

A STUDY OF ELECTRICAL EQUIVALENT CIRCUIT BATTERY
PARAMETER AND SOC ESTIMATION METHODS, AND OPTIMUM
CHARGING STRATEGIES FOR SAFETY AND LONGEVITY

by

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To my wife and daughter

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ABSTRACT

A STUDY OF ELECTRICAL EQUIVALENT CIRCUIT BATTERY PARAMETER AND SOC ESTIMATION METHODS, AND OPTIMUM CHARGING STRATEGIES FOR SAFETY AND LONGEVITY

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Adviser: Dr. Manohar Das, Ph.D.

This dissertation investigates enhancements to battery charging techniques by addressing issues associated with Lithium-ion batteries, particularly those related to fast and safe charging. The research covers a series of interconnected topics that contribute to advancements in battery technology. One goal of this study is to improve the accuracy of battery modeling and parameter estimation, the other being development of new charging techniques, ultimately improving the reliability and longevity of Li-ion batteries.

One of the primary objectives focuses on investigating the estimation of time-varying parameters of the Electrical-Equivalent Circuit (EEC) model, which is essential for accurate modeling of battery's charging behavior. Two techniques were used for estimation: a direct continuous-time system identification technique based on State Variable Filtering (SVF) and the Indirect Discrete Time (IDT) technique. The results

from both methods are compared to determine the most effective approach for accurate modeling.

The second objective focuses on assessing the robustness of the parameter estimation techniques by integrating a dynamic, time-varying EEC model to enhance the accuracy of State-of-Charge estimation. Kalman filter in its various forms is utilized for SOC estimation.

The third objective of this research is to create an improved Multi-stage Charging Current (MCC) profile. The goal is to optimize the charging current at each stage to reduce the total charge time while ensuring the safety of the battery by carefully monitoring its temperature during charging. To ensure efficient and effective charging of this technique without compromising the battery health, careful control of the charging current at each stage is required.

The fourth objective of this study is to develop a continuous, exponentially decaying charging current profile. This technique addresses one of the shortcomings of MCC, by minimizing the stress fluctuations caused by abrupt shifts in current during transitions between stages. This optimized current profile offers a balance between fast charging and protecting the battery from potential damage.

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LIST OF ABBREVIATIONS

I	Battery Current
$I_1, I_2, \dots, I_N$	Charging currents in MCC stages
V_t	Battery terminal voltage
V_{OCV}	Open-circuit voltage
SOC	State of Charge
SOH	State of Health
Q	Battery capacity
R_0	Ohmic/internal resistance
R_1, R_2	Polarization resistances
C_1, C_2	Polarization capacitances
τ_1, τ_2	Time constants of the RC branches
V_1, V_2	RC branch voltages
T_c	Battery core temperature
T_s	Battery surface temperature
T_{amb}	Ambient temperature
P	Power
E	Energy
k	Discrete-time index
t	Time
λ	Forgetting factor

LIST OF ABBREVIATIONS—Continued

P_k	Covariance matrix
Q_k	Process noise covariance matrix
R_k	Measurement noise covariance matrix
Ah	Ampere-hour
BMS	Battery Management System
BoL	Beginning of Life
CC	Constante Current
CC-CV	Constante Current - Constante Voltage
DP	Dynamic Programming
EDC	Exponentially Decaying Current
EEC	Electrical Equivalent Circuit
EKF	Extended Kalman Filter
EV	Electric Vehicle
GA	Genetic Algorithm
IDT	Indirect Discrete-Time
KF	Kalman Filter
Li-ion	Lithium-ion
LKF	Linear Kalman Filter
MCC	Multistage Constant Current

LIST OF ABBREVIATIONS—Continued

OCV	Open Circuit Voltage
PCC	Pulse Charging Current
RLS	Recursive Least Squares
SEI	Solid Electrolyte Interphase
SOC	State of Charge
SOH	State of Health
SVF	State Variable Filtering
UKF	Unscented Kalman Filter

CHAPTER ONE

INTRODUCTION

The global transition toward cleaner energy has increased demand for electric vehicles. However, these vehicles still rely primarily on Lithium-ion batteries, which form the backbone of green electric mobility. Li-ion batteries' high energy density, low self-discharge, and long cycle life make them the battery of choice for both consumer electronics and transportation. Also, ongoing material improvements and cost reductions have further supported their adoption [1].

Many types of battery technologies are available in today's market, designed for specific applications. In the automotive sector, for example, major car manufacturers are phasing out internal combustion engines and moving to full electric vehicles (EVs) to meet strict clean air regulations imposed by governments around the world [2]. Among applicable battery chemistries for EVs, Li-ion batteries are the most promising solution to advance powertrain electrification. In addition to offering higher energy efficiency and power density, Li-ion batteries offer the desirable feature of fast charging [3]. This characteristic is fundamental for car manufacturers to overcome EV concerns, often expressed by consumers, compared to traditional gasoline vehicles, e.g., longer charging times and limited charging infrastructure, which are often cited as issues that need to be addressed to build customer confidence toward the acceptance of battery electric vehicles. Battery materials research has focused on improving energy density to extend

battery range, but in recent years, however, fast charging capability has also become a priority for scientists and manufacturers worldwide. Fast charging offers several advantages, including reduced total charge time, reduced consumer concerns about limited range, and support the adoption of higher-capacity batteries in the automotive sector [3]. While fast charging technology is widely accepted as a key feature for the mass deployment of EVs in today's market, it is critical to address the imminent battery degradation and early cell aging caused by these charging techniques. Fast charging at higher current can negatively impact the function and battery cell longevity by diminishing its performance and capacity through resistive heating and internal temperature rise. Poor battery temperature management during charging process can lead to serious safety issues [3], therefore, developing a sophisticated and dependable battery management system (BMS) is vital in ensuring battery safety during operation. A BMS typically has the capability to monitor and evaluate key parameters like voltage, current, internal temperature variations, in addition to State of Charge (SOC) and State of Health (SOH) estimation [4]. This data is then consumed by the BMS to provide optimal operating conditions, which include proper cooling methods, cell balancing and power control during charging and discharging. With this systematic approach, the battery is made to operate with full potential over its expected lifecycle, while prioritizing safety.

Before proceeding with the design phase of a BMS, it is crucial to develop an accurate battery model. Battery models in general provide predictions of the overall performance and understanding of its behavior. These models can also predict the physical limitation of batteries during severe operating conditions. Literature presents many types of models that are categorized based on their accuracy and complexity.

Depending on the application of interest, a specific type of battery model is required to generate desired outputs. For example, while Electrochemical models [5], [6] provide an accurate reading of the physical process and battery dynamics, they are, however, complex models that require higher computational cost. Thus, it only makes sense that such models are used for the long-term study and development of batteries. Empirical models [7] come with a simple representation of the system. They often provide a poor picture since these models rely on approximations of battery operations that are usually based on experimental data rather than mathematical modeling. Analytical models, on the other hand, provide a balance between accuracy and simplicity. These models can capture battery performance at a higher resolution without compromising computation since the overall battery reaction is modeled using less mathematical equations. Compared to electrochemical models, analytical models describe the battery behavior at a higher level of abstraction [8]. While many types of analytical models exist, the Electrical Equivalent Circuit (EEC) model is the most suitable for battery management systems applications.

Additionally, an effective Battery Management System (BMS) requires an optimized charging approach that improves charging efficiency without compromising the battery's long-term performance. Zhang et al [9] classify charging strategies into four main categories, namely: those focusing on enhancing battery materials [10], [11], polarization-based techniques [12,13], methods focused on improving charging current [14], [15], [16], [17], and approaches that focus on the improvement of battery models [18], [19], [20], [21], [22]. Each approach has strengths and weaknesses. For example, methods focusing on improvement of battery materials show promising results as ongoing research explores new cell materials for next-generation Li-ion batteries to

achieve higher specific energy and lower costs. However, extensive testing is required to confirm they are safe and reliable for use. Similarly, polarization-based methods show promise but require further development and tend to be more complex than approaches focused on improving charging current or battery models. Since these last two strategies are particularly relevant, the charging method discussed in this dissertation follows similar principles.

Among the methods aimed at enhancing charging currents, there is a clear distinction between older and newer approaches. The older ones are of a heuristic nature and lack mathematical rationality and basis. On the other hand, the newer approaches presented by studies such as [14], [15], [16], [17] focus on designing optimal current patterns to speed up charging currents and extend battery life. For instance, Guo et al [14] presents a method that utilizes an EEC model to create an optimal nonlinear current profile by utilizing genetic algorithms for optimization. This approach shows the potential to reduce charging time while increasing the longevity of the battery cycles. However, it has limitations, including a highly nonlinear current profile, which makes it less suitable for time-sensitive applications due to the computational demands of the genetic algorithm. Other methods [15], [16], [17] can be described as variations of multistage constant current charging techniques. Vo et al [15] proposed using the Taguchi method to search for the optimal MCC profile. This technique is more effective than conventional charging methods as it reduced charging time and temperature variation. However, the computational cost of this method might limit its adoption.

In contrast with static charging techniques that use pre-defined profiles, adaptive charging currents compensate for internal variations within the battery during its lifespan.

These methods, in general, can be classified into experimental and model-based techniques, which often combine with optimization techniques. Chen et al [16], for instance, developed a method to find the optimal charging current patterns for a multistage constant current profile using an EEC model and the Grey Wolf optimization technique. This approach can reduce charging time, losses, along with lowering temperature rise, therefore improving both charging efficiency and battery life. Similarly, Jiang et al [17] introduced a hybrid charging strategy with adaptive current control, known as the CC-CC-CV strategy, which also reduces maximum temperature rise and energy losses.

Among the methods aimed at enhancing battery models [18], [19], [20], [21], [22], most approaches proposed for fast charging rely on either simplified electrochemical model [18], [19], [20] or EEC models [21], [22] to develop rapid charging strategies. In this study, Liu et al [18] formulated fast charging as a dynamic optimization problem to determine the battery's charging current profile to reach a targeted SOC while constraining the rate of lithium plating. They used a physics-based model combined with the Gauss pseudospectra method to optimize the current trajectory. Xu et al [19] developed an optimal MCC charging strategy to minimize charging time and capacity fading caused by Solid Electrolyte Interphase (SEI) buildup, lithium plating, and temperature rise. The strategy used an electrochemical battery model and dynamic programming (DP) for optimization.

Liu et al [20] introduced a comparable method but employed a Genetic Algorithm (GA) for optimization. The charging methods discussed here offer significant advantages for specific applications but have as the main disadvantage the high computational

demands of both the electrochemical model and the optimization methods. This drawback makes them less suitable for real-time applications requiring the fast determinations of optimal charging patterns. In contrast, equivalent circuit model-based methods, as developed in [21], [22], offer simple implementation and are promising for real-time applications. Both Hu et al [21] and Zhang et al [22] introduced EEC model-based fast charging strategies, that are based on solving a multi-objective optimization problem to minimize both charging time and losses simultaneously. The main difference between these approaches is the optimization method utilized; that is, [21] made use of large-scale Sequential Quadratic Programming (SQP), while [22], applied Dynamic Programming (DP). However, both SQP and DP come with a large computational requirement, thus the need for computationally more efficient optimization approaches.

The dissertation is organized into seven chapters. Chapter 1 introduces the motivation of research, problem statements, a literature review, and main contributions. Chapter 2 presents some background materials related to Li-ion batteries and their models, focusing mainly on the Electrical-Equivalent Circuit (EEC) model used throughout the dissertation. Chapter 3 presents two methods of parameter estimation of a time-varying EEC model, a direct continuous-time system identification technique based on State Variable Filtering (SVF) and an Indirect Discrete Time (IDT) method. It also includes a comparative evaluation of their performance. Chapter 4 presents two methods of SOC estimation of Li-ion batteries using Kalman filtering strategies, a technique based on Extended Kalman Filter (EKF) and a method based on Unscented Kalman Filter (UKF). It also includes a comparative evaluation of their performance. Chapter 5 presents a fast optimal charging scheme based on a Multi-stage Constant Current (MCC) profile

and points out one of its potential drawbacks resulting from the stress introduced by abrupt changes in charging current from one stage to another. Chapter 6, on the other hand, presents a fast optimal charging scheme based on an Exponentially Decaying Charging current profile, which attempts to mitigate the above drawback of MCC charging scheme. Finally, Chapter 7 summarizes the conclusions and outlines future research directions.

This dissertation makes the following new contributions. First, it introduces time-varying parameter estimation using SVF and IDT techniques to achieve improved battery modeling accuracy. Second, it delivers enhanced SOC estimation by integrating the dynamic EEC model with Kalman filters-based methods. Third, it proposes an optimized multi-stage current (MCC) charging strategy to reduce charging time while ensuring thermal safety, using a simplified optimization search with guaranteed convergence. Fourth, it develops a continuous exponentially decaying charging profile to minimize current-induced stress and improve battery longevity.

CHAPTER TWO

BACKGROUND

2.1. Introduction

A definition of a battery would be an energy storage device containing one or more electrochemical cells capable of converting chemical energy into electricity. For the intended applications, battery cells are usually connected in series and or in parallel configurations. It contains the following components: an anode, a cathode, an electrolyte, and a separator. When the battery is in use, a chemical reaction takes place inside the cell, making electrons flow from one electrode to the other, which generates electricity.

2.2. Types of Batteries

Batteries are mainly classified into two main types: primary and secondary. Primary (non-rechargeable) cells like Alkaline. These cells are made from chemical materials that prevent cells from reverting to their original condition once their chemical reaction is converted into electricity. While recharging primary cells is not possible, thus they are disposable when drained, these cells however offer advantages such as high specific energy, long storage time and immediate readiness for use [23].

Unlike primary batteries, secondary batteries are rechargeable batteries which, to a certain extent, can be restored to their original state before discharge by conversion of electrical energy into chemical reactions. This process is achieved by applying reverse charging current to normal cell operation. Rechargeable batteries are currently the focus

in the research, particularly Lithium-ion batteries and their applications in wide varieties of applications like renewable power storage and the automotive sector.

2.3. Why Lithium-Ion Cell

Compared with other battery chemistries, Lithium-ion batteries have many advantages such as high energy/power density, high charge/discharge cycles, low self-discharge, and fast-charge capabilities. On the other hand, Li-ion cells require careful monitoring and protection from overcharge and discharge to avoid capacity fading, lithium plating, and thermal runaway.

As shown in Figure 1, four main elements are contained within the casing of a battery cell. These elements are made of materials selected specifically as their chemical reactions create a certain level of cell performance in terms of voltage and current.

The first layer contains Anode materials (negative terminal) which often consist of Graphitic carbon powder since graphite improves the conductivity of the mixture as well as increases energy density [24].

Next is a porous separator or a barrier, which keep the anode and Cathode from coming into direct contact. This is very important to prevent self-discharge and short circuiting the cell. The electrolyte, which consists of a salt dissolving organic solvent, allows ion movement between the two electrodes. Lastly, the cathode layer, which is the primary source of active Lithium, remains a hot subject for research as it limits the energy density and keeps battery cost high [25]. As a result, this material must integrate a considerable amount of Lithium to give high energy capacity. Furthermore, to improve cycle life, cathode materials must be able to transfer Lithium with minimal structural changes reversibly [26].

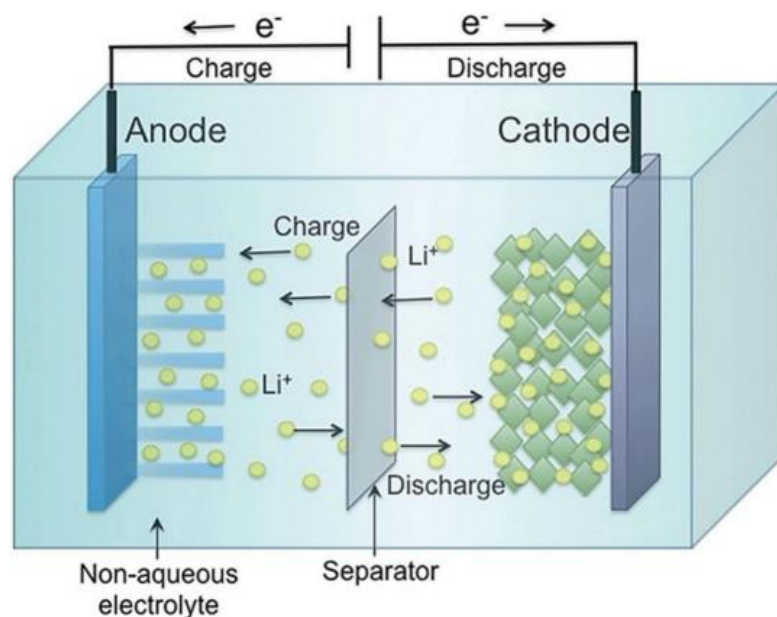


Figure 2.1. Lithium-ion intercalation during charge/discharge [27].

Based on the intercalation mechanism, defined as the reversible inclusion or insertion of a molecule/ion into compounds with layered structures, Lithium is stored between layers of carbon graphite within the anode when fully charged. Once the battery cell is cycled, electrons are separated from Lithium, causing a build-up of positively charged Lithium (Lithium-ion) to leave the graphite toward the cathode via the electrolyte. In contrast, the electrons travel throughout an external circuit. At the cathode, the lithium-ion and the electrons are recombined and again intercalate within the layers of the transition metal oxide of the cathode. The opposite reaction occurs when charging the Battery, moving the Lithium back toward the anode.

Different terms and variables are used to describe the state and properties of Li-ion battery cells. Summary of variables and terms used in this research are defined as follows:

2.4. Battery Terminology

2.4.1. Cell Capacity

Cell capacity is a measure of the total energy that can be stored or extracted from a fully charged battery. Its unit is expressed in ampere- hours (Ah). It refers to the total charge available when a battery is discharged at a certain C-rate from 100% State-of-charge till it reaches a pre-defined cut-off-voltage [28]. Cell capacity is strongly affected by internal impedances which can vary significantly based on previous operating conditions. It is calculated by multiplying the discharge current (in amperes) by the discharge duration (in hours)

2.4.2. C-rate

The C-rate measures the rate at which a battery charges or discharges compared to its maximum capacity. A 1C rate means that the battery can deliver a current equal to its capacity in Amps for a duration of one hour. For example: if a battery has a capacity of 1000 mAh, charging or discharging at 1C rate would correspond to current of 1000 mA. Likewise, at a 2C rate, the current would be 2000mA, and the 0.5C rate would correspond to a current of 500 mA. C-rate is essential to understand the charging/discharging capabilities of a battery [28].

2.4.3. Open Circuit Voltage

Open Circuit Voltage (OCV) of a battery refers to the unloaded battery terminal voltage, when and only when the battery has attained a state of equilibrium. While OCV is primarily a function of SOC as pointed out by numerous studies [29], it is also affected,

to a lesser extent, by temperature. However, studies have shown that temperature effect on OCV is relatively insignificant compared to that of the SOC.

To establish the relationship between OCV and SOC, experimental data is obtained by measuring the steady-state open-circuit terminal voltage at various SOC levels [30]. This process involves taking measurements over extended periods and could take days to collect data, which is sometimes affected by hysteresis since cells follow different charge-discharge curves caused by long-term aging. Understanding the relationship between OCV and SOC is essential for optimal battery usage and SOC estimation.

2.4.4. State Of Charge

Battery State of Charge (SOC) represents how much of energy is currently stored in a battery, giving an indication on its current capacity. It is calculated as the ratio of the cell's remaining capacity to its total capacity, where total capacity refers to the maximum ampere-hours a fully charged cell in its beginning-of-life could deliver. While residual capacity would be defined as the number of ampere-hours that could be extracted from a cell in current capacity till its fully discharged. Accurate estimation of a batteries' SOC will help gauge the available energy withing the battery for use. Monitoring SOC level is critical in automotive applications, it allows for accurate estimation of the remaining driving range, battery health management, and optimizes the overall performance [31].

2.4.5. State of Health

State of Health (SOH) measures a cell's current condition relative to its original state. SOH is expressed in percentage, ranging from the beginning-of-life (BoL) to end-of-life (EoL), indicating how much of a battery's life remains before replacement is needed.

SOH is typically “measured” using various methods, but the internal resistance and capacity fade estimation being the two primary indicators [32].

2.5. Review of Some Available Charging Protocols

The main goal for fast charging techniques is to minimize total charging time while optimizing efficiency and durability of the battery. Charging with high current would increase the loss of active materials, Solid-Electrolyte Interphase (SEI) at the surface of the electrodes, increase of Lithium plating (thus loss of lithium inventory) [33], etc.... which eventually causes an increase of battery internal impedance resulting in degradation and capacity fading. Therefore, it is a must to come up with a high-quality charging protocol with high efficiency aimed at reducing total charge time but also improving performance and lifespan of the battery.

Literature presents different charging techniques which were developed for the improvement of batteries performance and reliability. These techniques can be categorized into three groups which will be discussed separately.

2.5.1. Constant Current

Constant Current trickle charge consists of setting a battery to charge with a single rated uniform constant current until full charge is reached. Implementing such charging methods remain simple and inexpensive that relies on user to start and to terminate charging. While this technique helps with cell balancing within the battery pack, feedback based on critical battery parameters such as cell voltage, temperature, etc. is not considered, therefore risk of cell damaging due to overcharge and potential dendrite.

2.5.2. Fast Constant Current Charging Technique

Following the same principle as above, fast constant current charging consists of setting a battery to charge with a large constant current profile. Such method could improve total charge time, however implementing requires careful charge control and cell monitoring to terminate charging once cell is full. While this technique significantly improves total charge time, it does however, result to a negative long-term effect including capacity fading.

2.5.3. Constant Current Constant Voltage Passive Charging

Passive charging Constant Current-Constant Voltage (CC-CV) is the commonly used charging method for most applications. It includes two phases, a constant current (CC) phase where the Battery is charged using a constant current until a cut-off voltage reaches a pre-set value, followed by a constant voltage (CV) phase where the Battery maintains the cut-off voltage as the current drops to a near-zero. However, during CV stage, the charging time is significantly increasing waiting for the current to drop while maintaining the cut-off voltage [3]. CC-CV simplicity and ease of implementation has made it the standard charging protocol for most battery applications; however, other charging protocols have been proven to achieve better performance including reduced charging time and better efficiency.

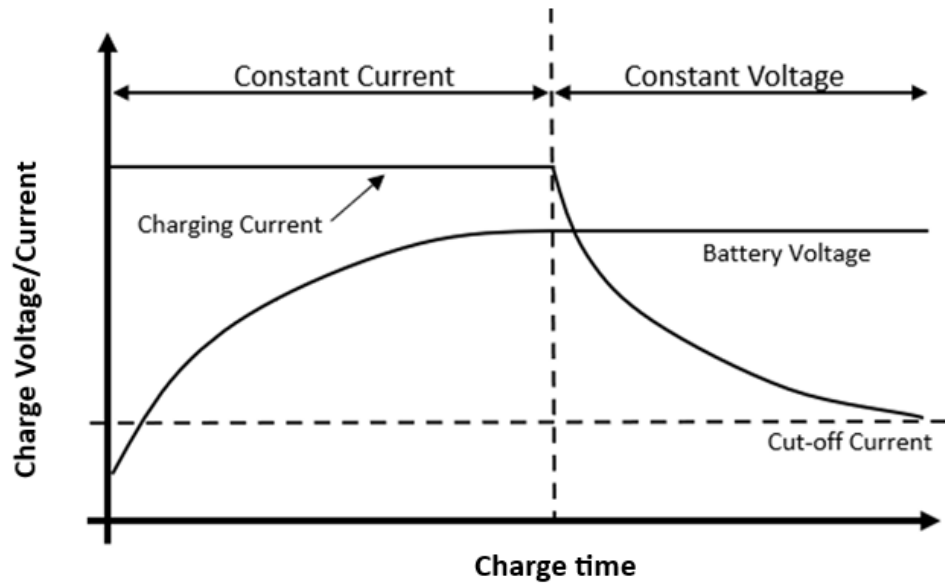


Figure 2.2. CC-CV charging method

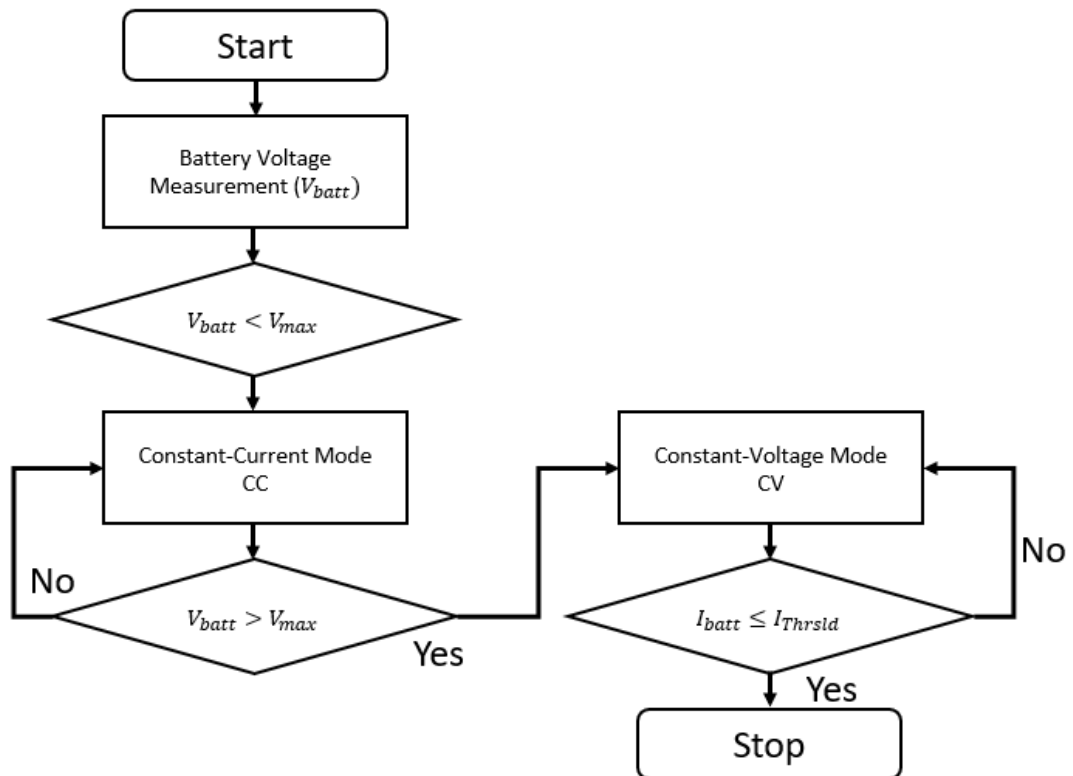


Figure 2.3. Constant Current Constant Voltage charging method flowchart

2.5.4. Multistage Constant Current Charging (MCC)

Multi-Stage Constant current (MCC) charging profile consists of N (≥ 2) constant current charging stages with different current levels until the Battery's cutoff voltage is reached [34]. Typically, the charging current starts from a high initial value, which is reduced gradually in a stepwise fashion for each successive stage. In other words, the successive charging currents can be denoted by I_k , $1 \leq k \leq N$ and $I_k > I_{k+1}$. Since CV mode is not utilized during MCC charge, it presents a significant advantage over conventional CC-CV. However, the performance of a MCC charging method is governed by many factors, such as the number of stages, N , charging currents, I_k , $1 \leq k \leq N$, and the total charging time.

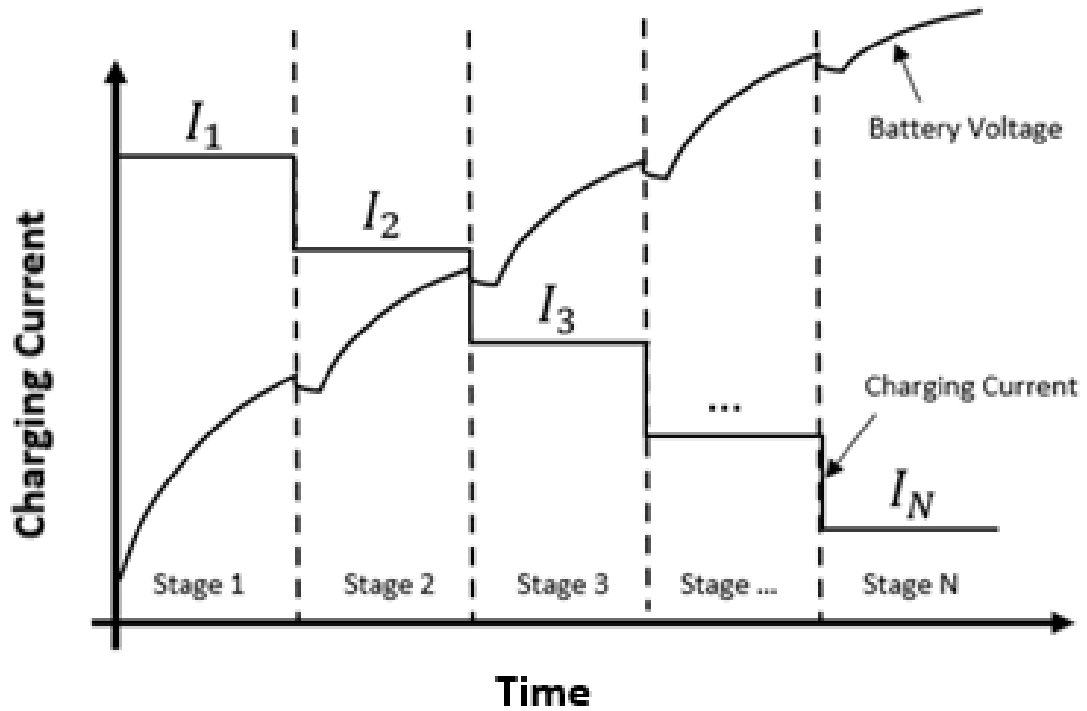


Figure 2.4. Multistage Constant Current charging method

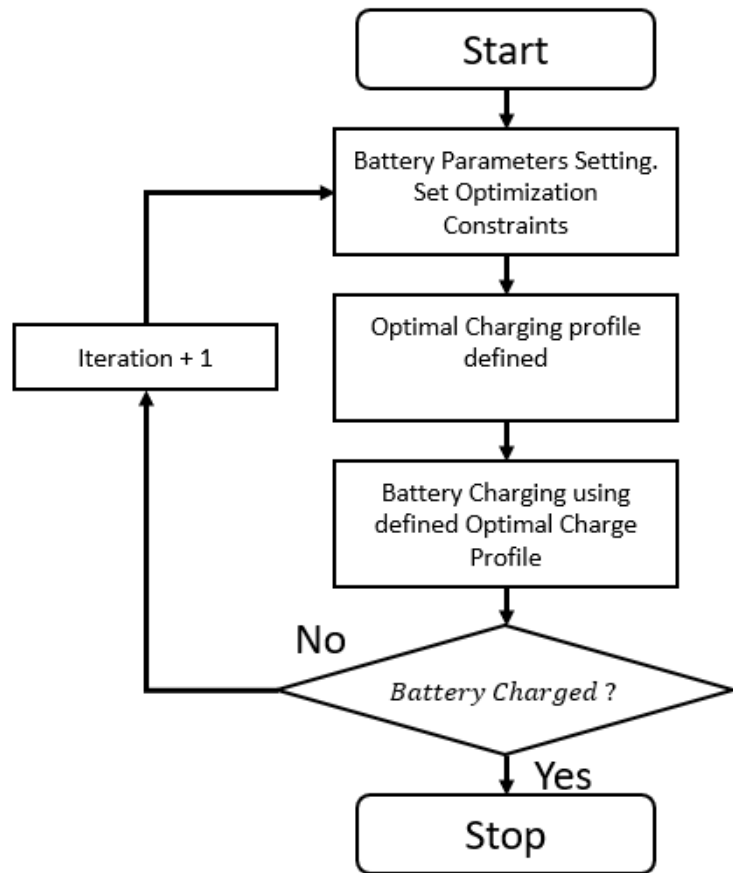


Figure 2.5. Multistage Constant Current Charging Method Flowchart

2.5.5. Pulse Charging Current

Pulse Charging Current (PCC) method, also widely used for fast charge battery applications as an alternative to CC-CV and MCC, it consists of multiple high-level current stages that are periodically interrupted by short rest periods [35], [36]. This cycle of charge-rest is carried out on a repetitive basis till the Battery is fully charge. Contrary to CC-CV, where constant stress is continuously applied to the battery during CV step while waiting for the cut-off current to be reached, it was found that the relaxation period implemented in PCC leads to the reduction of polarization and interfacial resistance.

