Analysis of Gear Tooth Profiles for Use in a Mechanical Clock

Submitted by:
Jacob Cowdrey

Mechanical Engineering

To
The Honors College
Oakland University

In partial fulfillment of the
requirement to graduate from
The Honors College

Mentor: Dr. Michael Latcha, Professor of Engineering
Department of Engineering
Oakland University

December 5, 2019

Abstract

This project explores the differences in two gear teeth profiles, involute and cycloidal, and determines which profile is more advantageous for use in a gear train like that found in a mechanical clock. Involute gear teeth have been the standard gear tooth design since the 1920's, before that all gear teeth were cycloids. Involute teeth keep the pressure between the teeth constant, are cheaper to produce, and allow for larger tolerances in design. However, a cycloidal gear tooth is stronger than an involute gear tooth, especially for very low numbers of teeth typically found in clocks. To achieve this, two sets of gears one with cycloidal teeth and one with involute are 3D printed and their operation is studied. This project benefits others faced with the decision of which gear tooth profile to use in their designs.

Table of Contents

Historical Significance

Throughout history, there have been many designs for the shape of a gear's teeth. The start of the 14th century signaled the emergence of mechanical clocks. At the time cycloidal gear teeth were common and were the choice for the gear trains found in mechanical clocks. A gear train is a series of connected gears designed to achieve an overall gear ratio. In clocks they are used to turn the hours, minutes, and seconds hands. Since then changes have been made to every other part of the clock to improve its function, but the gear teeth remain unchanged even to today [1].

With the invention of the involute gear tooth in the mid 1920's, the cycloidal gear tooth became obsolete for almost every use but mechanical clocks. Automobile manufacturers abandoned the cycloidal gear in favor of the involute gear because of its ability to handle high loads and high speeds effectively. In addition, the manufacturing process necessary to produce cycloidal gears led to profile errors in the gear shape that created high noise and wear in automobiles. Cycloidal gears don't handle high loads and high speeds well, but clocks are low speed, low load devices. This, in addition to the fact that involute gears don't work well for the low number of teeth on gears found in clock, didn't provide any reason for clockmakers to switch to involute gears [2]. But this doesn't mean they shouldn't have made the change.

In the 50's and 60's a debate formed about the gears used in the mechanical timers, or fuses, the US Army was making for bombs that were dropped out of airplanes. These timers were basically mechanical clocks designed to run for a certain period of time and then trigger detonation. Watch and clock-making companies made these fuses for the Army using the same cycloidal gears used in their clocks and watches [3,4]. Many, including engineers Louis Martin in the late 40's and early 50's and David A. Goldstein in the 60's, proposed changing over from cycloidal to involute gears for a variety of reasons. Goldstein argued that the manufacturing advantages of involute gears would allow for increased production of fuses, which was a problem during World War II. Involute gears with any number of teeth could be cut as along as the pressure angle and diametral pitch was the same whereas cycloidal gears are dependent on the gear they mesh with, which means a new cutter would have to be made if a design was changed. This often led to reusing gears that already had cutters made and in turn led to less than optimal designs [5].

Louis Martin, an engineer for the Eastman Kodak Company at the time, advocated for this switch to involute gearing in fuses during World War II. Eastman Kodak used involute gearing in their timers for camera shutters. When he proposed this change to those in the Army, they would not permit deviations from the drawings which used cycloidal gears. They didn't care about the ability to increase production or save money. They knew the old way worked and didn't want to change. Martin went as far as to make enlarged templates showing how all the gears in the fuse could be replaced with involute gears with no loss in performance or function, but the Army still would not budge. After the war, in the late 50's Martin produced a working copy of the fuse with all cycloidal gears replaced with involute gears and found it performed better than the original. His experiment was repeated independently in the 60's and no difference in performance was found [2].

All throughout this time period hearings were held before the Senate, where watch and clockmakers of the American Horological Industry argued that no one could replace their gear making skills with mass produced gears effectively. These debates carried through the 50's and 60's. Involute gears never fully replaced cycloidal gears in these fuses despite the efforts of outspoken individuals. Even with the advantages outlined, the Army was reluctant to switch from what they know worked. Cycloidal gears can still be found in fully mechanical clocks and watches today as the industry never made the switch to involute gears [3,4].

Introduction to Gear Tooth Design

Gears are designed by the number of teeth they have, the spacing of their teeth, and the shape of their teeth. The number of teeth a gear will have is determined by the designer based on a desired speed ratio between the two meshing gears. The spacing of a gear's teeth can be defined by the gear's diametral pitch or module. The diametral pitch is defined by the number of teeth per diameter of the gear. The module is the inverse of the diametral pitch. From Figure 1 below it can be seen that the larger the module the larger the gear tooth and the larger the overall gear would be to compensate [6].

