 a_3^2 a_4^6 a_5 a_6 + a_1 a_3 a_4^8 a_6 - a_2^6 a_6^5 \\
& - 3 a_2^5 a_3 a_5 a_6^4 - 9 a_2^5 a_4 a_5^2 a_6^3 + a_2^5 a_5^4 a_6^2 - 14 a_2^4 a_3^2 a_4 a_6^4 \\
& - 4 a_2^4 a_3^2 a_5^2 a_6^3 - 10 a_2^4 a_3 a_4^2 a_5 a_6^3 - 4 a_2^4 a_3 a_4 a_5^3 a_6^2 - 9 a_2^4 a_4^3 a_5^2 a_6^2 \\
& + 2 a_2^4 a_4^2 a_5^4 a_6 - a_2^3 a_4^4 a_6^4 + 2 a_2^3 a_3^3 a_4 a_5 a_6^3 + 4 a_2^3 a_3^2 a_4^3 a_6^3 \\
& + 2 a_2^3 a_3^2 a_4^2 a_5^2 a_6^2 + 8 a_2^3 a_3 a_4^4 a_5 a_6^2 - 7 a_2^3 a_3 a_4^3 a_5^3 a_6 - a_2^3 a_4^5 a_5^2 a_6 \\
& - 4 a_2^2 a_4^3 a_5^2 a_6^3 - 2 a_2^2 a_3^3 a_4^3 a_5 a_6^2 + a_2^2 a_3^2 a_4^5 a_6^2 + 8 a_2^2 a_3^2 a_4^4 a_5^2 a_6 \\
& + 2 a_2^2 a_3 a_4^6 a_5 a_6 + 3 a_2 a_3^4 a_4^4 a_6^2 - 3 a_2 a_3^3 a_4^5 a_5 a_6 - a_2 a_3^2 a_4^7 a_6 \\
& + 38 a_0^4 a_5 a_6^5 + 92 a_0^3 a_1 a_4 a_6^5 + 226 a_0^3 a_1 a_5^2 a_6^4 + 80 a_0^3 a_2 a_3 a_6^5 \\
& + 390 a_0^3 a_2 a_4 a_5 a_6^4 - 93 a_0^3 a_2 a_5^3 a_6^3 + 72 a_0^3 a_3^2 a_5 a_6^4 + 144 a_0^3 a_3 a_4^2 a_6^4 \\
& + 266 a_0^3 a_3 a_4 a_5^2 a_6^3 + 22 a_0^3 a_3 a_5^4 a_6^2 + 429 a_0^3 a_4^3 a_5 a_6^3 - 28 a_0^3 a_4^2 a_5^3 a_6^2 \\
& - a_0^3 a_4 a_5^5 a_6 + a_0^3 a_5^7 - 24 a_0^2 a_1^2 a_3 a_6^5 - 36 a_0^2 a_1^2 a_4 a_5 a_6^4 \\
& - 34 a_0^2 a_1^2 a_5^3 a_6^3 + 240 a_0^2 a_1 a_2^2 a_6^5 - 220 a_0^2 a_1 a_2 a_3 a_5 a_6^4 + 94 a_0^2 a_1 a_2 a_4^2 a_6^4 \\
& + 229 a_0^2 a_1 a_2 a_4 a_5^2 a_6^3 + 22 a_0^2 a_1 a_2 a_5^4 a_6^2 - 108 a_0^2 a_1 a_3^2 a_4 a_6^4 - 88 a_0^2 a_1 a_3^2 a_5^2 a_6^3 \\
& - 58 a_0^2 a_1 a_3 a_4^2 a_5 a_6^3 - 18 a_0^2 a_1 a_3 a_4 a_5^3 a_6^2 - 12 a_0^2 a_1 a_3 a_5^5 a_6 - 11 a_0^2 a_1 a_4^4 a_6^3 \\
& + 58 a_0^2 a_1 a_4^3 a_5^2 a_6^2 + 15 a_0^2 a_1 a_4^2 a_5^4 a_6 + a_0^2 a_1 a_4 a_5^6 - 33 a_0^2 a_2^3 a_5 a_6^4 \\