PCC profiles come in different shapes, the most common constant amplitude- constant rest period is shown in figure 2.6 (a), Constant amplitude with different rest period figure 2.6 (b), decaying current magnitude with constant rest period figure 2.6 (c) and pre-defined high-low current magnitude with constant rest period figure 2.6 (d)

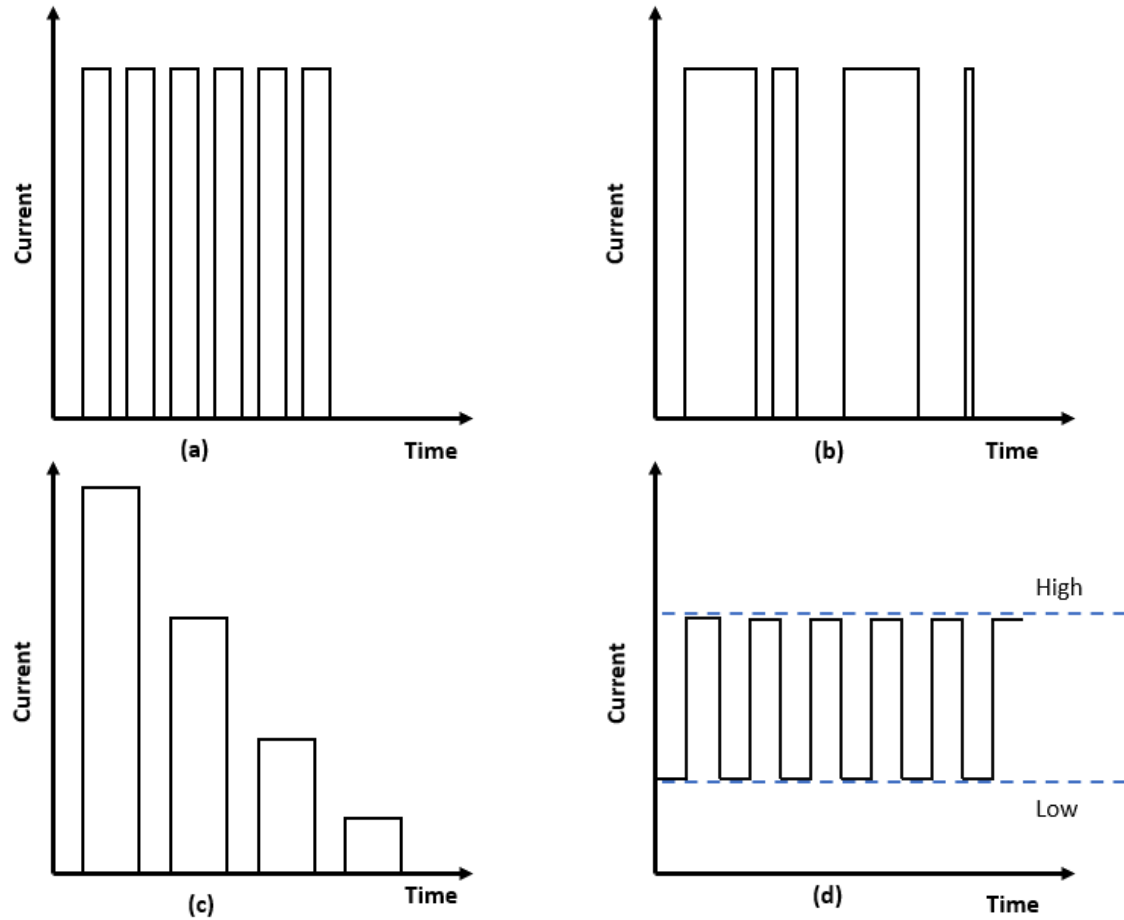


Figure 2.6. Various forms of Pulse Charging Current

2.5.6. Fast Charging Methods with Time-Varying Currents

As highlighted by Keil et al [37] and Lu et al [38], the process of transitioning between stages in the PC and MCC methods results in stress undulation caused by sudden changes in current, which can be detrimental to battery's overall stability. To overcome

this, the authors have proposed using a continuously time-varying exponential current (EC) charge profile, which has been shown to reduce battery stress. An improved EC approach, that is inspired by MCC optimization, is introduced in chapter 6.

2.6. Battery Models

Literature presents two main approaches that model battery cells which include Electrochemical models and Electrical-Equivalent-circuit models. These models are essential for cell performance analysis, aiding to predict, and optimizing battery charging in various applications. In this section, a detailed review of both electrochemical and electrical equivalent circuit models is discussed below, highlighting their strengths and limitations.

2.6.1. Electrical Equivalent Circuit Model

These models use basic electrical components like resistor and capacitors to represent charging and discharging behavior of a cell, as a result, they can be easily integrated, therefore enabling real time monitoring, SOC/SOH estimation and performance prediction. EEC model's primary objective is to model key electrical dynamics while also being computationally efficient. These models could be categorized into different types based on their complexity and accuracy in representing battery behavior.

2.6.2. R_{int} Battery Model

The R_{int} model [39], is recognized as the simplest EEC model. It models a battery with an open-circuit voltage (OCV) source and an internal resistance R_0 to model voltage drops and power losses. The model is simple, easy to implement, and can run with

minimal computational power. However, because of its simplicity, it fails to capture cell dynamics and polarization, making it unsuitable for detailed modeling. However, it can still be used for applications when computational efficiency outweighs accuracy.

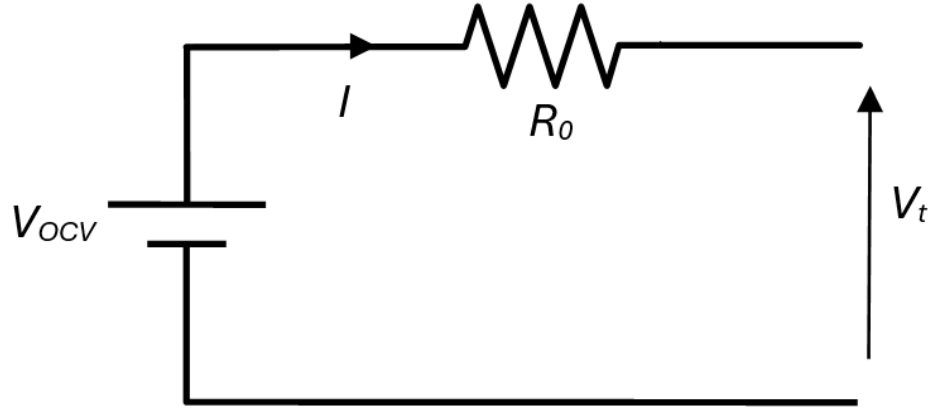


Figure 2.7. R_{int} Equivalent Circuit Model

The mathematical representation of the model is given by:

$$V_t = V_{OCV} - IR_0 \quad (2.1)$$

where V_t represent the terminal voltage of the battery, V_{OCV} is the open circuit voltage, I represent the battery charging or discharging current and R_0 is the battery's internal resistance.

2.6.3. Thevenin Electrical Model

Thevenin battery [40] model is another type of EEC model, which, similarly to R_{int} , consists of an ideal voltage source V_{ocv} in series with an internal resistance R_0 . However, unlike the R_{int} model, Thevenin models improve cell accuracy by adding one or more parallel RC branch to capture cell dynamic response, making it more suitable for applications that require precise voltage and transient behavior modeling.

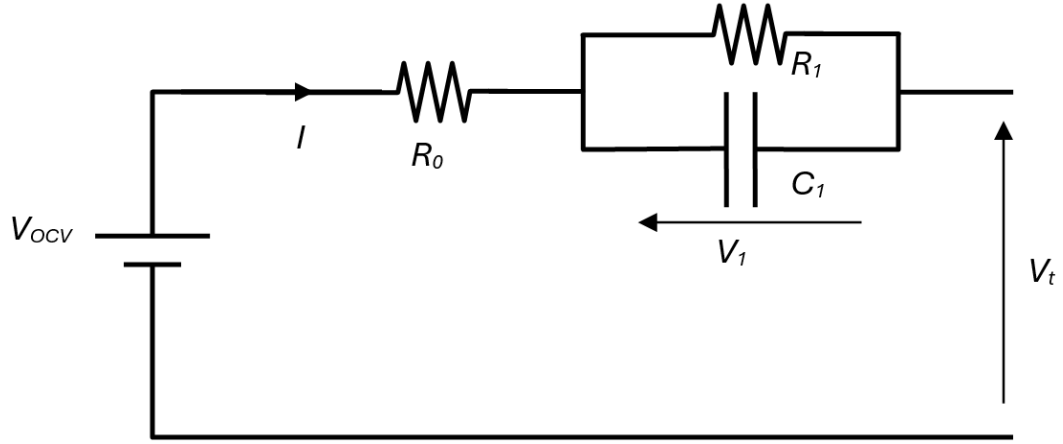


Figure 2.8. Thevenin Battery Model

The terminal voltage is given by $V_t = V_{OCV} - IR_0 - V_1$

where V_1 represent the voltage drop across the RC branch. Although Thevenin model offers improved accuracy over simple models, it has limitations. The model fails to capture full electrochemical dynamics of a battery, specifically effects of temperature on its parameters, aging, and degradation, which can affect long-term cell performance [41].

2.6.4. Dual Polarization Model (2nd Order RC Model)

The Dual Polarization model provides an improved accuracy to the dynamic model by introducing a second set of RC branches to the Thevenin model. It captures cell polarization effects, a crucial data in battery modeling, caused by the battery's slow and fast responses during charge and discharge cycles. This model demonstrates high accuracy under dynamic load conditions, therefore, suitable for EV and renewable energy applications. However, implementing this model will require increased computational cost and careful parameter identification. Despite these challenges, it is still a commonly used model in BMS for real-time SOC and SOH estimations.

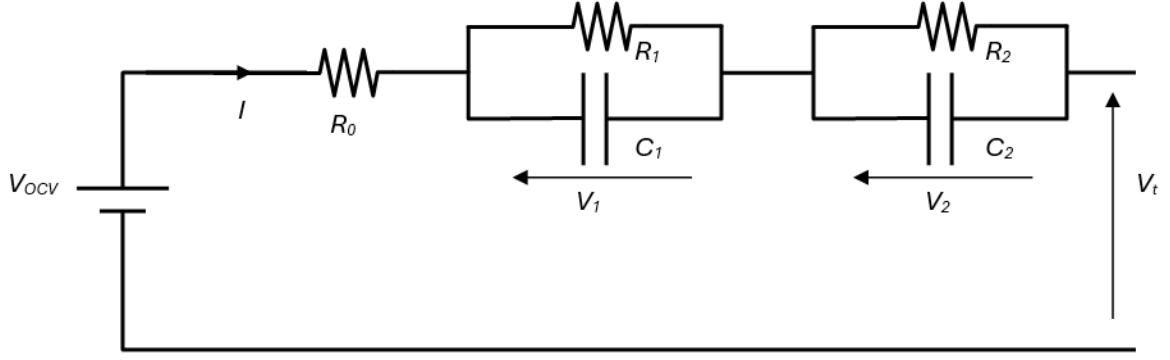


Figure 2.9. Dual Polarization Battery Model

The terminal voltage is given by

$$V_t = V_{OCV} - IR_0 - V_1 - V_2 \quad (2.2)$$

where V_1 and V_2 are voltage drop across the RC branches, governed by:

$$\dot{V}_1 = -\frac{1}{R_1 C_1} V_1 + \frac{1}{C_1} I \quad (2.3)$$

$$\dot{V}_2 = -\frac{1}{R_2 C_2} V_2 + \frac{1}{C_2} I \quad (2.4)$$

2.6.5. Enhanced Self-Correcting Model

The enhanced self-correcting model is designed to predict battery response with a high accuracy for dynamic applications. It expands on dual polarization model by adding a voltage hysteresis source, and in some cases, temperature dependent parameters to better represent cell dynamics [42]. The hysteresis voltage source will account for the voltage difference between charging and discharging cycles, which is important to model for Li-ion batteries that show noticeable hysteresis. This model's high accuracy makes it suitable for online applications; but it also adds complexity and will require collecting more data to accurately estimate its parameters.

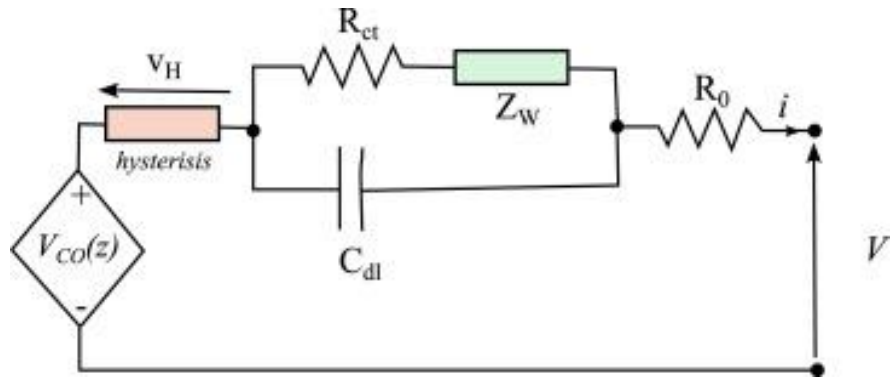


Figure 2.10. Enhanced Self-Correcting Model

Model	Components	Accuracy	Complexity	Typical Use cases
R_{int} model	V_{oc}, R_0	Low	Very Low	Basic SOC estimation, low-power systems
Thevenin Model	$V_{oc}, R_0, 1 RC$ branch	Moderate	Low	Dynamic load applications, portable electronics
Dual polarization	$V_{oc}, R_0, 2 RC$ branch	High	Moderate	Electric vehicles, renewable energies
Enhanced Model	V_{oc}, R_0, RC branch and Hysteresis	Very High	High	High-precision BMS, Aerospace...

Table 2.1. Performance and application comparison of battery models

In conclusion, selecting the most suitable EEC model for an application requires balancing accuracy with simplicity. Although Thevenin and dual polarization are considered simple models, they are still suitable for real-time applications use such as BMS. However, more advanced models will be needed to accurately represent complex battery behavior for high precision applications such as lab battery design.

2.6.6. Electrochemical Models

Electrochemical battery models are physics-based models used to describe the internal cell process such as ion transport, Lithium diffusion, and electrode reactions [43], using electrochemical fundamentals. They usually consist of a series of partial differential equations like Fick's laws of diffusion and the Butler-Volmer equation to describe battery performance, degradation, and thermal dynamics. Electrochemical models are very accurate but computationally intensive, so they cannot be used for real-time applications. However, they are extensively used in research and battery design labs, where high accuracy in performance is needed.

One common approach to electrochemical battery modeling is the lumped parameter model [43], which uses a small set of differential-algebraic equations (DAEs) to describe the time-dependent electrochemical interaction between the electrodes, while assuming a uniform spatial distribution of chemical species. The model relies on Nernst's equation to define the open-circuit potential, and the Butler-Volmer equation for representing the charge transfer kinetics at the electrode-electrolyte interface.

Nernst's equation is expressed as:

$$\phi(t) = \phi^0 - \frac{RT}{nF} \ln(\prod_k (m_k(t) \gamma_k)^{v_k}) \quad (2.5)$$

where k denotes the k th reactant, m represents molality, γ is the activity coefficient, n is the number of electrons, ν is the stoichiometric coefficient, and ϕ^0 refers to the electromotive force under standard conditions. The Butler-Volmer equation that relates the current density $J(t)$ to the overpotential $\eta(t)$ which represents the deviation from equilibrium due to electrochemical reactions is expressed as:

$$J(t) = J_0 \left[\exp \left(\frac{\alpha F}{RT} \eta(t) \right) - \exp \left(\frac{(1-\alpha) F}{RT} \eta(t) \right) \right] \quad (2.6)$$

Where J_0 is the exchange current density, α is the charge transfer coefficient, F is Faraday's constant, R is the universal gas constant, and T is the temperature.

2.6.7. Pseudo-Two-Dimensional (P2D) Model

A pseudo-two-dimensional (P2D) battery model, or Newman model [44], is a simplified approach for Lithium-ion battery simulation. The model relies on the porous electrode theory, considering the structure of battery electrodes formed by an array of conductive and electrochemically active materials with pores filled with electrolyte. The models describe Li-ion migrate via the electrolyte-filled pores, diffusion in solid active particles, and participation in the electrochemical reactions at the electrode-electrolyte interface. Unlike Lumped-parameter modeling, pseudo-two-dimensional model uses partial differential equations (PDEs) to represent electrochemical process. The equations are typically based on Fick's law of diffusion for active material concentration, Ohm's law for electrical potential distribution, and the Nernst and Butler-Volmer equations [42], [45].

2.7. Optimization Approach

The ultimate goal of battery charging optimization is to improve efficiency, ensure safety, and extend battery life. These optimization strategies are categorized into various classifications based on the designed role intended to serve in the charging process. There are many reported methods of charging in the literature, with the objective of minimizing charging time, minimizing temperature rise, and enhancing battery durability. Battery charging optimization techniques can be broadly divided into four main categories [9]: methods based on improved charging current waveforms, methods utilizing battery models, polarization-based methods, and methods focused on enhanced battery materials.

2.7.1. Improved Charging-current Waveform-based Methods

These techniques are usually favored for their simplicity and ease of implementation. In this regard, a charging strategy for Lithium batteries was proposed in [15], which combines the Taguchi method with state-of-charge estimation to search for the optimal charging current profile. It was achieved by applying different current profiles while considering the battery's current state of charge and the reliability of the process.

In [22], an effective charging method is presented to minimize charging time and energy dissipation via dynamic programming. The author in [5] introduced a framework aimed at enhancing battery health by optimizing charging protocols while considering the limitations imposed by side reactions. This health-conscious control strategy aims to enhance both the battery life and performance of batteries. In summary, these methods

typically involve adjusting the charging current to maximize the charging process efficiency. These methods, though, are heuristic in nature, they rely on empirical knowledge and iterative experiments instead of a solid theoretical foundation. These techniques, therefore, lack the accuracy required to optimally determine the charging current for different battery types and conditions, resulting in less-than-ideal or non-optimal performance.

2.7.2. Battery Model-based Methods

Use mathematical models and simulations to improve charge performance, efficiency and life of the battery. They rely on various approaches such as lumped EEC model, AC-impedance models, and electrochemical models. These models integrate external electrical behavior (like voltage, current and impedance) with internal reaction mechanisms [9] (such as electrochemical process) to predict battery response under different charging conditions [46].

2.7.3. Polarization-based Methods

Polarization is the voltage variation in a battery due to electrochemical reactions and internal resistance that can influence its efficiency and life during charging and discharging cycles. Polarization-based methods offer an efficient solution to achieve an acceptable charging current while meeting the desired voltage target. These methods play a key role in enhancing the performance by controlling voltage fluctuations occurring during charging. Effective management of polarization can resolve issues such as excessive heat generation and capacity degradation, eventually prolonging the batteries

life. Yet the complex relationship between polarization and battery aging is still a subject for research investigations [47].

2.7.4. Methods Focused on Enhanced Battery Materials

Lastly, enhanced battery material-based methods contribute to better battery charging by including additives to electrode materials, or the use of novel materials [48]. This improvement can potentially improve the overall performance by increasing conductivity, reducing internal resistance, therefore, enabling faster charging without compromising battery health. However, despite these potential advantages, there are still concerns about the safety and stability of these batteries, as new materials can introduce unknown risks such as thermal runaway and long-term structural degradation. Therefore, extensive research and testing are required to solve these issues prior to mass adoption.

2.8. Estimation of Dynamic Battery Parameters

Parameter estimation can be complex, but it is a crucial task for accurately modeling and predicting battery behavior. It involves the identification of key elements, in the case of EEC model, such as internal resistance and capacitance. Accurate estimation of these elements will ensure precise BMS applications, simulations and real-time implementation. Li-ion batteries parameters, including internal resistance, capacitance and OCV are found to be temperature, SOC and aging dependent [50].

2.8.1. Parameters Depended on SOC and Temperature

The parameterization process in [50] was done through experimental measurement and analysis of a five element EEC model, in which the author conducted tests at various SOC and temperature. It was found that the internal resistance R_0 shows

minimal dependence on SOC between 10 and 90% but it strongly depends on temperature during charge and discharge. However, R_1 , R_2 , C_1 and C_2 showed a strong dependence on both SOC and temperature.

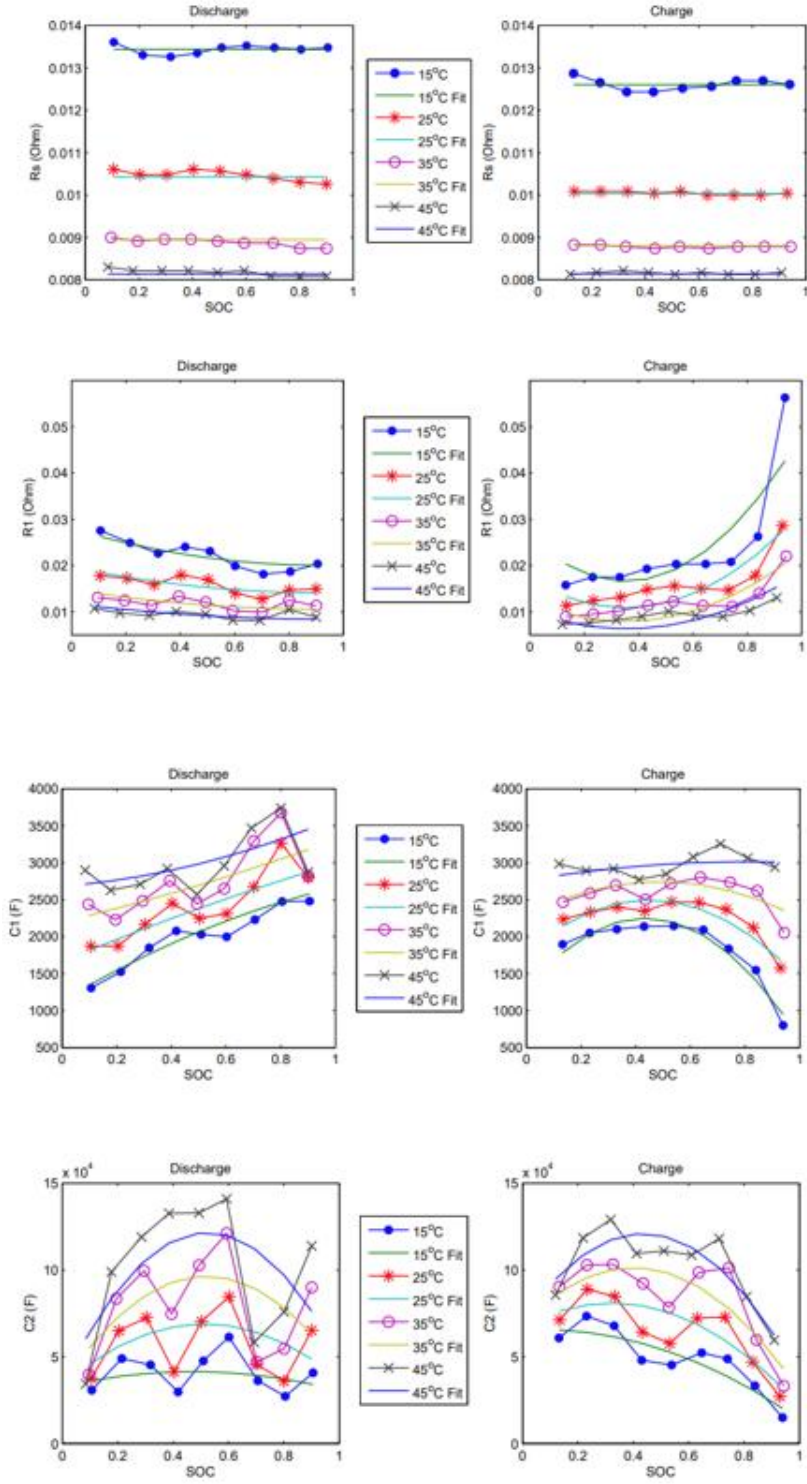


Figure 2.11. Variation of Equivalent Circuit Parameters with state of charge under different temperatures conditions during charge and discharge [50]

2.8.2. Parameters Depended on Battery Aging

Batteries age after many charge/discharge cycles, causing several of its internal parameters to vary directly impacting its performance. In [51], a non-destructive approach was developed to detect and characterize the effect of temperature on cycling aging rate of Li-ion batteries operating in elevated temperatures. The author found that in these conditions, a faster degradation of the cell's various components includes charge storage capacity, charge transfer rate and resistance (both electrode and electrolyte).

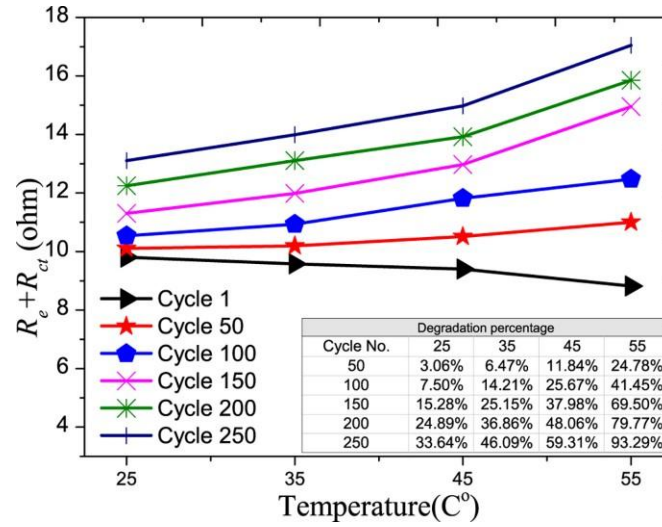


Figure 2.12. Cell total resistance aging vs. Temperature [51].

From the above, one could conclude that as battery ages, it would lead to a degradation in its performance, therefore it is mandatory to account for this decline, by accurately estimating parameters change to maintain reliable battery operations, especially for long term applications.

2.9. EEC Model Parameter Estimation Techniques

In [52], a parameter identification technique for a coupled electro-thermal model was presented. The author extended the work from [53] by allowing the parameter to be

temperature and SOC depended. This resulted in a model with several unknown parameters, where identification was carried out using a multi-step Genetic Algorithm (GA) for large-scale optimization. In [54], the author proposed an online parameter estimation technique for an EEC model, based on Least Square Estimation, which then fed to a linear observer to estimate the battery's SOC. The nonlinear relationship between SOC and OCV was approximated with piece-wise linear functions. This method helped simplify the model while maintaining a good level of SOC estimation.

The author in [55] had presented an EEC parameter estimation method based on Electrochemical Impedance Spectroscopy (EIS), which is a proven method used to measure a cell's impedance at different frequencies, analyzing internal resistance and electrochemical properties. The method used is based on fast Fourier transformation, feature extraction, curve fitting and least-squares estimation. It improved parameter estimation accuracy compared to existing methods.

In this work, we will focus on the investigation and the comparison of two system identification techniques: State Variable Filtering (SVF) and Indirect Discrete-Time (IDT) to achieve accurate estimation of the time varying parameters of the EEC model while the battery is charging or discharging. The main difference between the two methods lies in how data is handled during the modeling process. For instance, the direct continuous-time approach will operate in the continuous time domain and estimate the parameters of a continuous-time model using identification methods such as Least Squares Estimation. On the other hand, the Indirect Discrete-Time approach will first involve the discretization of the original continuous time model by applying discrete time

estimation techniques. Afterward, the DT model is converted back into its desired CT representation.

A linear, discrete-time noise-free system could be represented by the following equation:

$$y(t_k) + a_1 y(t_{k-1}) + \dots + a_n y(t_{k-n}) = b_1 u(t_{k-1}) + \dots + b_m u(t_{k-m}) \quad (2.7)$$

where $u(t_k)$ and $y(t_k)$ denote the sampled input and output signals, respectively, for $k = 1, 2, \dots, N$

Equation (2.7) can be expressed using the backward shift operator q^{-1} as shown in (2.8), or equivalently in the transfer-function form given in (2.9)

$$A(q^{-1})y(t_k) = B(q^{-1})u(t_k) \quad (2.8)$$

$$y(t_k) = \frac{B(q^{-1})}{A(q^{-1})} u(t_k) \quad (2.9)$$

where:

$$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \dots + b_m q^{-m} \quad (2.10)$$

$$A(q^{-1}) = a_1 q^{-1} + a_2 q^{-2} + \dots + a_n q^{-n} \quad (2.11)$$

q^{-1} is the backward shift operator, such as $q^{-1}y(t_k) = y(t_{k-1})$.

For identification purposes, the discrete-time model is rewritten in the linear regression form of (2.12), where ρ^T is the regression vector, that is constructed from past input and output samples, and θ is the vector of unknown parameters defined in (2.13) and (2.14) respectively.

$$y(t_k) = \rho^T(t_k) \theta \quad (2.12)$$

$$\rho^T = [-y(t_{k-1}) \dots y(t_{k-n}) \quad u(t_{k-1}) \dots u(t_{k-m})] \quad (2.13)$$

$$\theta = [a_1 \dots a_n \ b_1 \dots b_m]^T \quad (2.14)$$

The parameter vector θ is then estimated using discrete-time identification methods as least squares or recursive least squares. This identified discrete-time model is then converted back into an equivalent continuous-time representation.

2.9.1. Continuous-time Equation Model

As previously mentioned, this method of system identification refers to estimating a system's parameters directly in its continuous-time differential equation form, without the need to discretizing the model.

A Continuous-time system noise-free is represented by the following equation:

$$\begin{aligned} \frac{d^n}{dt} y(t) + a_1 \frac{d^{n-1}}{dt} y(t) + \dots + a_n y(t) = \\ b_0 \frac{d^m}{dt} u(t) + \dots + b_m u(t) \end{aligned} \quad (2.15)$$

which represents a linear system with constant coefficients relating to a system's output and input signals and their derivatives. Equation 2.15 may equivalently be expressed in the derivative notation shown in 2.16:

$$\begin{aligned} y^{(n)}(t) + a_1 y^{(n-1)}(t) + \dots + a_n y(t) = \\ b_0 u^{(m)}(t) + \dots + b_m u(t) \end{aligned} \quad (2.16)$$

where $y^{(n)}(t)$ denotes the n th time derivative of the continuous-time output. Equations (2.15) and (2.16) can be written using polynomial differential operators as shown in equation (2.17) and the corresponding transfer function representation is (2.18).

$$A(D)y(t) = B(D)u(t) \quad (2.17)$$

$$y(t) = \frac{B(D)}{A(D)}u(t) \quad (2.18)$$

where:

$$B(D) = b_0 D^m + b_1 D^{m-1} + \dots + b_m \quad (2.19)$$

$$A(D) = D^n + a_1 D^{n-1} + \dots + a_n \quad (2.20)$$

D is the differential operating such as: $Dy(t) = \frac{d}{dt}y(t)$ and $n \geq m$

For parameter estimation purposes, the system equation can be rearranged into a regression form that is linear in the unknown parameters, as shown in 2.21.

$$y^{(n)}(t) = \rho^T(t) \theta \quad (2.21)$$

In this formulation, ρ^T is the known regression vector composed of measured input and output signals and their derivatives, while θ is the parameter vector contains the unknown system coefficients as defined in (2.22) and (2.23), respectively.

$$\rho^T = [-y^{(n-1)}(t) \dots -y(t) \ u^{(m)}(t) \dots u(t)] \quad (2.22)$$

$$\theta = [a_1 \dots a_n \ b_1 \dots b_m]^T \quad (2.23)$$

This regression representation forms the basis for applying continuous-time parameter estimation algorithms, such as Recursive least squares to directly identify the system parameters.

In practice, real world physical systems do not necessarily behave as their model predicts.

This is mainly due to several factors, such as measurement noise from sensor imperfections and electrical interference; process noise which includes environmental disturbances; and finally, unmodeled/neglected dynamics which may include nonlinearities that are approximated as linear relationships, in the case of batteries, variation of parameters due to aging.

Incorporating this effect, the continuous-time model becomes:

$$A(D)y(t) = B(D)u(t) + \omega(t) \quad (2.24)$$

or equivalently

$$y^{(n)}(t) = \rho^T(t)\theta + \omega(t) \quad (2.25)$$

where $\omega(t)$ is the process noise, typically modeled as a zero-mean with covariance Q .