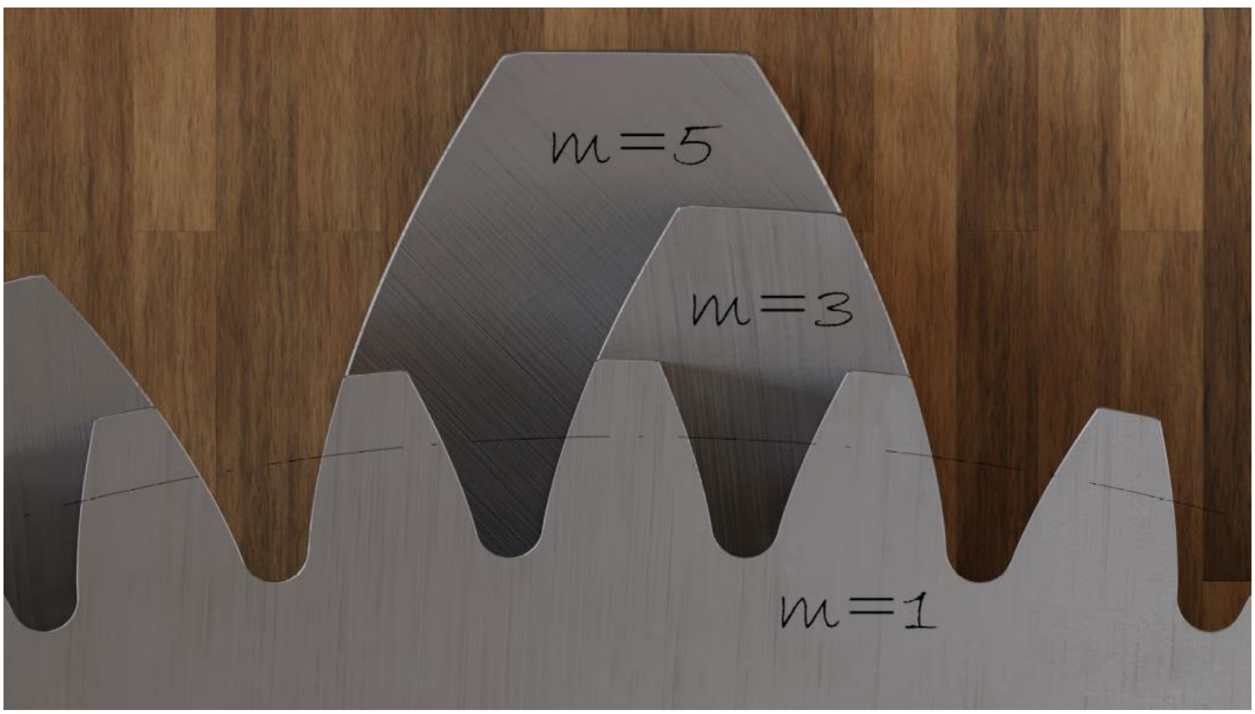

Figure 1: Comparison of gear tooth modules [7]

If the module is referred to by the letter m, the diametral pitch is referred to by DP, their relationship can be given the equation,

$$m = \frac{1}{DP} \tag{1}$$

Using the module, or the diametral pitch, and the number of teeth, which will be referred to by the letter z, the size of the gear can be defined. The pitch diameter, referred to by the letter D, of any gear can then be given by,

$$D = m * z = \frac{z}{DP} \tag{2}$$

The pitch diameter is the diameter of the pitch circle, the circle where two gears will mesh [6]. Two gear tooth types will be discussed in this project, involute and cycloidal gear teeth. The design of both gear teeth will be discussed in further detail later.

Cycloidal Gear Tooth Design

Cycloidal gear teeth are drawn by connecting two curves, an epicycloid, and a hypocycloid. The addendum of the cycloid tooth is an epicycloid and the dedendum of the cycloid tooth is a hypocycloid. An epicycloid curve is the result of rolling a circle around the outside of another circle. Conversely, a hypocycloid curve is the result of rolling a circle around the inside of another circle. The addendum and dedendum of a cycloid gear are the distance of the tooth above and below the pitch circle respectively. In the case of gears, the circle the other circles are rolled along is the pitch circle [8]. This can be seen in Figure 2 below,

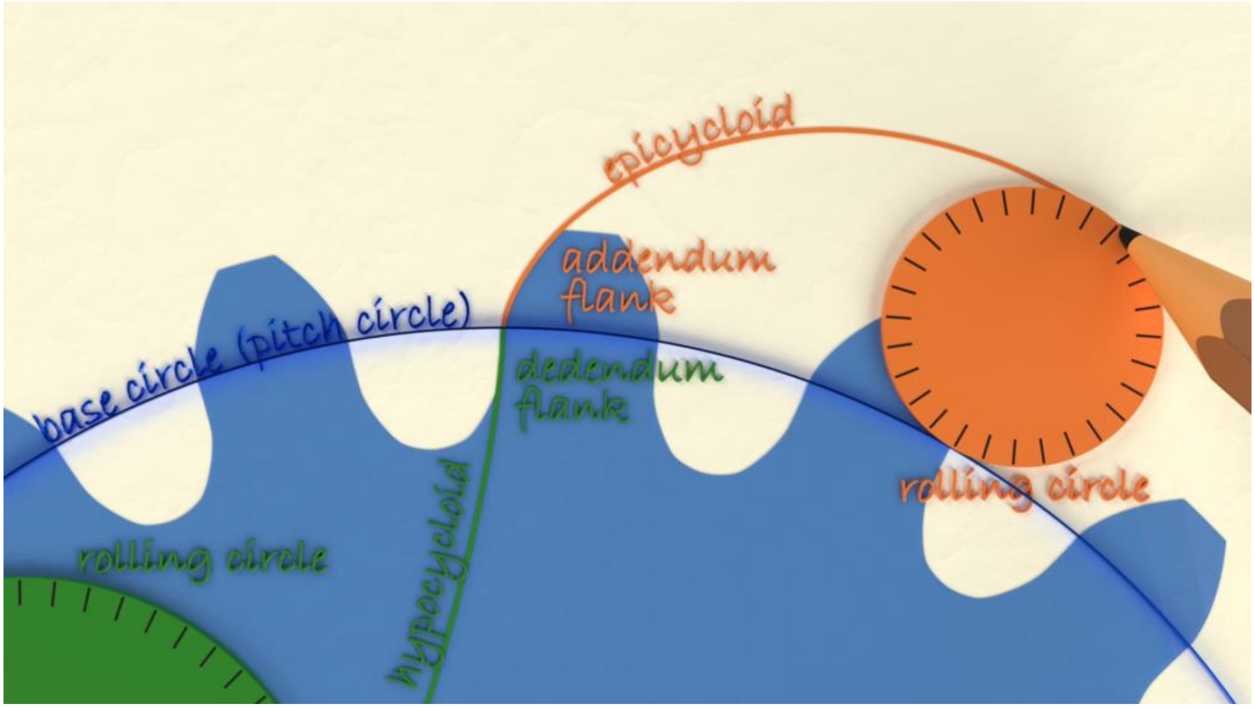

Figure 2: Design of a cycloidal gear tooth using rolling circles [8]

For two cycloidal gears to mesh, the addendum of one gear must be the same shape as the dedendum of the other gear and vice versa. This means that the design of a cycloidal gear is dependent on the gear it meshes with and that cycloidal gears cannot be interchanged to change gear ratios [9].