& + 15 a_0^2 a_2^2 a_3 a_4 a_6^4 + 147 a_0^2 a_2^2 a_3 a_5^2 a_6^3 + 342 a_0^2 a_2^2 a_4^2 a_5 a_6^3 - 94 a_0^2 a_2^2 a_4 a_5^3 a_6^2 \\
& + 2 a_0^2 a_2^2 a_5^5 a_6 - 88 a_0^2 a_2 a_3^3 a_6^4 - 250 a_0^2 a_2 a_3^2 a_4 a_5 a_6^3 - 14 a_0^2 a_2 a_3^2 a_5^3 a_6^2 \\
& - 237 a_0^2 a_2 a_3 a_4^3 a_6^3 + 124 a_0^2 a_2 a_3 a_4^2 a_5^2 a_6^2 + 17 a_0^2 a_2 a_3 a_4 a_5^4 a_6 - a_0^2 a_2 a_3 a_5^6 \\
& + 171 a_0^2 a_2 a_4^4 a_5 a_6^2 - 34 a_0^2 a_2 a_4^3 a_5^3 a_6 - 2 a_0^2 a_2 a_4^2 a_5^5 - 16 a_0^2 a_3^4 a_5 a_6^3 \\
& + 16 a_0^2 a_3^3 a_4^2 a_6^3 - 48 a_0^2 a_3^3 a_4 a_5^2 a_6^2 - 12 a_0^2 a_3^2 a_4^3 a_5 a_6^2 - 24 a_0^2 a_3^2 a_4^2 a_5^3 a_6 \\
& + 51 a_0^2 a_3 a_4^5 a_6^2 + 35 a_0^2 a_3 a_4^4 a_5^2 a_6 + 4 a_0^2 a_4^6 a_5 a_6 - a_0^2 a_4^5 a_5^3 \\
& - 136 a_0 a_1^3 a_2 a_6^5 + 48 a_0 a_1^3 a_3 a_5 a_6^4 + 60 a_0 a_1^3 a_4 a_5^2 a_6^3 + 6 a_0 a_1^3 a_5^4 a_6^2 \\
& + 152 a_0 a_1^2 a_2^2 a_5 a_6^4 + 104 a_0 a_1^2 a_2 a_3 a_4 a_6^4 - 72 a_0 a_1^2 a_2 a_3 a_5^2 a_6^3 - 120 a_0 a_1^2 a_2 a_4^2 a_5 a_6^3 \\
& - 64 a_0 a_1^2 a_2 a_4 a_5^3 a_6^2 - 5 a_0 a_1^2 a_2 a_5^5 a_6 + 32 a_0 a_1^2 a_3^3 a_6^4 + 16 a_0 a_1^2 a_3^2 a_4 a_5 a_6^3 \\
& + 8 a_0 a_1^2 a_3^2 a_5^3 a_6^2 + 72 a_0 a_1^2 a_3 a_4^3 a_6^3 - 40 a_0 a_1^2 a_3 a_4^2 a_5^2 a_6^2 - 20 a_0 a_1^2 a_3 a_4 a_5^4 a_6 \\
& + 76 a_0 a_1^2 a_4^4 a_5 a_6^2 + 4 a_0 a_1^2 a_4^3 a_5^3 a_6 - a_0 a_1^2 a_4^2 a_5^5 + 40 a_0 a_1 a_2^3 a_4 a_6^4 \\
& - 11 a_0 a_1 a_2^3 a_5^2 a_6^3 - 32 a_0 a_1 a_2^2 a_3^2 a_6^4 - 136 a_0 a_1 a_2^2 a_3 a_4 a_5 a_6^3 + 10 a_0 a_1 a_2^2 a_3 a_5^3 a_6^2 \\
& + 65 a_0 a_1 a_2^2 a_4^3 a_6^3 + 104 a_0 a_1 a_2^2 a_4^2 a_5^2 a_6^2 + 25 a_0 a_1 a_2^2 a_4 a_5^4 a_6 + a_0 a_1 a_2^2 a_5^6
\end{aligned}$$

$$\begin{aligned}
& + 40a_0a_1a_2a_3^3a_5a_6^3 - 80a_0a_1a_2a_3^2a_4^2a_6^3 - 32a_0a_1a_2a_3^2a_4a_5^2a_6^2 + 12a_0a_1a_2a_3^2a_5^4a_6 \\