Let the measurement output be denoted by $y_m(t)$. The measurement equation is defined as:

$$y_m(t) = y(t) + v(t) \quad (2.26)$$

where $v(t)$ is the measurement noise, commonly assumed to also be zero-mean with covariance R .

Substituting the noisy measurement $y_m(t)$ into the regression form yields

$$y_m^{(n)} = \rho^T(t)\theta + \omega(t) + v^{(n)}(t) \quad (2.27)$$

where $v^{(n)}(t)$ denotes the $n - th$ derivative of the measurement noise.

This expression itself is problematic and highlights key challenges in continuous time system identification since the differentiation of noisy measurements would amplify high-frequency noise [56], [57], making the estimation sensitive to measurement noise.

To overcome the issue of noise amplification caused by differentiating measured signals in continuous-time system identification, literature presents several approaches that could be employed. A preferred method would be one to avoid explicit differentiation. State-variable filtering (SVF): is a robust technique that is widely used in continuous-time parameter estimation that also mitigates noise amplification resulting from direct differentiation of measured signals. The idea behind this technique is to apply a stable, linear and properly designed low-pass filter to both sides of the system's differential

equation, which will generate a filtered version of the signals and their derivatives without necessary computing time derivatives. By applying this dynamic filtering to the system, SVF guarantees suppression of high-frequency measurement noise while maintaining the linear structure of the continuous time model.

The state variable filter (SVF) approach employs a stable filter of the form:

$$F(D) = \frac{1}{(D+\lambda)^n} \quad (2.28)$$

where $D = \frac{d}{dt}$ and $\lambda > 0$ is a design parameter controlling the filter bandwidth [58].

applying this filter to both sides of equation (4.17) maintains linearity in the parameters while avoiding direct numerical differentiation of the measured signals. Defining the filtered output and the regressor as:

$$z(t) = \frac{p^n}{(p+\lambda)^n} y_m(t) \quad (2.29)$$

$$\phi(t) = \frac{1}{(p+\lambda)^n} \rho(t) \quad (2.30)$$

The model is transformed into the linear regression form

$$z(t) = \phi^T(t)\theta + \eta(t) \quad (2.31)$$

where $\eta(t)$ represents the filtered disturbance and noise terms.

Based on this SVF-based regression model, Recursive Least Squares (RLS) with exponential forgetting is used to estimate θ . The estimation error is defined as:

$$\varepsilon(t) = z(t) - \phi^T(t)\hat{\theta}(t) \quad (2.32)$$

and the exponentially weighted cost function is minimized to obtain the recursive equations. In continuous time:

$$\dot{P}(t) = \lambda P(t) - P(t) \phi(t) \phi^T(t) P(t) \quad (2.33)$$

$$\hat{\theta}(t) = P(t)\phi(t)\varepsilon(t) \quad (2.34)$$

For implementation, the continuous-time estimation equations are discretized using Euler or Zero-order hold, the most common discrete-time RLS update is given by:

$$K_k = \frac{P_{k-1}\phi_k}{\lambda + \phi_k^T P_{k-1} \phi_k} \quad (2.35)$$

$$\varepsilon_k = z_k - \phi_k^T \hat{\theta}_{k-1} \quad (2.36)$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k \varepsilon_k \quad (2.37)$$

$$P_k = \lambda^{-1}(P_{k-1} - K_k \phi_k^T P_{k-1}) \quad (2.38)$$

2.9.2. Discrete-time RLS Algorithm

Just like the above, the discrete-time Recursive Least Squares (RLS) algorithm estimates unknown parameters from noisy measurements by minimizing an exponentially weighted least squares cost function. After discretization, the algorithm can be expressed in recursive form, making it suitable for real-time implementation. The final RLS equations are:

Kalman gain

$$K_k = \frac{P_{k-1}\phi_k}{\lambda + \phi_k^T P_{k-1} \phi_k} \quad (2.39)$$

Parameter update

$$\hat{\theta}_k = \hat{\theta}_{k-1} - K_k(\phi_k^T \hat{\theta}_{k-1} - z_k) \quad (2.40)$$

and the covariance update

$$P_k = \frac{1}{\lambda} (P_{k-1} - K_k \phi_k^T P_{k-1}) \quad (2.41)$$

A detailed derivation of these equations applied to the 5-elements EEC battery model is presented in chapter 3.

2.10. State of Charge Estimation Methods

SOC estimation is critical in achieving maximum battery performance. If effectively integrated with a BMS, it reduces the risk of overcharging and discharging of a battery, avoiding early degradation while maintaining efficiency. It is a challenging task to estimate SOC, due to several influencing factors mainly operating temperature (hot/cold ambient), measurement error originated from noisy sensor data, voltage variations, and aging (internal resistance increase, capacity fading). These factors need to be accounted for to ensure accurate estimation. Literature presents several SOC estimation techniques. The authors in [41] have presented an overview of various SOC estimation methods suitable for control-oriented applications.

2.10.1. Open Loop-based SOC Estimation

This category of SOC estimation depends primarily on battery models and external measurements with no feedback corrections or observations. It typically includes voltage-relationship between OCV and SOC once the battery reaches equilibrium after a long rest. SOC is then “estimated” from OCV using battery specific parameter table and measurements. As straightforward as this method could be, its accuracy is affected by various factors, mainly aging and operating temperatures.

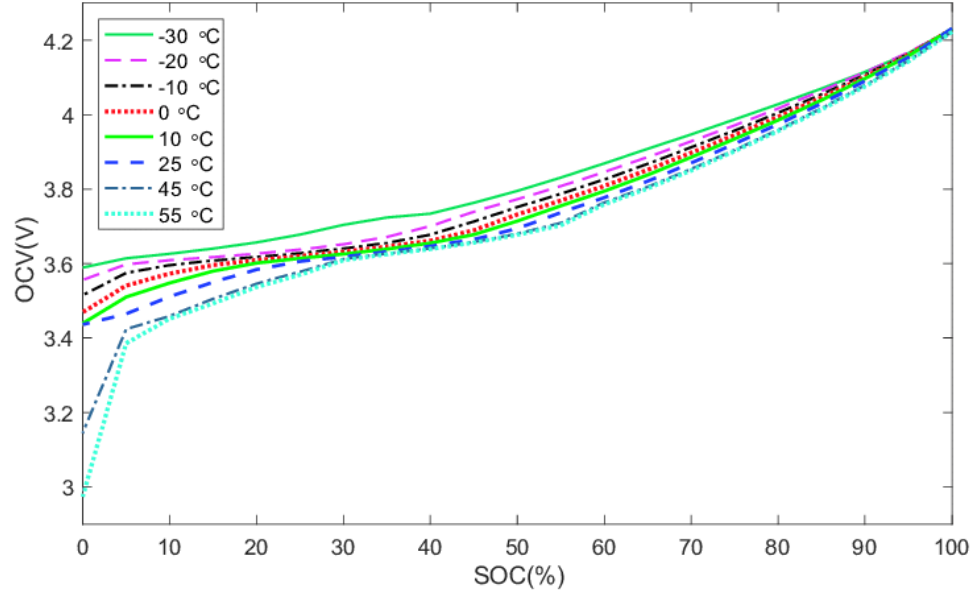


Figure 2.13. Battery SOC-OCV relationship at different temperatures [59].

Electrochemical Impedance Spectroscopy (EIS) is also part of the open-loop-based SOC estimation methods [60]. This method analyzes a battery's impedance response to applied alternating current across a spectrum of frequencies while measuring the corresponding voltage response [61]. Applying EIS results in generating a transfer function that represents batteries impedance response describing the variation of impedance with frequency [62]. The generated transfer function, that is the ratio between voltage and current as describe in equation 2.42, can then be “linked” to an EEC model to estimate internal electrochemical properties, which varies as SOC changes. EEC total impedance is expressed by equation 2.43, where R_s is cell's internal resistance, RC pairs capture cell's dynamics, and the Warburg impedance represents the diffusion process of lithium.

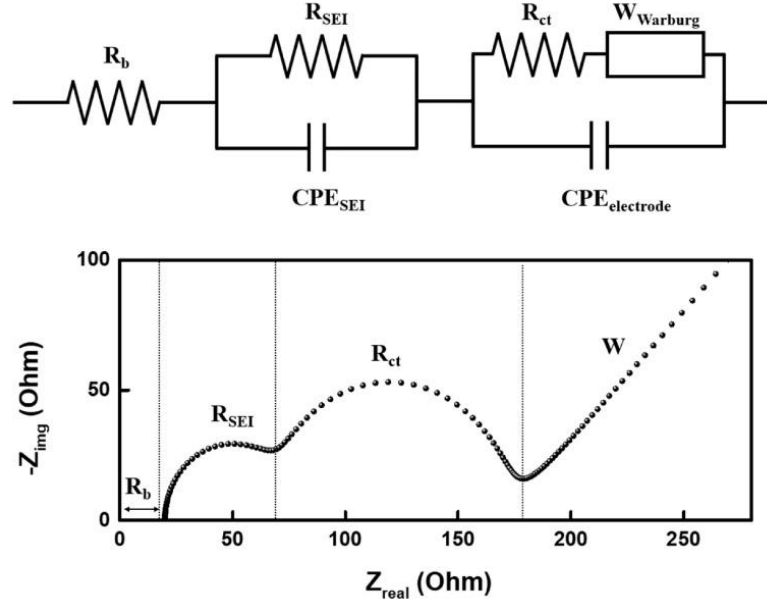


Figure 2.14. Simplified equivalent circuit represents the internal impedance [63]

$$z(\omega) = \frac{v(\omega)}{i(\omega)} = \text{Real}(z) - j\text{Imag}(z) \quad (2.42)$$

$$z(\omega) = z_s(\omega) + z_1(\omega) + z_2(\omega) + z_w(\omega) \quad (2.43)$$

2.10.1.1. Amper-hour or Coulomb Counting

Also referred to as book-keeping method, it is a commonly used technique for SOC estimation. It determines a battery's SOC by integrating the current flow during charge or discharge over time, continuously summing up the charge added or removed. It is mathematically expressed as:

$$\text{SOC}(t) = \text{SOC}(0) - \frac{1}{Q} \int_{t_0}^t (\eta \cdot i_{\text{bat}}(t) - S_d) dt \quad (2.44)$$

Where Q is the nominal capacity, η is the coulombic efficiency, S_d is the self-discharge rate, i_{bat} is the charging current. Although this method is easily implementable and requires low computational resources, its key challenges lie in the uncertainty of the

initial SOC [58] and the cumulative errors due to accuracy of the sensors and temperature variations, which could affect its performance.

2.10.1.2. Data Driven-Based Estimation

Data-driven based SOC estimation approach employs machine learning and artificial intelligence (AI) for the estimation of batteries' SOC. This method relies on identifying patterns within large datasets enabling accurate predictions. In the literature, Data-driven SOC estimation is primarily classified into Fuzzy Logic and Neural Networks [64].

2.10.1.3. Fuzzy Logic

Fuzzy logic is a powerful tool utilized for modeling complex and non-linear systems, such as Li-ion batteries, without relying on accurate mathematical modeling [65]. it employs a set of Fuzzy-rule base to establish the relationship between variables in the form of “If-then” logic to process input data and to generate outputs. These rules handle decision making based on imprecise or uncertain information. A fuzzy rule base for battery applications might look like:

- **If** (voltage is low) and (Current is high), **then** (SOC is decreasing **rapidly**).
- **If** (SOC is low) and (temperature is high), **then** (charging speed is **slow**)
- **If** (voltage is low) and (charge cycle is high), **then** (SOH is **low**)

Figure 2.15 illustrates the four main steps of fuzzy logic

- **Fuzzification** step converts crisp input (real values) into fuzzy values using membership functions which define how much an input belongs to a specific fuzzy set.

- **Fuzzy Rule base** that describes the system's behavior by defining the input and output relationship. These rules are defined based on previous knowledge of system behavior.
- **Inference engine** evaluates the rules based on the fuzzified inputs, and generates fuzzy outputs through aggregation.
- **Defuzzification**: converts the fuzzy output into crisp.

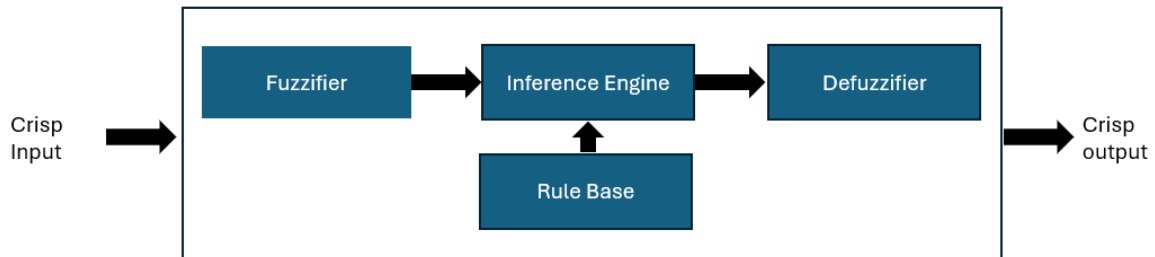


Figure 2.15. A complete Fuzzy interface system [65]

The author in [66] proposed a combined approach to improve SOC estimation by integrating Fuzzy logic with Coulomb counting. This approach was applied to overcome the shortcomings of coulomb counting that is prone to accumulated errors due to initial SOC, sensor inaccuracies and battery aging. This combined approach effectively compensates for these errors. The author in [67] presented a combined approach to accurately estimate SOC for Li-ion batteries by incorporating fuzzy logic with sliding mode observer to improve its performance. Results of this technique showed that the fuzzy logic sliding mode observer delivered better SOC accuracy and convergence rate and did outperform traditional techniques in robustness to measurement noise and parameter disturbance.

Fuzzy logic for SOC estimation brings advantages such as flexibility to handle uncertainty, imprecision and nonlinearity of a battery model, making it attractive approached to use. However, it also has drawbacks, mainly having a requirement for careful design of rule base and membership function, which can be subjective and time consuming. Besides, it requires more computational demand compared to simpler methods like coulomb counting.

2.10.2. Closed Loop based SOC Estimation

Closed loop-based SOC estimation continuously corrects and compensates for SOC value based on real-time feedback from battery systems [60]. Close-loop estimation differs from open-loop methods (e.g., Coulomb counting), that only depend on initial conditions and current integration. It relies on additional measurements like voltage, current and temperature to implement advanced algorithms for a better SOC estimation. However, just like any other method, it comes with a trade-off of complexity and computational demand.

Various techniques are used in this estimation category, as described in [60], and can be classified into filter-based estimation and observer-based methods. Filter-based includes Kalman filter (KF) and its variants (Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF)), that predict SOC based on system model and continuously updating the estimation based on real-time feedback. Observer-based methods that contain Sliding Mode Observer (SMO), Luenberger Observer, which estimates SOC by modeling battery dynamics.

2.10.2.1. Linear Kalman Filter (LKF)

Linear Kalman Filter is a state estimation technique that is commonly used for SOC estimation. It assumes that both the system model and the noise are linear Gaussian [68]. For SOC estimation, the battery is typically modeled using an Electrical Equivalent Circuit (EEC) in a linear state-space representation, where SOC is a state variable. LKF recursively updates SOC estimation in two steps: the prediction step, during which SOC is estimated using the system model, and the correction step, where the estimation is refined using Kalman gain while updating the error covariance to improve estimation accuracy [69]. Although LKF is computationally efficient and suitable for linear applications, it cannot be applied to Li-ion batteries mainly due to the OCV-SOC relationship is highly non-linear and temperature dependent [70], which potentially could lead to significant estimation errors. Therefore, Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) are generally employed since they both account for the non-linearity of the battery behavior to provide accurate SOC estimation.

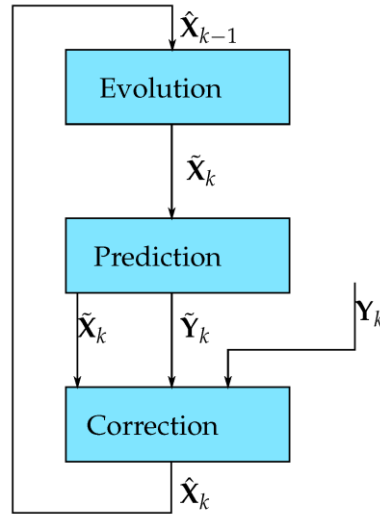


Figure 2.16. The Kalman Filter Recursive Process [69]

Step	Kalman Filter	Real System
Evolution	$\tilde{\mathbf{X}}_{k+1} = [\mathbf{A}]\hat{\mathbf{X}}_k + [\mathbf{B}]\mathbf{U}_k$ $[\hat{\mathbf{P}}_{k+1}] = [\mathbf{A}][\hat{\mathbf{P}}_k][\mathbf{A}]^T + [\mathbf{Q}]$	$\mathbf{X}_{k+1} = [\mathbf{A}]\mathbf{X}_k + [\mathbf{B}]\mathbf{U}_k + \mathbf{V}_k$
Prediction/measurement	$\tilde{\mathbf{Y}}_k = [\mathbf{C}]\tilde{\mathbf{X}}_k + [\mathbf{D}]\mathbf{U}_k$	$\mathbf{Y}_k = [\mathbf{C}]\mathbf{X}_k + [\mathbf{D}]\mathbf{U}_k + \mathbf{W}_k$
Correction	$\mathbf{E}_k = \mathbf{Y}_k - \tilde{\mathbf{Y}}_k$ $[\mathbf{K}_k] = [\hat{\mathbf{P}}_k][\mathbf{C}]^T ([\mathbf{C}][\hat{\mathbf{P}}_k][\mathbf{C}]^T + [\mathbf{R}])^{-1}$ $\hat{\mathbf{X}}_k = \tilde{\mathbf{X}}_k + [\mathbf{K}_k]\mathbf{E}_k$ $[\hat{\mathbf{P}}_k] = (\mathbf{I} - [\mathbf{K}_k][\mathbf{C}])[\hat{\mathbf{P}}_k]$	

Table 2.2: Kalman Filter Processing steps [69]

2.10.2.2. Extended Kalman Filter (EKF)

The Extended Kalman Filter (EKF) is a nonlinear extension for Kalman filter that is widely used for SOC estimation effectively addressing the nonlinear OCV-SOC relationship. Unlike LKF, which is only applicable to linear system dynamics, EKF linearizes the system using the first-order Taylor series approximation [70]. EKF also recursively updates SOC estimation in two steps: the prediction step, during which SOC is estimated using the system model (Coulomb counting), and the correction step, where the estimation is refined by linearizing the system at each time step, while updating the error covariance to improve estimation accuracy. The Jacobian matrix consists of partial derivatives that include OCV-SOC and the parameters of the states of the model. Calculating the linear approximation at each step increases the computational cost of EKF and could introduce a significant truncation error [71] [72], which eventually reduces the estimation accuracy. To solve this problem, the Unscented Kalman Filter (UKF) is often used for highly non-linear systems as it eliminates the need for Jacobian matrix calculations [73].

The discrete-time nonlinear state equations of the 5-element batter model are defined as:

$$SOC_{k+1} = SOC_k + \frac{T_s}{3600C_k} I_k + \omega_{soc,k} \quad (2.45)$$

$$V_{1,k+1} = e^{-T_s/(R_{1,k}C_{1,k})} V_{1,k} + R_{1,k}(1 - e^{-T_s/(R_{1,k}C_{1,k})}) I_k + \omega_{V_1,k} \quad (2.46)$$

$$V_{2,k+1} = e^{-T_s/(R_{2,k}C_{2,k})} V_{2,k} + R_{2,k}(1 - e^{-T_s/(R_{2,k}C_{2,k})}) I_k + \omega_{V_2,k} \quad (2.47)$$

where T_s is the sampling time, I_k is the applied current and ω_k denotes zero-mean Gaussian process noise.

The terminal voltage measurement equation is expressed as:

$$V_{t,k} = h(x_k, I_k) + v_k \quad (2.48)$$

$$V_{t,k} = V_{ocv,k}(SOC) + R_{0,k}I_k + V_{1,k} + V_{2,k} + v_k \quad (2.49)$$

where $V_{ocv,k}(SOC)$ denotes the nonlinear open-circuit voltage – SOC relationship, $R_{0,k}$ is the ohmic resistance which, in this study, is identified online via SVF-RLS, Finally, v_k is the measurement noise. Because of the existing nonlinearity in both the state equation (specifically the inverse dependance of the SOC on capacity) and the measurement equation ($V_{ocv} = fct(SOC)$), the EKF linearizes the system around the current state using the Jacobian matrix. The state transition Jacobian F_k is derived by partially differentiating the vector function with respect to each state variable. The state transition Jacobian is given by:

$$F_k = \begin{bmatrix} 1 & 0 & 0 \\ 0e^{-T_s/(R_{1,k}C_{1,k})} & 0 & 0 \\ 0 & 0 & e^{-T_s/(R_{2,k}C_{2,k})} \\ 0 & 0 & 0 \end{bmatrix} \quad (2.50)$$

The measurement Jacobian is:

$$H_k = \begin{bmatrix} \left. \frac{dOCV}{dSOC} \right|_{SOC_k} & 1 & 1 \end{bmatrix} \quad (2.51)$$

The EKF prediction step is given by:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1}, I_{k-1}) \quad (2.52)$$

$$P_{k|k-1} = F_k P_{k-1} F_k^T + Q \quad (2.53)$$

where P is the state estimation error covariance matrix that represents the filter's current confidence in the estimated states. Q is the process noise covariance that models the uncertainty in the system dynamics and unmodeled disturbances.

The measurement update step is performed as:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R)^{-1} \quad (2.54)$$

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k (V_{t,k} - h(\hat{x}_{k|k-1}, I_k)) \quad (2.55)$$

$$P_k = (I - K_k H_k) P_{k|k-1} \quad (2.56)$$

R denotes the measurement noise covariance matrix, it quantifies the uncertainty of measurement process, a large R means noisier measurement which causes filter to rely more on the model prediction, a smaller R indicates more reliable measurement.

2.10.2.3. Unscented Kalman Filter (UKF)

The Unscented Kalman Filter (UKF) provides an advanced solution for SOC estimation of Li-ion batteries. Its main benefit over EKF is that it eliminates the need for the Jacobian linearization to approximate nonlinear system dynamics at each step, which maintains accuracy without the extra computational cost and error introduction risk. UKF algorithm steps are identical to that of other forms of Kalman filter except for the method used to perform the approximation of nonlinear transformation. UKF uses a different approach based on Unscented transformation (UT) that computes the statistical properties (mean and covariance) of a random variable that undergoes a nonlinear transformation

[73]. Sigma Points are then deterministically identified using this transformation, which guarantees the accuracy of the mean and the covariance matrix of these points up to the third order of the Taylor series [74]. These sigma points are then passed through the model, allowing the UKF to predict its states. When UKF is applied [75], a $2n + 1$ sigma point is generated from the current state estimate $\hat{x}_k$ and covariance P_k , where n denotes the state dimension. A scaling parameter is defined as:

$$\lambda = \alpha^2(n + k) - n \quad (2.57)$$

where α determines the spread of the sigma points from the mean, k is a secondary scaling parameter (usually $k = 0$) and the sigma points are calculated as follows:

$$\chi_k^0 = \hat{x}_k \quad (2.58)$$

$$\chi_k^{(i)} = \hat{x}_k + (\sqrt{(n + \lambda)P_k})_i, i = 1, \dots, n \quad (2.59)$$

$$\chi_k^{(i+n)} = \hat{x}_k - (\sqrt{(n + \lambda)P_k})_i, i = 1, \dots, n \quad (2.60)$$

$\sqrt{(n + \lambda)P_k}$ denotes the matrix square root which is obtained using Cholesky decomposition [75].

These Sigma points are then recursively propagated through the non-linear battery model, which includes the OCV-SOC function. The predicted state and covariance are reconstructed from the transformed sigma points with applied weighting factors. This mechanism allows UKF to better capture nonlinearities in the system and provides improved estimation performance compared to first-order linearization methods adopted by EKF.

CHAPTER THREE

TIME-VARYING EEC AND PARAMETER ESTIMATION

In this chapter, we discuss the findings of a comparative study of the effectiveness of recursive techniques used for robust identification and parameter estimation of a five-element electrical equivalent circuit model of Li-ion batteries with time-varying characteristics. The study examines two specific methods: a direct continuous time system identification method based on State Variable Filtering (SVF), and an Indirect Discrete Time (IDT) technique. The results of both techniques are analyzed in detail; however, it was found that while both methods performed well for identifying time-invariant circuit models, the direct continuous method surpasses the indirect discrete time techniques when it comes to identifying time-varying circuit models.

3.1 System Identification Procedure

Two distinct time-domain approaches are available to obtain a continuous-time (CT) model of a system based on its sampled input-output data [58]:

- Direct Continuous-time System Identification
- Indirect Discrete-time System Identification

The direct approach estimates the continuous-time parameters – resistances and capacitances – directly from the sampled data. The indirect approach first estimates a discrete-time model and then converts it to continuous-time. Comparing the two helps us

determine which method gives more accurate and reliable parameters estimates for the battery equivalent circuit, particularly under time-varying conditions.

The continuous EEC model represented in figure 3.1 is derived by applying Kirchhoff's law. The resulting equations are defined as follows:

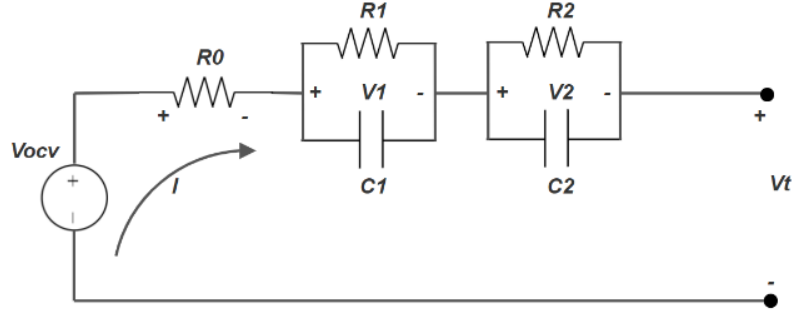


Figure 3.1. Five-element battery Electric Equivalent Circuit (EEC) model

$$\dot{V}_1(t) = -\frac{1}{R_1 C_1} V_1(t) + \frac{1}{C_1} I(t) \quad (3.1)$$

$$\dot{V}_2(t) = -\frac{1}{R_2 C_2} V_2(t) + \frac{1}{C_2} I(t) \quad (3.2)$$

$$V_t(t) = V_{ocv}(t) - V_1(t) - V_2(t) - I(t)R_0 \quad (3.3)$$

V_1 and V_2 represent the voltage across the capacitors C_1 and C_2 respectively. The Current I is the input, and V_t is the output of the circuit.

$$V_t(t) = V_{ocv}(\text{SOC}(t)) - V_1(t) - V_2(t) - I(t)R_0 \quad (3.4)$$

Defining the time constants $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$, the impedance representation in the Laplace domain becomes:

$$Z(s) = R_0 + \frac{R_1}{s\tau_1 + 1} + \frac{R_2}{s\tau_2 + 1} \quad (3.5)$$

After polynomial expansion, the input-output relationship can be written as:

$$(1 + a_1 s + a_2 s^2)V_t(s) = (b_0 + b_1 s + b_2 s^2)I(s) \quad (3.6)$$

where

$$a_1 = \tau_1 + \tau_2, a_2 = \tau_1 \tau_2 \quad (3.7)$$

$$b_0 = R_0 + R_1 + R_2 \quad (3.8)$$

$$b_1 = R_0(\tau_1 + \tau_2) + R_1 \tau_2 + R_2 \tau_1 \quad (3.9)$$

$$b_2 = R_0 \tau_1 \tau_2 \quad (3.10)$$

Applying the inverse Laplace transform yields the continuous-time regression form

$$V_t(t) = -a_1 \dot{V}_t(t) - a_2 \ddot{V}_t(t) + b_0 I(t) + b_1 \dot{I}(t) + b_2 \ddot{I}(t) \quad (3.11)$$

3.2 Parameter Estimation Using SVF Method

To avoid direct differentiation of measured signals, a state filter is introduced:

$F(s) = 1/(s + \omega_f)^2$ where ω_f is the filter cutoff frequency is chosen to attenuate the measurement noise. Filtering both sides of the equation yields:

$$F(s) = \frac{1}{(s + \omega_f)^2} = \frac{1}{(s^2 + 2\omega_f s + \omega_f^2)} \quad (3.12)$$

Filtering both sides results in

$$y_f(t) = -a_1 y_f^{(1)}(t) - a_2 y_f^{(2)}(t) + b_0 u_f(t) + b_1 u_f^{(1)}(t) + b_2 u_f^{(2)}(t) \quad (3.13)$$

where

$$y_f(t) = F(s)V_t, \quad y_f^{(1)}(t) = sF(s)V_t, \quad y_f^{(2)}(t) = s^2F(s)V_t \quad (3.14)$$

$$u_f(t) = F(s)I, \quad u_f^{(1)}(t) = sF(s)I, \quad u_f^{(2)}(t) = s^2F(s)I$$

This formulation eliminates direct numerical differentiation while preserving linear parameter dependence. The model is rewritten in compact regression form:

$$y_f(t) = \phi_f^T(t)\theta \quad (3.15)$$

where

$$\phi_f(t) = \begin{bmatrix} -y_f^{(1)}(t) \\ -y_f^{(2)}(t) \\ u_f(t) \\ u_f^{(1)}(t) \\ u_f^{(2)}(t) \end{bmatrix} \quad \theta = \begin{bmatrix} a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix} \quad (3.16)$$

- Recursive Least Squares Estimation

Discretizing at sampling time T_s yields

$$y_f[k] = \phi_f^T[k]\theta + \eta[k] \quad (3.17)$$

where $\eta[k]$ represents the measurement noise.

A recursive Least Squares (RLS) algorithm with forgetting factor $\lambda \in (0,1)$ is employed for online estimation.

The recursive equations are:

$$K[k] = \frac{P[k-1]\phi_f[k]}{\lambda + \phi_f^T[k]P[k-1]\phi_f[k]} \quad (3.18)$$

$$\hat{\theta}[k] = \hat{\theta}[k-1] + K[k]e[k] \quad (3.19)$$

$$P[k] = \frac{1}{\lambda} (P[k-1] - K[k]\phi_f^T[k]P[k-1]) \quad (3.20)$$

- Recovery of the physical Parameters

After convergence, the time constants are obtained solving:

$$s^2 - a_1s + a_2 = 0 \quad (3.21)$$

The ohmic resistance is computed as:

$$R_0 = \frac{b_2}{\tau_1\tau_2} \quad (3.22)$$

The remaining parameters are derived from linear relations:

$$R_1 + R_2 = b_0 - R_0 \quad (3.23)$$

$$R_1\tau_2 + R_2\tau_1 = b_1 - R_0(\tau_1 + \tau_2) \quad (3.24)$$

$$C_1 = \frac{\tau_1}{R_1}, C_2 = \frac{\tau_2}{R_2} \quad (3.25)$$

3.3 Parameter Estimation Using IDT Method

The indirect discrete-time (IDT) approach estimates the continuous-time parameters by first converting the continuous-time model into a discrete-time representation using a discretization method. In this work, the bilinear (Tustin) transformation is employed, which maps the continuous-time Laplace operator, s , into discrete-time z -domain as:

$s = \frac{2}{T} \left(\frac{z-1}{z+1} \right)$ where T denotes the sampling period. Applying this transformation gives mapping from continuous time to discrete-time.