The cycloidal gears used in clocks are a special condition of a cycloidal gears. In clock gears the diameter of the rolling circle of the hypocycloid is one half the diameter of the pitch circle. This results in the dedendum flanks of the cycloidal tooth being a straight line instead of a semi-circle [10]. This design is appropriately called “clock toothing” and can be seen in Figure 3 below.

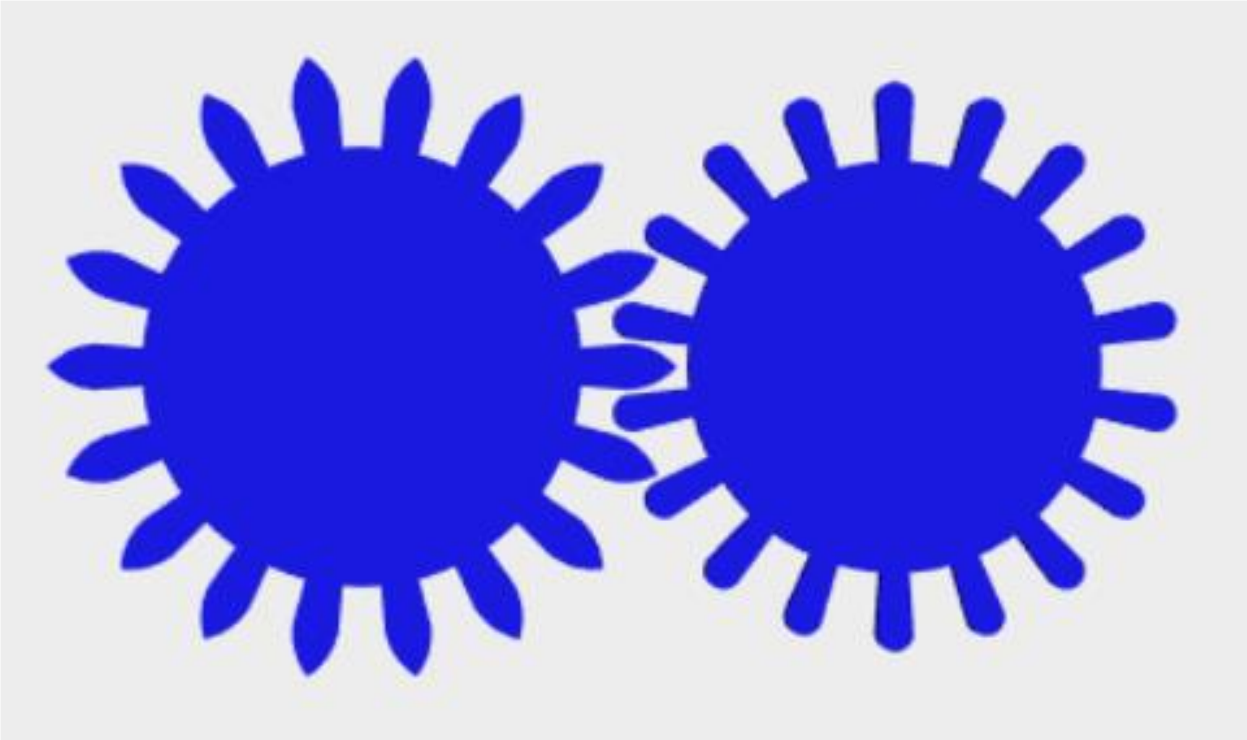

Figure 3: Cycloidal clock gears showing straight flanks and tooth modifications [11]

Because of this design, other design changes must be made. On the pinions, the smaller of the two meshing gears, of a clock the addendum is rounded off so that it is not full length to ensure smooth meshing of the gears [10]. This can be seen on the gear to the right in Figure 3 above.

Involute Gear tooth Design

An involute gear design depends on two parameters the diametral pitch and the pressure angle. Any two involute gears can mesh regardless of the number of teeth if they have the same diametral pitch and pressure angle. The pressure angle is the angle between the line of action and the normal line of centers. The line of action is the line tangent to the base circle of both gears that goes through the point of contact of the two pitch circles. The normal line of centers is a vertical line drawn through the point of contact of the two gears. A visual representation of these lines can be seen in Figure 4 below [7].