& - 196a_0a_1a_2a_3a_4^3a_5a_6^2 + 14a_0a_1a_2a_3a_4^2a_5^3a_6 - 42a_0a_1a_2a_4^5a_6^2 - 17a_0a_1a_2a_4^4a_5^2a_6 \\
& + 4a_0a_1a_2a_4^3a_5^4 - 16a_0a_1a_3^4a_4a_6^3 - 48a_0a_1a_3^3a_4^2a_5a_6^2 - 54a_0a_1a_3^2a_4^4a_6^2 \\
& - 40a_0a_1a_3^2a_4^3a_5^2a_6 + 12a_0a_1a_3a_4^5a_5a_6 - 2a_0a_1a_3a_4^4a_5^3 + 10a_0a_1a_4^7a_6 \\
& + a_0a_1a_4^6a_5^2 - 12a_0a_2^4a_3a_6^4 + 37a_0a_2^4a_4a_5a_6^3 - 10a_0a_2^4a_5^3a_6^2 \\
& + 14a_0a_2^3a_3^2a_5a_6^3 - 24a_0a_2^3a_3a_4^2a_6^3 + 80a_0a_2^3a_3a_4a_5^2a_6^2 - a_0a_2^3a_3a_5^4a_6 \\
& + 62a_0a_2^3a_4^3a_5a_6^2 - 29a_0a_2^3a_4^2a_5^3a_6 - a_0a_2^3a_4a_5^5 + 24a_0a_2^2a_3^3a_4a_6^3 \\
& - 8a_0a_2^2a_3^3a_5^2a_6^2 - 32a_0a_2^2a_3^2a_4^2a_5a_6^2 - 16a_0a_2^2a_3^2a_4a_5^3a_6 - 3a_0a_2^2a_3a_4^4a_6^2 \\
& + 54a_0a_2^2a_3a_4^3a_5^2a_6 + a_0a_2^2a_3a_4^2a_5^4 + 18a_0a_2^2a_4^5a_5a_6 - 3a_0a_2^2a_4^4a_5^3 \\
& + 16a_0a_2^2a_5^5a_6^3 + 48a_0a_2a_3^4a_4a_5a_6^2 + 60a_0a_2a_3^3a_4^3a_6^2 + 24a_0a_2a_3^3a_4^2a_5^2a_6 \\
& - 9a_0a_2a_3^2a_4^4a_5a_6 - 9a_0a_2a_3a_4^6a_6 + 4a_0a_2a_3a_4^5a_5^2 - a_0a_2a_4^7a_5 \\
& - a_0a_3^2a_4^6a_5 + a_0a_3a_4^8 + 16a_1^5a_6^5 - 32a_1^4a_2a_5a_6^4 \\
& - 32a_1^4a_3a_4a_6^4 - 8a_1^4a_3a_5^2a_6^3 + 16a_1^4a_4^2a_5a_6^3 - 20a_1^3a_2^2a_4a_6^4 \\
& + 18a_1^3a_2^2a_5^2a_6^3 + 16a_1^3a_2a_3^2a_6^4 + 48a_1^3a_2a_3a_4a_5a_6^3 + 16a_1^3a_2a_3a_5^3a_6^2 \\
& - 16a_1^3a_2a_4^3a_6^3 + 12a_1^3a_2a_4^2a_5^2a_6^2 + 3a_1^3a_2a_4a_5^4a_6 + 8a_1^3a_3^2a_4^2a_6^3 \\
& + 8a_1^3a_3^2a_4a_5^2a_6^2 - 24a_1^3a_3a_4^3a_5a_6^2 - 8a_1^3a_3a_4^2a_5^3a_6 + 8a_1^3a_4^5a_6^2 \\
& - 4a_1^3a_4^4a_5^2a_6 - a_1^3a_4^3a_5^4 + 24a_1^2a_2^3a_3a_6^4 + 16a_1^2a_2^3a_4a_5a_6^3 \\
& - 6a_1^2a_2^3a_5^3a_6^2 - 16a_1^2a_2^2a_3^2a_5a_6^3 + 18a_1^2a_2^2a_3a_4^2a_6^3 - 62a_1^2a_2^2a_3a_4a_5^2a_6^2 \\