From the continuous-time impedance transfer function

$$Z(s) = R_0 + \frac{R_1}{s\tau_1+1} + \frac{R_2}{s\tau_2+1} \quad (3.26)$$

where τ_1 and τ_2 are the time constants $\tau_1 = R_1C_1$ and $\tau_2 = R_2C_2$

Applying the bilinear transformation yields:

$$H_j(z) = \frac{\beta_j(1+z^{-1})}{1+\alpha_j z^{-1}} \quad (3.27)$$

where $\alpha_j = \frac{1-a_j}{1+a_j}$, $\beta_j = \frac{R_j}{a_j+1}$ and $a_j = \frac{2}{T}\tau_j$

Combining all components, the total battery/cell impedance in discrete time becomes:

$$Z(z) = R_0 + \frac{\beta_1(1+z^{-1})}{1+\alpha_1 z^{-1}} + \frac{\beta_2(1+z^{-1})}{1+\alpha_2 z^{-1}} \quad (3.28)$$

After polynomial expansion, the voltage-current relationship is expressed as second-order difference equation:

$$V_t(k) = -(\alpha_1 + \alpha_2)V_t(k-1) - \alpha_1\alpha_2V_t(k-2) + \gamma_0I(k) + \gamma_1I(k-1) + \gamma_2I(k-2) \quad (3.29)$$

where

$$\gamma_0 = (R_0 + \beta_1 + \beta_2) \quad (3.30)$$

$$\gamma_1 = R_0(\alpha_1 + \alpha_2) + \beta_1(1 + \alpha_2) + \beta_2(1 + \alpha_1) \quad (3.31)$$

$$\gamma_2 = R_0\alpha_1\alpha_2 \quad (3.32)$$

This yields the regression model

$$V_t(k) = \phi^T(k)\theta + \eta(k) \quad (3.33)$$

with

$$\phi(k) = \begin{bmatrix} -V_t(k-1) \\ -V_t(k-2) \\ I(k) \\ I(k-1) \\ I(k-2) \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} \alpha_1 + \alpha_2 \\ \alpha_1\alpha_2 \\ \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} \quad (3.34)$$

- Recursive Least Squares Estimation

The parameter vector is estimated online using RLS with forgetting factor $\lambda \in (0,1)$:

$$e(k) = V_t(k) - \phi^T(k)\hat{\theta}(k-1) \quad (3.35)$$

which represents the prediction error.

$$K(k) = \frac{P(k-1)\phi(k)}{\lambda + \phi^T(k)P(k-1)\phi(k)} \quad (3.36)$$

The Kalman gain determines how much weight to give to the new measurement versus the previous one.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)e(k) \quad (3.37)$$

The parameters are updated based on prediction error.

$$P(k) = \frac{1}{\lambda} [P(k-1) - K(k)\phi^T(k)P(k-1)] \quad (3.38)$$

- Recovery of the physical Parameters

The original discrete-time parameters are to be recovered next. After convergence, the discrete poles $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are obtained from

$$x^2 - \hat{\theta}_1 x + \hat{\theta}_2 = 0 \quad (3.39)$$

The continuous-time parameters are recovered using the inverse Tustin mapping:

$$\hat{\tau}_j = \frac{T}{2} \frac{1 - \hat{\alpha}_j}{1 + \hat{\alpha}_j} \quad (3.40)$$

The ohmic resistance is computer as

$$\hat{R}_0 = \frac{\hat{\gamma}_2}{\hat{\alpha}_1 \hat{\alpha}_2} \quad (3.41)$$

Parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ are obtained from the linear relation defined by $\hat{\gamma}_0$ and $\hat{\gamma}_1$, after which

$$\hat{R}_1 = \hat{\beta}_1(\hat{\alpha}_1 + 1) \quad \text{and} \quad \hat{R}_2 = \hat{\beta}_2(\hat{\alpha}_2 + 1) \quad (3.42)$$

And the capacitances are obtained from the time constant definitions:

$$\hat{C}_1 = \frac{\hat{\tau}_1}{\hat{R}_1} \quad \text{and} \quad \hat{C}_2 = \frac{\hat{\tau}_2}{\hat{R}_2} \quad (3.43)$$

3.4 Time Varying Parameters of a Battery Model

The parameters of the equivalent battery model cannot be assumed static constants, they might be treated as constants for a fresh battery, however, this assumption cannot hold in real life applications. In practice, these parameters are time-varying due to battery aging, repeated charge/discharge cycling, temperature variation, etc. This variation reflects the continuous degradation of internal electrochemical processes such as Solid-electrolyte interphase (SEI) [75], as well as active material loss. Therefore, assuming a model with fixed parameters would not only lead to inaccurate results over

the battery lifetime but also fail to capture the real dynamic behavior of the cell under real operational conditions. Figure 3.2 shows the variation of the EEC parameters with SOC.

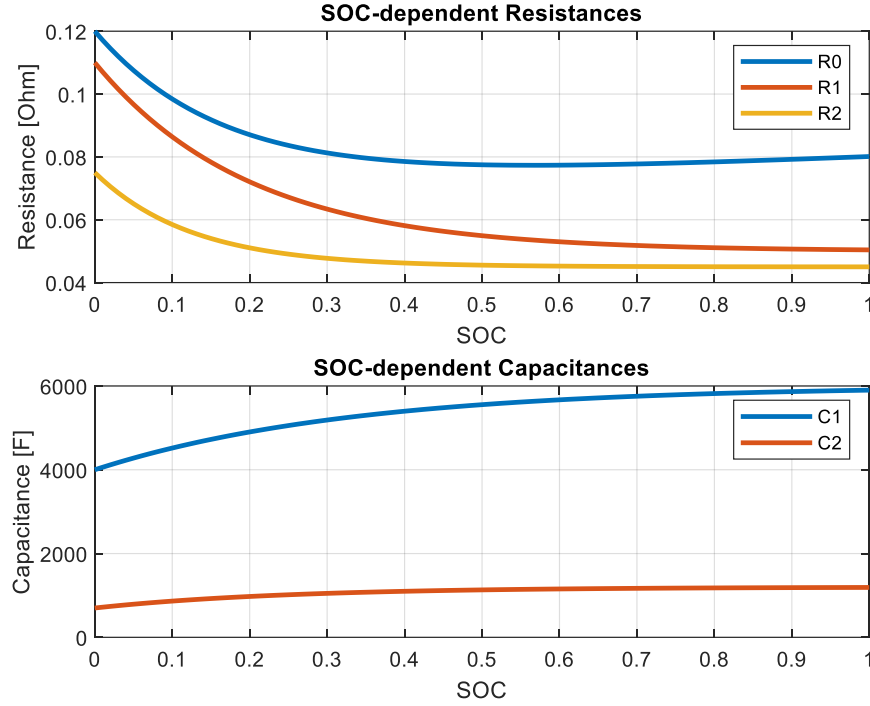


Figure 3.2. Time-varying resistances and capacitance example

To achieve high-fidelity parameters and SOC/SOH estimation, it is necessary to employ adaptive identification techniques that can track time-varying model parameters in real time. An effective approach would be using Recursive Least Squares (RLS) with Variable Forgetting Factor (VFF). The adoption of a variable forgetting factor enhances the estimator's ability to track rapid parameters variation while reducing sensitivity to measurement noise [76].

Mathematically, the Variable Forgetting Factor (VFF) recursive least squares algorithm can be expressed as:

$$e(k) = V_t(k) - \phi^T(k)\hat{\theta}(k-1) \quad (3.44)$$

$$K(k) = \frac{P(k-1)\phi(k)}{\lambda(k) + \phi^T(k)P(k-1)\phi(k)} \quad (3.45)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)e(k) \quad (3.46)$$

$$P(k) = \frac{1}{\lambda(k)} [P(k-1) - K(k)\phi^T(k)P(k-1)] \quad (3.47)$$

where $\lambda(k)$ is the time-varying forgetting factor, which varies based on the magnitude of the prediction error. The idea behind VFF is to adjust $\lambda(k)$ online based on feedback from the prediction error $e(k)$. If the error is large, it would indicate a possible model mismatch due to parameter variation, $\lambda(k)$ is then reduced to discard old data. On the other hand, when $e(k)$ is small, this would suggest stable conditions with slowly varying parameters, $\lambda(k)$ is then allowed to increase toward unity, to reduce noise sensitivity.

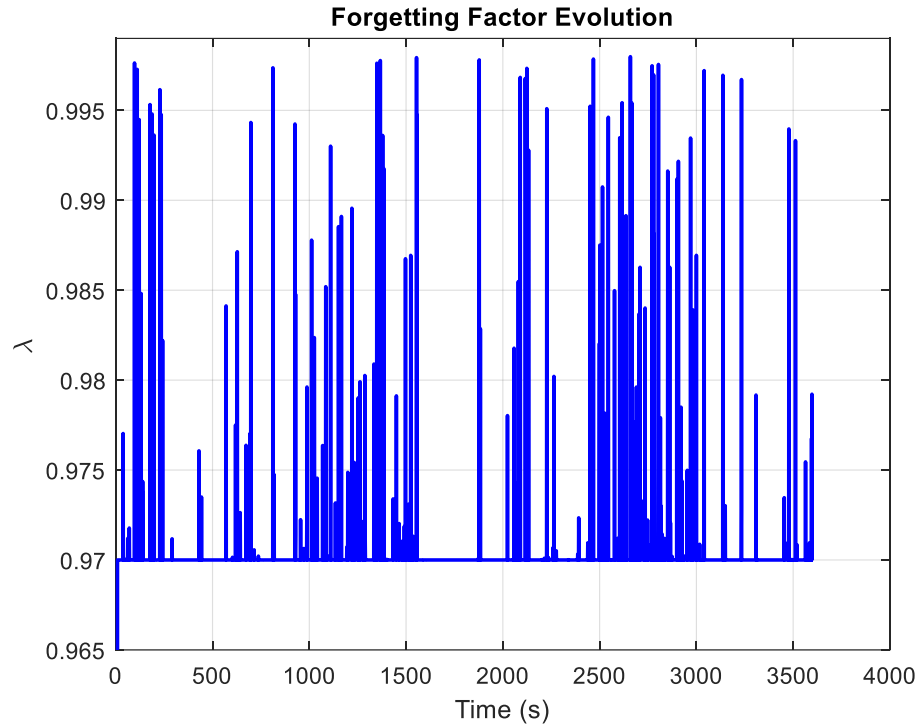


Figure 3.3. Variable Forgetting Factor

In this work, the variable forgetting factor is defined as:

$$\lambda(k) = \lambda_{min}(k) + (1 - \lambda_{min}) * \exp(-\alpha e^2(k)) \quad (3.48)$$

where λ_{min} denotes the lower bound for estimation stability, α is a positive tuning parameter to control sensitivity for the forgetting factor to the prediction error $e(k)$.

The behavior of $\lambda(k)$ can be derived directly from the properties of the exponential function:

- When prediction error is small, $e(k) \rightarrow 0$, using the Taylor expansion of the exponential function around 0 yield:

$$\exp(-\alpha e^2(k)) \approx 1 - \alpha e^2(k) \text{ for } e(k) \approx 0 \quad (3.49)$$

The VFF becomes:

$$\lambda(k) \approx \lambda_{min}(k) + (1 - \lambda_{min}) * (1 - \alpha e^2(k)) \quad (3.50)$$

taking the limit as $e(k) \rightarrow 0$

$$\lim_{e(k) \rightarrow 0} \lambda(k) = 1 \quad (3.51)$$

- When the prediction error is large, $\exp(-\alpha e^2(k)) \rightarrow 0$

which yields

$$\lim_{e(k) \rightarrow \infty} \lambda(k) = \lambda_{min}(k) \quad (3.52)$$

$e(k) \rightarrow 0 \Rightarrow \lambda(k) \rightarrow 1 \Rightarrow$ noise-robust estimation.

Large $e(k) \Rightarrow \lambda(k) \rightarrow \lambda_{min} \Rightarrow$ fast parameter tracking.

From the covariance update equation

$$P(k) = \frac{1}{\lambda(k)} [P(k-1) - K(k)\phi^T(k)P(k-1)] \quad (3.53)$$

A smaller $\lambda(k)$ would increase the effective covariance $P(k)$, which also would increase the Kalman gain $K(k)$ and accelerating parameter adaptation. On the other hand, as $\lambda(k)$ increase towards unity, $P(k)$ would decrease, resulting in a reduction of the Kalman gain $K(k)$.

3.5 Simulation Results and Discussion

The proposed estimation methods are evaluated using simulated five-element battery model. The current profile is designed to excite the circuit dynamics while also allowing parameters to converge. Measurement noise was added to the input current and to terminal voltage to create realistic conditions. For the SVF-based estimation, a state variable filter is employed to avoid direct differentiation of noisy measurements. The recursive least square (RLS) is used for parameters estimating, using either a constant or variable forgetting factor depending on the scenario.

3.5.1. SVF-Based Time – Invariant Parameter Estimation

Figures 3.4. to 3.9 show the estimated parameters using the SVF-based RLS algorithm and assuming a time-invariant system parameter. Overall, the results show acceptable identification as all five estimated parameters $\hat{R}_0$, $\hat{R}_1$, $\hat{R}_2$, $\hat{C}_1$ and $\hat{C}_2$ converge to their true values after an initial transient period.

Figure 3.4 shows the excitation current, OCV-SOC relationship and the SOC evolution. The estimated terminal voltage tracks well the measured voltage, which suggests that the identified parameters did capture the battery dynamics.

Figure 3.5 shows the convergence of the discrete parameter vector θ_k , which they all did, indicating good conditioning of the regression model.

Figure 3.6 to 3.8 show physical parameters recovery estimates. The estimated time constants τ_1, τ_2 , the resistances R_0, R_1 and R_2 and the capacitance C_1 and C_2 all converge to their true values after a short transient period. The estimation remains stable and unbiased except for some small variation during steady state.

Figure 3.9 shows the covariance matrix trace decreases which indicate an increase in estimation confidence.

These results confirm the effectiveness of SVF approach in mitigating noise amplification associated with differentiation, making this method suited for continuous-time parameter identification under constant operating conditions.

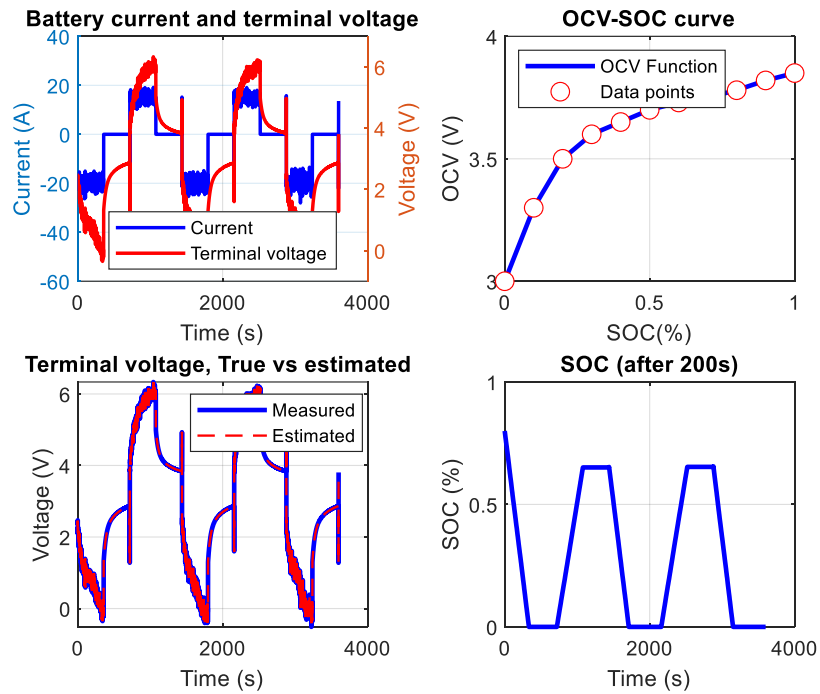


Figure 3.4. Battery Model Identification and Validation using SVF method: Current Excitation and voltage, OCV-SOC Relationship, Voltage Estimation, and SOC evolution (for SVF)

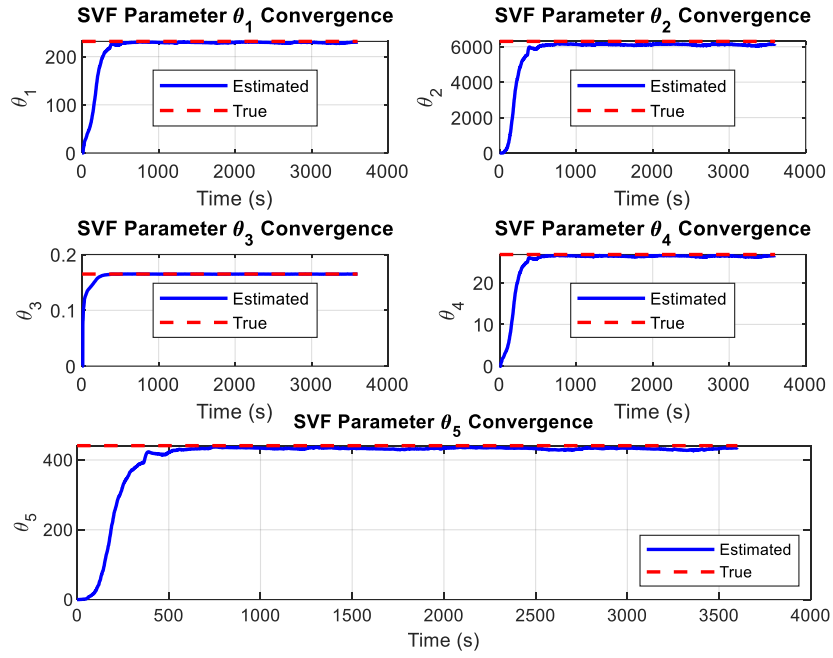


Figure 3.5. Time Evolution of Convergence of Parameter Estimates $\theta_1 - \theta_5$ (for SVF)

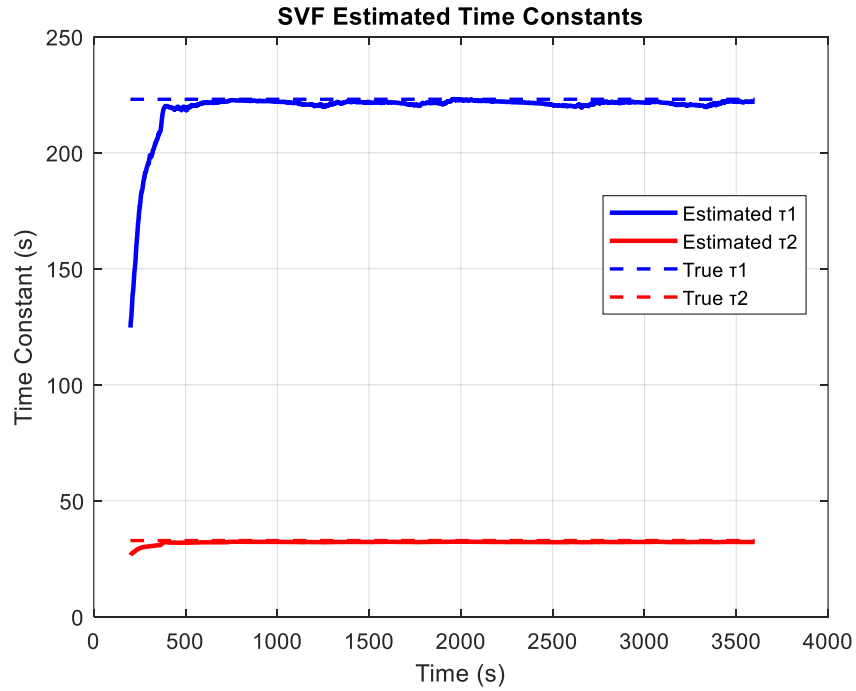


Figure 3.6. Time Evolution of Convergence of Estimated and True Time Constante τ_1 and τ_2 (for SVF)

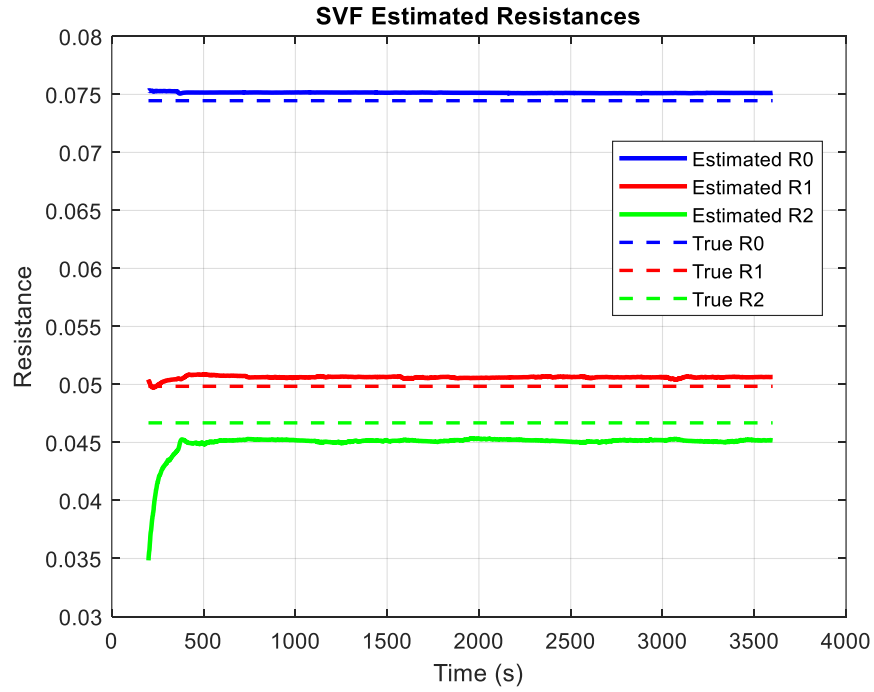


Figure 3.7. Time Evolution of Convergence of Estimated and True Resistances R_0 , R_1 and R_2 (for SVF)

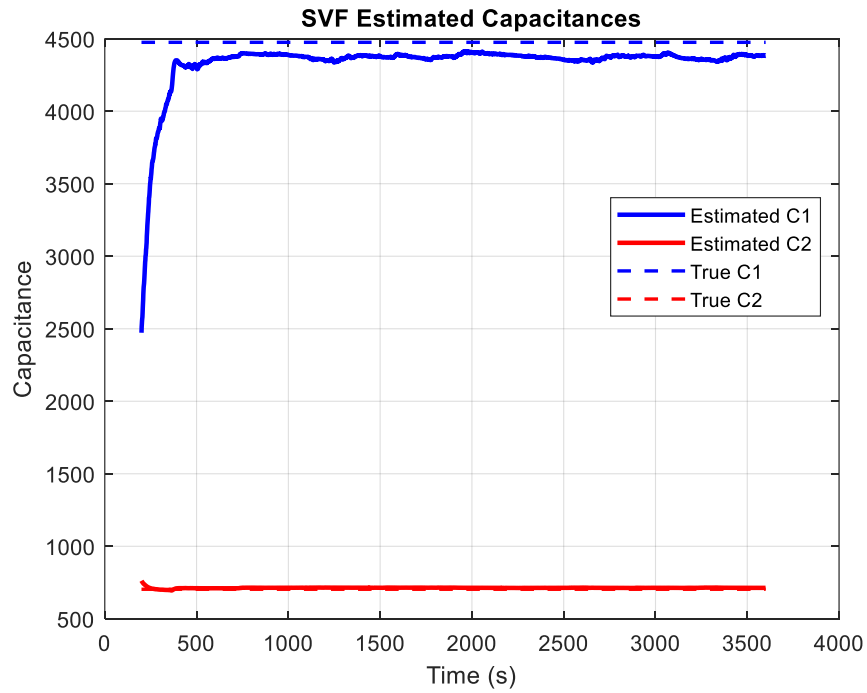


Figure 3.8. Time Evolution of Convergence of Estimated and True Capacitances C_1 and C_2 (for SVF)

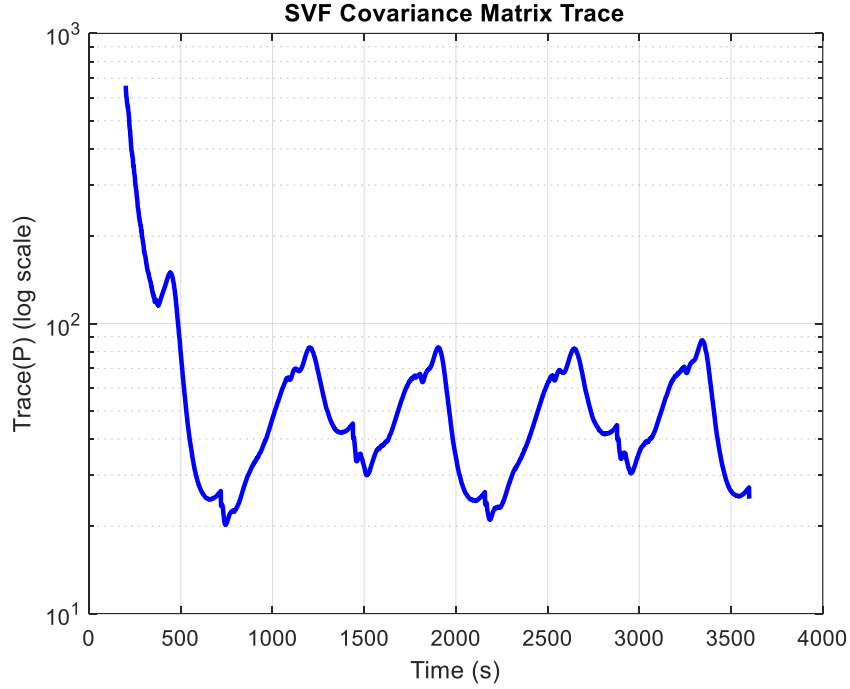


Figure 3.9. Evolution of the trace of Covariance (for SVF)

3.5.2. IDT- based Time – Invariant Parameter Estimation

Figures 3.10 to 3.14 show the estimation results obtained using IDT approach, assuming time-invariant parameters. Overall, the results indicate that the estimation performance is moderate and does not show strong convergence characteristics. Although the parameter vector θ_k in figure 3.11 shows fast convergence, the corresponding actual physical parameters such as R_1 do not demonstrate acceptable estimation accuracy, demonstrating significant fluctuations and deviation from its true value.

Figure 3.10 shows the excitation current, OCV-SOC relationship and the SOC evolution. The estimated terminal voltage tracks well the measured voltage. The regression vector θ_k converges fast (figure 3.11), which indicates numerical stability of the RLS algorithm in the discrete-time domain.

Figure 3.14. shows the decrease in the trace of the covariance matrix, which indicates an increase in the estimator's confidence. However, this confidence is not with respect to true extracted physical parameters, but instead in the discrete parameters θ_k . Overall, from these results we can highlight the limitations of the IDT approach. The discretization and inverse mapping (to extract physical parameters) introduce conditioning causing identification issues, which reflects in poor physical parameters estimation.

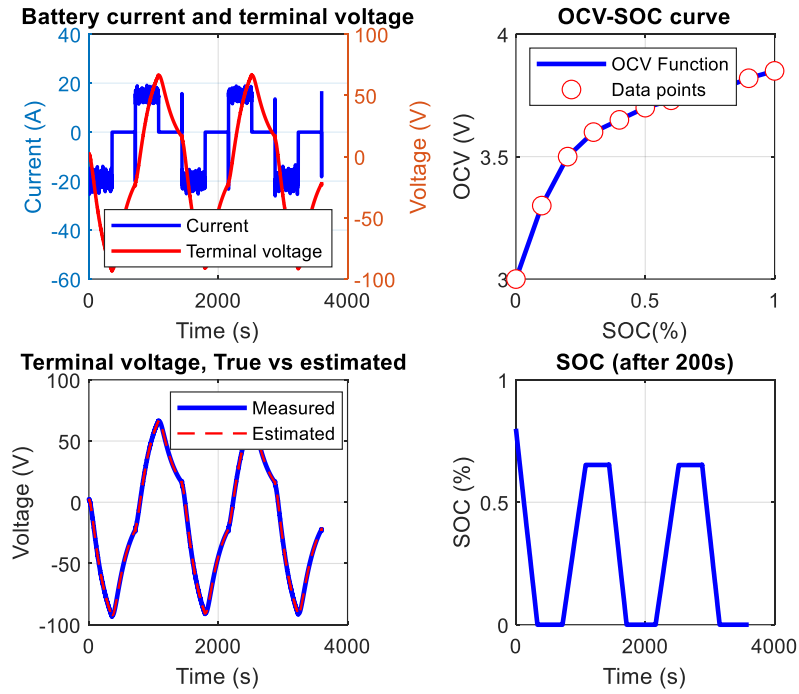


Figure 3.10. Battery Model Identification and Validation using IDT method: Current Excitation and voltage, OCV-SOC Relationship, Voltage Estimation, and SOC evolution (for IDT)

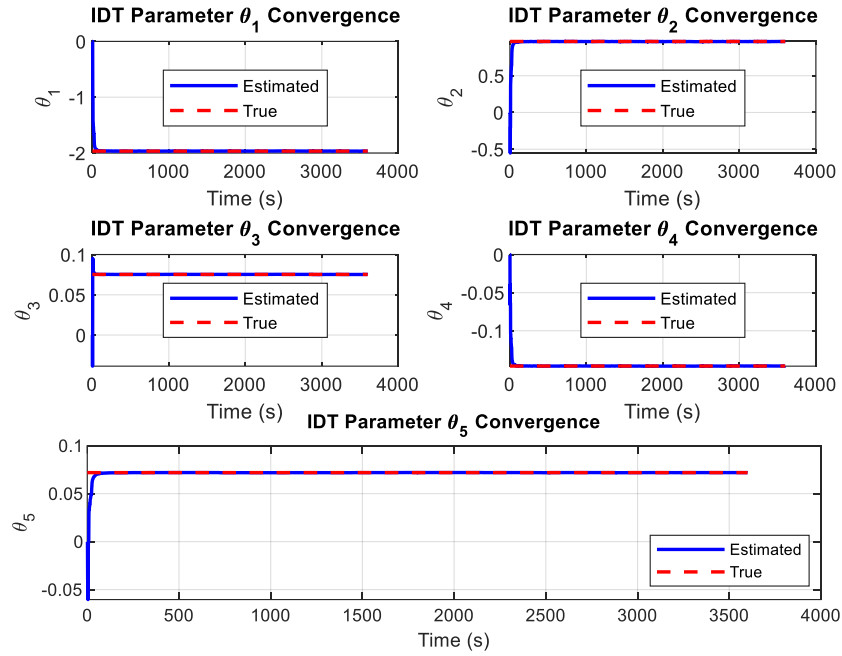


Figure 3.11. Time Evolution of Convergence of Parameter Estimates $\theta_1 - \theta_5$ (for IDT)

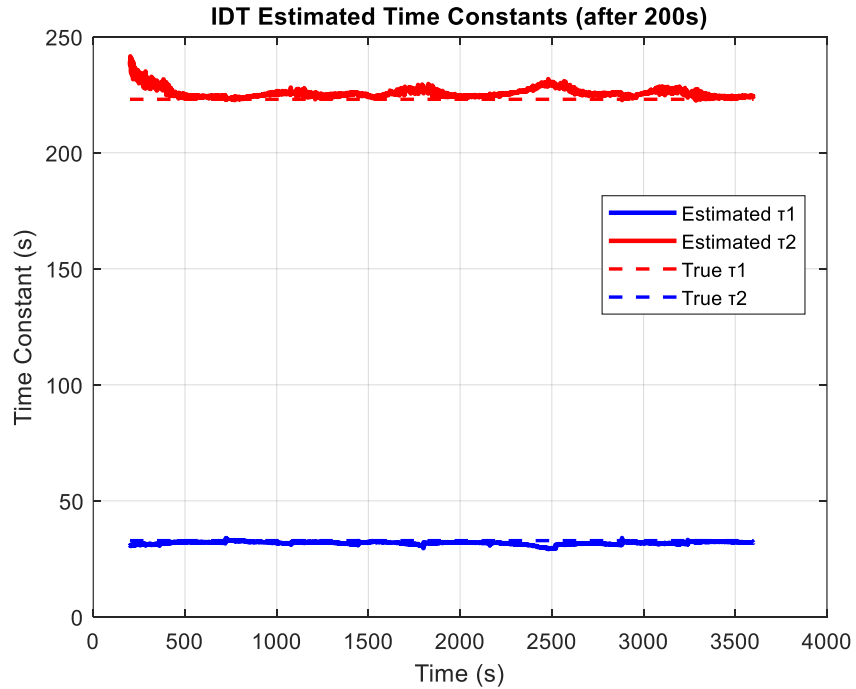


Figure 3.12. Time Evolution of Convergence of Estimated and True Time Constante τ_1 and τ_2 (for IDT)

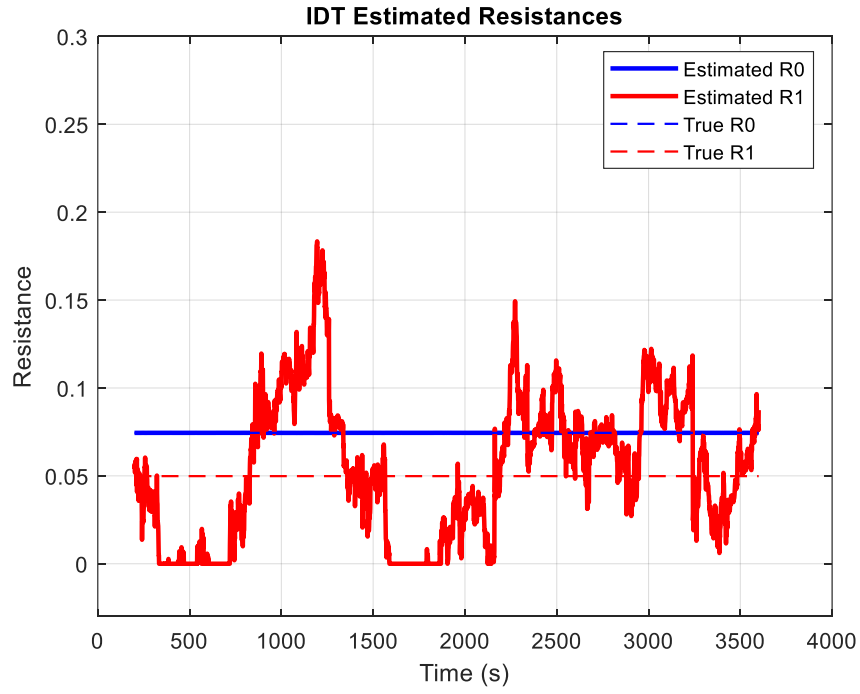


Figure 3.13. Time Evolution of Convergence of Estimated and True Resistances R_0 and R_1 (for IDT)

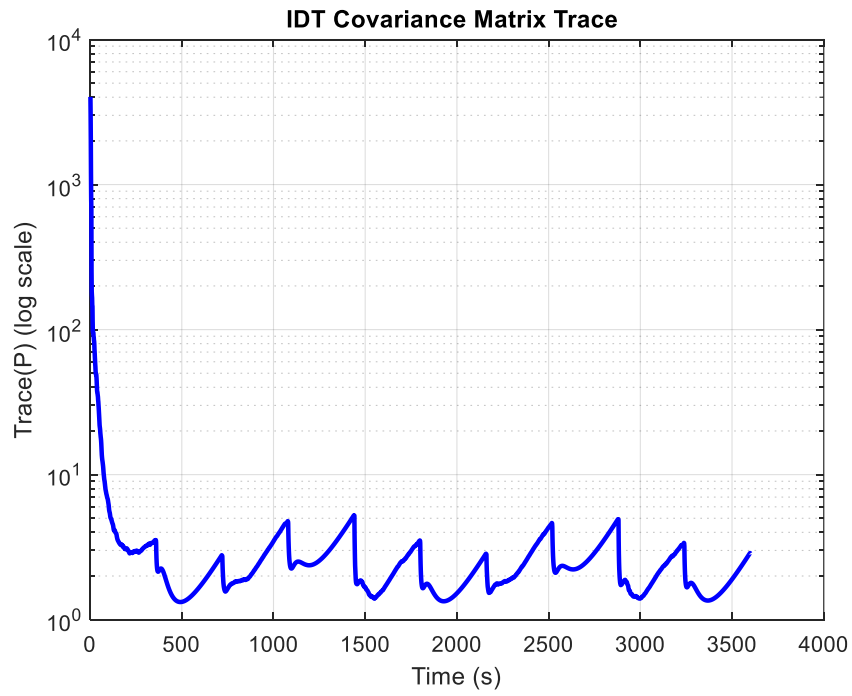


Figure 3.14. Evolution of the trace of Covariance (for IDT)

3.5.3. SVF- based Time –varying (SOC-Dependent) Parameter Estimation

To evaluate the tracking performance of the proposed SVF-based RLS, the circuit parameters are now modeled as SOC-dependent and vary continuously over time. Figures 3.15 to 3.21 present the corresponding estimation results.