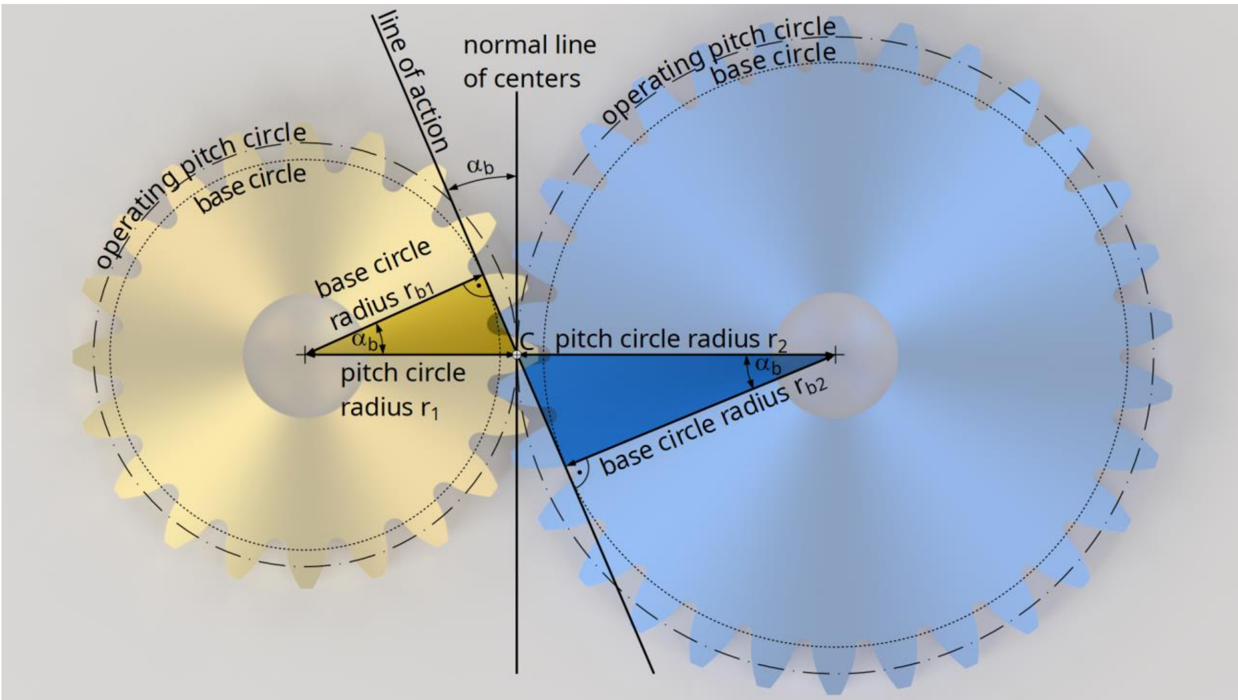

Figure 4: Line of action and pressure angle description [7]

For most applications, including in this report, the pressure angle is 20° but pressure angles of 14.5° and 25° are also commonly used.

The flanks of an involute gear are made up of two involute curves. An involute curve is a special case of an epicycloid. The rolling circle of the epicycloid is made larger and larger until it is infinitely large and can be represented by a straight line. Rolling this straight line around the outside of the base circle results in the involute curve as shown in Figure 5 below [7].

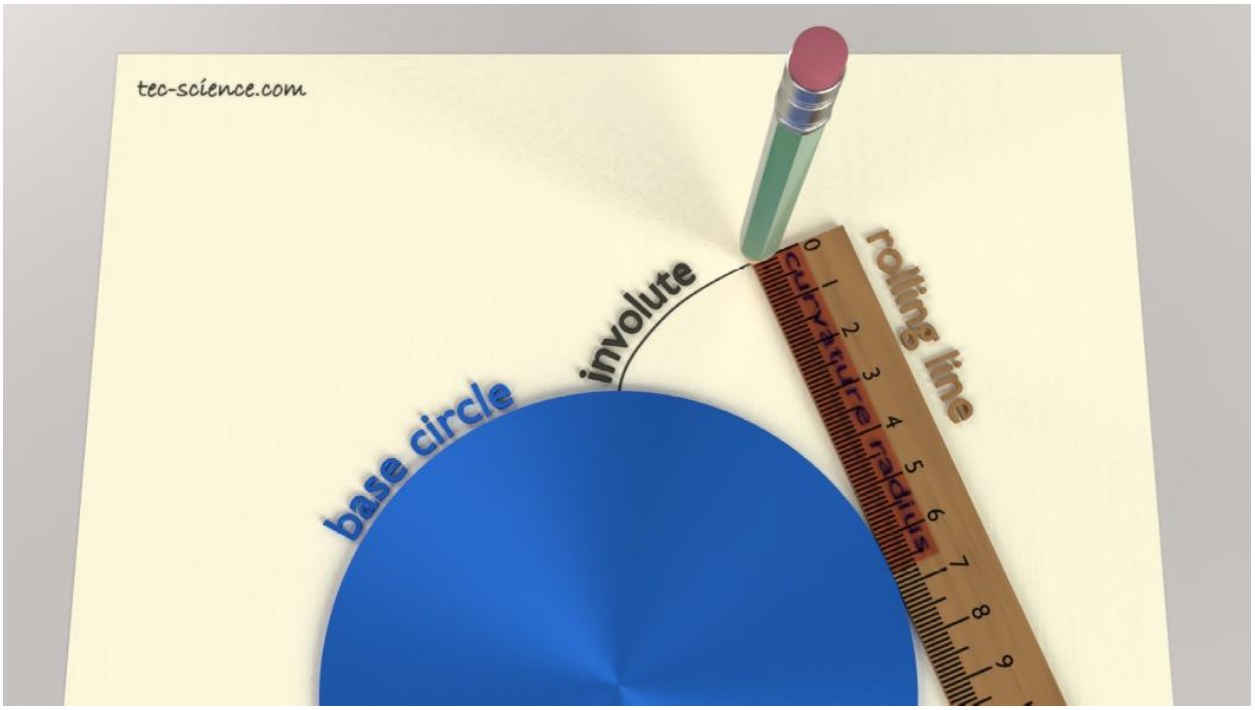

Figure 5: Generation of an involute curve from a rolling straight line [7]

An involute gear tooth consists of two of these involutes that are generated from the base circle and a series of 4 concentric circles, the addendum, pitch, base, and dedendum circles, that help generate the rest of the involute shape. Recall from equation 2, that the pitch diameter is,

$$D = m * z = \frac{z}{DP} \tag{2.1}$$

The addendum diameter, AD, is equal to the pitch diameter plus twice the module.