& - 10a_1^2a_2^2a_3a_5^4a_6 - 2a_1^2a_2^2a_4^3a_5a_6^2 - 8a_1^2a_2^2a_4^2a_5^3a_6 - a_1^2a_2^2a_4a_5^5 \\
& + 8a_1^2a_2a_3^3a_4a_6^3 - 8a_1^2a_2a_3^3a_5^2a_6^2 + 56a_1^2a_2a_3^2a_4^2a_5a_6^2 + 8a_1^2a_2a_3^2a_4a_5^3a_6 \\
& + 12a_1^2a_2a_3a_4^4a_6^2 + 22a_1^2a_2a_3a_4^3a_5^2a_6 + a_1^2a_2a_3a_4^2a_5^4 + 4a_1^2a_2a_4^5a_5a_6 \\
& + 2a_1^2a_2a_4^4a_5^3 - 16a_1^2a_3^2a_4^4a_5a_6 - 8a_1^2a_3a_4^6a_6 - 2a_1^2a_3a_4^5a_5^2 \\
& + 4a_1a_2^5a_6^4 - 8a_1a_2^4a_3a_5a_6^3 - 7a_1a_2^4a_4^2a_6^3 + 8a_1a_2^4a_4a_5^2a_6^2 \\
& + 3a_1a_2^4a_5^4a_6 - 26a_1a_2^3a_3^2a_4a_6^3 + 4a_1a_2^3a_3^2a_5^2a_6^2 - 20a_1a_2^3a_3a_4^2a_5a_6^2 \\
& + 12a_1a_2^3a_3a_4a_5^3a_6 + a_1a_2^3a_4^4a_6^2 + 3a_1a_2^3a_4^3a_5^2a_6 + 2a_1a_2^3a_4^2a_5^4 \\
& - 16a_1a_2^2a_3^4a_6^3 - 16a_1a_2^2a_3^3a_4a_5a_6^2 - 12a_1a_2^2a_3^2a_4^3a_6^2 - 32a_1a_2^2a_3^2a_4^2a_5^2a_6 \\
& - 14a_1a_2^2a_3a_4^4a_5a_6 - 4a_1a_2^2a_3a_4^3a_5^3 + a_1a_2^2a_4^6a_6 - a_1a_2^2a_4^5a_5^2 \\
& + 16a_1a_2a_3^3a_4^3a_5a_6 + 11a_1a_2a_3^2a_4^5a_6 + 2a_1a_2a_3^2a_4^4a_5^2 + 2a_1a_2a_3a_4^6a_5 \\
& - a_1a_3^2a_4^7 + a_2^6a_5a_6^3 + 11a_2^5a_3a_4a_6^3 + 8a_2^5a_4^2a_5a_6^2 \\
& - 4a_2^5a_4a_5^3a_6 + 4a_2^4a_3^3a_6^3 + 2a_2^4a_3^2a_4a_5a_6^2 - 2a_2^4a_3a_4^3a_6^2 \\
& + 8a_2^4a_3a_4^2a_5^2a_6 + 2a_2^4a_4^4a_5a_6 - a_2^4a_4^3a_5^3 + 2a_2^3a_3^3a_4^2a_6^2 \\
& - 2a_2^3a_3^2a_4^3a_5a_6 - 2a_2^3a_3a_4^5a_6 + 3a_2^3a_3a_4^4a_5^2 - 2a_2^2a_3^3a_4^4a_6 \\
& - 3a_2^2a_3^2a_4^5a_5 + a_2a_3^3a_4^6 - 68a_0^4a_4a_6^4 + 200a_0^4a_5^2a_6^3 \\
& - 32a_0^3a_1a_3a_6^4 - 516a_0^3a_1a_4a_5a_6^3 - 92a_0^3a_1a_5^3a_6^2 + 134a_0^3a_2^2a_6^4 \\
& - 308a_0^3a_2a_3a_5a_6^3 - 418a_0^3a_2a_4^2a_6^3 + 320a_0^3a_2a_4a_5^2a_6^2 + 8a_0^3a_2a_5^4a_6
\end{aligned}$$

$$\begin{aligned}
& -48a_0^3a_3^2a_4a_6^3 - 56a_0^3a_3^2a_5^2a_6^2 - 320a_0^3a_3a_4^2a_5a_6^2 - 16a_0^3a_3a_4a_5^3a_6 \\
