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SPECIAL ISSUE ON ALGEBRA AND COMPUTATIONAL ALGEBRAIC GEOMETRY

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Algebraic geometry is one of the main branches of modern mathematics with roots from classical Italian geometers. Its modern flavor started with Grothendieck and continued with many illustrious algebraic geometers of the second half of the 20-th century. During the last twenty years, the subject has changed drastically due to developments of new computational techniques and access to better computing power. Such changes have spurred a new direction of algebraic geometry, the so called *computational algebraic geometry*. While there is no universal agreement among mathematicians that what exactly is computational algebraic geometry, loosely stated it includes the areas of algebraic geometry where computer algebra can be used to obtain explicit results. It is obvious that such area will be of deep impact and importance in the future mathematics. Furthermore, such new developments have made possible applications of algebraic geometry in areas such as coding theory, computer security and cryptography, computer vision, mathematical biology, and many more.

This special issue contains papers that cover classical mathematical problems from a computational viewpoint and problems in newer developments. We intentionally did not limit the papers in a narrow area. Instead, we tried to present a wide variety of topics. This special issue consists of ten papers on the following topics:

The paper by D. Haran and M. Jarden studies the embedding problem. Let K be an ample field, G a finite group, and L a finite Galois extension of K such that $Gal(L/K)$ is isomorphic to a subgroup of G. They prove that $K(x)$ has a Galois extension F which is regular over L such that $Gal(F/K(x)) \cong G$ and F has a K-place ϕ such that $\phi(x) \in K$ and $\phi(F) = L \cup \{\infty\}$.

The paper by M. Joswig, B. Sturmfels, and J. Yu explores the relationship between convexity in tropical geometry and notions of convexity in the theory of affine buildings, from a combinatorial and computational perspective. Results include a convex hull algorithm for the Bruhat–Tits building of $SL_d(K)$ and techniques for computing with apartments and membranes.

The paper by J. Hakim is on discrete series representations of p-adic groups associated to symmetric spaces. The purpose of this paper is study the natural symmetric space analogues of various notions related to discrete series representations of a p-adic group such as Schur's orthogonality relations and formal degrees.

A. Elezi in his paper focuses on toric fibrations and mirror symmetry. The relation between the quantum $\mathcal{D}$-modules of a smooth variety X and a toric bundle is studied. The author describes the relation completely when X is a semi-ample complete intersection in a toric variety. In this case, all the relations in the small quantum cohomology ring of the bundle are obtained.

Q. Gashi studies toric varieties associated with root systems. Consider a root system R and the corresponding toric variety V_R whose fan is the Weyl fan and whose lattice of characters is given by the root lattice for R. The author proves the vanishing of the higher cohomology groups for certain line bundles on V_R by proving a purely combinatorial result for root systems. These results are related to a converse to Mazur's Inequality for split reductive groups.

In their paper A. Elkin and R. Pries show there exists a hyperelliptic curve of genus $g \geq 3$ with p-rank $g-3$ and a-number 1 in characteristic p when $p=3$ or $p=5$. The method of proof is to show that a generic point of the moduli space of hyperelliptic curves of genus 3 and p-rank 0 has a-number 1. When $p=3$, it is also shown that this moduli space is irreducible.

There are two papers on theta functions of algebraic curves in this special issue. The first paper by E. Previato, T. Shaska, and S. Wijesiri studies relations among the classical thetanulls of cyclic curves, namely curves $\mathcal{X}$ (of genus $g(\mathcal{X}) > 1$) with an automorphism σ such that σ generates a normal subgroup of the group G of automorphisms, and $g\left(\mathcal{X}/\langle\sigma\rangle\right) = 0$. Relations between thetanulls and branch points of the projection are the object of much classical work, especially for hyperelliptic curves, and of recent work, in the cyclic case. In this paper the authors determine the curves of genus 2 and 3 in the locus $\mathcal{M}_g(G, \mathbf{C})$ for all G that have a normal subgroup $\langle\sigma\rangle$ as above, and all possible signatures $\mathbf{C}$, via relations among their thetanulls.

In the second paper about theta functions, Y. Kopeliovich describes modular equations of order prime p and theta functions. Let p be a prime integer and $\mathbf{H_g}$ be a collection of complex $g \times g$ matrices τ such that: i) $\tau = \tau^t$ i.e. τ is symmetric and ii) $Im\tau$ is a positive definite quadratic form.

Denote by $p\tau$ the multiplication of τ by p. In this paper it is described an explicit process to obtain algebraic identities between theta functions with integral characteristics evaluated at τ and $p\tau$. For $g=1$ this produces modular equations between $\lambda(\tau), \lambda(p\tau)$ where $\lambda(\tau)$ is the invariant associated with elliptic curve generated by τ, described by the equation: $y^2 = x(x-1)\left(x - \lambda(\tau_1)\right)$. Consequently, if $g > 1$ The algebraic identities we obtain might serve as a higher dimensional generalization for the one dimensional modular equations.

V. Ustimenko focuses on the extremal regular directed graphs without commutative diagrams and their applications in coding theory and cryptography. There are methods from computational algebra via Groebner basis which can be applied in the study of such graphs, even though they are not fully explored in this paper.

In the last paper, T. Shaska re-visits an old list of open problems in the area of computer algebra and algebraic curves. That list first appeared in 2003, in the ACM, *SIGSAM Bulletin*, *Comm. Comp. Alg.*. It was a list of problems on algebraic curves which could be approached computationally. Some of those problems were solved and many papers from various authors were written based on that modest paper. The author has updated the list with new problems and has included some problems on higher dimensional varieties.

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