$$AD = D + 2m \tag{3}$$

The dedendum diameter, DD, is equal to the pitch diameter minus a little more than twice the module. In this case 2.5 times the module. This allows for clearance in the gear teeth when they mesh.

$$DD = D - 2.5m \tag{4}$$

Finally, the base diameter, BD, can be found by multiplying the pitch diameter by the cosine of the pressure angle, α, in radians.

$$BD = Dcos(\alpha) \tag{5}$$

The involute curves start at the base circle and extend out until they are cutoff by the addendum circle. This makes up the addendum portion of the involute. The dedendum portion of the involute tooth is made by taking out enough material to allow for the gear tooth to fit into that space. This can be done by drawing radial lines to the points where the two involutes start and removing that material or by drawing a tangent arc that connects the points where the two involutes start. Figure 6 below gives a visual representation of how all these circles play a part in the gear took design [7].

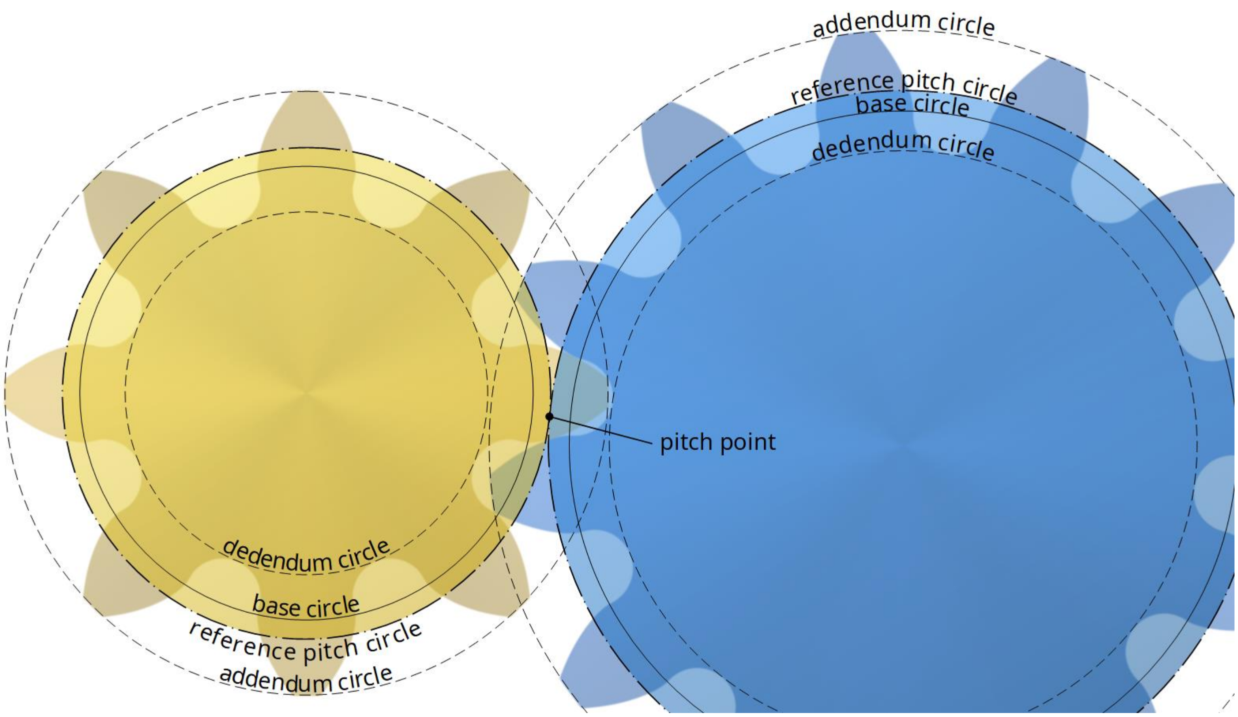

Figure 6: The design of an involute gear tooth [12]

Benefits of Gear Tooth Designs

Cycloidal gears have many benefits for use in gear trains such as those found in mechanical clocks. Cycloidal gears can be made to have as few as two teeth and can still function properly. This is advantageous because clocks will use pinions, a special name for a gear with a very small number of teeth found in clocks, to achieve large gear reduction ratios and thus lowering the overall number of gears needed and size of those gears [10]. Involute gears are limited to a minimum of 18 teeth with a pressure angle of 20° to avoid undercutting. Undercutting is when the dedendum portion of the gear cuts into the gear tooth. This severely weakens the strength of the gear and must be avoided [6]. This minimum number of teeth means that either more gear reductions must be used or larger gears with more teeth must be used to achieve the necessary gear reductions. Another benefit of cycloidal gearing is that only one or two teeth are ever in contact at a time while an involute gear will always have two to three teeth in contact. If the amount of friction per tooth is the same for both gear sets then the involute gears will experience more friction [10]. This claim will come in to question later. Finally, cycloidal gearing has historically always been the gearing used for clocks. They have the benefit of years of proven experience working in clocks [10].

Involute gears have many benefits that lend themselves to be useful in mechanical clocks. Involute gears are easier to manufacture and inspect than cycloidal gears. Gears are usually cut out of pieces of metal using cutters called hobs. The shape of the involute tooth, mostly straight edges, makes it easier to make hobs for than the cycloidal gear tooth which has no straight lines and only curves with changing radii. Inspection of an involute gear is at least ten times more accurate than a cycloidal gear because involute gears can be tested on gear roll testers while cycloidal gears need to be inspected using an optical projector [2].

Involute gears also allow for variation in their center to center distance when they mesh. The center to center distance is the sum of the two pitch radii of the two gears in mesh. With cycloidal gearing, if the center to center distance is not exactly the sum of the two pitch radii then the gears will not maintain a constant speed ratio. However, with involute gearing a constant speed ratio is independent of the center to center distance of the gears. This allows for slight variations in assembly of the gears in the gear train [13].