Figure 3.15 shows the excitation current, OCV-SOC relationship, terminal voltage estimation and SOC evolution. The estimated terminal voltage tracks the measured voltage, with some error, which suggests that the identified parameters, to some extent, captured the battery dynamics.

Figure 3.16 shows the evolution of the discrete parameter vector θ_k . Unlike the time-invariant condition (figure 3.5), the parameters no longer converge to constant values, but instead they continuously vary, reflecting the fact that: 1- the estimator can track parameter variation, and 2- the SOC dependency behavior of the system. While the estimator can follow the general trend of parameter variation, the tracking performance is not great and some noticeable errors are present, particularly during period of parameter change.

Figures 3.17 to 3.19 compare the estimated and true values of time constants, resistances and capacitance. Although the estimator can track general trends of some SOC-dependent parameters, the estimation quality is not satisfactory. Huge errors with tracking delays and oscillations are observed, particularly during rapid parameter change. To address this issue and to improve tracking smoothness, an IIR filtering stage is introduced to reduce the estimation error and to improve stability under time-varying conditions.

Figure 3.20 shows the evolution of the variable forgetting factor, and figure 3.21 shows the evolution of the covariance matrix trace.

Overall, the results of SVF-based RLS with variable forgetting factor can track SOC-depended parameter change while keeping the estimation parameters bounded. However, estimation accuracy is only moderate, with noticeable errors. Improvement by integrating filtering helped enhance estimation accuracy.

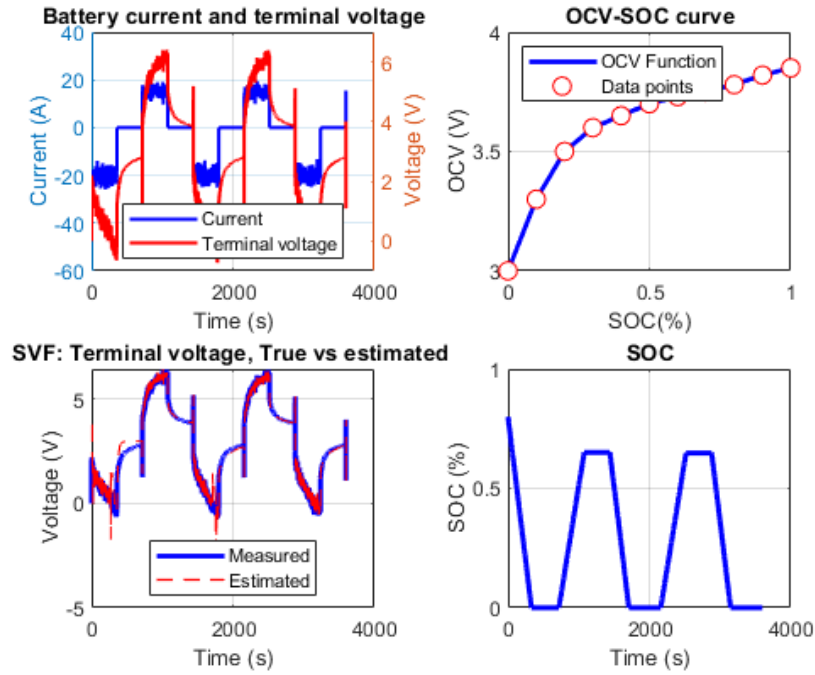


Figure 3.15. Battery Model Identification and Validation using SVF method: Current Excitation and voltage, OCV-SOC Relationship, Voltage Estimation, and SOC evolution (for SVF)

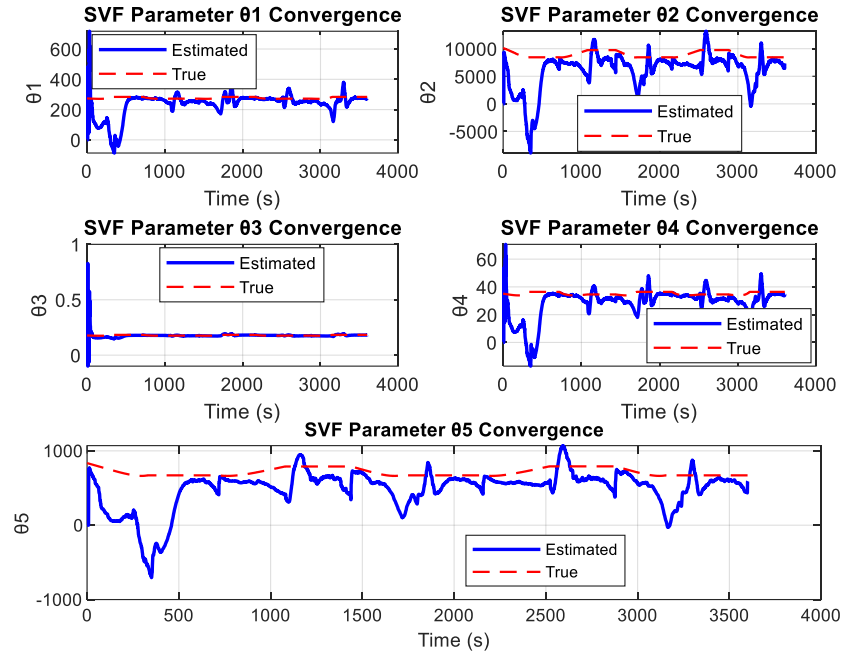


Figure 3.16. Time Evolution of Convergence of Parameter Estimates $\theta_1 - \theta_5$ (for SVF)

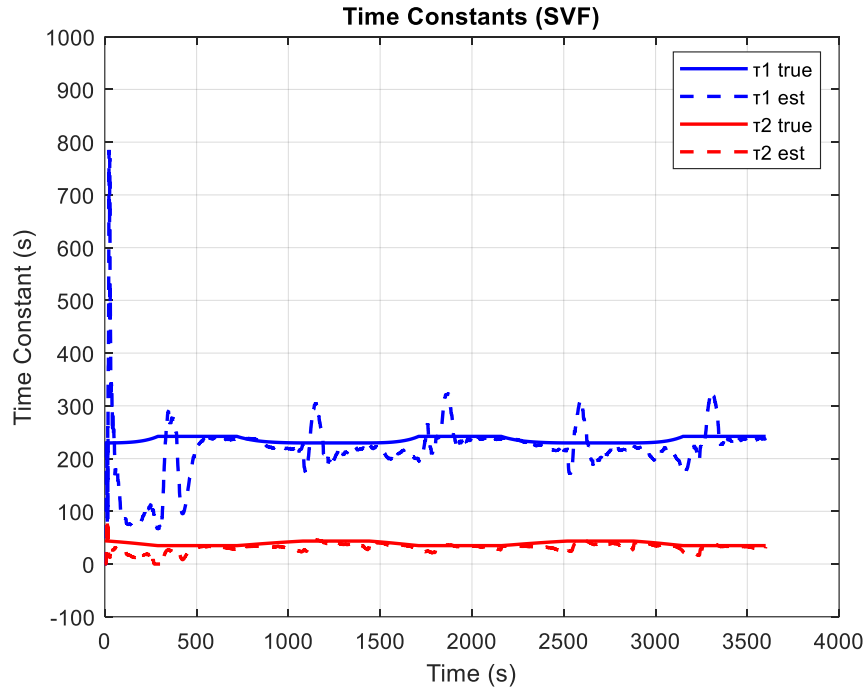


Figure 3.17. Time Evolution of Convergence of Estimated and True Time Constante τ_1 and τ_2 (for SVF)

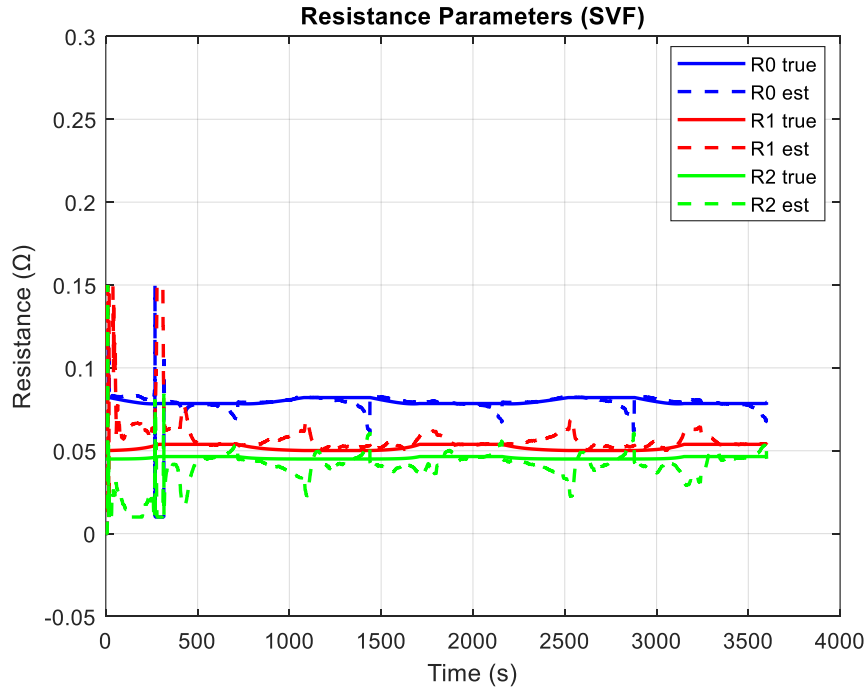


Figure 3.18. Time Evolution of Convergence of Estimated and True Resistances R_0 , R_1 and R_2 (for SVF)

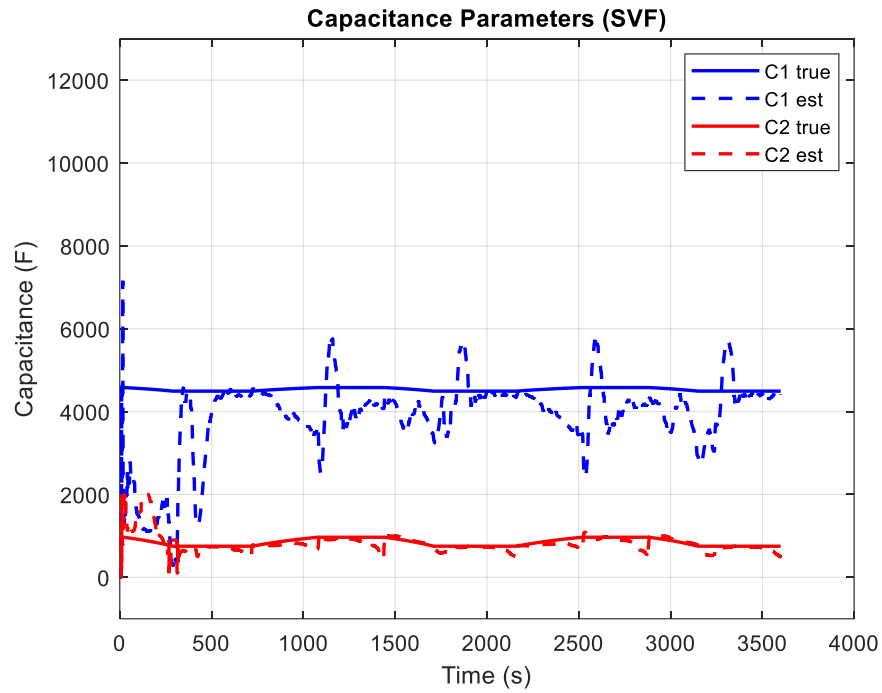


Figure 3.19. Time Evolution and Convergence of Estimated and True Capacitances C_1 and C_2 (for SVF)

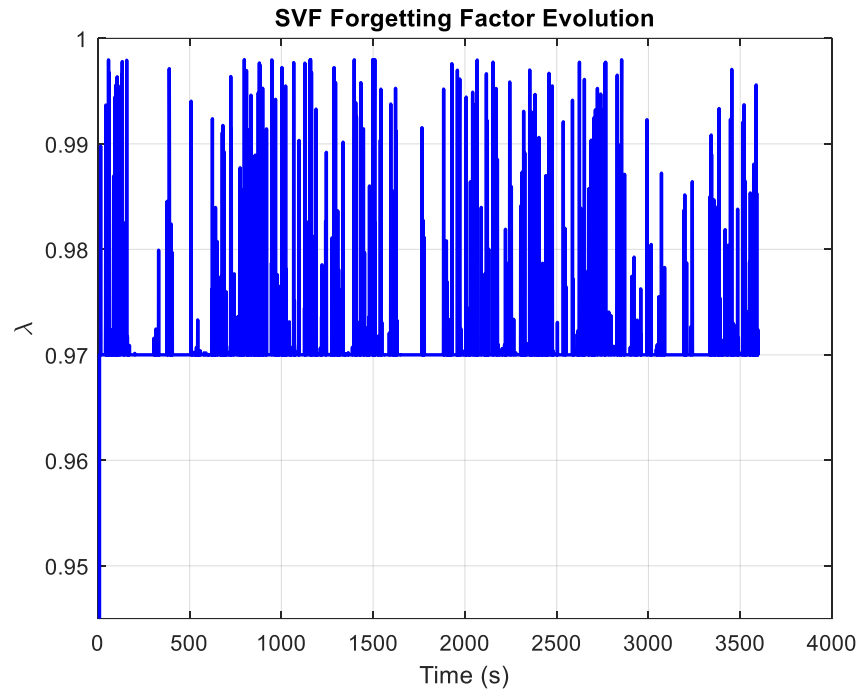


Figure 3.20. Evolution of Variable Forgetting Factor (for SVF)

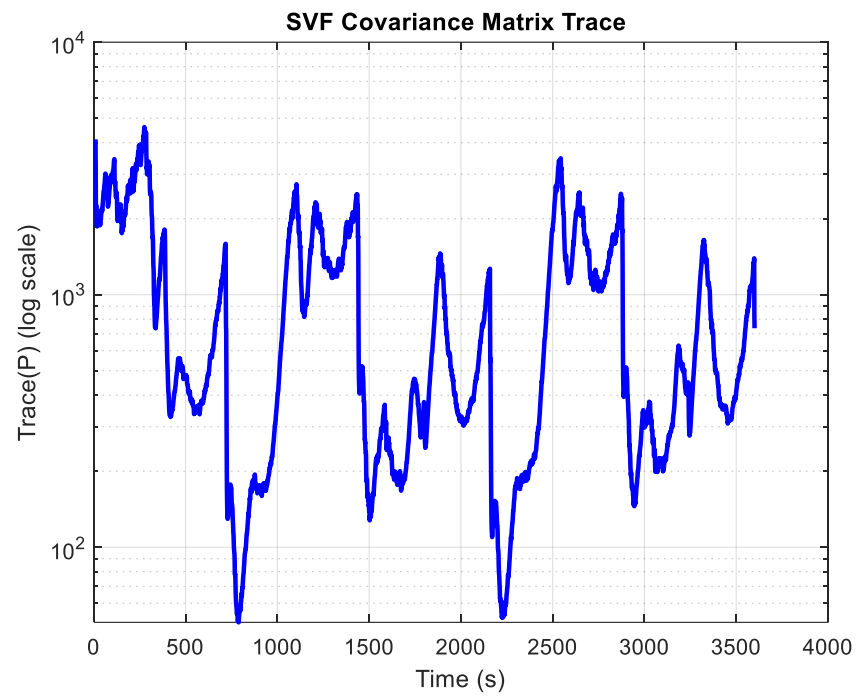


Figure 3.21. Evolution of the trace of Covariance (for SVF)

3.5.4. IDT- based Time –varying (SOC-Dependent) Parameter Estimation

Figure 3.22 – 3.25 presents the results of IDT-based estimation under time-varying conditions. The circuit parameters are now modeled as SOC-dependent and vary continuously over time.

Figure 3.22 shows the excitation current, OCV-SOC relationship, terminal voltage estimation and SOC evolution. Although the estimated terminal voltage generally tracks the measured voltage, one can notice some deviations. This means that the model does maintain some consistency, but the identification accuracy of some physical parameters worsens under time-varying conditions. As shown in figure 3.23 and 3.24, while R_0 is estimated relatively well, R_1 , C_1 and C_2 demonstrate fluctuations and poor tracking of the true values.

Unlike SVF-based method, IDT approach does not perform well in this environment, further reflecting sensitivity to discretization and inverse mapping.

Overall, these results show that the IDT method only provides limited tracking capability in time-varying environments.

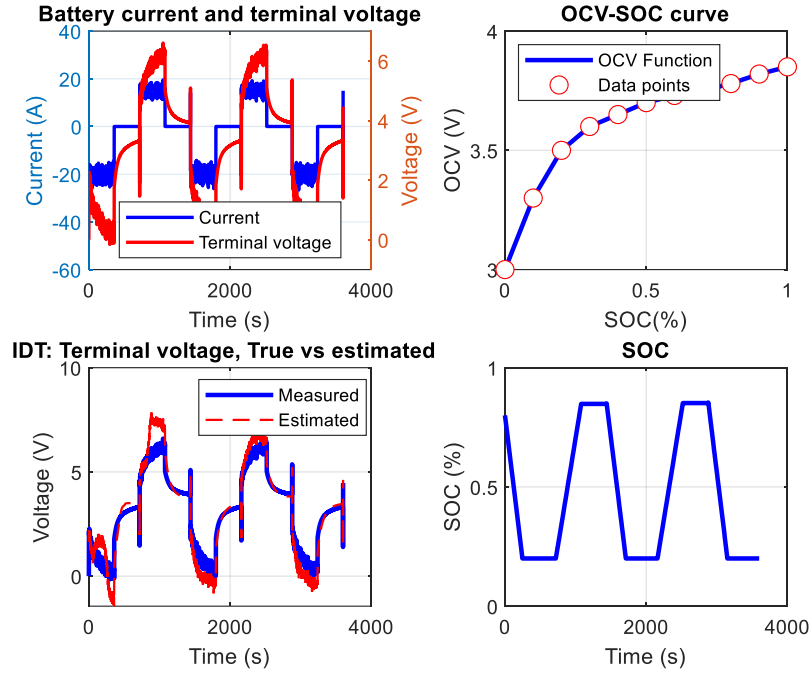


Figure 3.22. Battery Model Identification and Validation using IDT method: Current Excitation and voltage, OCV-SOC Relationship, Voltage Estimation, and SOC evolution (for IDT)



Figure 3.23. Time Evolution of Convergence of Estimated and True Resistances R_0 and R_1 (for IDT)

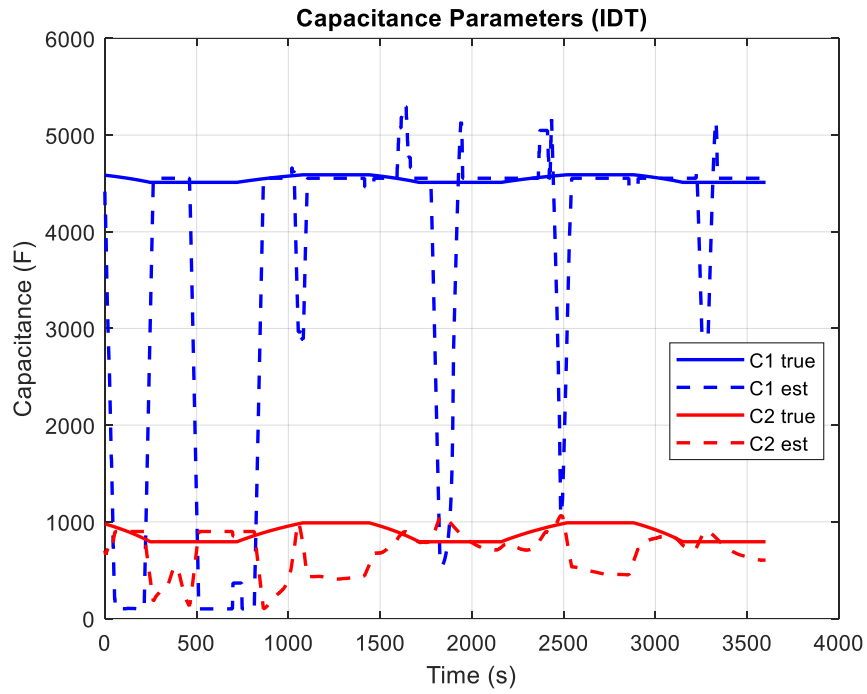


Figure 3.24. Time Evolution of Convergence of Estimated and True Capacitances C_1 and C_2 (for IDT)

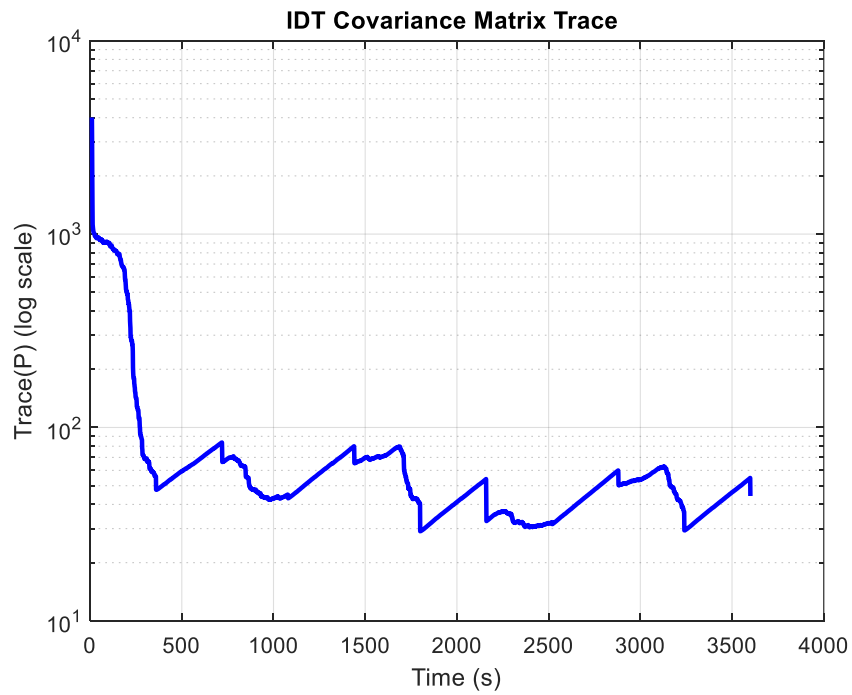


Figure 3.25. Evolution of the trace of Covariance (for IDT)

3.5.5. Improved SVF- based Time –varying Parameter Estimation – IIR filter Implementation

To further improve the parameter estimation accuracy while using the SVF-based estimation technique under time-varying conditions, a first order infinite impulse response (IIR) filter shown in equation (3.54) is applied to the RLS estimates parameter vector.

$$\theta_f[k] = \alpha\theta_f[k - 1] + (1 - \alpha)\hat{\theta}[k] \quad (3.54)$$

where $0 < \alpha < 1$

Applying the Z-transform yields:

$$H(z) = \frac{1-\alpha}{1-\alpha z^{-1}} \quad (3.55)$$

The filter has one pole at $z = \alpha$ and one zero at $z = 0$.

The magnitude response satisfies: $|H(e^{j\omega})| = \begin{cases} 1, & \text{for } \omega = 0 \\ \frac{1-\alpha}{1+\alpha}, & \text{for } \omega = \pi \end{cases}$

So, slow SOC-dependent parameters variations will be preserved, while high-frequency noise will be attenuated.

Figures 3.26 – 3.30 show the estimation results obtained using SVF-based RLS and the first-order IIR filter in time-varying conditions.

Figure 3.26 shows the excitation current, OCV-SOC relationship, terminal voltage estimation and SOC evolution. Compared to the unfiltered case (figure 3.15), the estimated terminal voltage shows improved smoothness while maintaining accurate tracking of the measured terminal voltage. This confirms that the integration of the IIR filter did not affect system dynamics.

Figure 3.27 – 3.28 show the evolution of the filtered discrete parameter vector θ_1 and θ_3 .

The high frequency oscillation observed previously (figure 3.16) is attenuated. Although a tracking delay is noticed due to the low pass nature of the filter, it follows the overall SOC-dependent variation.

Figure 3.29 – 3.30 show the recovered physical parameters. The Resistances and capacitances show a noticeable reduced oscillation compared to non-filtered case (figure 3.18 – 3.19).

The implementation of the first-order filter did improve the parameter estimation of time-varying operation. Although a small lag is introduced, the reduction in noise-induced oscillations leads to a more reliable physical parameters estimation.