& -12a_0^3a_3a_5^5 - 86a_0^3a_4^4a_6^2 + 30a_0^3a_4^3a_5^2a_6 + 10a_0^3a_4^2a_5^4 \\
& -264a_0^2a_1^2a_2a_6^4 + 168a_0^2a_1^2a_3a_5a_6^3 + 72a_0^2a_1^2a_4^2a_6^3 + 24a_0^2a_1^2a_4a_5^2a_6^2 \\
& +12a_0^2a_1^2a_5^4a_6 - 70a_0^2a_1a_2^2a_5a_6^3 + 452a_0^2a_1a_2a_3a_4a_6^3 - 372a_0^2a_1a_2a_4^2a_5a_6^2 \\
& -82a_0^2a_1a_2a_4a_5^3a_6 - 2a_0^2a_1a_2a_5^5 + 32a_0^2a_1a_3^3a_6^3 + 120a_0^2a_1a_3^2a_4a_5a_6^2 \\
& +48a_0^2a_1a_3^2a_5^3a_6 - 48a_0^2a_1a_3a_4^3a_6^2 - 72a_0^2a_1a_3a_4^2a_5^2a_6 - 12a_0^2a_1a_3a_4a_5^4a_6^4 \\
& -28a_0^2a_1a_4^4a_5a_6 - 2a_0^2a_1a_4^3a_5^3 - 41a_0^2a_2^3a_4a_6^3 + 78a_0^2a_2^3a_5^2a_6^2 \\
& +4a_0^2a_2^2a_3^2a_6^3 - 330a_0^2a_2^2a_3a_4a_5a_6^2 - 28a_0^2a_2^2a_3a_5^3a_6 - 136a_0^2a_2^2a_3^2a_4^2a_6^2 \\
& +155a_0^2a_2^2a_4^2a_5^2a_6 + 7a_0^2a_2^2a_4a_5^4 + 72a_0^2a_2a_3^3a_5a_6^2 + 264a_0^2a_2a_3^2a_4^2a_6^2 \\
& -16a_0^2a_2a_3^2a_4a_5^2a_6 + 12a_0^2a_2a_3^2a_5^4 - 94a_0^2a_2a_3a_4^3a_5a_6 + 10a_0^2a_2a_3a_4^2a_5^3 \\
& -15a_0^2a_2a_4^5a_6 + 5a_0^2a_2a_4^4a_5^2 + 48a_0^2a_3^3a_4^2a_5a_6 - 48a_0^2a_3^3a_4^4a_6 \\
& -2a_0^2a_3a_4^5a_5 - a_0^2a_4^7 + 64a_0a_1^4a_6^4 - 16a_0a_1^3a_2a_5a_6^3 \\
& -144a_0a_1^3a_3a_4a_6^3 - 48a_0a_1^3a_3a_5^2a_6^2 - 48a_0a_1^3a_4^2a_5a_6^2 - 156a_0a_1^2a_2^2a_4a_6^3 \\
& -4a_0a_1^2a_2^2a_5^2a_6^2 - 48a_0a_1^2a_2a_3^2a_6^3 + 288a_0a_1^2a_2a_3a_4a_5a_6^2 + 40a_0a_1^2a_2a_3a_5^3a_6 \\
& +88a_0a_1^2a_2a_4^3a_6^2 + 36a_0a_1^2a_2a_4^2a_5^2a_6 + 6a_0a_1^2a_2a_4a_5^4 - 16a_0a_1^2a_3^3a_5a_6^2 \\
& +48a_0a_1^2a_3^2a_4^2a_6^2 + 64a_0a_1^2a_3^2a_4a_5^2a_6 - 16a_0a_1^2a_3a_4^3a_5a_6 + 8a_0a_1^2a_3a_4^2a_5^3 \\
& -32a_0a_1^2a_4^5a_6 - 4a_0a_1^2a_4^4a_5^2 + 84a_0a_1a_2^3a_3a_6^3 - 76a_0a_1a_2^3a_4a_5a_6^2 \\
& -14a_0a_1a_2^3a_5^3a_6 + 40a_0a_1a_2^2a_3^2a_5a_6^2 + 40a_0a_1a_2^2a_3a_4^2a_6^2 - 104a_0a_1a_2^2a_3a_4a_5^2a_6 \\