The final advantage of involute gearing over cycloidal is that the teeth roll on each other without sliding [6]. This is the most important aspect of involute gearing and it will be explored in further depth later.

The gears in clocks have two important functions to ensure the smooth running of a clock. The gears must maintain constant speed ratios between each other and must have as little as friction as possible. Ideally, they will have no friction between the teeth. The constant speed ratios ensure that the clock keeps accurate time. It is important for clock gear to have no friction so that they don’t wear. Clock gears generally cannot be lubricated since they are open to environment. Dust and other particles would stick to the lubricant and damage the gears [6]. While both gears, if mounted properly, can maintain constant speed ratios, involute gears will have less friction and should therefore be the gear of choice in mechanical clocks.

Design of Gear Assemblies

To test if cycloidal gears have more sliding than involute gears two gear assemblies were designed using the CAD program SolidWorks and then 3D printed. The assemblies consist of two partial gears mounted on to a base. These assemblies were then tested, and their performance was compared to determine if one set experienced more sliding than the other. For consistency in the testing, a diametral pitch of 4 was used, along with 18 teeth for both gears in both assemblies. In the involute gear set a pressure angle of 20° was used. This gives both gear sets a center to center distance of 4.5 inches.

To draw the involute gear set in SolidWorks, two gear slices, and a base must be designed. First, the gear slices must be made. When making the gear slices, instead of drawing the gear teeth, the spaces between the gear teeth will be drawn and cut out of the gear. This is the way gears are made in production facilities. Using the equations in the involute gear section and the fact that $DP = 4$ and $z = 18$, the involute gear can start to be designed. The first step is to calculate the pitch diameter,

$$D = \frac{z}{DP} - 0.01 = \frac{18}{4} - 0.01 \ = 4.49\ inches \tag{2.2}$$

Which comes out to 4.49 inches. The true pitch diameter is 4.5 inches, but 0.01 inches were removed from the pitch diameter for backlash and additional clearance for 3D printing. Backlash is a clearance built in to mating gear teeth to prevent the gears from binding. Next the addendum diameter, AD, dedendum diameter, DD, and the base diameter, BD can be calculated,

$$AD = D + \frac{2}{DP} = 4.49 + \frac{2}{4} = 4.99\ inches \tag{3.1}$$

$$DD = D - \frac{2.5}{DP} = 4.49 - \frac{2.5}{4} = 3.865\ inches \tag{4.1}$$

$$BD = Dcos(\alpha) = 4.49 \cos\left(\frac{30\pi}{180}\right) = 4.21922\ inches \tag{5.1}$$

After calculating these four diameters, the involute curve that will shape the side of the tooth will need to be found. An involute curve for the parameters given above can be given by a set of parametric equations,

$$x(t) = t * \cos\left(0.07236 + \tan\left(\cos^{-1}\left(\frac{BD}{2t}\right)\right) - \cos^{-1}\left(\frac{BD}{2t}\right)\right) \tag{6}$$

$$y(t) = t * \sin\left(0.07236 + \tan\left(\cos^{-1}\left(\frac{BD}{2t}\right)\right) - \cos^{-1}\left(\frac{BD}{2t}\right)\right) \tag{7}$$

The parametric equations start at the base circle, $t_1 = \frac{BD}{2}$, and end at the addendum circle, $t_2 = \frac{AD}{2}$. All the pieces of information to make the involute gear are now known.

The following are the steps to make the involute gear slices [6]:

1. Draw the addendum circle centered at the origin with diameter AD.
2. Pad the circle to create a very short cylinder. This is a gear blank. The teeth will be cut out of this cylinder.
3. Start a sketch on top of the cylinder to draw the cutout between the gear teeth. One cutout will be drawn and then a pattern will be created to cutout all the teeth.
4. To start drawing the cutout, draw and dimension the addendum circle, pitch circle, base circle, and dedendum circle. Make all these circles concentric at the origin of the cylinder. All these circles can be made into construction elements.
5. Next, using the parametric equations function in SolidWorks, draw the involute curve using equations 6 and 7 above. The curve starts at base circle diameter and ends at the addendum circle diameter, t_1 and t_2 respectively.
6. Draw a horizontal construction line from the origin to the addendum circle in the direction of the involute curve.
7. Draw a radial line from the origin to the point where the involute curve intersects the base circle. Trim this line so that it is only between the dedendum circle and the base circle.
8. Mirror the involute curve and the line created in step 7 about the construction line created in step 6.
9. The tips of the involute curve and the new mirrored involute curves can be connected using the tangent arc function.
10. The bases of the two lines that end at the dedendum circle can then be connected with a straight line between the two points.
11. Fillets can then be added to these newly created corners for stress relief.

12. Now that the cutout sketch is complete. The extruded cut function can be used to cut this section out of the gear blank.
13. After one cutout is created. The circular pattern function can be used to cutout the remaining gear teeth.
14. By drawing an appropriate sketch and using the extruded cut function again the complete gear can be cut into a gear slice containing just 3 teeth.
15. Cut a small circle through the center of the gear to allow the gear slice to be mounted on to a base for testing purposes.
16. The final step in the gear design is to cut spokes into the gear slice to reduce the overall mass of the gear slice. This will reduce cost when 3D printing. The pitch circle and dedendum circles were also marked on the gear slices using cuts that don't go through the whole gear.

After the involute gear slices are designed, the cycloidal gear slices, and a common base can be designed before testing starts. The cycloidal gear slices were designed using an online cycloidal gear calculator and instructions found online. Like the involute gears diametral pitch of 4 was used and since cycloidal gear design depends on the number of teeth of both mating gears, 18 teeth were used for both gears. Plugging this information into the cycloidal gear calculator gives the results seen below [14].