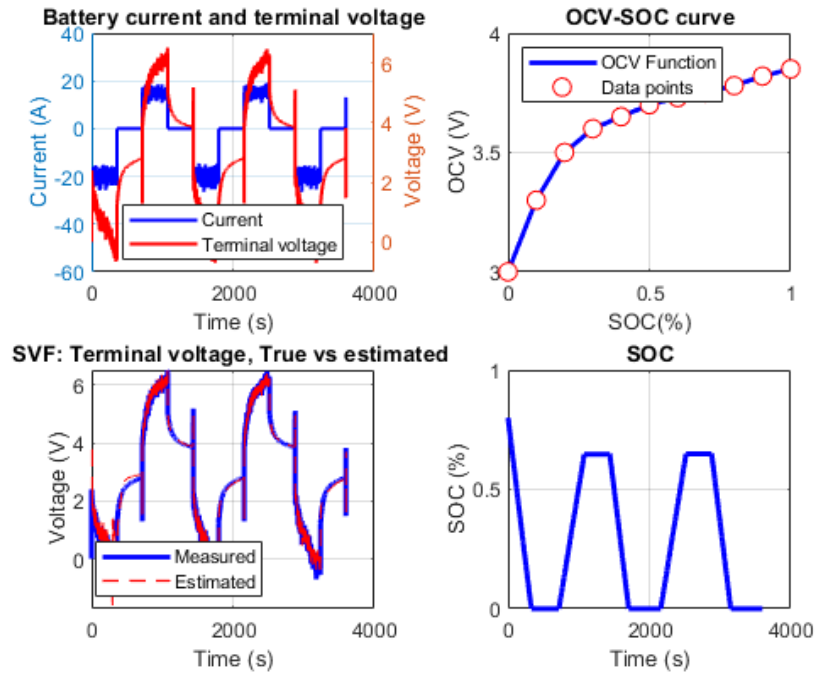


Figure 3.26. Battery Model Identification and Validation using SVF method + IRR Filter: Current Excitation and voltage, OCV-SOC Relationship, Voltage Estimation, and SOC evolution (for SVF + IRR)

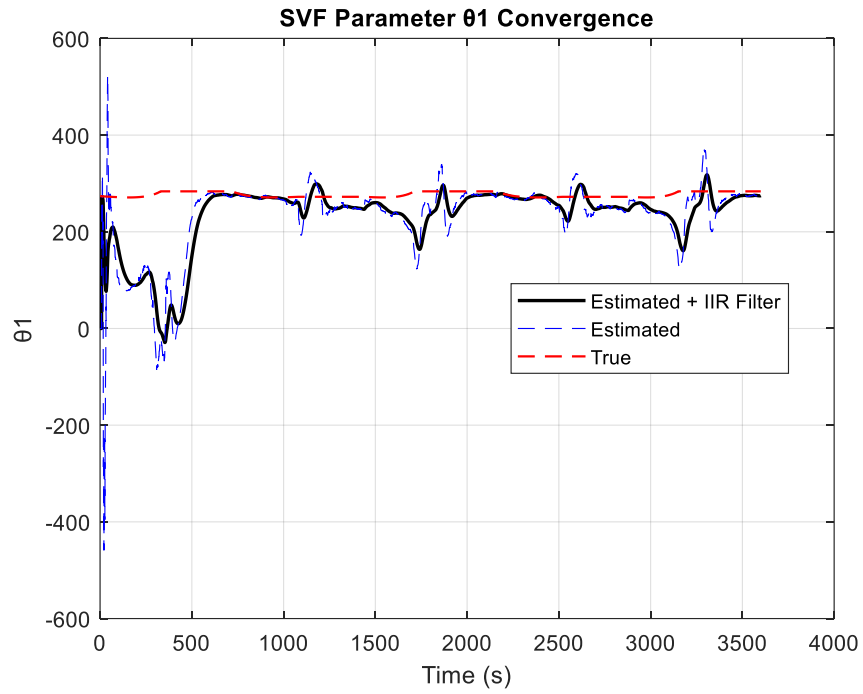


Figure 3.27. Time Evolution of Convergence of Parameter Estimates θ_1 (for SVF + IRR)

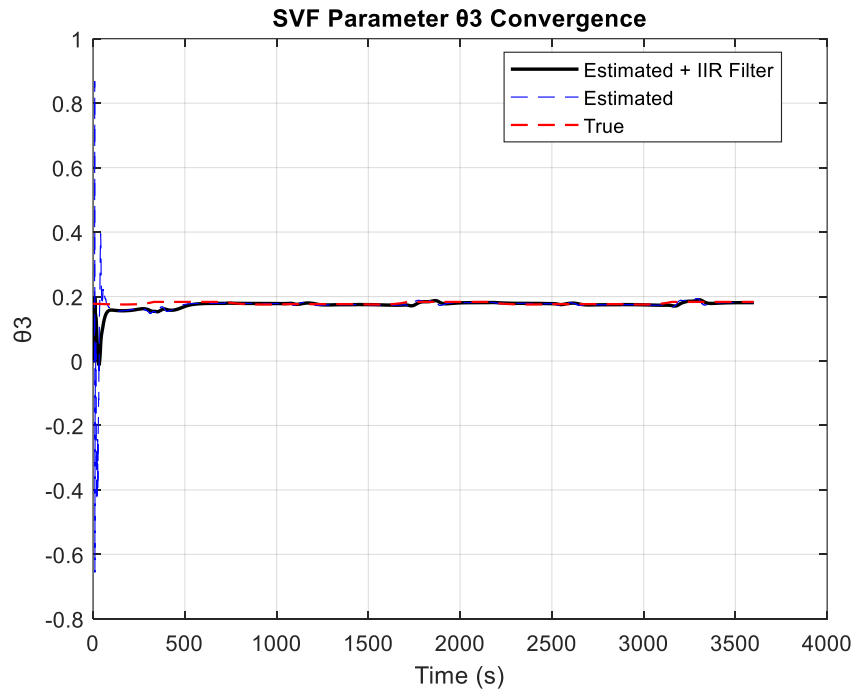


Figure 3.28. Time Evolution of Convergence of Parameter Estimates θ_3 (for SVF + IRR)

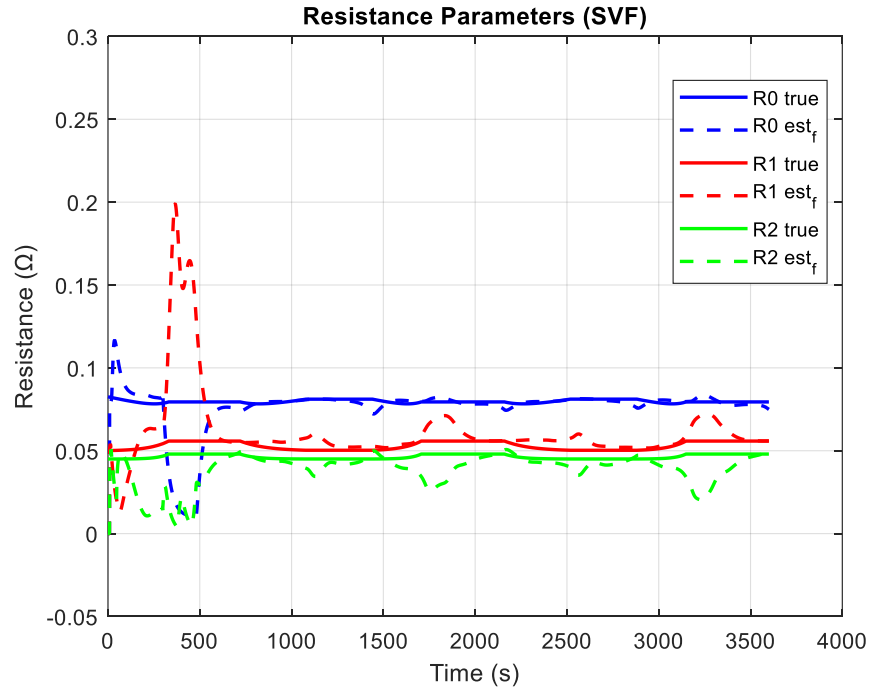


Figure 3.29. Time Evolution of Convergence of Estimated and True Resistances R_0 , R_1 and R_2 (for SVF + IIR)

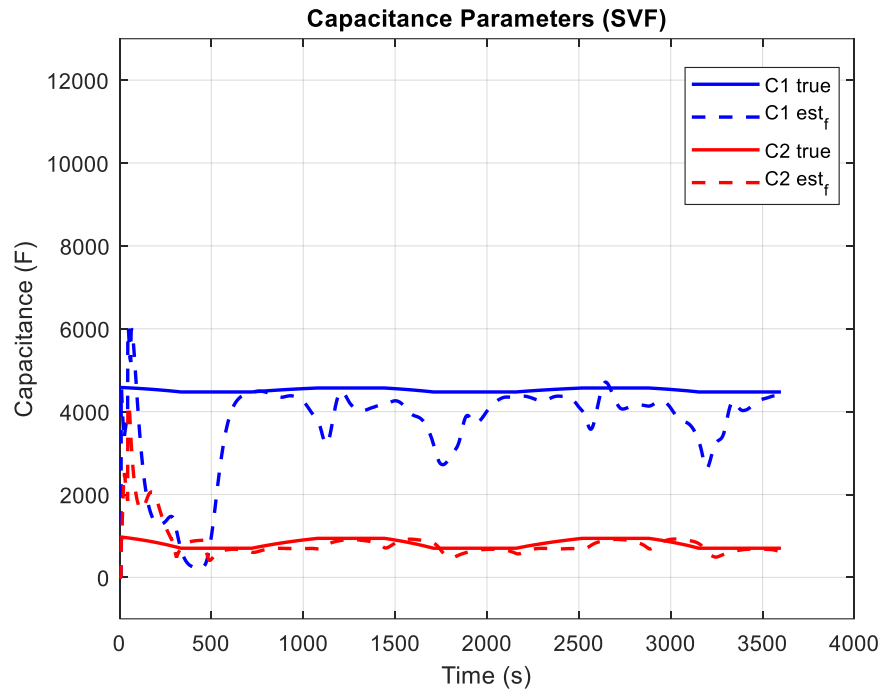


Figure 3.30. Time Evolution of Convergence of Estimated and True Capacitances C_1 and C_2 (for SVF + IIR)

CHAPTER FOUR

SVF-BASED SOC ESTIMATION WITH TIME-VARYING PARAMETERS

To further evaluate the estimated parameters robustness using SVF method, they are employed in the simultaneous, real-time estimation of the State of Charge of the cell. This is achieved by implementing both the Extended Kalman Filter (EKF) and the Unscented Kalman filter (UKF); each use the output of the SVF-RLS estimated parameters. The EKF state vector includes SOC, the RC branches polarization voltages V_1 and V_2 . V_1 and V_2 will update following the discrete-time RC model parameterized by the continuous updated SVF-RLS estimates. Similarly, UKF utilizes the same state-space formulation, but avoids the local linearization step (adopted by EKF) by using a set of sigma points through the nonlinear system dynamics. Overall, this integrated SVF-RLS-EKF/UKF framework allows real-time tracking of SOC under realistic conditions, providing a reliable solution for advanced battery management strategies.

4.1 Extended Kalman Filter (EKF) Formulation

As previously mentioned, the SVF-RLS-EKF/UKF framework is employed in this study to estimate the SOC, V_1 and V_2 of the battery cell. The EKF state vector in this case is defined as:

$$x_k = [SOC_k \ V_{1,k} \ V_{2,k}]^T \quad (4.1)$$

The discrete-time nonlinear state equations of the 5-element batter model are defined as:

$$SOC_{k+1} = SOC_k + \frac{T_s}{3600C_k} I_k + \omega_{soc,k} \quad (4.2)$$

$$V_{1,k+1} = e^{-T_s/(R_{1,k}C_{1,k})} V_{1,k} + R_{1,k} (1 - e^{-T_s/(R_{1,k}C_{1,k})}) I_k + \omega_{V_1,k} \quad (4.3)$$

$$V_{2,k+1} = e^{-T_s/(R_{2,k}C_{2,k})} V_{2,k} + R_{2,k} (1 - e^{-T_s/(R_{2,k}C_{2,k})}) I_k + \omega_{V_2,k} \quad (4.4)$$

where T_s is the sampling time, I_k is the applied current and ω_k denotes zero-mean Gaussian process noise.

The terminal voltage measurement equation can be expressed as:

$$V_{t,k} = h(x_k, I_k) + v_k \quad (4.5)$$

$$V_{t,k} = V_{ocv,k}(SOC) + R_{0,k} I_k + V_{1,k} + V_{2,k} + v_k \quad (4.6)$$

where $V_{ocv,k}(SOC)$ denotes the nonlinear open-circuit voltage – SOC relationship, $R_{0,k}$ is the ohmic resistance which is already identified online via SVF-RLS, Finally, v_k is the measurement noise.

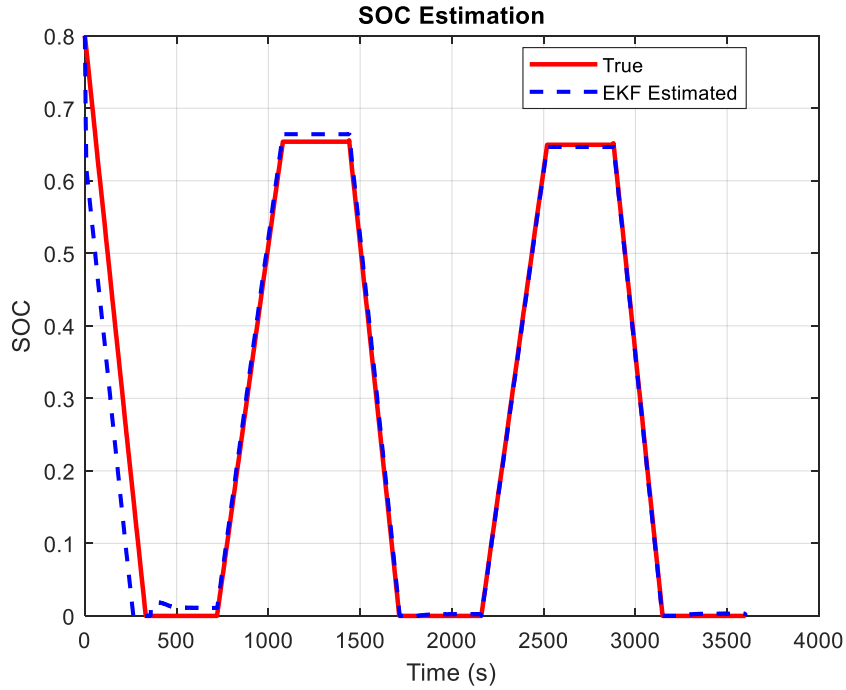


Figure 4.1. State of Charge Estimation Using EKF Compared with True SOC

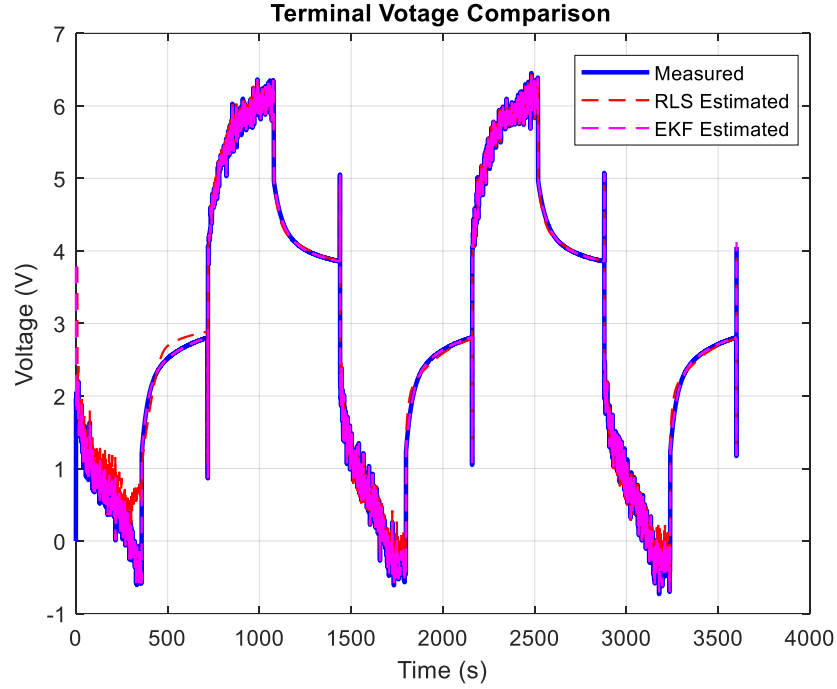


Figure 4.2 Comparison of Measured and Model-Estimated Terminal Voltage using RLS and EKF

The SOC estimation results (figure 4.1 and 4.2) show that the proposed SVF-RLS-EKF framework can accurately track the true SOC change that is obtained from Coulomb counting with some deviation that is expected due to increased model uncertainty and voltage polarization effects during charge/discharge pulses [76]. However, the EKF corrects the SOC estimation once the current enters the rest period. It is known in literature that including the RC polarization voltages V_1 and V_2 as EKF states significantly improves SOC accuracy by capturing the cell's transient voltage dynamics, however, the continuous update of these states using time-varying parameters further increase the SOC estimation accuracy by accounting for the nonlinearities in battery dynamics that vary with SOC, temperature and aging affects [76].

4.2 Unscented Kalman Filter (UKF) Formulation

Because the battery displays nonlinear behavior triggered by OCV-SOC relationship, as well as the cell's internal dynamics, we also implemented the Unscented Kalman Filter (UKF) to improve SOC estimation. Unlike the standard EKF, UKF does not rely on linear approximation. Instead, it uses chosen “sigma points” to capture how the battery's nonlinearity affects the estimation of SOC, leading to better accuracy. A brief derivation and implementation of the UKF are presented in chapter 2.

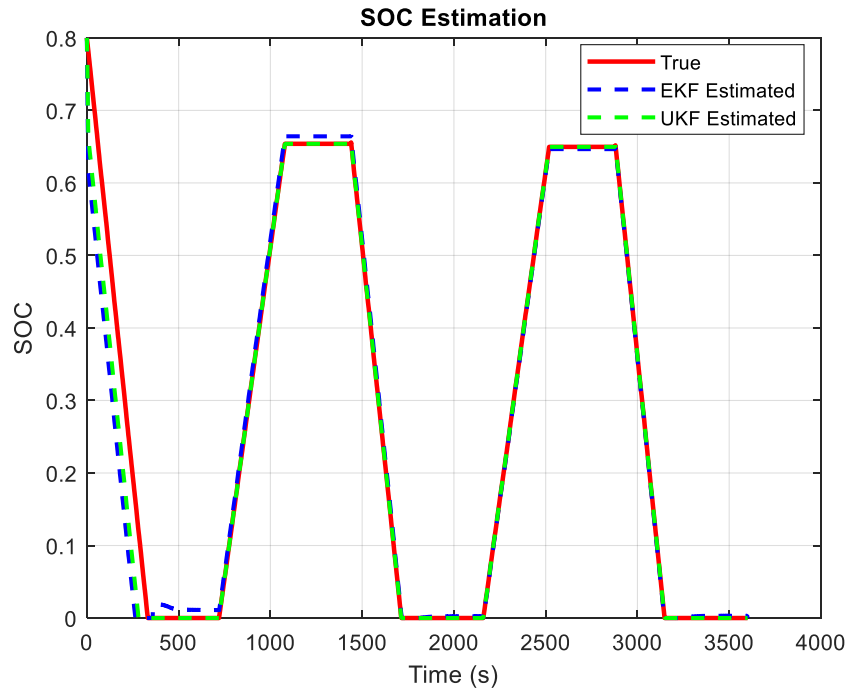


Figure 4.3. State of Charge Estimation Using EKF and UKF Compared with True SOC

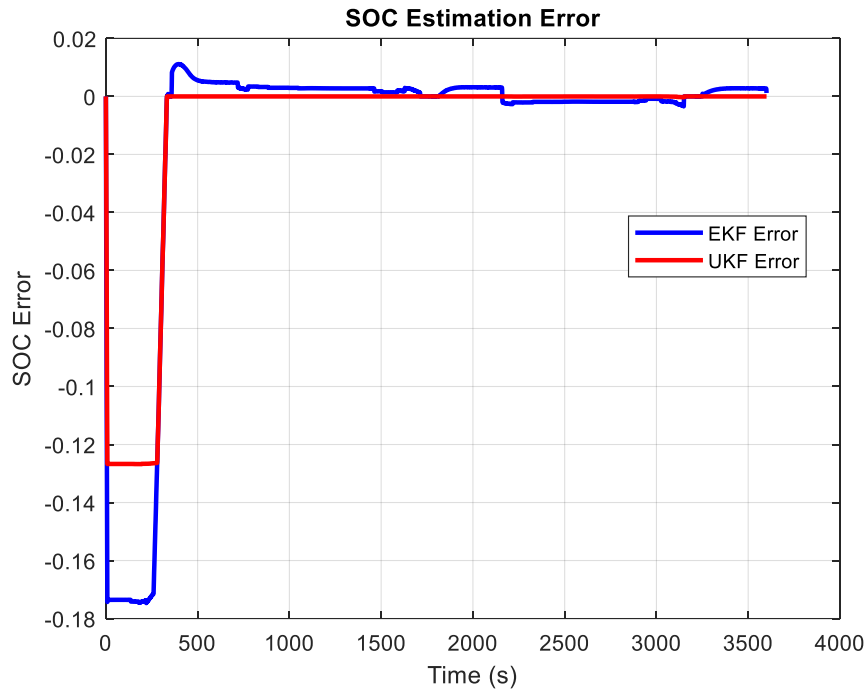


Figure 4.4. SOC Estimation Error Comparison Between EKF and UKF

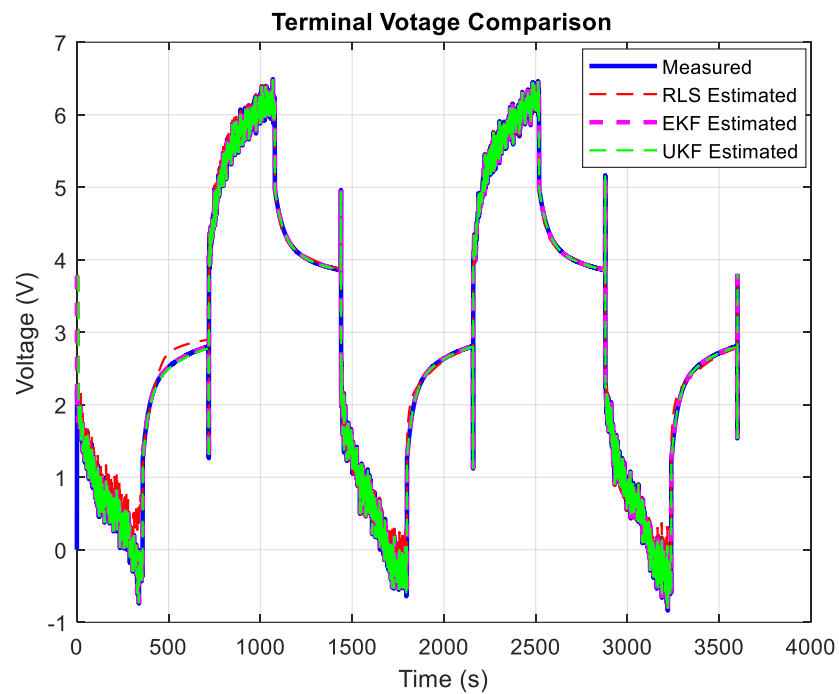


Figure 4.5. Comparison of Measured and Model-Estimated Terminal Voltage using RLS and EKF – UKF

Figure 4.3 shows the comparison between the true SOC and the SOC estimated using the EKF (blue dashed line) and UKF (green dashed line) over the simulation cycle. Overall, both filters are able to track the true SOC profile with good accuracy. This indicates that the dynamic battery model and the filters designed are properly implemented.

At the beginning of the simulation (0 ~ 400s), one can observe a small deviation between true SOC and the estimated values. This is expected because of the initialization uncertainty of both filters. The UKF does, however, converge slightly faster with no overshoot compared to EKF. This proves UKF handles better OCV-SOC nonlinear function.

During the flat SOC regions, both filters show very small steady-state error. However, the UKF estimation aligns more closely with the true SOC compared to EKF. This could be attributed to the UKF ability to propagate sigma points through the nonlinear OCV-SOC function without the need for linearization. In summary, these results confirm the effectiveness of both filters for SOC estimation, but also the robustness and reliability of the SVF-RLS estimated parameters when applied in real-time state estimation.

CHAPTER 5

FAST OPTIMAL CHARGING SCHEME USING MCC CHARGING CURRENT PROFILE

This chapter discusses the use of an electro-thermal model of a Li-ion battery to search for an optimized multi-stage constant-current charging method that minimizes the total charge time while keeping the temperature rise at each stage within acceptable limits. The model consists of a battery electrical equivalent circuit model that is coupled with a thermal model. The EEC model would continuously predict the terminal voltage while the thermal model predicts its temperature rise during charging. The proposed optimization employed a novel, single variable search algorithm (for both methods) that is simple to implement and guarantees convergence. Simulation results of the proposed charging techniques show a fast, safe and practical alternative to many existing optimal charging methods which are often computationally demanding.

5.1. Integrated Electro-Thermal Model of a Li-ion Battery

The integrated electrothermal model studied here is based on a coupled electrical equivalent circuit (EEC) model and a thermal battery model. The EEC model is used to predict the battery's terminal voltage during charge or discharge operations, while the thermal model estimates the temperature rise of the battery [77]. The following sections first detail the EEC model, followed by a summary of the thermal model.

5.1.1. Electrical Equivalent Circuit (EEC) Model

As mentioned before, Electrical Equivalent Circuit models offer relatively accurate simulation results of a battery by capturing electrochemical cell dynamics through electrical components. In this work, the second-order RC model, that is proposed in [77], was selected to capture the effects of concentration polarization more effectively. This second order EEC battery model, characterized by five lumped parameters, has been widely used in literature to model the voltage-current relationship of a Li-ion battery during charge and discharge operations.

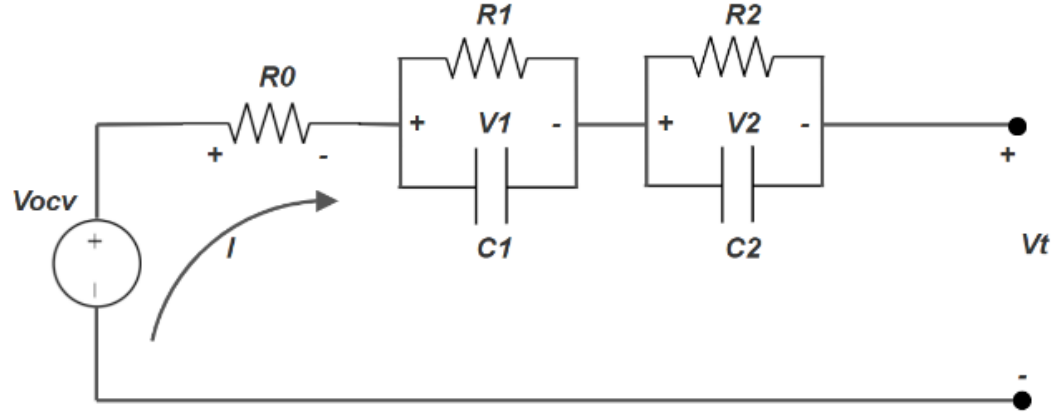


Figure 5.1. Electrical Equivalent Circuit model

This circuit models the transient response of the battery when connected to an external load. The voltage-current response of a battery due to a step change in the load current exhibits both instantaneous characteristics as well as short-term and long-term transient characteristics. The instantaneous response is modeled by a series resistor, R_0 , which basically represents the internal series resistance of the battery, whereas the short-term and long-term transients are modeled by a pair of parallel RC circuits (R_1 ,

C_1, R_2, C_2). V_{OCV} denotes the battery's open circuit voltage which is a voltage-controlled voltage source dependent on the battery state of charge and temperature.

The equations of the EEC model represented in figure 3.1 are derived by applying Kirchhoff's law.

$$\frac{d(soc)}{dt} = -\frac{I}{C_n} \quad (5.1)$$

$$\dot{V}_1(t) = -\frac{1}{R_1 C_1} V_1(t) + \frac{1}{C_1} I(t) \quad (5.2)$$

$$\dot{V}_2(t) = -\frac{1}{R_2 C_2} V_2(t) + \frac{1}{C_2} I(t) \quad (5.3)$$

$$V_t(t) = V_{ocv}(SOC(t)) - V_1(t) - V_2(t) - I(t)R_0 \quad (5.4)$$

where C_n denotes the battery nominal capacity, V_t denotes the terminal voltage across the battery. V_1 and V_2 are state variables representing the voltages across capacitors C_1 and C_2 , and IR_0 term represents cell voltage drop across the internal resistance. Although V_{ocv} is primarily a function of SOC, as pointed out by several researchers recently [29], [78], it also depends (to a much lesser extent) on temperature. In fact, this temperature coefficient of voltage (TCV) is only of the order 0.1 mV/°C for Li-carbon anodes [78], but still for a high-capacity battery, such as a 75 AH battery studied in [28], it may cause a V_{ocv} difference of about 25 mV for a temperature variation from 0 °C to 45 °C. Since this study focusses on a low capacity (2.3 AH) battery, and the temperature variation is typically restricted to below 10 °C, we ignore the temperature coefficient of voltage here and assume V_{ocv} is essentially a function of SOC. However, the results presented here can easily be extended to high-capacity batteries and battery packs by adding a correction term due to TCV.

5.1.2. Thermal Model

As Li-ion batteries charge, heat is produced due to Joule heating from its internal resistance and by the entropic effect from the ongoing chemical reaction. Thus, continuously monitoring the temperature of the battery while charging is critical, as excessive heat generated could lead to degradation in performance, accelerated aging, thermal stress as well as safety concerns. Because Li-ion batteries performance is mainly affected by temperature, limiting the rise of temperature during operation is essential. For this purpose, battery thermal models are adopted since they are found to be useful in enabling the estimation of temperature rise during charging. The thermal model [50] selected in this work is a two-state lumped-parameter system that is designed to estimate both the core and the surface (casing) temperatures during charge/discharge. A state representation of this model is given by:

$$\dot{T}_c(t) = \frac{Q}{C_c} + \frac{T_s(t) - T_c(t)}{R_c C_c} \quad (5.5)$$

$$\dot{T}_s(t) = \frac{T_f - T_s(t)}{R_u C_s} - \frac{T_s(t) - T_c(t)}{R_c C_s} \quad (5.6)$$

where C_c and C_s denote the lumped heat capacities of the core and surface, respectively.

Also, R_c and R_u denote the heat conduction and convection resistances, respectively, and T_f , T_c and T_s denote the ambient, core, and surface temperatures, respectively.

Moreover, Q is the heat generated during the chemical reaction of the battery, which includes the Joule heating over-potential and the heat generated due to entropy change. It can be expressed as:

$$Q = I(V_{OCV} - V_T) + IT_c \frac{dV_{OCV}}{dT_c} \quad (5.7)$$

This model helps quantify the internal thermal gradients which are important and critical to accurately assess battery performance and thermal behavior under different operational conditions.

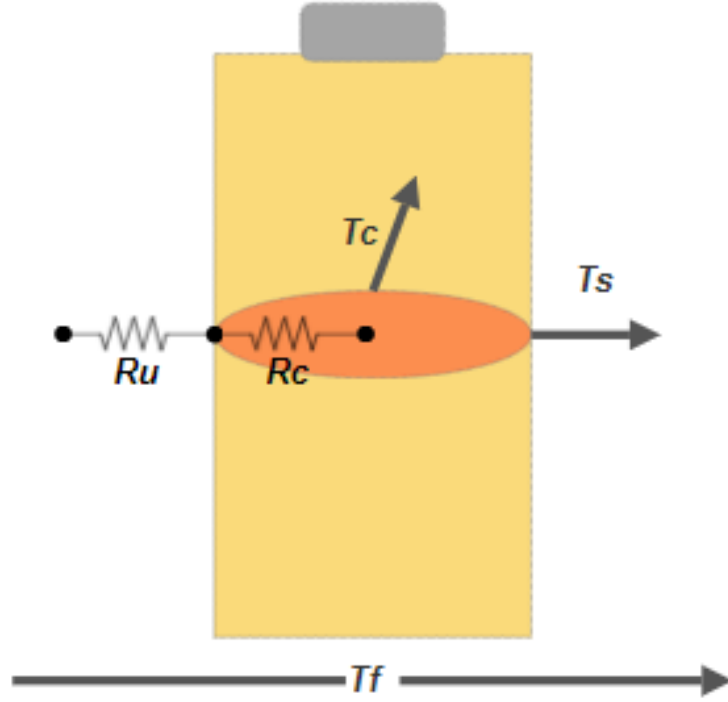


Figure 5.2. A simplified Thermal Model

5.1.3. Coupled Electro-Thermal Model

An interconnection emerges between the EEC (Electrical equivalent Circuit) and the thermal models when the heat generated by the electrical model, denoted by Q , serves as an input to the thermal model. The core and surface temperatures are predicted by the thermal model, and its resultant mean value, T_m , is used to update the values of the EEC model parameters [77]. The integrated electro-thermal model can be mathematically expressed as:

$$\dot{X} = A * X + B * U \quad (5.8)$$

where

$$X = [\text{SOC}(t) \quad V_1(t) \quad V_2(t) \quad T_c(t) \quad T_s(t)]^T \quad (5.9)$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_c C_c} & \frac{1}{R_c C_c} \\ 0 & 0 & 0 & \frac{1}{R_c C_s} & -(\frac{1}{R_c C_s} + \frac{1}{R_u C_s}) \end{bmatrix} \quad (5.10)$$

$$B = \begin{bmatrix} -\frac{1}{C_n} & 0 \\ \frac{1}{C_1} & 0 \\ \frac{1}{C_2} & 0 \\ \frac{1}{C_c} (V_{OCV} - V_t(t)) & 0 \\ 0 & \frac{1}{R_u C_s} \end{bmatrix} \quad (5.11)$$

$u = \begin{bmatrix} I(t) \\ T_f \end{bmatrix}$ is the input.

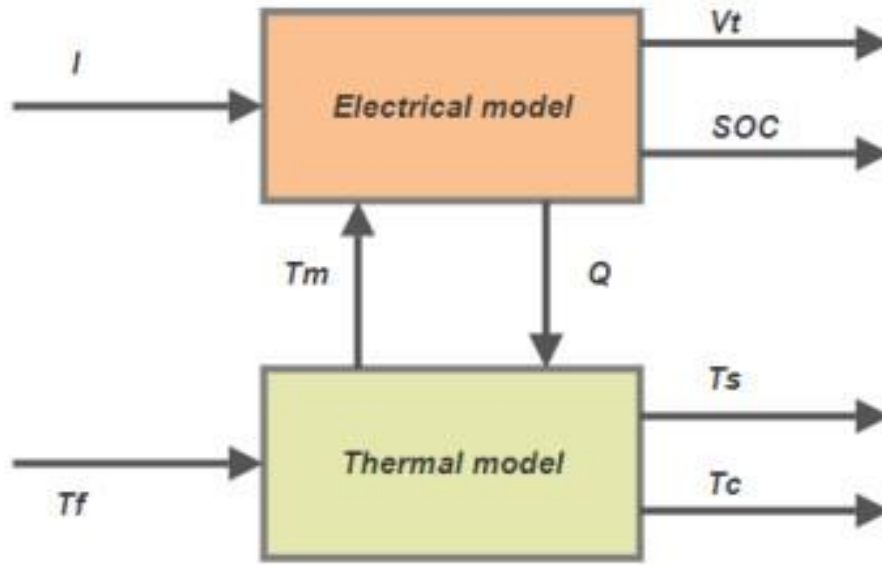


Figure 5.3. Coupled electro-thermal model

Table 5.1. Electro-thermal Model Parameters

C_1, C_2	Capacity of the two RC circuits.
C_c, C_s	Core and surface lumped heat capacities.
C_n	Battery capacity.
I	Charging current.
Q	Heat generated by electrical circuit.
R_0	Internal resistance.
R_1, R_2	Resistance of the two RC circuits.
R_c, R_u	heat conduction and convection resistances.
T_c, T_s	Core and surface temperatures.
T_f	Ambient temperature.
T_m	Core and surface temperature mean value.
V_t	Terminal voltage.
V_1, V_2	Voltage across the two RC circuits

Table 5.2. Thermal Model Parameters and values

Parameters	Description	Values
C_c	Internal heat capacity	62.7 (J/K)
C_s	Surface hear capacity	4.5 (J/K)
R_c	Heat conduction resistance	1.94 (K/W)
R_u	Convection resistance	3.19 (K/W)

5.2. MCC Formulation of optimal Charge Control

The MCC charge pattern optimization method is presented in this section. Here the objective of optimization is to find a suitable charging pattern that minimizes the total charging time, while limiting the Battery's temperature rise.