& -12a_0a_1a_2^2a_3a_5^4 - 56a_0a_1a_2^2a_4^3a_5a_6 - 18a_0a_1a_2^2a_4^2a_5^3 - 64a_0a_1a_2a_3^3a_4a_6^2 \\
& -48a_0a_1a_2a_3^3a_5^2a_6 + 56a_0a_1a_2a_3^2a_4^2a_5a_6 + 80a_0a_1a_2a_3a_4^4a_6 - 8a_0a_1a_2a_3a_4^3a_5^2 \\
& +4a_0a_1a_2a_4^5a_5 + 48a_0a_1a_3^3a_4a_6 + 8a_0a_1a_3^2a_4^4a_5 - 8a_0a_1a_3a_4^6 \\
& -6a_0a_2^5a_6^3 - 29a_0a_2^4a_3a_5a_6^2 - 11a_0a_2^4a_4^2a_6^2 + 39a_0a_2^4a_4a_5^2a_6 \\
& +a_0a_2^4a_5^4 - 4a_0a_2^3a_3^2a_4a_6^2 + 8a_0a_2^3a_3^2a_5^2a_6 - 58a_0a_2^3a_3a_4^2a_5a_6 \\
& +8a_0a_2^3a_3a_4a_6^3 - 19a_0a_2^3a_4^4a_6 + 14a_0a_2^3a_4^3a_5^2 - 16a_0a_2^2a_3^4a_6^2 \\
& +32a_0a_2^2a_3^3a_4a_5a_6 - 8a_0a_2^2a_3^2a_4^3a_6 - 8a_0a_2^2a_3^2a_4^2a_5^2 - 14a_0a_2^2a_3a_4^4a_5 \\
& +a_0a_2^2a_4^6 - 48a_0a_2a_3^4a_4^2a_6 + 2a_0a_2a_3^2a_4^5 + 48a_1^4a_2a_4a_6^3 \\
& +16a_1^4a_2^3a_6^3 - 16a_1^4a_4^3a_6^2 - 40a_1^3a_2^2a_3a_6^3 - 16a_1^3a_2^2a_4a_5a_6^2 \\
& -32a_1^3a_2a_3^2a_5a_6^2 - 32a_1^3a_2a_3a_4^2a_6^2 - 24a_1^3a_2a_3a_4a_5^2a_6 - 16a_1^3a_3^3a_4a_6^2 \\
& +16a_1^3a_3^2a_4^2a_5a_6 + 16a_1^3a_3a_4^4a_6 + 8a_1^3a_3a_4^3a_5^2 - 7a_1^2a_2^4a_6^3 \\
& +24a_1^2a_2^3a_3a_5a_6^2 + 4a_1^2a_2^3a_4^2a_6^2 + 10a_1^2a_2^3a_4a_5^2a_6 + a_1^2a_2^3a_5^4 \\
& +40a_1^2a_2^2a_3^2a_4a_6^2 + 32a_1^2a_2^2a_3^2a_5^2a_6 + 32a_1^2a_2^2a_3a_4^2a_5a_6 + 8a_1^2a_2^2a_3a_4a_5^3 \\
& -4a_1^2a_2^2a_4^4a_6 - 2a_1^2a_2^2a_4^3a_5^2 + 16a_1^2a_2a_3^4a_6^2 - 16a_1^2a_2a_3^3a_4a_5a_6 \\
& -40a_1^2a_2a_3^2a_4^3a_6 - 8a_1^2a_2a_3^2a_4^2a_5^2 - 8a_1^2a_2a_3a_4^4a_5 + 8a_1^2a_3^2a_4^5 \\
& -5a_1a_2^4a_3a_4a_6^2 - 16a_1a_2^4a_3a_5^2a_6 - 13a_1a_2^4a_4^2a_5a_6 - 4a_1a_2^4a_4a_5^3 \\
& +18a_1a_2^3a_3a_4^3a_6 - 4a_1a_2^3a_3a_4^2a_5^2 + 2a_1a_2^3a_4^4a_5 + 16a_1a_2^2a_3^3a_4^2a_6
\end{aligned}$$

$$\begin{aligned}