Cycloidal Gear Calculator

British Standard 978, Part 2

Module	.25	mm
Number of wheel teeth	18	
Number of pinion teeth	18	
Circular pitch	0.7854	mm
Dedendum	0.3927	mm
Gear ratio	1	
Addendum factor	1.62882	
Addendum	0.40721	mm
Addendum Radius	0.60009	mm
Pitch diameter of wheel	4.5	mm
Pitch diameter of pinion	4.5	mm

Figure 7: Cycloidal gear calculations [14]

The results in the figure above are given in units of millimeters but the numbers can be used with units of inches without trouble. Inches are used in this project to make the gears big enough to be able to see sliding easily. The information contained above is enough to draw a cycloidal gear and pinion set like those seen in clocks.

The following are the steps to draw the set of cycloidal gear slices:

1. Draw the pitch circle as a construction element with a diameter of Module * Number of Wheel teeth (in this case 4.5 inches) from Figure 7 above.
2. Draw the dedendum circle by taking the pitch diameter and subtracting 2 times the dedendum value from Figure 7 above.
3. Draw the addendum circle as a construction element by taking the pitch diameter and adding 2 times the addendum value from Figure 7 above.
4. Draw a horizontal construction line from the origin out to the addendum circle.
5. Draw the dedendum line for one side of the gear tooth by drawing a line from the origin to the pitch circle at an angle of $\frac{360}{4 * Number\ of\ wheel\ teeth}$ from the horizontal.
6. Trim the dedendum line so that it is only from the dedendum circle to the pitch circle.
7. Draw an arc connecting the point where the dedendum line intersects the pitch circle to the point where the horizontal construction line intersects the addendum circle. The radius of this arc is equal to the value in the addendum radius box from Figure 7 above.
8. The dedendum line and the addendum arc are then mirrored about the horizontal construction line to create a full gear tooth.
9. This tooth can then be patterned around the gear dedendum circle using the circular sketch pattern function.
10. Trim the dedendum circle where it is between gear teeth flanks.
11. The extruded boss function can then be used to make the gear 3 dimensional.
12. By drawing an appropriate sketch and using the extruded cut function again the complete gear can be cut into a gear slice containing just 3 teeth.
13. Cut a small circle through the center of the gear to allow the gear slice to be mounted on to a base for testing purposes.

14. To draw the cycloidal pinion, start by repeating steps 1 – 6 above.
15. This time mirror only the dedendum line about the horizontal construction line.
16. Draw a tangent arc connecting the two dedendum flanks.
17. Finish the cycloidal pinion by repeating steps 9 – 13 above.

These instructions resemble those found online but have been edited for clarity [15]. Now that all four gear sections are designed a base can be designed for testing.

The following are the steps to draw the base for testing:

1. Draw a 2” wide by 6” long rectangle.
2. Make it 3 dimensional using the extruded boss function.
3. Draw two circles on top of the base with a center to center distance of 4.5 inches. It is important for the circles cut into the gear slices to be 0.06 inches larger in diameter than the circular posts on the base for clearance when 3D printing.
4. Make these posts 3 dimensional using the extruded boss function.
5. Cut holes in the base to reduce mass for 3D printing to save cost. Avoid cutting holes where the gears will mesh so that it is easier to see their meshing interaction.

Now that all the parts are created, two assemblies can be created for 3D printing. One assembly contains the testing base and two involute gear slices and the other contains the testing base and two cycloidal gear slices. In the assemblies, make the gear slices slide over the posts on the base and rest 0.03 inches above the base. It is important that none of the parts are touching in the SolidWorks assembly. If any parts are touching in the assembly, they will be fused together when 3D printed, and the assemblies will not move. It is also possible to print all parts separately to make sure nothing is fused but printing as an assembly allows for testing of the assembly before printing to ensure it functions properly. Figure 8 below shows the two completed, printed assemblies that are used for testing.

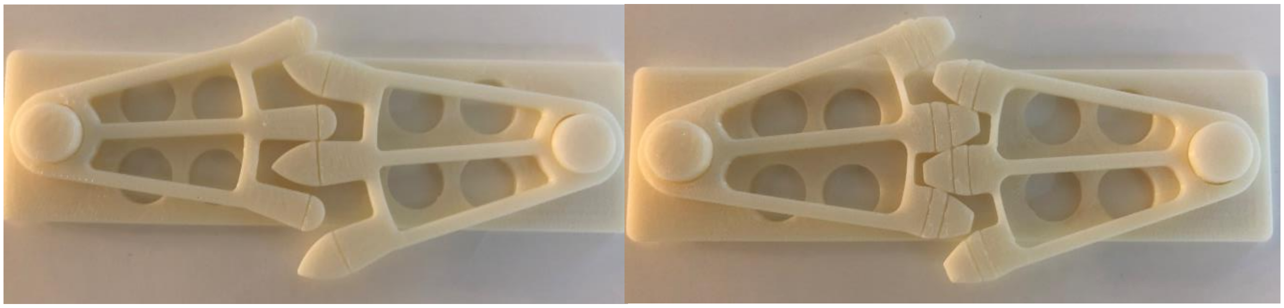

Figure 8: Cycloidal (Left) and Involute (Right) gear assemblies for testing

Test Setup

Before testing the gears, they were run back and forth to see if any visible sliding occurred. Just from observation, it was apparent that sliding occurred in the cycloidal gear set. The sliding action was most noticeable when the gear, on the right in the figure above, was first making contact and ending its contact with the pinion. For the involute gear set, it was much harder to tell if sliding occurred just from visual inspection, but if any sliding occurred it was a very small amount. To test the gears for sliding, a piece of paper was placed between the gears where they meshed and tucked into the corner of one of the gears, and the gear assemblies were turned. If there is sliding, the force from the sliding will pull the paper up out of the corner along the flank of the tooth.