The MCC charge pattern optimization problem can be stated as follows.

Optimization Problem 1

Find optimal values of $\{N, \Delta t_k \text{ and } I_k, 1 \leq k \leq N\}$ that minimize the total charge time, t_N , subject to the constrains,

$$\sum_{k=1}^N \Delta T_k \leq \Delta T_{\max} \quad (5.12)$$

$$\sum_{k=1}^N \Delta SOC_k = SOC_{\text{Goal}} \quad (5.13)$$

where N denotes the number of stages, I_k is the charging current and t_k denotes the charge termination time for stage k , and $\Delta t_k = t_k - t_{k-1}$ represents the charging duration for stage k . Also, ΔT_k and ΔSOC_k denote the temperature-rise and SOC gain for stage k , $\Delta T_{\max}$ is the maximum allowable temperature rise of the Battery during charging, and SOC_{Goal} denotes the targeted SOC level.

The above optimization problem is difficult to solve unless some simplifying assumptions are introduced. In fact, most researchers assume that N is known a priori, and the allowable values of charge current levels, I_k , are assumed to be known fractions or multiples of the C-rate charge current, such as 0.5C, 1C, 1.5C, etc. Thus, a simplified version of the above problem can be expressed as:

Optimization Problem 2

Find optimal values of $\{t_k, I_k, 1 \leq k \leq N\}$ that minimize the total charge time, t_N , subject to the constraints (5.12) -(5.13).

Optimization problem 2 could be solved using several methods such as multi-objective optimization algorithm, like minmax or goal programming, or evolutionary algorithms such as Genetic Algorithm (GA). In fact, GA has been used in previous study [5] [21]. However, results provided by GA should not be considered as optimal, but only quasi-optimal due to the following facts:

- The number of stages N is assumed to be known a priori.
- The presumed charging profile may not be optimal.
- A solution provided by GA is not guaranteed to be the global optimum. It can at best be a very good local optimum solution.

Although GA can produce a good solution, it is rather slow to execute because it relies on numerous random searches. This makes GA and similar evolutionary algorithms unsuitable for real-time fast charging applications.

Based on the above discussion, a practical fast-charging algorithm requires a simple form of Optimization Problem 2. To simplify the problem, we retain constraint (5.13) as it is,

but replace constraint (5.12) by a new one related to temperature-rise in each stage, resulting in the following simplified optimization problem:

Optimization problem 3

For a given set of charge currents, $\{I_k, 1 \leq k \leq N\}$, find optimal values of $\{t_k, 1 \leq k \leq N\}$ that minimize the total charge time, t_N , subject to the constraints:

$$\Delta T_k \leq \Delta T_{k,max} \quad (5.14)$$

$$\sum_{k=1}^N \Delta SOC_k = SOC_{Goal} \quad (5.15)$$

where $\Delta T_{k,max}$ represents the maximum (permitted) temperature rise for stage k , $1 \leq k \leq N$.

Note that optimization problem 3 introduces a novel approach. While it requires some additional priori information – the knowledge of I_k and $\Delta T_{k,max}$ $1 \leq k \leq N$ – this trade-off significantly reduces the computational time and the complexity of the optimization process.

A solution to optimization problem 3 would require a complex multivariable minimization technique. However, these methods are slow and do not guarantee to find the true global optimal solution, therefore they are not suitable for our goal. Instead, we propose a practical quasi-optimal search strategy consisting of successive single variable searches to find the optimal values of $\{t_k, 1 \leq k \leq N\}$. Specifically, for the k^{th} charging stage, we find the optimal value of the charge termination time, t_k , by using a technique similar to interval-halving method [79], which is described below. Before describing the algorithm, first we introduce a few notations that are shown in Table 5.3 below.

Table 5.3. Notations used in the proposed fast charging

Notation	Definition
N	No. of (MCC) stages.
I_k	Charging current for stage k .
t_0	Initial time.
t_k	Charge termination time for stage k .
t_{kL}, t_{kU}	Interval of uncertainty (IOU) for t_k , where t_{kL} and, t_{kU} denote the lower and upper bounds, respectively, of t_k .
t_{ol}	Tolerance (> 0), used to terminate search for t_k ; for instance, $t_{ol} = 30/60$ sec.
t_M	Middle point of the interval of uncertainty.
t_N	Total charge time.
$SOC_{Initial}$	Initial SOC level.
SOC_{Goal}	Targeted SOC level.
SOC_{Accu}	SOC accumulated from the previous stages of MCC charge.
ΔSOC_k	Incremental SOC gain during stage k .
$\Delta T(I, dt)$	Battery's temperature rise due to charging at a constant current, I , for a duration, dt .
$\Delta T_{k,max}$	Maximum allowable temperature rise for stage k .

Fast charging algorithm with thermal constraint:

The proposed fast charging strategy determines the optimal charging duration at each stage of the MCC charge profile while satisfying a temperature rise constraint.

Initialization

Let $SOC_{Initial}$ and SOC_{Goal} denote the initial and target states of charge, respectively.

Define the number of stages N , the stage charging currents I for $k = 1, 2, \dots, N$ and

the corresponding maximum allowable temperature rise $\Delta T_{k,max}$.

Initialize: $SOC_{Accu} = SOC_{Initial}$, $k = 1$, $t_0 = 0$

Stage-wise optimization Procedure:

At each stage k , the optimal charging duration t_k is determined such that the SOC progresses toward SOC_{Goal} while satisfying the thermal constraint.

If $SOC_{Accu} \geq SOC_{Goal}$, the charging process ends. Otherwise, an upper bound for the charging time is first computer as:

$$t_{kU} = \frac{(SOC_{Goal} - SOC_{Accu})}{I_k} * C_n$$

This represents the time required to reach the target SOC at charging current I_k , assuming no thermal limitation.

The thermal model is then evaluated over the interval (t_{k-1}, t_{kU}) :

$$\Delta T(I_k, t_{kU} - t_{k-1})$$

- If

$$\Delta T(I_k, t_{kU} - t_{k-1}) \leq \Delta T_{k,max}$$

Then $t_k = t_{kU}$.

- Otherwise, the feasible charging duration lies within the interval $(0, t_{kU})$, and a bisection (interval-halving) search is performed to determine the maximum admissible t_k satisfying:

$$\Delta T(I_k, t_{kU} - t_{k-1}) \leq \Delta T_{k,max}$$

The search continues until the interval width is smaller than a predefined tolerance.

Stage Update

once t_k is determined, the electro-thermal model is simulated over (t_{k-1}, t_k) and the SOC is updated as:

$$\Delta SOC_k = \frac{I_k(t_k - t_{k-1})}{C_n}$$

$$SOC_{accu} = SOC_{accu} + \Delta SOC_k$$

the stage index is incremented ($k = k + 1$), and the procedure repeats until SOC_{Goal} is reached.

The main object of this algorithm is to make sure the maximum allowable charging duration is defined and selected while ensuring that the thermal constraint is strictly satisfied.

For better clarity, the above optimization procedure is also illustrated in the following flow chart.

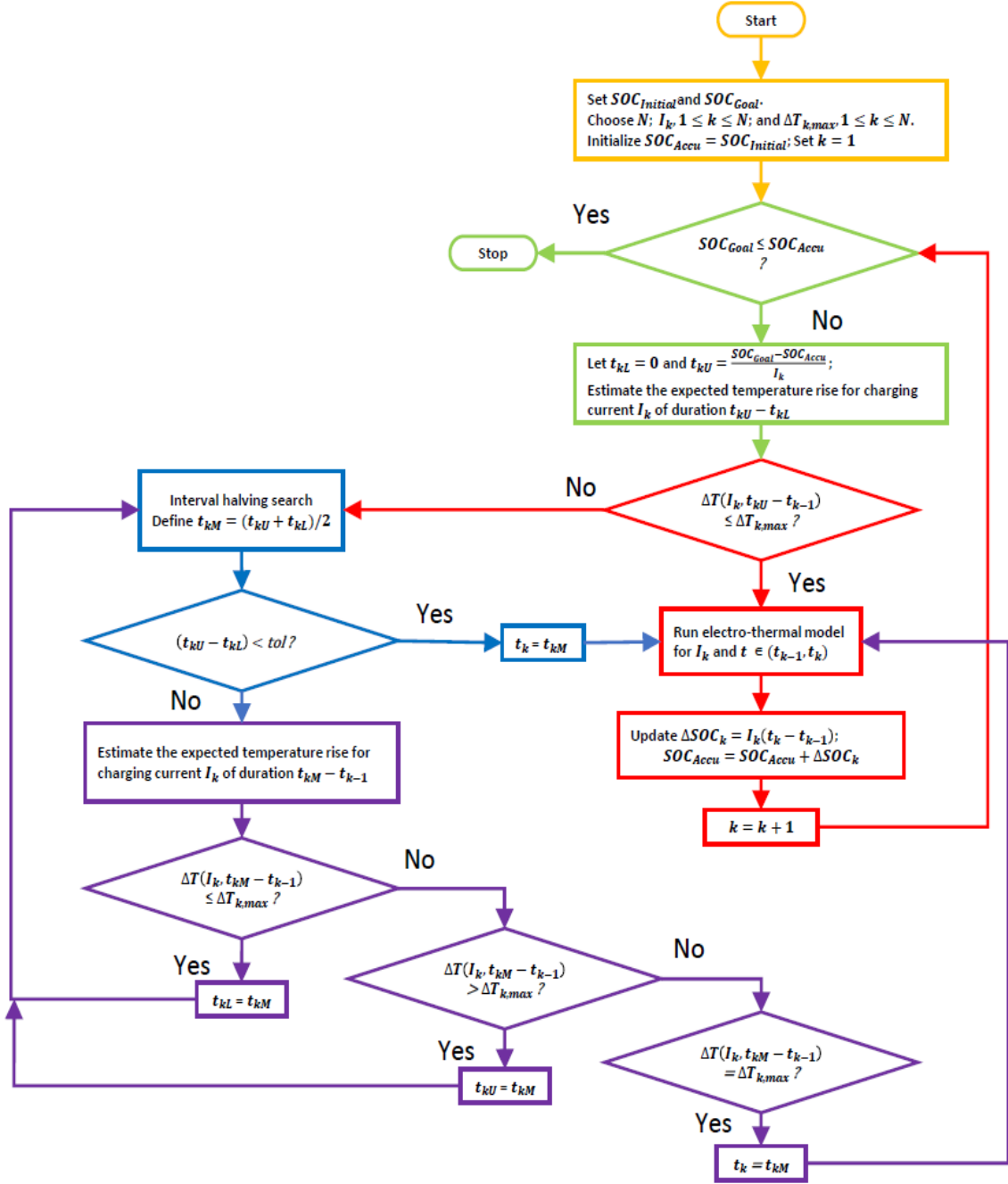


Figure 5.4. Flow chart of the proposed fast charging algorithm

5.2.1. Time Complexity of the Proposed MCC Fast Charging Algorithm

The proposed optimization search algorithm is computationally simple because it reduces the optimization to a single-variable search at each charging stage. Therefore, the optimal charge profile can be obtained in a very short time compared with more complex optimization methods, such as genetic algorithms.

The actual time taken to compute an optimal charge profile depends on several factors, such as the length of the initial search interval in each stage, L_0 , the desired tolerance for termination of search in each stage, ϵ , the time required for each function evaluation (i.e., the time taken for each simulation run of the electrothermal model), t_s , and the number of stages, N . The time taken to compute an optimal charge profile, T , is approximately given by [79]:

$$T \approx (Nt_s) \log_2\left(\frac{L_0}{\epsilon}\right) \quad (5.16)$$

where $\log_2\left(\frac{L_0}{\epsilon}\right)$ denotes the approximate number of function evaluations required for optimization using interval halving method. As an example, if we assume $N = 5$, $t_s = 1ms$, $L_0 = 10min$, and $\epsilon = 0.5 min$, then an approximate estimate of the time taken to compute a charge profile is $T \approx 0.02s$.

5.3. Optimization Results and Discussion

The effectiveness of the proposed quasi-optimal MCC fast charge pattern was validated by extensive simulations using an integrated electro-thermal battery model designed for a 2.3 Ah A123-26650 Li-ion battery. It is worth noting that this model has been experimentally verified to be reasonably accurate and was used by several researchers for fast charging and other applications. Although an extensive set of

simulation studies was conducted to evaluate the performance of the proposed optimization algorithm, the results of only 4 such studies are summarized here to illustrate its key properties.

Each one of the case studies presented here, the ambient temperature is assumed to be at 25 °C, $SOC_{Initial} = 0\%$ and $SOC_{Goal} = 100\%$. Additionally, the number of MCC stages is chosen to be $N = 5$.

For the initial simple case study, we opt for a maximum allowed overall temperature rise of 9°C. The selected values for the charging currents C-rates and the maximum allowed temperature rise for the five MCC stages are as follows:

$$\{I_k, 1 \leq k \leq 5\} = \{3.5 \ 3 \ 2.5 \ 2 \ 1.5\} \text{ (In C-rates)}$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{2 \ 2 \ 2 \ 2 \ 1\} \text{ (In } ^\circ\text{C)}$$

According to the simulation results, shown in blue color in Figures 5.5 – 5.9, it is observed that the core temperature of the battery increases to 33.7°C and the total charge time is 1746 seconds.

For the second sample case study, we opt for $\Delta T_{max} = 7^\circ\text{C}$. The values of the charging C-rate current and the maximum allowed temperature rise for the five MCC stages are chosen to be as follows:

$$\{I_k, 1 \leq k \leq 5\} = \{3 \ 2.5 \ 2 \ 1.5 \ 1.2\} \text{ (In C-rates)}$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{1 \ 1 \ 1 \ 2 \ 2\} \text{ (In } ^\circ\text{C)}$$

According to the simulation results, shown in red color in Figures 5.5 – 5.9, it is observed that the core temperature of the battery increases to 31.1°C and the total charge time is 2307 seconds.

For the third sample case study, we opt for $\Delta T_{max} = 5^\circ\text{C}$. The values of the charging C-rate current and the maximum allowed temperature rise for the five MCC stages are chosen to be as follows:

$$\{I_k, 1 \leq k \leq 5\} = \{2.5 \ 2 \ 1.7 \ 1.5 \ 1.2\} \text{ (In C-rates)}$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{1 \ 1 \ 0.5 \ 0.5 \ 1\} \text{ (In } ^\circ\text{C)}$$

According to the simulation results, shown in yellow color in Figures 5.5 – 5.9, it is observed that the core temperature of the battery increases to 29.2°C and the total charge time is 2735 seconds.

A comparison of the three above-mentioned case studies highlights the trade-off between ΔT_{max} and the total charge time t_N . Specifically, as ΔT_{max} gradually decreases, the total charge increases accordingly. This outcome aligns with our expectations because a lower ΔT_{max} would necessitate lower heat dissipation in the battery, or equivalently, lower charging currents, which would increase the total charge time.

In the final sample case study, we prove that a considerably faster charging rate can be attained, but at the expense of a higher overall temperature rise. For this case study, we selected $\Delta T_{max} = 12^\circ\text{C}$. The chosen values for charging current levels and the maximum allowed temperature rise for the five MCC stages are as follows:

$$\{I_k, 1 \leq k \leq 5\} = \{4 \ 3.5 \ 3 \ 2.5 \ 2\} \text{ (In C-rates)}$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{2 \ 2 \ 2 \ 2 \ 2\} \text{ (In } ^\circ\text{C)}$$

According to the simulation results, shown in purple color in Figures 10-14, it is observed that the core temperature of the battery increases to 37°C and the total charge time is now reduced to 1331 seconds.

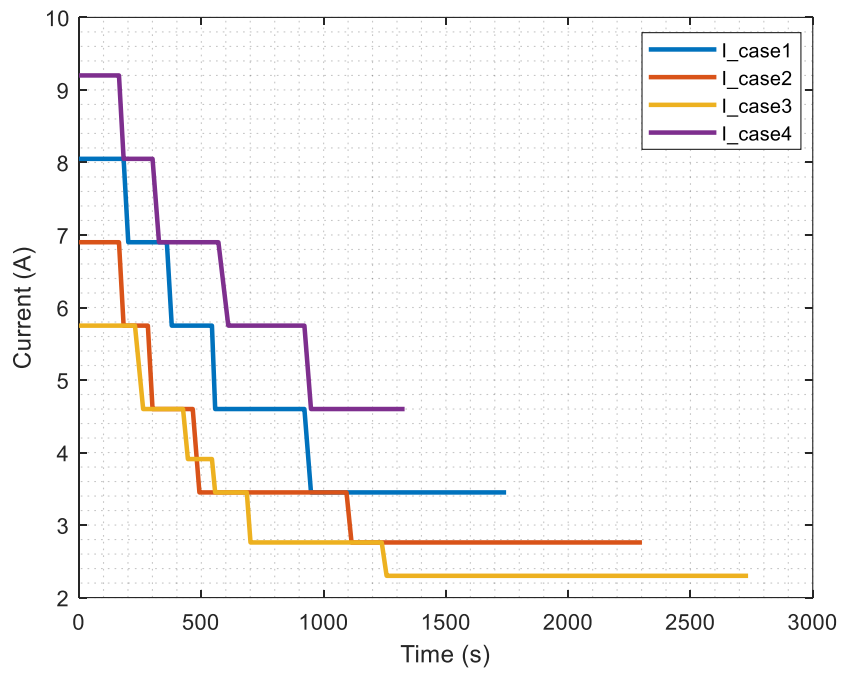


Figure 5.5. Optimum MCC charge profiles four case studies

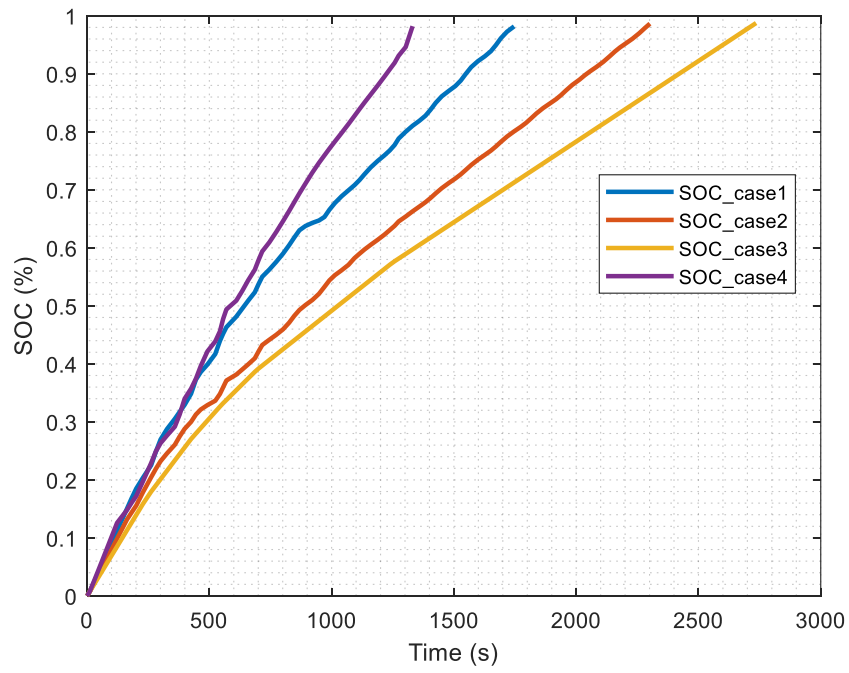


Figure 5.6. Evolution of battery's SOC for four case studies

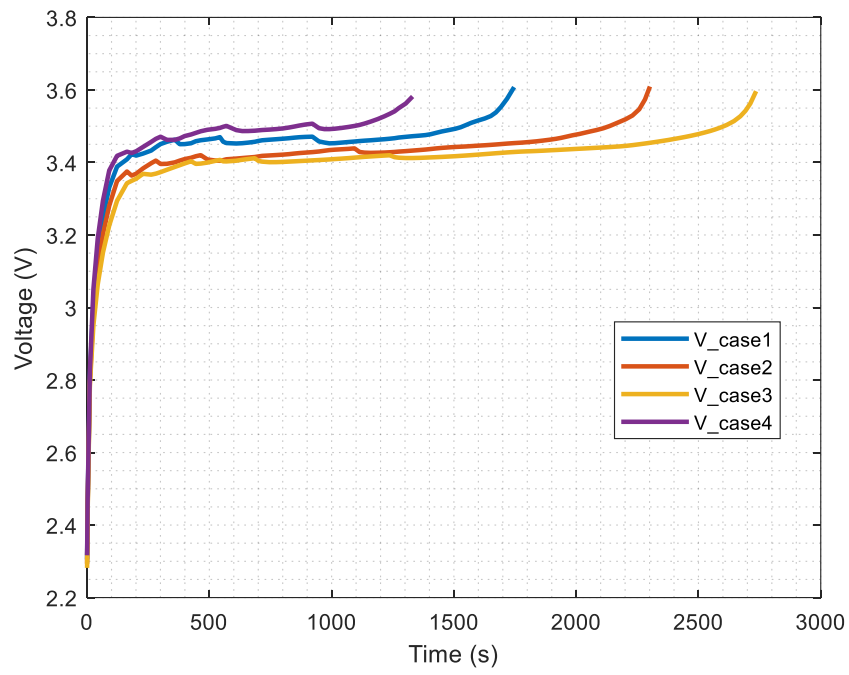


Figure 5.7. Evolution of battery terminal voltage for four case studies

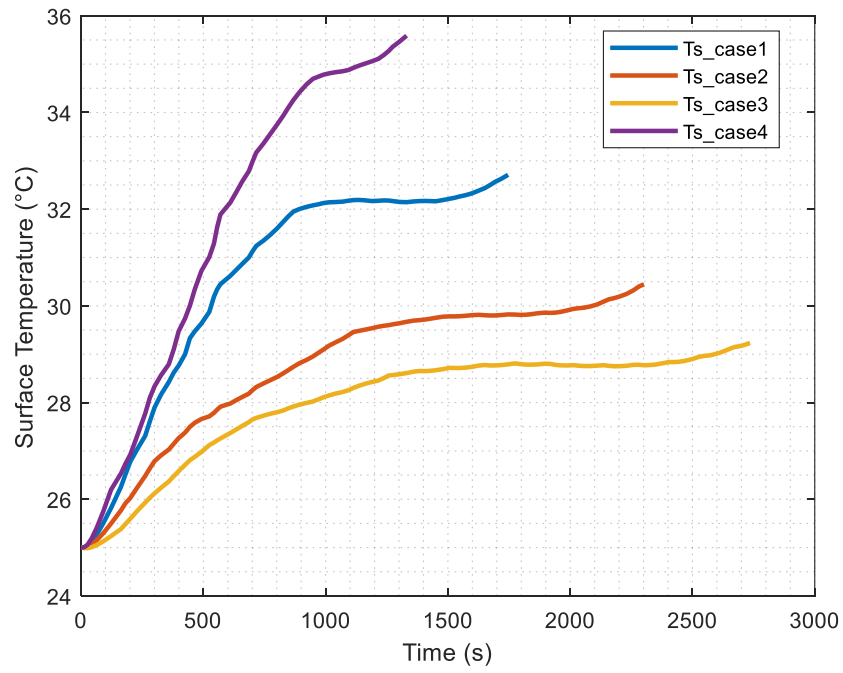


Figure 5.8. Evolution of battery's surface (T_s) temperatures for four case studies

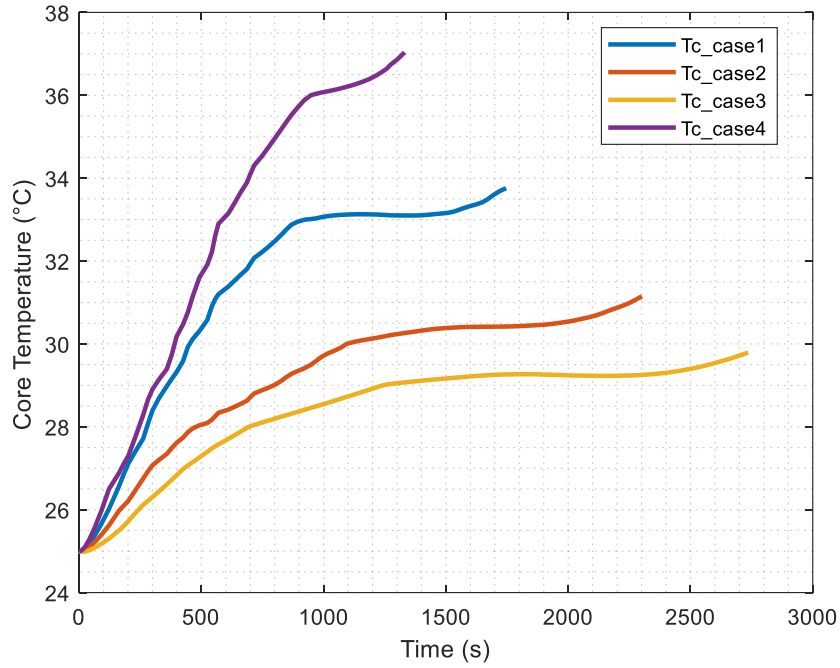


Figure 5.9. Evolution of battery's core (T_c) temperatures for four case studies

5.3.1. Results of a Partial Validation Study of the Proposed MCC Fast Charging Algorithm

A partial validation of the proposed MCC Fast Charging algorithm was carried out by Olatoye Onabiyi. The goal of the study is to investigate the effectiveness of the charging algorithm in reducing the charging time while limiting the battery temperature within a safe operating limit. The electro-thermal battery model and the MCC charging strategy were implemented in both simulation and laboratory experiments, after which the simulation and experimental results were compared to evaluate the performance of the proposed charging strategy. The results of this validation study are presented in Appendix to chapter 5. Overall, the experimental results show agreement with the

simulation results, providing a partial validation of the MCC fast charging algorithm. However, some differences were observed, mainly with the battery temperature. The experimental battery showed a lower temperature rise than that of the simulation, that is due to some limitations in the experimental setup and battery cyclers configuration, but still, the battery temperature remained within the safe operating limits throughout the charging process.

CHAPTER SIX

FAST OPTIMAL CHARGING SCHEME USING EXPONENTIALLY DECAYING CHARGING CURRENT PROFILE

As previously mentioned, the MCC and PC strategies are prone to induce unavoidable stress undulations due to sudden transitions in the charge current between stages [37]. The sudden drop in the charging current rate introduces cyclic variation in the electrochemical, thermal and mechanical stress [80]. Moreover, charging at a higher current rate for an extended time will eventually cause rapid capacity fading. This prompts the need for a charging approach that incorporates a continuous, time-varying charging current to reduce overall degradation. By eliminating sudden current changes, stress undulation is effectively reduced.

6.1. EDC Charge Pattern Optimization Problem

Inspired by the MCC charging optimization method, the exponentially decaying charge pattern proposed here consists of N consecutive exponentially decaying charge profiles, I_k , $1 \leq k \leq N$ of unequal durations, $\Delta t_k = t_k - t_{k-1}$, $1 \leq k \leq N$, where the charge profile in stage k is determined by an exponentially decaying function of the form:

$$I_k = I_{k-1} e^{-\alpha_k t}, \quad t_{k-1} < t \leq t_k, \quad (6.1)$$

where α_k is a dimensionless decaying factor at stage k . The charge pattern optimization problem can be stated as follows: Find optimal values of $\{N; \Delta t_k, \alpha_k, 1 \leq k \leq N\}$ that minimize the total charge time, t_N , subject to the constraints:

$$\sum_{k=1}^N \Delta T_k \leq \Delta T_{\max} \quad (6.2)$$

$$\sum_{k=1}^N \Delta SOC_k = SOC_{\text{Goal}} \quad (6.3)$$

where ΔT_k and ΔSOC_k denote the temperature-rise and SOC gain during the k^{th} stage of charge, and $\Delta T_{\max}$ denotes the maximum allowable rise in core temperature of the battery.

Again, solving the optimization problem above would be difficult without certain simplifying assumptions. Therefore, we assume that both N (number of stages) and the allowable values of α_k are known in advance. So, a simplified version of the above problem becomes: Find optimal values of $\{t_k, \alpha_k, 1 \leq k \leq N\}$ that minimize the total charge time, t_N , subject to the constraints:

$$\sum_{k=1}^N \Delta T_k \leq \Delta T_{\max} \quad (6.4)$$

$$\sum_{k=1}^N \Delta SOC_k = SOC_{\text{Goal}} \quad (6.5)$$

Similar to MCC proposal, an optimal solution to the above problem can be obtained using either a multi-objective optimization algorithm, such as minmax or goal programming, or an evolutionary optimization method like genetic algorithm (GA). Nevertheless, it is crucial to acknowledge that the outcome derived from these approaches are again considered quasi-optimal rather than true optimal solutions due to two factors. Firstly, N and α_k presumed to be known in advance. Additionally, while the solution produced by GA follows a systematic random search technique, it does not guarantee optimality.

In view of above, we propose utilizing a quasi-optimal search strategy that is much faster, easy-to-implement and can provide a near optimal solution. To further simplify the

problem, we introduce an additional constraint related to temperature-rise at each stage.

As a result, constraint (24) is modified to the following simplified form:

$$\Delta T_k \leq \Delta T_{k,max} \quad (6.6)$$

where $\Delta T_{k,max}$ denotes the maximum temperature-rise allowed for the k^{th} stage. With prior knowledge of $\Delta T_{k,max}$, $1 \leq k \leq N$, the above optimization problem reduces to finding the optimal values of successive stage-to-stage transition times, $\{t_k, 1 \leq k \leq N\}$, which only requires successive one-dimensional searches for each stage. For instance, a simple Interval-Halving method [33] for stage k may consist of the following steps:

Fast charging algorithm

The proposed fast charging strategy determines the optimal charging duration at each stage while satisfying a temperature rise constraint. Each stage k is characterized by a charging current I_k , a stage-dependent parameter α_k , and a maximum allowable temperature rise $\Delta T_{k,max}$

Initialization

Let $SOC_{Initial}$ and SOC_{Goal} denote the initial and target states of charge, respectively.

Define the number of stages N , the stage charging currents I for $k = 1, 2, \dots, N$, the stage parameter α_k and the corresponding maximum allowable temperature rise $\Delta T_{k,max}$.

Initialize: $SOC_{Accu} = SOC_{Initial}$, $k = 1$, $t_0 = 0$

Stage-wise optimization Procedure:

At each stage k , the optimal charging duration t_k is determined such that the SOC progresses toward SOC_{Goal} while satisfying the thermal constraint.

If $SOC_{Accu} \geq SOC_{Goal}$, the charging process ends. Otherwise, an upper bound for the charging time is first computer as:

$$t_{kU} = \frac{(SOC_{Goal} - SOC_{Accu})}{I_k} * C_n$$

This represents the time required to reach the target SOC at charging current I_k , assuming no thermal limitation.

The thermal model is then evaluated over the interval (t_{k-1}, t_{kU}) :

$$\Delta T(I_k, \alpha_k, t_{kU} - t_{k-1})$$

- If

$$\Delta T(I_k, \alpha_k, t_{kU} - t_{k-1}) \leq \Delta T_{k,max}$$

Then $t_k = t_{kU}$.

- Otherwise, the feasible charging duration lies within the interval $(0, t_{kU})$, and a bisection (interval-halving) search is performed to determine the maximum admissible t_k satisfying:

$$\Delta T(I_k, \alpha_k, t_{kU} - t_{k-1}) \leq \Delta T_{k,max}$$

The search continuous until the interval width is smaller than a predefined tolerance.

Stage Update

once t_k is determined, the electro-thermal model is simulated over (t_{k-1}, t_k) and the SOC is updated as:

$$\Delta SOC_k = \frac{I_k(t_k - t_{k-1})}{C_n}$$

$$SOC_{accu} = SOC_{accu} + \Delta SOC_k$$

the stage index is incremented ($k = k + 1$), and the procedure repeats until SOC_{Goal} is reached.

6.2. Optimization results and discussion

Extensive simulations were conducted to evaluate the effectiveness of the proposed fast charging technique. These simulations were carried out using the same integrated electro-thermal model in the search for MCC charge profile previously discussed. To demonstrate the effectiveness of the proposed method, figures 16 show different optimized charge profiles that are compared with a traditional 2.5C constant-current charging method. The ambient temperature is assumed to be 25°C, $SOC_{Initial} = 0\%$ and $SOC_{Goal} = 100\%$, and the number of the consecutive exponentially decaying stages is $N = 5$.