& + 16a_1a_2^2a_3^2a_4^3a_5 - 2a_1a_2^2a_3a_4^5 - 8a_1a_2a_3^3a_4^4 - 3a_2^6a_4a_6^2 \\
& + 2a_2^6a_5^2a_6 - a_2^5a_3^2a_6^2 + a_2^5a_3a_4a_5a_6 - a_2^5a_4^3a_6 \\
& + 3a_2^5a_4^2a_5^2 - 4a_2^4a_3^2a_4^2a_6 - 6a_2^4a_3a_4^3a_5 + 3a_2^3a_3^2a_4^4 \\
& + 32a_0^4a_3a_6^3 - 360a_0^4a_4a_5a_6^2 - 72a_0^4a_5^3a_6 + 104a_0^3a_1a_2a_6^3 \\
& + 352a_0^3a_1a_3a_5a_6^2 + 320a_0^3a_1a_4^2a_6^2 + 80a_0^3a_1a_4a_5^2a_6 + 8a_0^3a_1a_5^4 \\
& - 316a_0^3a_2^2a_5a_6^2 + 416a_0^3a_2a_3a_4a_6^2 + 56a_0^3a_2a_3a_5^2a_6 - 196a_0^3a_2a_4^2a_5a_6 \\
& - 36a_0^3a_2a_4a_5^3 + 80a_0^3a_2^2a_4a_5a_6 + 48a_0^3a_3^2a_5^3 + 64a_0^3a_3a_4^3a_6 \\
& - 56a_0^3a_3a_4^2a_5^2 + 4a_0^3a_4^4a_5 + 16a_0^2a_1^3a_6^3 + 96a_0^2a_1^2a_2a_5a_6^2 \\
& - 240a_0^2a_1^2a_3a_4a_6^2 - 96a_0^2a_1^2a_3a_5^2a_6 + 48a_0^2a_1^2a_4^2a_5a_6 + 192a_0^2a_1a_2^2a_4a_6^2 \\
& + 8a_0^2a_1a_2^2a_5^2a_6 - 336a_0^2a_1a_2a_3^2a_6^2 + 312a_0^2a_1a_2a_3a_4a_5a_6 + 16a_0^2a_1a_2a_3a_5^3 \\
& + 96a_0^2a_1a_2a_4^3a_6 + 24a_0^2a_1a_2a_4^2a_5^2 - 64a_0^2a_1a_3^3a_5a_6 + 48a_0^2a_1a_2^3a_4^2a_6 \\
& + 48a_0^2a_1a_2^2a_4a_5^2 + 8a_0^2a_1a_3a_4^3a_5 + 8a_0^2a_1a_4^5 + 144a_0^2a_2^3a_3a_6^2 \\
& - 170a_0^2a_2^3a_4a_5a_6 - 6a_0^2a_2^3a_5^3 + 80a_0^2a_2^2a_3^2a_5a_6 + 20a_0^2a_2^2a_3a_4^2a_6 \\
& - 32a_0^2a_2^2a_3a_4a_5^2 - 22a_0^2a_2^2a_4^3a_5 - 80a_0^2a_2a_3^3a_4a_6 - 48a_0^2a_2a_3^3a_5^2 \\
& - 8a_0^2a_2a_3^2a_4^2a_5 + 20a_0^2a_2a_3a_4^4 + 96a_0a_1^3a_3^2a_6^2 + 32a_0a_1^3a_4^3a_6 \\
& - 48a_0a_1^2a_2^2a_3a_6^2 + 32a_0a_1^2a_2^2a_4a_5a_6 - 80a_0a_1^2a_2a_3^2a_5a_6 - 176a_0a_1^2a_2a_3a_4^2a_6 \\
& - 48a_0a_1^2a_2a_3a_4a_5^2 - 64a_0a_1^2a_3^3a_4a_6 - 16a_0a_1^2a_3^2a_4^2a_5 + 16a_0a_1^2a_3a_4^4 \\
& - 18a_0a_1a_2^4a_6^2 + 56a_0a_1a_2^3a_3a_5a_6 + 68a_0a_1a_2^3a_4^2a_6 + 16a_0a_1a_2^3a_4a_5^2 \\