Results and Conclusion

The results of the testing confirmed that the cycloidal gears have much more sliding in their movement than the involute gears. Every time the cycloidal gear set was tested the paper would lift out the corner as the two teeth would reach the end of their meshing. When the involute gear set was tested the paper remained in the corner of the gear tooth through the entire engagement and disengagement of the two gear teeth. This shows that if any sliding occurred between the teeth it was so minimal that it didn’t provide enough force to pull the paper out of the corner of the gear. The results of the testing along with the observation of the meshing gear sets clearly show that the cycloidal gear set has more sliding than the involute gear set.

Overall, involute gears should replace cycloidal gears in clock like they replaced them in almost every other use of gears. Involute gears have many more advantages for use in a clock than cycloidal gears. Cycloidal gears have the advantage of being able to be made with a very small number of teeth. This is very beneficial for clock which utilize multiple gear reductions to transmit speeds and turn the hands accurately. The small number of teeth cycloidal gears can be made with allow for larger gear reductions to be used reducing the number of gear reductions that are necessary and the overall size of the clock [10]. A clock with involute gears would need to use larger gears and possibly more gear reductions to achieve the same thing. However, the advantages of involute gears far outweigh the drawback of increasing the size of the clock. As mentioned earlier, involute gears have manufacturing and inspection benefits over cycloidal gears that allow for better more accurate gears to be made and mass produced. Involute gears are also tolerable of deviations in their center-to-center distance while still maintaining constant speed ratios. This allows for more tolerance in the building of the clock while keeping one of the two important aspects of gears in a clock [13]. Finally, as the results of the testing show involute gears have far less sliding than cycloidal gears which reduces the wear on the gear teeth over time. Since clock gears generally shouldn’t or can’t be lubricated because they are exposed to the air, reducing sliding is the only way to keep the wear down [6]. When gear teeth wear in a clock, they will stop maintaining constant speed ratios and need to be replaced. In conclusion, involute gear teeth are a better choice for mechanical clocks than cycloidal gear teeth.

References

[1] M. Bellis, “Clocks Through the Years: Mechanical Pendulum and Quartz Clocks,” *ThoughtCo*, 12-Apr-2018. [Online]. Available: https://www.thoughtco.com/history-of-mechanical-pendulum-clocks-4078405. [Accessed: 20-Nov-2019].

[2] R. Thoen, “Cycloidal Gearing is Obsolete - A Personal View,” *Cycloidal Gearing is Obsolete - A Personal View*, 24-Feb-2004. [Online]. Available: https://www.csparks.com/watchmaking/CycloidalGears/RichardThoen.xhtml. [Accessed: 20-Nov-2019].

[3] *Essentiality to the National Defense of the Domestic Horological Industry*. Washington D.C.: U.S. Government Printing Office, 1954.

[4] *American Horological Industry*. Washington D.C.: U.S. Government Printing Office, 1964.

[5] D. A. Goldstein, “Involute and Cycloidal Tooth Forms For Use In Mechanical Escapement Systems,” in *Proceedings of the Timers for Ordinance Symposium*, 1966, vol. II, pp. 33–53.

[6] M. Latcha, “Mechanical Clock,” Oakland University, 2018

[7] Tec-Science, “Geometry of involute gears,” *Tec-Science*, 19-Oct-2019. [Online]. Available: https://www.tec-science.com/mechanical-power-transmission/involute-gear/geometry-of-involute-gears/. [Accessed: 20-Nov-2019].

[8] Tec-Science, “Geometry of cycloidal gears,” *Tec-Science*, 19-Oct-2019. [Online]. Available: https://www.tec-science.com/mechanical-power-transmission/cycloidal-gear/geometry-of-cycloidal-gears/. [Accessed: 20-Nov-2019].

[9] Tec-Science, “Meshing of cycloidal gears,” *Tec-Science*, 19-Oct-2019. [Online]. Available: https://www.tec-science.com/mechanical-power-transmission/cycloidal-gear/meshing-of-cycloidal-gears/. [Accessed: 20-Nov-2019].

[10] H. Sparks, “Designing Cycloidal Gears,” *Designing Cycloidal Gears*, 26-Apr-2013. [Online]. Available: https://www.csparks.com/watchmaking/CycloidalGears/index.jxl. [Accessed: 20-Nov-2019].

[11] R. Hessmer, “Cycloidal Gear Builder,” *Cycloidal Gear Builder,* 2013. [Online]. Available: http://www.hessmer.org/gears/CycloidalGearBuilder.html. [Accessed: 20-Nov-2019].

[12] Tec-Science, “Meshing of involute gears,” *Tec-Science*, 19-Oct-2019. [Online]. Available: https://www.tec-science.com/mechanical-power-transmission/involute-gear/meshing-line-action-contact-pitch-circle-law/#Influence_of_the_centre_distance_on_the_transmission_ratio. [Accessed: 20-Nov-2019].

[13] “Comparison between Involute and Cycloidal Gear Tooth Profiles,” *ME Mechanical Engineering*, 15-Apr-2018. [Online]. Available: https://me-mechanicalengineering.com/comparison-between-involute-and-cycloidal-tooth-profiles/. [Accessed: 20-Nov-2019].

[14] H. Sparks, “Cycloidal Gear Calculator,” *Cycloidal Gear Calculator*. [Online]. Available: https://www.csparks.com/watchmaking/CycloidalGears/CycloidCalculator.html. [Accessed: 20-Nov-2019].

[15] S. Winther, “Cycloidal Gear Design,” *Cycloidal Gear Design*, 02-Jan-2017. [Online]. Available: https://grabcad.com/library/cycloidal-gear-design-1. [Accessed: 20-Nov-2019].