For the initial simple case study, we opt for a maximum allowed overall temperature rise of 12°C. The selected value for the initial charging current is 4C (or 9.2 A). Also, the values of the time constant α_k for the five stages are chosen to be as follows:

$$\{\alpha_k, 1 \leq k \leq 5\} = \{0.025 \ 0.03 \ 0.035 \ 0.04 \ 0.045\},$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{4 \ 2 \ 2 \ 2 \ 2\} \text{ (in } ^\circ\text{C)},$$

The simulation results are shown in three parts, as blue colored graphs (purple in the case of CC method) in Figures 6.1 – 6.5. The blue graph in Figure 6.1 shows the optimal current profile for case study 1, whereas Figure 6.2 shows the SOC as a function of time. Also, the blue graph in Figure 6.3 shows the evolution of the battery's terminal voltage for case study 1 as a function of time, whereas the one in Figure 6.4 – 6.5 shows the rise

of the battery's surface and core temperatures as a function of time. As evident in Figure 6.4, the battery's surface temperature reaches 36.4 °C after a charge duration of 1205 seconds. Compared with the CC method, where the surface temperature reached 35.5 °C in 1350 seconds, the total charge time was improved by 145 seconds while nearly maintaining the total temperature rise.

For the second case study, the maximum allowed temperature rise ΔT_{max} is set to 11°C, which is lower than case 1, and the initial charging current is set at 3.5C (or 8.05A). Also, the values of the time constant α_k for the five stages are chosen to be as follows:

$$\{\alpha_k, 1 \leq k \leq 5\} = \{0.025 \ 0.03 \ 0.035 \ 0.04 \ 0.045\},$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{4 \ 2 \ 2 \ 2 \ 1\} \text{ (in } ^\circ\text{C)},$$

The simulation results are shown again in three parts, as red-colored graphs in Figures 6.1 – 6.5. As evident in Figure 6.4, the battery's surface temperature reaches 35.5 °C after a charge duration of 1275 seconds. In this case, the total charge time becomes a little longer because the temperature rise was restricted to 11°C instead of 12°C in case study 1. This trade-off between total charge time and maximum temperature rise is consistent with our expectations from theory. Also, compared to the CC charging method (total charge time 1350 seconds at 35.5 °C), both charge time and temperature rise were improved by 75 seconds while maintaining the same peak temperature of 35.5 °C.

The above tradeoff between total charge time and maximum temperature rise is further demonstrated in an additional case study. For the third case study, the maximum allowed temperature rise ΔT_{max} is set to 10°C and the initial charging current is chosen to be 3C (or 6.9A). The values of the time constant α_k for the five stages are chosen to be as follows:

$$\{\alpha_k, 1 \leq k \leq 5\} = \{0.025 \ 0.03 \ 0.035 \ 0.04 \ 0.045\},$$

$$\{\Delta T_{k,max}, 1 \leq k \leq 5\} = \{4 \ 2 \ 2 \ 1 \ 1\} \text{ (in } ^\circ\text{C)},$$

The simulation results are shown again in three parts, as yellow-colored graphs in Figures 6.1 - 6.5. As evident in Figure 6.4, the battery's surface temperature reaches 34.4 °C after a charge duration of 1473 seconds. However, in this case, compared with CC method, while total temperature rise was further reduced by 1.1 °C, it still needed 123 seconds more to reach the total charge.

A comparison of the above case studies consistently demonstrates the expected tradeoff between the total charge time and the battery's temperature rise, namely, charging with a slower exponentially decaying charge profiles (such as blue or red) significantly decreases the total charge time, but they also cause more heat generation within the cell, which results in higher temperature rise in the battery.

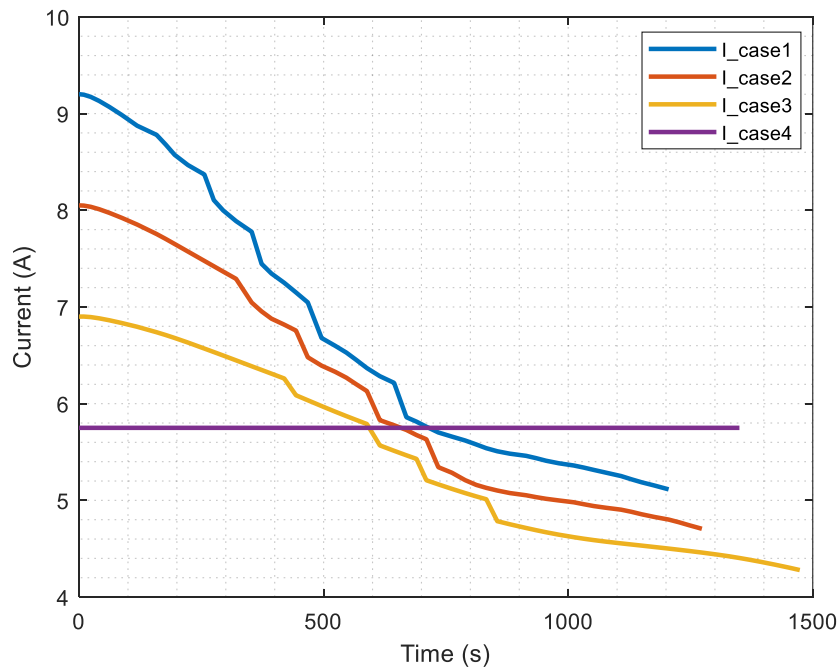


Figure 6.1. Charging current profiles for the three case studies

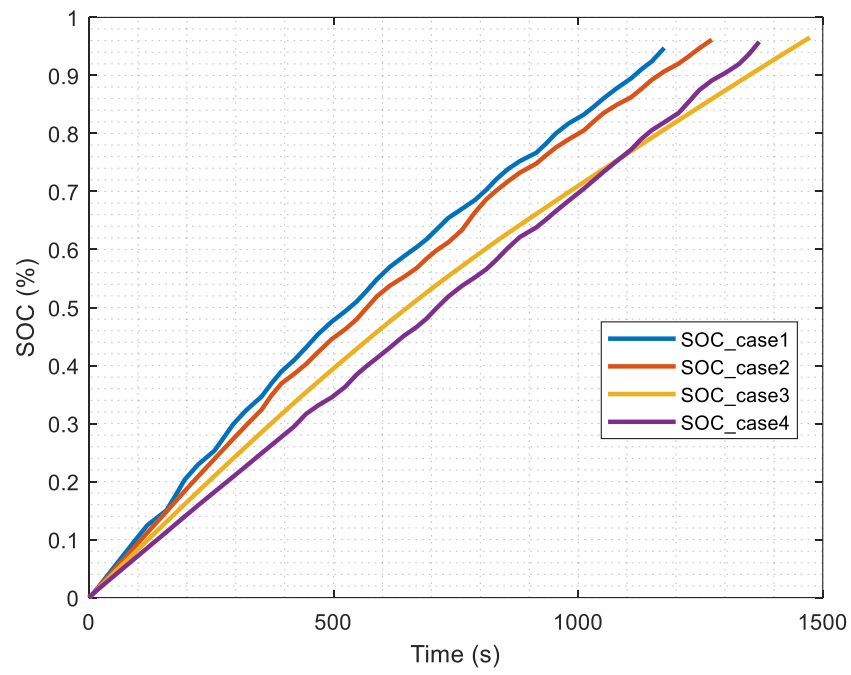


Figure 6.2. SOC profiles for the three case studies

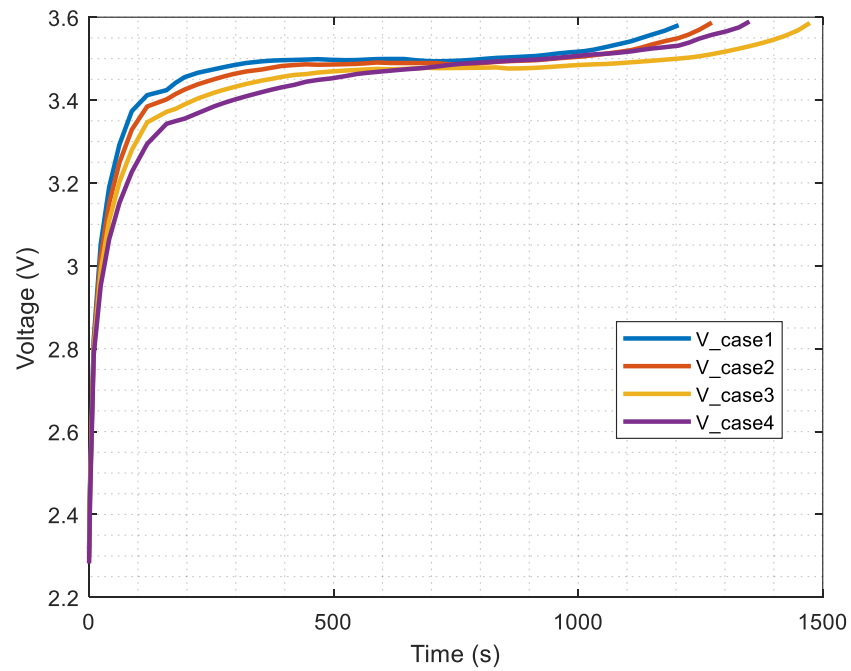


Figure 6.3. Evolution of battery terminal voltage for four case studies

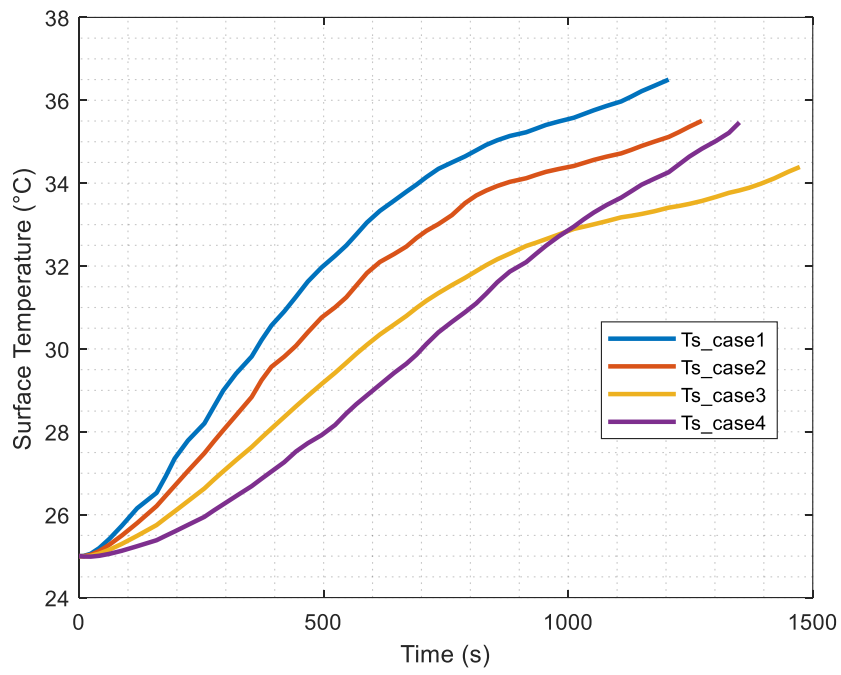


Figure 6.4. Evolution of battery's surface temperature (T_s) for the three case studies

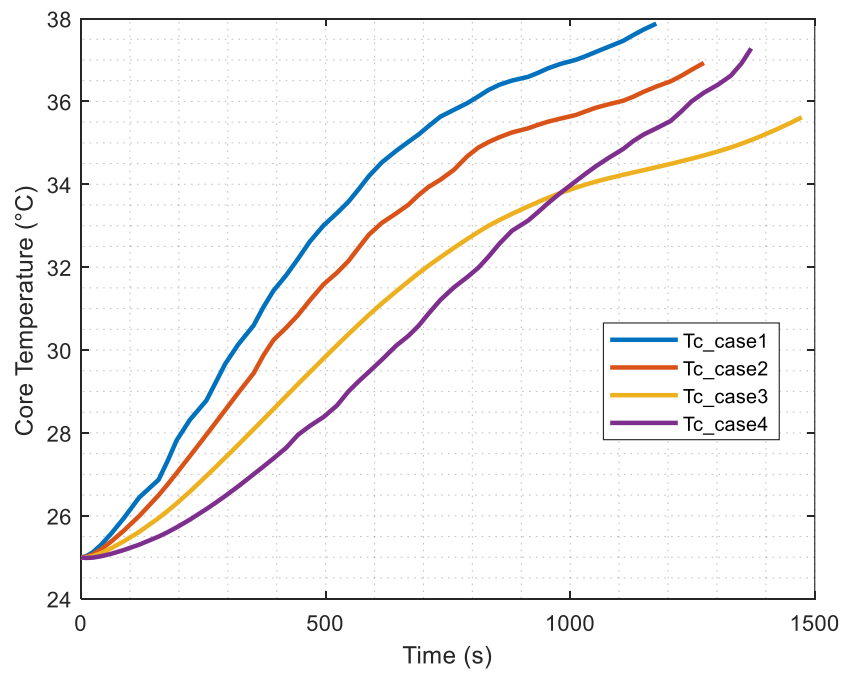


Figure 6.5. Evolution of battery's core temperature (T_c) for the three case studies

CHAPTER SEVEN

CONCLUSIONS AND FUTURE WORK

7.1. Conclusions

The main objective of this dissertation is to improve the charging performance of Lithium-ion batteries through enhanced modeling, parameter estimation, and charging strategies for fast and safe charging. The contribution of this research can be summarized as follows:

- A study of continuous-time State Variable Filtering (SVF) and Discrete-Time (IDT) techniques for the online identification of time-varying parameters of a second-order electrical equivalent circuit (EEC) battery model.
- Improving state-of-charge estimation accuracy by integrating time-varying parameter estimation with Kalman filter-based methods (EKF/UKF).
- Design and optimization of a multi-stage charging current (MCC) profile for fast charging while ensuring battery safety.
- Design and optimization of a continuously exponentially decaying charging current profile to reduce stress from stage-to-stage current changes and improve battery life.

Chapter 3 presents a comparative study of parameter estimation techniques for a time-varying five-element electrical equivalent circuit model (EEC). Two techniques are

investigated there: a direct continuous-time method based on State Variable Filtering (SVF), and an indirect discrete-time (IDT) method, both these techniques were implemented using Recursive Least Squares (RLS). The results show that the SVF-RLS approach performed well, delivering better parameter estimation accuracy, especially under time-varying conditions. The integration of a variable forgetting factor and IIR filtering further improved the estimation accuracy.

Chapter 4 highlights the use of the SVF-RLS estimated parameters for real-time SOC estimation using EKF and UKF. The results show that both filters accurately track true SOC. Comparing the two, UKF did provide better performance due to its way of handling nonlinear battery dynamics. Overall, based on simulation results, the combined SVF-RLS-EKF/UKF framework delivers reliable real-time SOC estimation.

Chapter 5 presents a fast-charging optimization strategy using a coupled electro-thermal battery model to capture both the electrical and thermal behavior of the battery during charging. A Multi-stage Constant Current (MCC) profile is optimized to reduce charging time while limiting temperature rise using a simplified search algorithm. The simulation results show a clear trade-off between charging speed and temperature constraints, providing a fast and efficient solution for battery application.

Chapter 6 introduces an exponentially decaying current (EDC) charging strategy that aims to reduce stress caused by sudden current changes during stages. This method also optimizes charging speed, while limiting temperature rise using the same search algorithm previously developed in chapter 5.

Overall, this work addressed key challenges associated with battery modeling, parameter estimation, SOC estimation and fast charging optimization. SVF-RLS

proposed framework performed well in tracking time-varying battery parameters in real-time. When integrated with EKF and UKF for SOC estimation, these estimated parameters enabled reliable SOC tracking with improved accuracy under nonlinear conditions. Also, the developed charging optimization strategies including MCC and EDC provide efficient and safe fast charging solutions for battery applications.

7.2. Recommendations for Future Work

As always, more work is needed to improve and extend the results presented in this dissertation. My future work will focus on the following:

- The EEC model parameters used to evaluate this framework are currently SOC dependent only. For future work, these parameters should be extended to account for temperature and battery aging, which will improve modeling accuracy.
- Also, the proposed estimation technique needs to be evaluated using real battery data in order to confirm its effectiveness.
- Implementing Machine Learning techniques is an area of interest for future research. I am currently investigating the integration of data-driven methods with physics-based models in order to improve parameter adaptation to temperature, SOC, SOH and other varying operating conditions.

For the charge optimization portion of this research, future work will focus on

- Extension of the initial experimental validation through additional testing using real battery systems to further verify the effectiveness of the proposed method.

- In order to optimize the charging process in real-time application, we need to develop adaptive charging strategies that can adjust dynamically based on operating conditions such as ambient temperature and SOC. This should help improve safety and extend battery life.
- As previously discussed in chapter 5 and chapter 6, the optimal search algorithm evaluates the trade-off between fast charging and temperature rise. However, the long-term impact on battery health should also be accounted for in search for the optimized charge profile. Therefore, depending on the intended application, we need to incorporate a type of degradation-aware optimization algorithm, that balances charge performance with long-term battery durability.

APPENDIX TO CHAPTER 5

RESULTS OF A PARTIAL VALIDATION STUDY OF THE PROPOSED MCC FAST CHARGING ALGORITHM.

This appendix presents simulation and experimental results that were obtained to validate the proposed MCC profile. Testing was conducted by Olatoye Onabiyi [81], including complete charging profiles and the corresponding measured battery responses. The values of the charging C-rate current and the maximum allowed temperature rise for this experiment are chosen to be as follows:

$$\{I_k, 1 \leq k \leq 5\} = \{2.17 \ 1.85 \ 1.52 \ 1.2 \ 0.87\} \text{ (In C-rates)}$$

$$\{T_{k,max}, 1 \leq k \leq 5\} = \{21.5 \ 22.5 \ 23 \ 24 \ 23.7\} \text{ (In } ^\circ\text{C)}$$

Figures A5.2 – A5.4 show the plots of experimentally measured values of battery terminal voltage, SOC and temperature, whereas Figures A5.5 and A5.6 show the same obtained from the simulation study. Overall, the experimental results show agreement with the simulation results, providing a partial validation of the MCC fast charging algorithm. However, some differences were observed, mainly with the battery temperature. The experimental battery showed a lower temperature rise than that of the simulation, that is due to some limitations in the experimental setup and battery cyler configuration, but still, the battery temperature remained within the safe operating limits throughout the charging process.

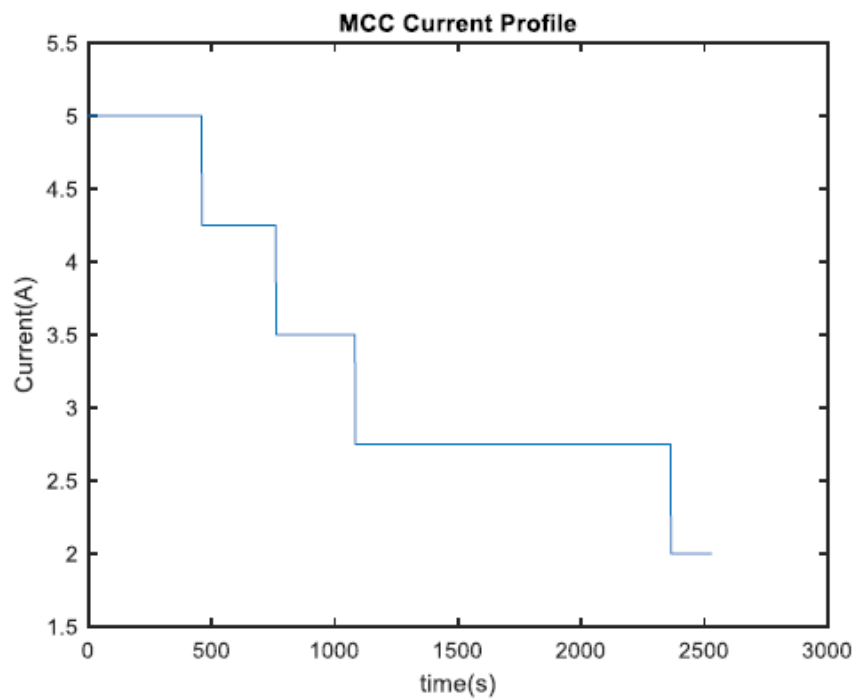


Figure A5.1. Experimental MCC charging current profile

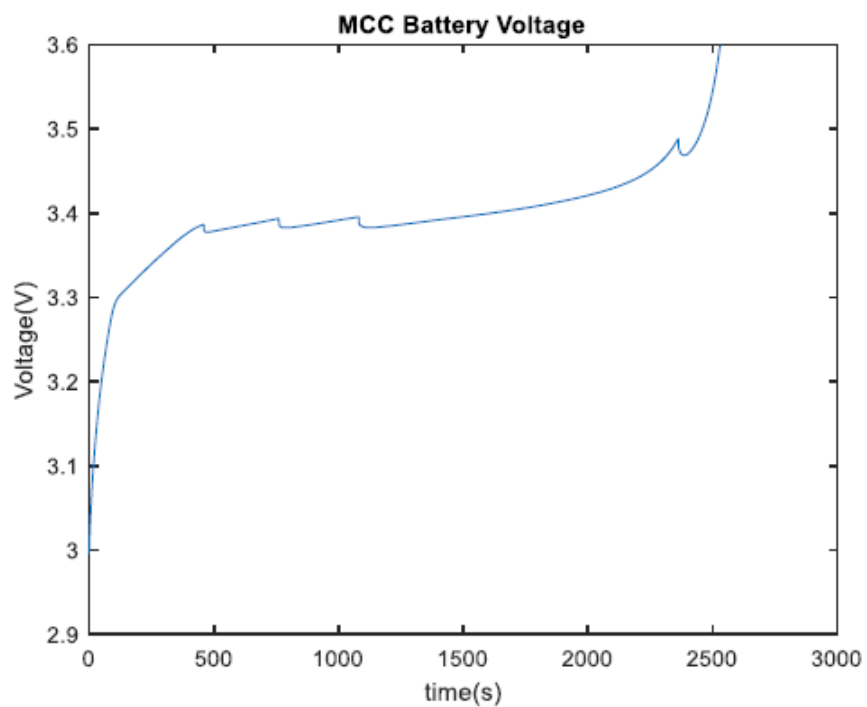


Figure A5.2. Experimental battery terminal voltage

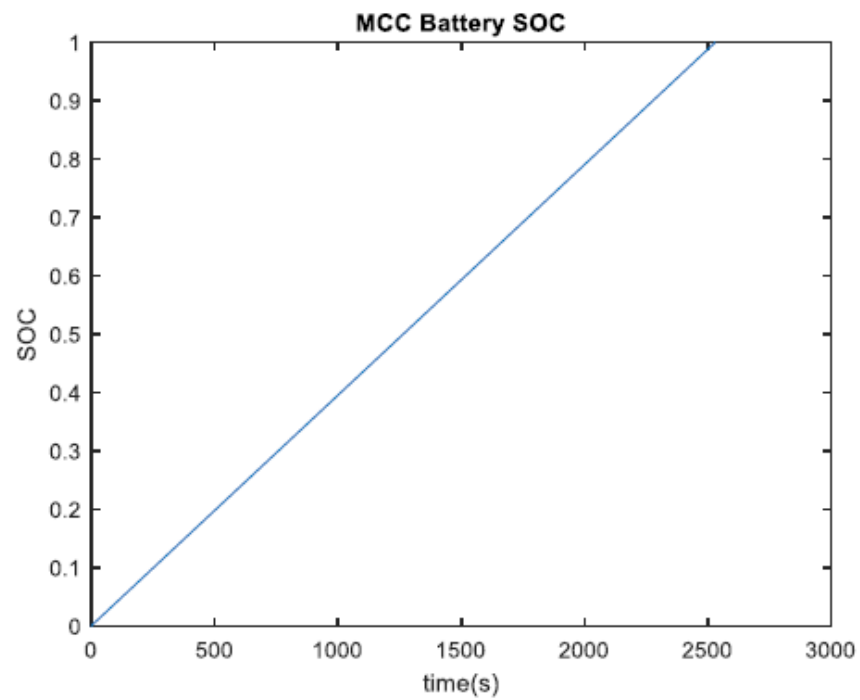


Figure A5.3. Experimental battery SOC profile

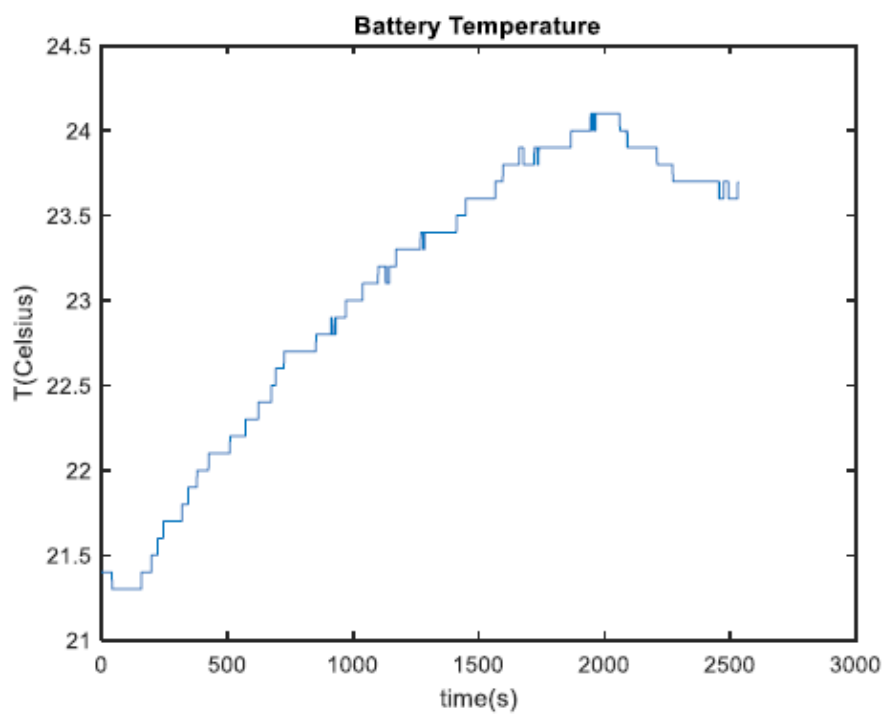


Figure A5.4. Experimental battery temperature

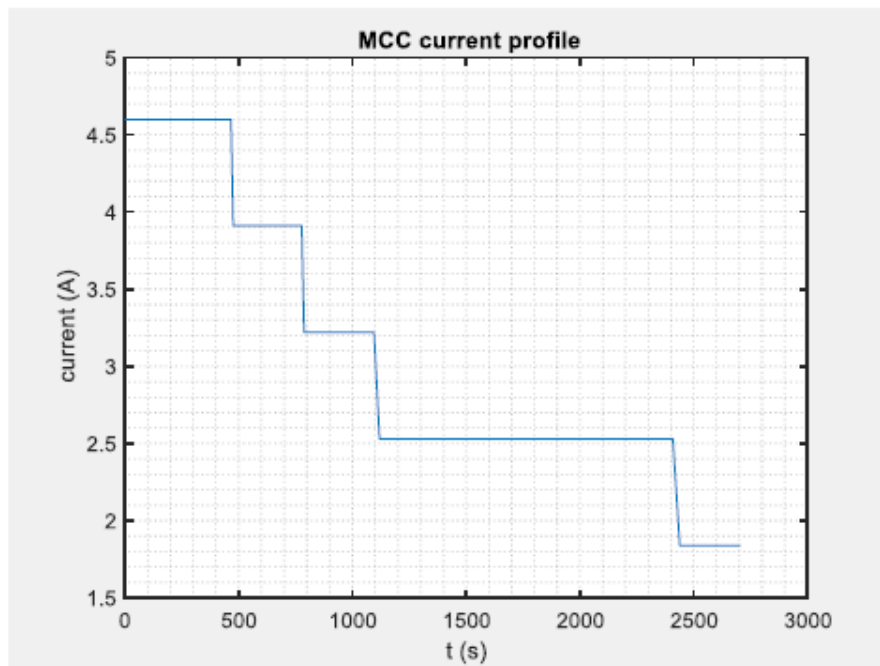


Figure A5.5. Simulated MCC charging current profile

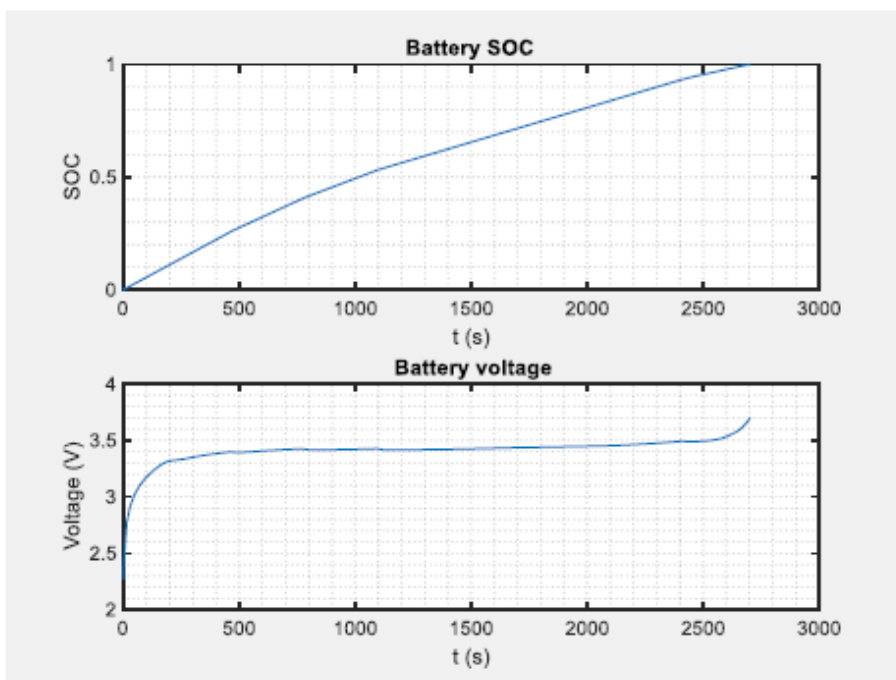


Figure A5.6. Simulated battery SOC and terminal voltage

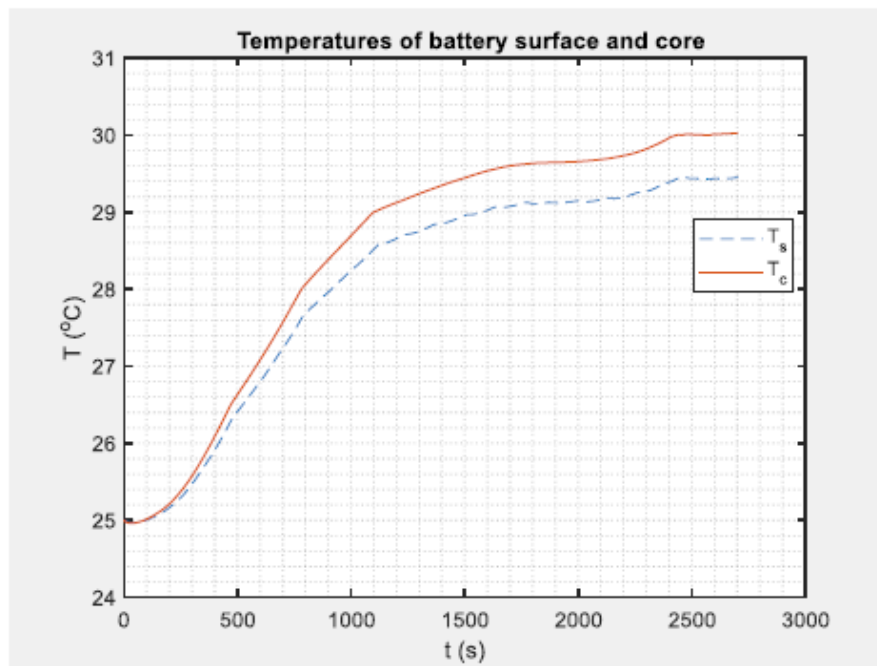


Figure A5.7. Simulated battery surface and core temperatures

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