& + 16a_0a_1a_2^2a_3^2a_4a_6 + 48a_0a_1a_2^2a_3^2a_5^2 + 72a_0a_1a_2^2a_3a_4^2a_5 - 4a_0a_1a_2^2a_4^4 \\
& + 64a_0a_1a_2a_3^4a_6 - 32a_0a_1a_2a_3^2a_4^3 - 17a_0a_2^5a_5a_6 + 24a_0a_2^4a_3a_4a_6 \\
& - 8a_0a_2^4a_3a_5^2 - 21a_0a_2^4a_4^2a_5 - 16a_0a_2^3a_3^3a_6 - 16a_0a_2^3a_3^2a_4a_5 \\
& + 12a_0a_2^3a_3a_4^3 + 16a_0a_2^2a_3^3a_4^2 + 16a_1^3a_2^3a_6^2 + 48a_1^3a_2a_3^2a_4a_6 \\
& - 16a_1^3a_3^3a_4^3 - 40a_1^2a_2^3a_3a_4a_6 - 8a_1^2a_2^3a_3a_5^2 - 32a_1^2a_2^2a_3^3a_6 \\
& - 16a_1^2a_2^2a_3^2a_4a_5 + 8a_1^2a_2^2a_3a_4^3 + 16a_1^2a_2a_3^3a_4^2 + 7a_1a_2^5a_4a_6 \\
& + 2a_1a_2^5a_5^2 + 16a_1a_2^4a_3^2a_6 + 16a_1a_2^4a_3a_4a_5 - a_1a_2^4a_4^3 \\
& - 16a_1a_2^3a_3^2a_4^2 - 2a_2^6a_3a_6 - 3a_2^6a_4a_5 + 3a_2^5a_3a_4^2 \\
& + 304a_0^4a_2a_6^2 + 288a_0^4a_3a_5a_6 + 48a_0^4a_4^2a_6 + 48a_0^4a_4a_5^2 \\
& - 160a_0^3a_1^2a_6^2 - 16a_0^3a_1a_2a_5a_6 - 256a_0^3a_1a_3a_4a_6 - 64a_0^3a_1a_3a_5^2 \\
& - 16a_0^3a_1a_4^2a_5 + 200a_0^3a_2^2a_4a_6 + 32a_0^3a_2^2a_5^2 - 352a_0^3a_2a_3^2a_6 \\
& + 144a_0^3a_2a_3a_4a_5 - 24a_0^3a_2a_4^3 - 64a_0^3a_3^3a_5 + 64a_0^3a_3^2a_4^2 \\
& - 144a_0^2a_1^2a_2a_4a_6 + 192a_0^2a_1^2a_3^2a_6 - 16a_0^2a_1^2a_4^3 - 16a_0^2a_1a_2^2a_4a_5 \\
& - 32a_0^2a_1a_2a_3^2a_5 - 80a_0^2a_1a_2a_3a_4^2 - 64a_0^2a_1a_3^3a_4 + 52a_0^2a_2^4a_6 \\
& + 24a_0^2a_2^3a_3a_5 + 16a_0^2a_2^3a_4^2 + 16a_0^2a_2^2a_3^2a_4 + 64a_0^2a_2a_3^4 \\
& - 32a_0a_1^2a_2^3a_6 + 96a_0a_1^2a_2a_3^2a_4 - 64a_0a_1a_2^3a_3a_4 - 64a_0a_1a_2^2a_3^3 \\
& + 10a_0a_2^5a_4 + 16a_0a_2^4a_3^2 + 16a_1^2a_2^3a_3^2 - 8a_1a_2^5a_3
\end{aligned}$$

$$\begin{aligned}
& + a_2^7 - 64a_0^4a_1a_6 - 32a_0^4a_2a_5 - 192a_0^4a_3a_4 \\
& + 96a_0^3a_1a_2a_4 + 128a_0^3a_1a_3^2 - 128a_0^3a_2^2a_3 + 16a_0^2a_1a_2^3 \\
& + 128a_0^5
\end{aligned} \tag{A.35}$$

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